High-Energy Limit of QCD beyond Sudakov Approximation

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Loops and Legs in Quantum Field Theory

St. Goar, Germany, May 4th, 2018

Beauty and Magic of QCD at High Energy

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Topics discussed

- Introduction
- QCD at high energy to $\mathcal{O}(m_q^2/Q^2)$
  - *non-Sudakov double logs in power suppressed amplitudes*
  - *soft quark emission, eikonal color charge nonconservation*
  - *factorization and resummation of non-Sudakov double logs*
  - *light quark effects in Higgs production*
  - *asymptotic formula for the form factors*
- *Moments of beauty and magic*

Based on

T. Liu, A.A. Penin, Phys.Rev.Lett. **119** *(2017) 262001*

previous work:

T. Liu, A.A. Penin and N. Zerf, Phys.Lett. B **771** *(2017) 492*

A.A. Penin and N. Zerf, Phys.Lett. B **760** *(2016) 816*

A.A. Penin, Phys.Lett. B **745** *(2015) 69*

Sudakov limit

- Sudakov limit
  - *on-shell*
  - *exclusive*
  - *high energy*
  - *fixed angle*
- Sudakov logarithms *(leading power in m_q^2/Q^2)*
  - *each external line gets* $e^{-\frac{\alpha}{4\pi}\frac{C_R}{2}\ln^2(Q^2)}$

    Sudakov (1956); Frenkel, Taylor (1976)
  - *subleading logs exponentiate as well*

    Mueller (1979); Collins (1980); Sen (1981), ...
- What about power suppressed logs?

High energy limit beyond Sudakov approximation

- Logarithmically enhanced power corrections
  - *phenomenologically crucial e.g. bottom loop in Higgs production*
  - *intriguing from QFT point of view*

High energy limit beyond Sudakov approximation

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- Power corrections and renormalization group

$$\Delta\mathcal{L} = \sum_i C_i O_i, \qquad \frac{d}{d\ln\mu} C_i = f_i(C_j)$$

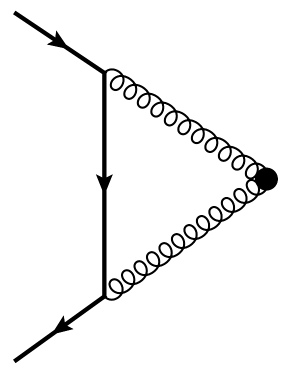
✔	*OPE, large mass expansion*	⇨	*local composite operators*
✔	*threshold nonrelativistic limit*	⇨	*spatially nonlocal potentials*
✘	*Sudakov ultrarelativistic limit*	⇨	*light-cone Wilson lines, etc.*

High energy limit beyond Sudakov approximation

- Light quark mass effects in Higgs production
  K.Melnikov and A.Penin, JHEP **1605** *(2016) 172*
- Next-to-eikonal soft gluon radiation
  D.Bonocore, E.Laenen, L.Magnea, L.Vernazza, C.D.White, JHEP **1612**, *121 (2016)*,
- Jets and jettiness
  R. Boughezal, X. Liu and F. Petriello, JHEP **1703**, *160 (2017)*
- Power corrections in SCET
  I.Moult, I.W.Stewart and G.Vita, JHEP **1707**, *067 (2017)*
  M.Beneke, M.Garny, R.Szafron, J.Wang, JHEP **1803** *(2018) 001*
- *and many other recent studies ...*

Power-suppressed amplitude

- Quark scattering by $(G^a_{\mu\nu})^2$ operator

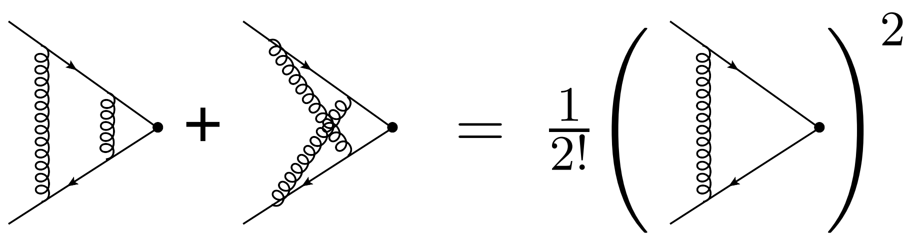


  - *helicity flip* ⇨ *mass suppression*
  - *soft quark* $S \approx m_q/l^2$ *eikonal gluons* $D \approx g_{\mu\nu}/(2p_i l)$

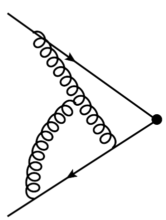
- Power suppressed double logs
  - *soft fermion exchange*
  - *eikonal color charge nonconservation*

Eikonal factorization (Sudakov)

- Eikonal identity $$\frac{1}{p_i l_1}\frac{1}{p_i(l_1+l_2)}+\frac{1}{p_i(l_1+l_2)}\frac{1}{p_i l_2}=\frac{1}{p_i l_1}\frac{1}{p_i l_2}$$
- Sum over vertex permutations (QED)

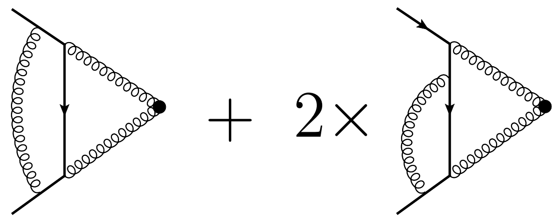


↪ $$\text{n-loop}=\frac{(1-\text{loop})^n}{n!}$$

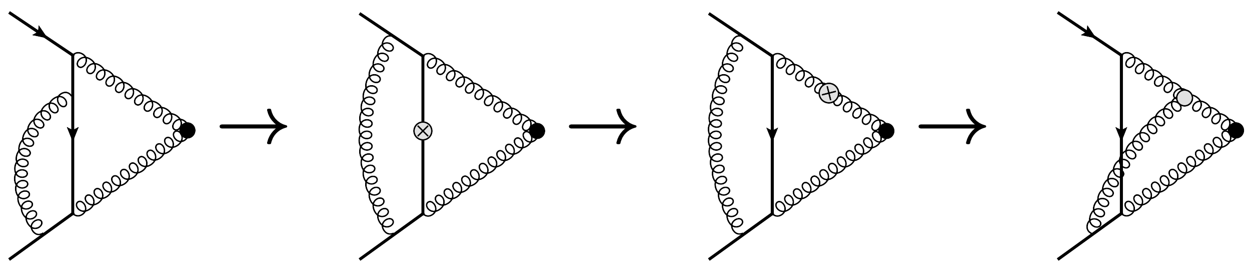
- *in QCD*  *cancels the non-Abelian part by Ward identity*

Power-suppressed amplitude

- Radiative corrections (QED)



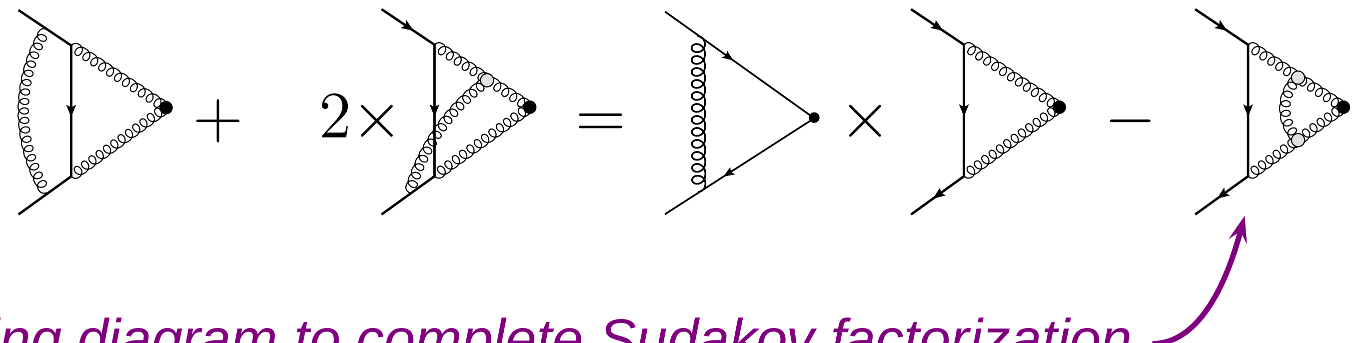
  - *Ward identities*



  - *crossed vertex:* $S(l) \to S(l) - S(l + l_g^+)$
  - *3-photon vertex:* $2e_q p_1^\mu$*, where* e_q *is the quark charge*

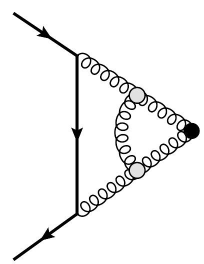
Power-suppressed amplitude

- Sudakov logs factorization



missing diagram to complete Sudakov factorization

- Non-Sudakov logs

 $\propto e_q^2$

  - *multiple soft photons exponentiate*

Power-suppressed amplitude

- In QCD $\qquad e_q^2 \to C_F - C_A$

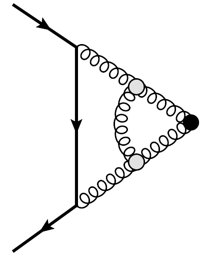
  ➥ *measures the eikonal color charge nonconservation*

- Factorization formula

$$\mathcal{G} = Z_q^2\, g(-z)\, \mathcal{G}^{(0)}$$

  - *quark Sudakov factor* $Z_q^2 = \exp\left[-C_F\left(\frac{\alpha_s}{2\pi}\frac{\ln(m_q^2/Q^2)}{\varepsilon} + x\right)\right]$
  - *non-Sudakov double-logarithmic function* $g(-z)$*,*
    where $z = (C_A - C_F)x,\ x = \frac{\alpha_s}{4\pi}\ln^2(Q^2/m_q^2),\ g(0) = 1$

Evaluation of $g(z)$

$$g(-z) \sim$$ 

- *New variables*
  - *Sudakov parameters* $l = up_1 + vp_2 + l_\perp$
  - *hypercube coordinates*

    $\eta = \ln v / \ln(m_q^2/Q^2),\ \xi = \ln u / \ln(m_q^2/Q^2),\ 0 < \eta,\ \xi < 1$
  - *strict ordering* $\eta_i < \eta_j,\ \ldots$, *onshell condition* $\eta_i + \xi_i < 1$
- *Integral representation*

$$g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta \quad e^{2z\eta\xi}$$

soft gluon loop exponent → $e^{2z\eta\xi}$

Result

- Exact formula

$$g(z) = {}_2F_2\,(1,1;3/2,2;z/2)$$

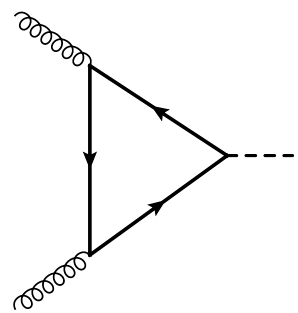
- Asymptotic behavior at $z \to +\infty$

$$g(-z) \sim \frac{\ln(2z)+\gamma_E}{z}, \qquad g(z) \sim \left(\frac{2\pi e^z}{z^3}\right)^{1/2},$$

- The amplitude $\mathcal{G} \propto g(-z)$
  - *in QCD* $z > 0$ ⇨ *power suppressed*
  - *in QED* $z < 0$ ⇨ *exponentially enhanced (!)*

Bottom loop in $gg \to H$

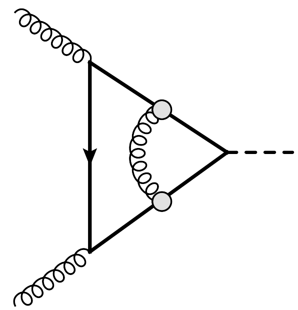
- Leading contribution



- *effective expansion parameter* $\alpha_s \ln^2(m_H^2/m_b^2) \sim 40\alpha_s$
- ➥ *resummation is mandatory (main source of uncertainty)*
- *factorization structure similar to* $\mathcal{G}$

Moment of beauty

- Non-Sudakov logs



  - *inverted color flow:* $C_F \leftrightarrow C_A$, $z \to -z$

- Factorization formula

$$\mathcal{M}^b_{gg\to H} = Z_g^2\, g(z)\, \mathcal{M}^{(0)}_{gg\to H}$$

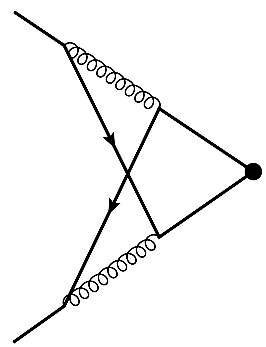
  - *gluon Sudakov factor* $Z_g^2 = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\frac{\mu^{2\varepsilon}}{m_H^{2\varepsilon}}\right]$
  - $g(z)$ *exponentially enhanced in QCD*

Dirac form factor beyond the leading power

- High-energy expansion

$$F_D = Z_q^2 \left(1 + \frac{m_q^2}{Q^2} F_D^{(1)} + \dots \right) ,$$

- Double logarithms in $F_D^{(1)}$



  - *induced by soft quark pair exchange*

A.A. Penin, Phys.Lett. B **745** (2015) 69

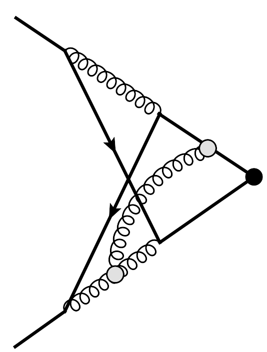
Dirac form factor beyond the leading power

- All-order result

$$F_1^{(1)} = \left[\frac{C_F(C_A-2C_F)}{6}x^2\right] \; f(-z)$$

two-loop contribution

- non-Sudakov double-logarithmic function

$f(-z) \sim$  $+$ *symmetric*

Result for $f(z)$

- Integral representation

$$f(z) = 12\int_0^1 \mathrm{d}\eta_1 \int_{\eta_1}^1 \mathrm{d}\eta_2 \int_0^{1-\eta_2} \mathrm{d}\xi_2 \int_{\xi_2}^{1-\eta_1} \mathrm{d}\xi_1 \, e^{2z\eta_1(\xi_1-\xi_2)} e^{2z\xi_2(\eta_2-\eta_1)}$$

- Taylor series $f(z) = \sum_n c_n z^n$
  - *high-order behavior* $c_n \sim \frac{\ln n}{n! 2^n n^{5/2}}$

- Asymptotic behavior at $z \to +\infty$

$$f(-z) \sim C_- \left(\frac{\ln z}{z}\right)^2, \quad f(z) \sim C_+ \ln z \left(\frac{e^z}{z^5}\right)^{1/2}$$

$C_- = 3.6\ldots, C_+ = 14.8\ldots$

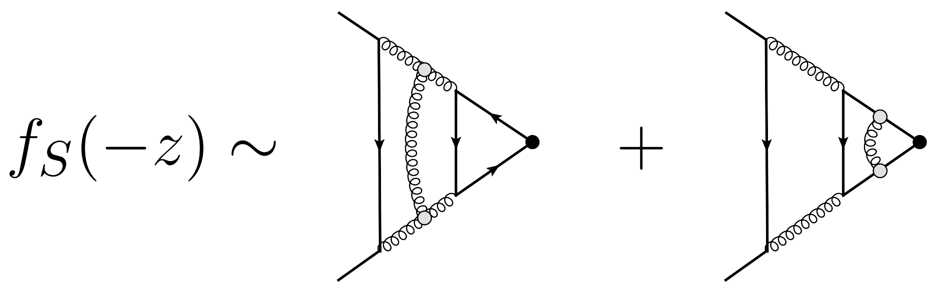
➥ *exponentially enhanced in QED, power suppressed in QCD*

Moment of magic

- Scalar form factor

$$F_S^{(1)} = \left[-\frac{C_F T_F}{3} x^2\right] f_S(-z)$$

- non-Sudakov double-logarithmic function



- *Amazing universality* $f_S(z) = f(z)$

  ➥ *the same function for axial and pseudoscalar form factors*

Summary

- The first LL resummation beyond leading power in QCD *(SCET free!)*
- Non-Sudakov logs
  - *induced by soft quark exchange*
  - *eikonal charge nonconservation*
  - *exponential enhancement*
- Beauty and magic
  - *universality* ⇨ *two functions* $g(z)$ *and* $f(z)$
  - *relations between different amplitudes and gauge groups*
- To be done
  - *Higgs plus jet production, amplitudes beyond two legs and beyond leading logs*