Non-planar two-loop Feynman integrals contributing to $H + j$ production

Hjalte Frellesvig

TTP - Institut für Theoretische Teilchenphysik,
KIT - Karlsruhe Institute of Technology

May 2, 2018

Karlsruher Institut für Technologie   Institut für Theoretische Teilchenphysik

Work with R. Bonciani, V. Del Duca, J. Henn, F. Moriello, V. Smirnov.

Higgs plus jet production at the LHC.

Higgs plus jet production at the LHC.

Three processes in one:

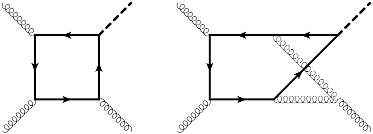
$pp \to Hj$	is important in its own right
$H \to 3j$	Higgs decay
$pp \to H$	Real radiation at next order

Higgs plus jet production at the LHC.

Three processes in one:

$pp \to Hj$	is important in its own right
$H \to 3j$	Higgs decay
$pp \to H$	Real radiation at next order

Leading order QCD is one-loop



NLO/two-loop is not yet known analytically
with full m_t dependence...

Karlsruher Institut für Technologie Institut für Theoretische Teilchenphysik

Previous work on NLO $H + j$ production

HEFT results:	R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze. (2013) arXiv:1302.6216 X. Chen, T. Gehrmann, E. W. N. Glover, M. Jaquier. (2015) arXiv:1408.5325 R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze. (2015) arXiv:1504.07922 R. Boughezal, C. Focke, W. Giele, X. Liu, F. Petriello. (2015) arXiv:1505.03893
Various other limits and expansions:	R. Harlander, T. Neumann, K. Ozeren, M. Wiesemann. (2012) arXiv:1206.0157 T. Neumann and M. Wiesemann. (2014) arXiv:1408.6836 T. Neumann and C. Williams. (2017) arXiv:1609.00367 R. Mueller and D. Öztürk. (2016) arXiv:1512.08570 K. Melnikov, L. Tancredi, C. Wever. (2016) arXiv:1610.03747 K. Melnikov, L. Tancredi, C. Wever. (2017) arXiv:1702.00426 J. Lindert, K. Melnikov, L. Tancredi, C. Wever. (2017) arXiv:1703.03886 K. Kudashkin, K. Melnikov, C. Wever. (2017) arXiv:1712.06549 * J. Lindert, K. Kudashkin, K. Melnikov, C. Wever. (2018) arXiv:1801.08226 * *: See the previous talk by Kirill Kudashkin.
Numerical results:	S. Jones, M. Kerner, G. Luisoni. (2018) arXiv:1802.00349 ** **: See Tuesday's talk by Matthias Kerner.
Planar integrals:	R. Bonciani, V. Del Duca, HF, J. Henn, F. Moriello, V. Smirnov. (2016) arXiv:1609.06685

Karlsruher Institut für Technologie   Institut für Theoretische Teilchenphysik

Why are analytical results useful?

- Valid in all corners of phase space
- Useful for sub-leading contributions (b-quarks, photons...)
- Allows for new physics applications (e.g. $m_{\tilde{H}} > m_t$)
- Useful for other processes (e.g. $Z + j$ production)

Why are analytical results useful?

- Valid in all corners of phase space
- Useful for sub-leading contributions (b-quarks, photons...)
- Allows for new physics applications (e.g. $m_{\tilde{H}} > m_t$)
- Useful for other processes (e.g. $Z + j$ production)
- May allow for new, deeper structures to be seen
- Analytical results feel more satisfying...

Why are analytical results useful?

- Valid in all corners of phase space
- Useful for sub-leading contributions (b-quarks, photons...)
- Allows for new physics applications (e.g. $m_{\tilde{H}} > m_t$)
- Useful for other processes (e.g. $Z + j$ production)
- May allow for new, deeper structures to be seen
- Analytical results feel more satisfying...

This talk will be about the calculation of the Feynman integrals - the first and (in this case) biggest step in a cross section calculation.

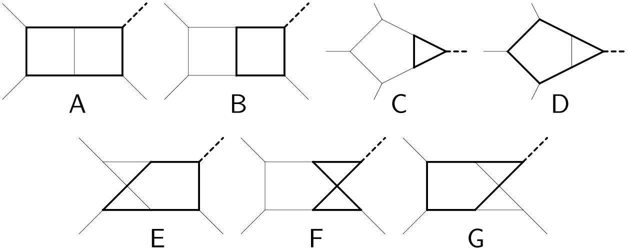
We evaluate the Feynman integrals with the "method of diff. equations". The Feynman integrals need to be expressed in terms of a minimal set of (canonical) "master integrals" and sorted into "integral families":

$$I^{f}_{a_1,\ldots,a_9} = \iint \frac{\mathrm{d}^d k_1}{i\pi^{d/2}} \frac{\mathrm{d}^d k_2}{i\pi^{d/2}} \frac{P_{f,8}^{-a_8}\ P_{f,9}^{-a_9}}{P_{f,1}^{a_1} P_{f,2}^{a_2} P_{f,3}^{a_3} P_{f,4}^{a_4} P_{f,5}^{a_5} P_{f,6}^{a_6} P_{f,7}^{a_7}}$$

We evaluate the Feynman integrals with the "method of diff. equations". The Feynman integrals need to be expressed in terms of a minimal set of (canonical) "master integrals" and sorted into "integral families":

$$I^{f}_{a_1,\ldots,a_9} = \iint \frac{\mathrm{d}^d k_1}{i\pi^{d/2}} \frac{\mathrm{d}^d k_2}{i\pi^{d/2}} \frac{P_{f,8}^{-a_8}\ P_{f,9}^{-a_9}}{P_{f,1}^{a_1} P_{f,2}^{a_2} P_{f,3}^{a_3} P_{f,4}^{a_4} P_{f,5}^{a_5} P_{f,6}^{a_6} P_{f,7}^{a_7}}$$

There are seven integral families:



Prepared for submission to JHEP MITP/16-088

Two-loop planar master integrals for Higgs→ 3 partons with full heavy-quark mass dependence

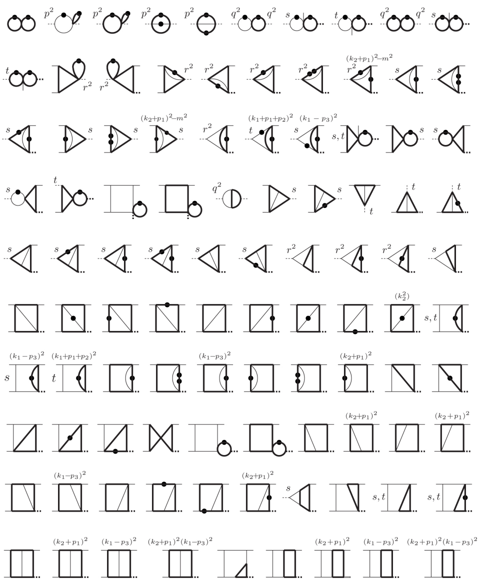
Roberto Bonciani,[a,b] **Vittorio Del Duca,**[c,d] **Hjalte Frellesvig,**[e] **Johannes M. Henn,**[f] **Francesco Moriello,**[a,b,c] **Vladimir A. Smirnov**[g]

[a]*Sapienza - Università di Roma, Dipartimento di Fisica, Piazzale Aldo Moro 5, 00185, Rome, Italy*
[b]*INFN Sezione di Roma, Piazzale Aldo Moro 2, 00185, Rome, Italy*
[c]*ETH Zurich, Institut fur theoretische Physik, Wolfgang-Paulistr. 27, 8093, Zurich, Switzerland*
[d]*INFN Laboratori Nazionali di Frascati, 00044 Frascati (Roma), Italy*
[e]*Institute of Nuclear and Particle Physics, NCSR Demokritos, Agia Paraskevi, 15310, Greece*
[f]*PRISMA Cluster of Excellence, Johannes Gutenberg University, 55099 Mainz, Germany*
[g]*Skobeltsyn Inst. of Nuclear Physics of Moscow State University, 119991 Moscow, Russia*

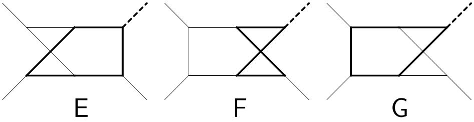
E-mail: roberto.bonciani@roma1.infn.it, delducav@itp.phys.ethz.ch, frellesvig@inp.demokritos.gr, henn@uni-mainz.de, fmoriell@phys.ethz.ch, smirnov@theory.sinp.msu.ru

Abstract: We present the analytic computation of all the planar master integrals which contribute to the two-loop scattering amplitudes for Higgs→ 3 partons, with full heavy-quark mass dependence. These are relevant for the NNLO corrections to fully inclusive Higgs production and to the NLO corrections to Higgs production in association with a jet, in the full theory. The computation is performed using the differential equations method. Whenever possible, a basis of master integrals that are pure functions of uniform weight is used. The result is expressed in terms of one-fold integrals of polylogarithms and elementary functions up to transcendental weight four. Two integral sectors are expressed in terms of elliptic functions. We show that by introducing a one-dimensional parametrization of the integrals the relevant second order differential equation can be readily solved, and the solution can be expressed to all orders of the dimensional regularization parameter in terms of iterated integrals over elliptic kernels. We express the result for the elliptic sectors in terms of two and three-fold iterated integrals, which we find suitable for numerical evaluations. This is the first time that four-point multiscale Feynman integrals have been computed in a fully analytic way in terms of elliptic functions.

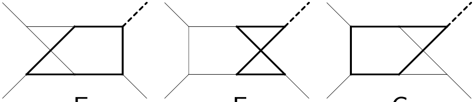
Planar integrals



The three non-planar families.



The three non-planar families.

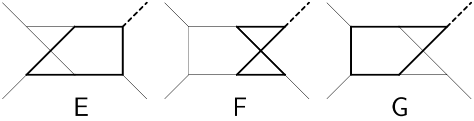


E F G

Most integrals overlap with the planar sectors.

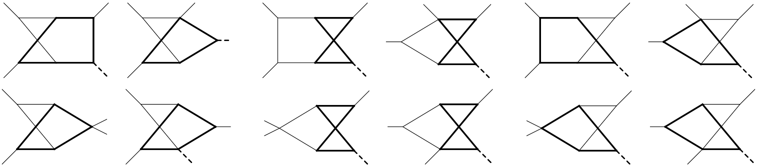
Only four genuinely non-planar sectors
possible for each family.

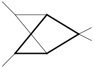
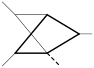
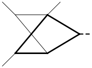
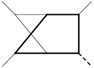
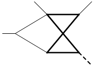
The three non-planar families.

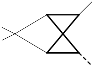
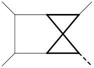
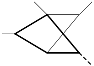
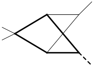
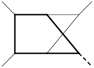


Most integrals overlap with the planar sectors.

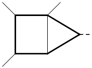
Only four genuinely non-planar sectors
possible for each family.

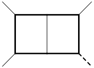


name	figure	nr.	comment
E_{60}		1	canonical form
E_{61}		2	canonical form
E_{63}		4	canonical form
E_{67}		5	canonical form
F_{58}		4×2	canonical form

name	figure	nr.	comment
F_{66}		2	elliptical
F_{68}		6	elliptical times 2
$G_{\text{"63"}}$		4×2	canonical form
$G_{\text{"95"}}$		1	canonical form
$G_{\text{"127"}}$		5	elliptical via A66

Elliptic sectors

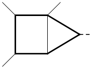
A_{66}  $I_{\text{max cut}} \propto \int \frac{dz}{s\sqrt{P_{4;1}(z)}}$

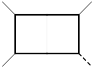
A_{70}  Max cut is log. Ellipticity enters through A_{66}.

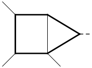
Cut: $\displaystyle\int_{-\infty}^{\infty} \frac{f(x)}{x} dx \;\rightarrow\; \oint \frac{f(x)}{x} dx$

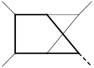
$$P_{4;1} = \left((m_H^2 + z)^2 - 4m_H^2 m_t^2\right)\left(4m_t^2 tu/s + (t+z)^2\right)$$

Elliptic sectors

$A_{66}/G_{\text{“}126\text{”}}$  $I_{\text{max cut}} \propto \int \frac{dz}{s\sqrt{P_{4;1}(z)}}$

A_{70}  Max cut is log. Ellipticity enters through A_{66}.

$G_{\text{“}123\text{”}}$  $I_{\text{max cut}} \propto \int \frac{dz}{s\sqrt{P_{4;2}(z)}}$. This is A_{66} in $t \leftrightarrow u$.

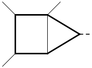
$G_{\text{“}127\text{”}}$  Max cut is log. Ell. enters through $G_{\text{“}123\text{”}}$ and $G_{\text{“}126\text{”}}$.

$$P_{4;1} = \left((m_H^2 + z)^2 - 4m_H^2 m_t^2\right)\left(4m_t^2 tu/s + (t+z)^2\right)$$
$$P_{4;2} = \left((m_H^2 + z)^2 - 4m_H^2 m_t^2\right)\left(4m_t^2 tu/s + (u+z)^2\right)$$

Karlsruher Institut für Technologie Institut für Theoretische Teilchenphysik

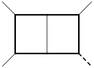
Elliptic sectors

$A_{66}/G_{\text{“}126\text{”}}$



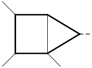
$I_{\text{max cut}} \propto \int \frac{dz}{s\sqrt{P_{4;1}(z)}}$

A_{70}



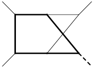
Max cut is log. Ellipticity enters through A_{66}.

$G_{\text{“}123\text{”}}$



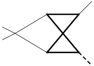
$I_{\text{max cut}} \propto \int \frac{dz}{s\sqrt{P_{4;2}(z)}}$. This is A_{66} in $t \leftrightarrow u$.

$G_{\text{“}127\text{”}}$



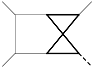
Max cut is log. Ell. enters through $G_{\text{“}123\text{”}}$ and $G_{\text{“}126\text{”}}$.

F_{66}



$I_{\text{max cut}} \propto \int \frac{dz}{(m_H^2-t)\sqrt{P_{4;3}(z)}}$

F_{68}



$I_{\text{max cut}} \propto \int \frac{dz}{t\,(z+u)\sqrt{P_{4;3}(z)}}$. Ell. also through F_{66}.

$$P_{4;1} = \left((m_H^2+z)^2 - 4m_H^2 m_t^2\right)\left(4m_t^2 tu/s + (t+z)^2\right)$$
$$P_{4;2} = \left((m_H^2+z)^2 - 4m_H^2 m_t^2\right)\left(4m_t^2 tu/s + (u+z)^2\right)$$
$$P_{4;3} = z(t-m_H^2-z)\left(4m_t^2 t - z(m_H^2-t+z)\right)$$

Representation

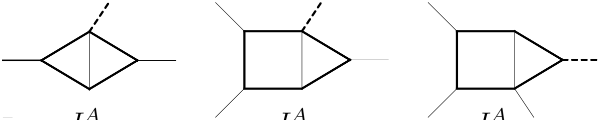
Three different "complexity classes":

1) Canonical form $\rightarrow$ $\log$, Li_n, $\mathrm{Li}_{2,2}$.

2) Canonical form $\rightarrow$ $\log + \mathrm{Li}_2$ at weigth 2, one-fold integrals at w.s 3 and 4.

3) No canonical form, elliptic integrals.

All cases present in family A



I^A_{32} I^A_{64} I^A_{66}

Representation

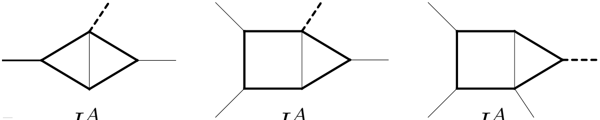
Three different "complexity classes":

1) Canonical form $\rightarrow$ $\log$, Li_n, $\mathrm{Li}_{2,2}$.

2) Canonical form $\rightarrow$ $\log + \mathrm{Li}_2$ at weigth 2, one-fold integrals at w.s 3 and 4.

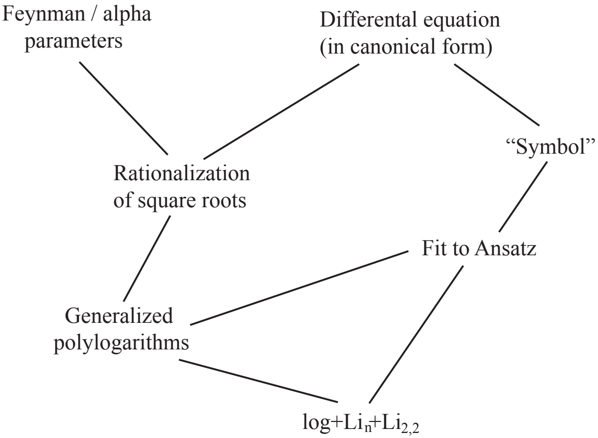
3) No canonical form, elliptic integrals.

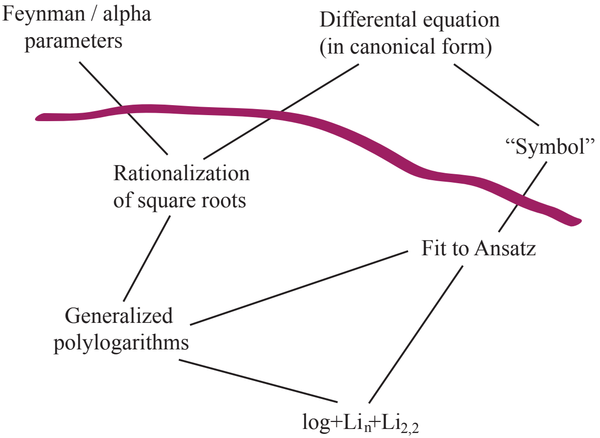
All cases present in family A



I^A_{32} I^A_{64} I^A_{66}

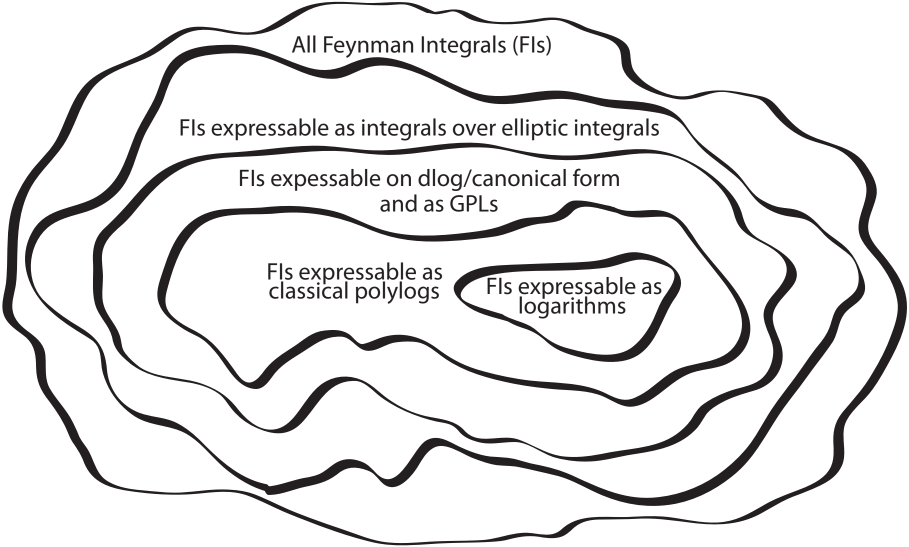
Elliptic cases from direct integration of diff.eq. or Feynman parameters.

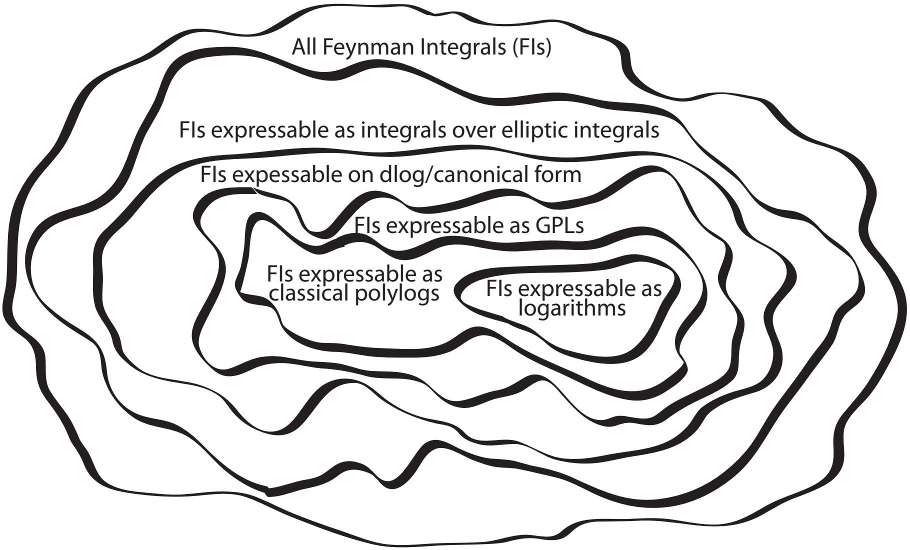

Feynman / alpha parameters
Differental equation (in canonical form)
"Symbol"
Rationalization of square roots
Fit to Ansatz
Generalized polylogarithms
log+Li$_n$+Li$_{2,2}$



Rationalization and argument finding are not fully algortihmic,
may not always be possible?

Discussion


All Feynman Integrals (FIs)
FIs expressable as integrals over elliptic integrals
FIs expessable on dlog/canonical form
and as GPLs
FIs expressable as
classical polylogs
FIs expressable as
logarithms


All Feynman Integrals (FIs)
FIs expressable as integrals over elliptic integrals
FIs expessable on dlog/canonical form
FIs expressable as GPLs
FIs expressable as classical polylogs
FIs expressable as logarithms

Conclusions

Non-planar integrals for $H + j$ ready in the not too distant future.

Conclusions

Non-planar integrals for $H + j$ ready in the not too distant future.

Other ingedients needed for the cross section:

- Analytical continuation to physical regions
- Real radiation + subtractions
- ...

Karlsruher Institut für Technologie Institut für Theoretische Teilchenphysik

Non-planar integrals for $H + j$ ready in the not too distant future.

Other ingedients needed for the cross section:

- Analytical continuation to physical regions
- Real radiation $+$ subtractions
- ...

Points to consider:

- Is the "intermediate class" (symbols but no GPLs) real?
- How to handle it?
- Common description elliptic and non-elliptic cases
- Method for quick evaluation
- ...

Non-planar integrals for $H + j$ ready in the not too distant future.

Other ingedients needed for the cross section:

- Analytical continuation to physical regions
- Real radiation $+$ subtractions
- ...

Points to consider:

- Is the "intermediate class" (symbols but no GPLs) real?
- How to handle it?
- Common description elliptic and non-elliptic cases
- Method for quick evaluation
- ...

Thank you for listening!

Karlsruher Institut für Technologie Institut für Theoretische Teilchenphysik