Sector-improved residue subtraction: Improvements and Applications

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Outline

NNLO subtraction schemes

Handling real radiation contribution in NNLO calculations
cancellation of infrared divergences

Increasing number of available NNLO calculations with a variety of schemes

- qT-slicing [Catani,Grazzini, '07], [Ferrera,Grazzini,Tramontano, '11], [Catani,Cieri,DeFlorian,Ferrera,Grazzini,'12], [Gehrmann,Grazzini,Kallweit,Maierhofer,Manteuffel,Rathlev,Torre,'14-15], [Bonciani,Catani,Grazzini,Sargsyan,Torre,'14-'15], [Grazzini,Kallweit,Wiesemann '17]
- N-jettiness slicing [Gaunt,Stahlhofen,Tackmann,Walsh, '15], [Boughezal,Focke,Giele,Liu,Petriello,'15-'16], [Boughezal,Campbell,Ellis,Focke,Giele,Liu,Petriello,'15-'16], [Campbell,Ellis,Williams,'16], [Moult,Rothen,Stewart,Tackmann,XingZhu '16-'17]
- Antenna subtraction [Gehrmann, GehrmannDeRidder,Glover,Heinrich,'05-'08], [Weinzierl,'08,'09], [Currie,Gehrmann,GehrmannDeRidder,Glover,Pires,'13-'17], [Bernreuther,Bogner,Dekkers,'11,'14], [Abelof,(Dekkers),GehrmannDeRidder,'11-'15], [Abelof,GehrmannDeRidder,Maierhofer,Pozzorini,'14], [Chen,Gehrmann,Glover,Jaquier,'15], [Cruz-Martinez,Gehrmann,Glover,Huss '18]
- Colorful subtraction [DelDuca,Somogyi,Trocsanyi,'05-'13], [DelDuca,Duhr,Somogyi,Tramontano,Trocsanyi,'15], [DelDuca,Duhr,Kardos,Somogyi,Trocsanyi '16], [Kardos,Kluth,Somogyi,Tulipant,Verbytskyi '17/'18]
- Sector-improved residue subtraction (STRIPPER) [Czakon,'10,'11], [Czakon,Fiedler,Mitov,'13,'15], [Czakon,Heymes,'14] [Czakon,Fiedler,Heymes,Mitov,'16,'17], [Boughezal,Caola,Melnikov,Petriello,Schulze,'13,'14], [Boughezal,Melnikov,Petriello,'11], [Caola,Czernecki,Liang,Melnikov,Szafron,'14], [Brucherseifer,Caola,Melnikov,'13-'14], [Caola, Melnikov, Röntsch,'17]
- Projection-to-Born [Cacciari,Dreyer,Karlberg,Salam,Zanderighi '15], [Currie,Gehrmann,Glover,Huss,Niehues,Vogt '18]

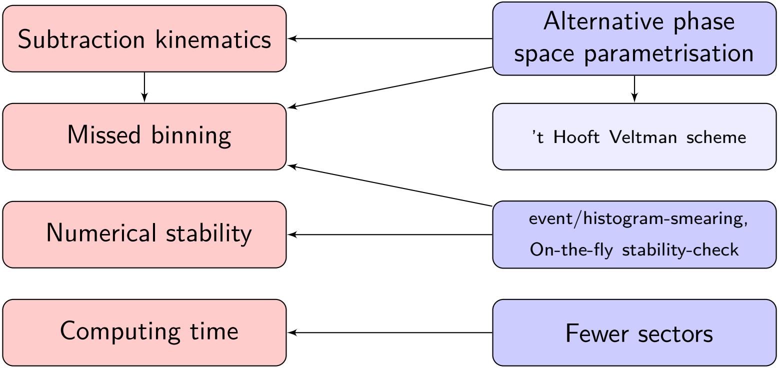
NNLO subtraction schemes

Handling real radiation contribution in NNLO calculations
cancellation of infrared divergences

Increasing number of available NNLO calculations with a variety of schemes

- "Torino" subtraction [Magnea, Maina, Torrielli, Uccirati '17/'18]
- Geometric IR subtraction [Herzog '18]
- ...

How to improve the STRIPPER subtraction scheme?



Idea: Optimisation through minimisation

Formulation

Hadronic cross section:

$$\sigma_{h_1 h_2}(P_1, P_2) = \sum \iint_0^1 \mathrm{d}x_1\, \mathrm{d}x_2\, f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

Partonic cross section:

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \hat{\sigma}_{ab}^{(1)} + \hat{\sigma}_{ab}^{(2)} + \mathcal{O}\left(\alpha_S^3\right)$$

Contributions with different final state multiplicities and convolutions:

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\mathrm{RR}} + \hat{\sigma}_{ab}^{\mathrm{RV}} + \hat{\sigma}_{ab}^{\mathrm{VV}} + \hat{\sigma}_{ab}^{\mathrm{C2}} + \hat{\sigma}_{ab}^{\mathrm{C1}}$$

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_{n}$$

Sector decomposition

Several layers of decomposition

Selector functions

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

Factorization of double soft limits

$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

Sector parametrisation

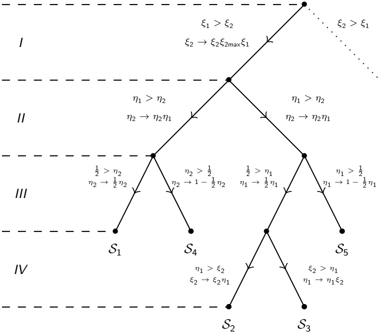
Parametrisation of u_i with respect to the reference parton r:

Angles: $\hat{\eta}_i = \frac{1}{2}(1 - \cos\theta_{ir}) \in [0, 1]$

Energies: $\hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0, 1]$

Triple collinear factorisation

originally: 5 sub-sectors



Extract singularities

$$x^{-1-b\epsilon} = \frac{-1}{b\epsilon} + \left[x^{-1-b\epsilon} \right]_+$$

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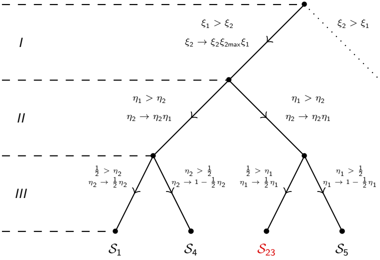
Triple collinear factorisation

now: 4 sub-sectors



[Caola, Melnikov, Röntsch '17]

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New phase space construction: Idea

Goal

Phase space construction with a minimal # of subtraction kinematics

Old construction

- Start with unresolved partons
- Fill remaining phase space with Born configuration

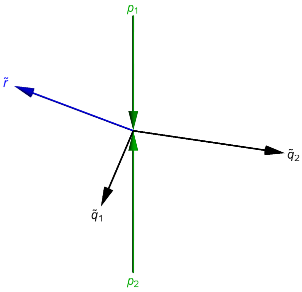
$\rightarrow$ Non-minimal # kinematic configurations
(e.g. single soft and collinear limits yield different configurations)

New construction

- Start with Born configuration
- Add unresolved partons (u_i)
- Cleverly adjust Born configuration to accommodate the u_i

New phase space construction

Mapping from $n+2$ to Born configuration: $\{P, r_j, u_k\} \to \{\tilde{P}, \tilde{r}_j\}$
Modification of [Frixione,Webber'02]/[Frixione,Nason,Oleari'07]



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i:
  $\{\tilde{P}, \tilde{r}_j, u_k\} \to \{P, r_j, u_k\}$
- Preserve $q^2 = \tilde{q}^2$, $\tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born phase space configuration
- Generate unresolved partons u_i
- Rescale reference momentum
- Boost non-reference momenta of the Born configuration

New phase space construction

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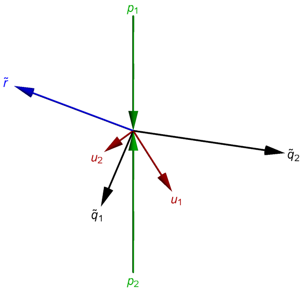

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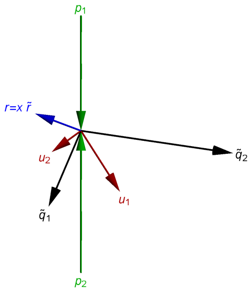

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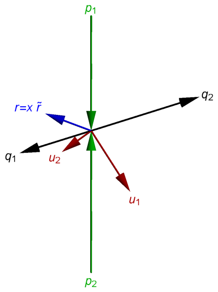

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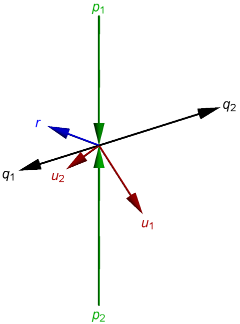
Behaviour in singular limits

Collinear limit of u_2

(sector 1, $\eta_2 \to 0$)



Soft limit of u_2

(sector 1, $\xi_2 \to 0$)

$\rightarrow$ Both singular limits approach the same kinematic configuration

Behaviour in singular limits

Collinear limit of u_2

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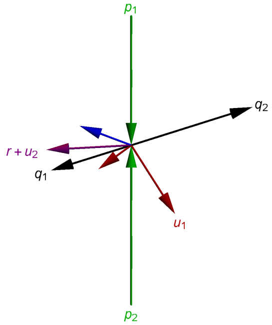

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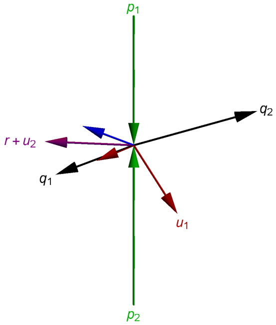

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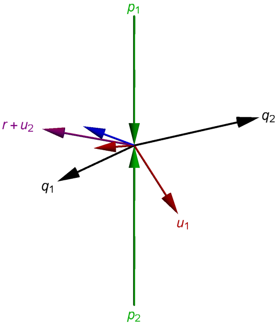

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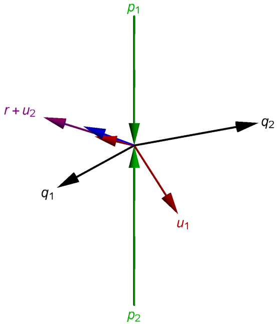

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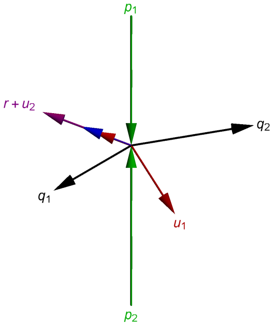

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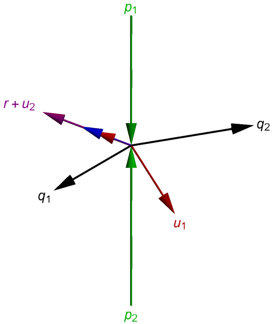

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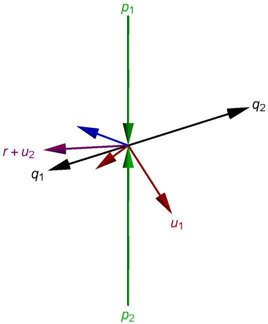

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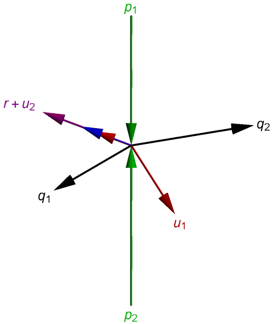

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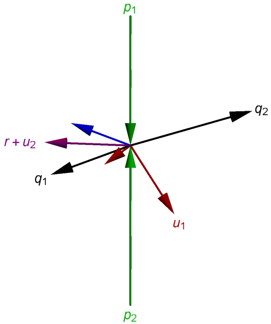

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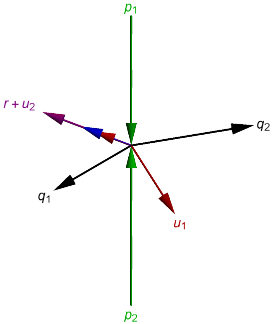

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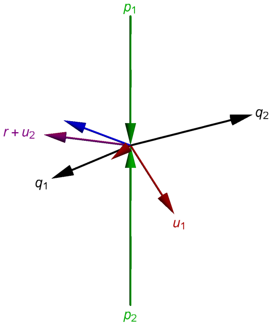

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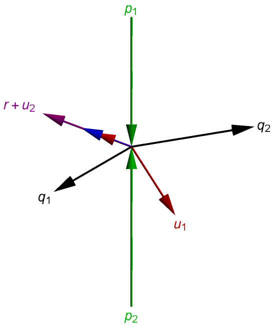

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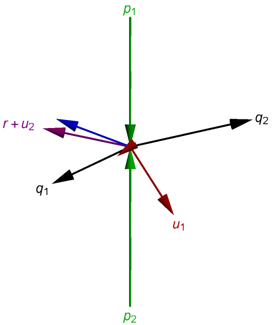

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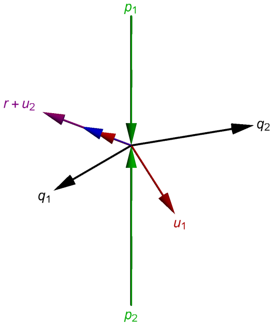

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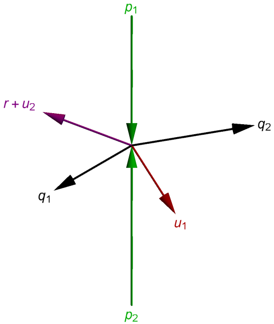

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Behaviour in singular limits

Triple collinear limit of u_1 & u_2

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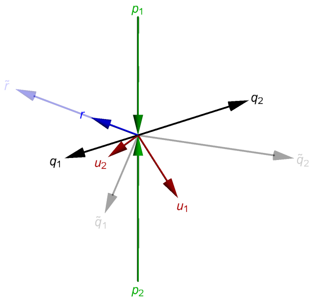

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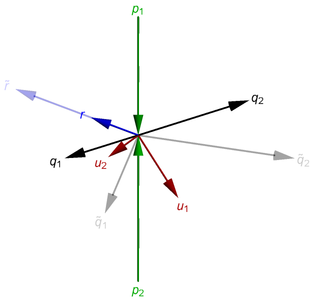

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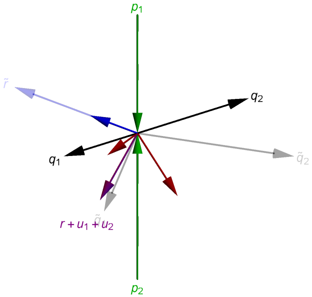

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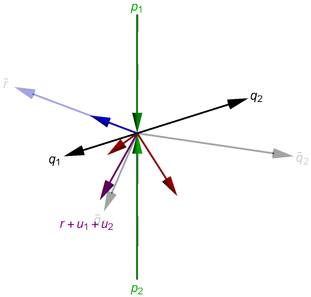

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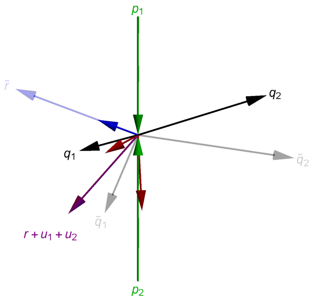

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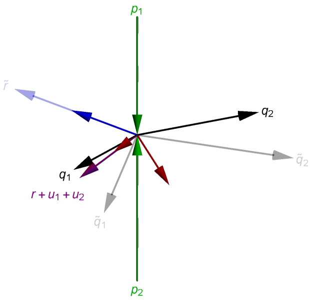

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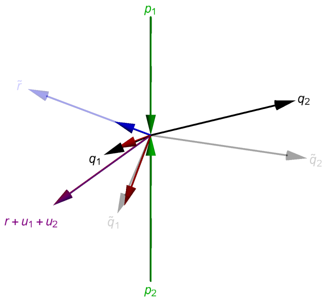

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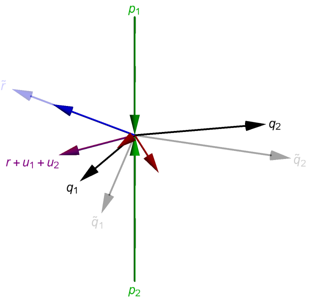

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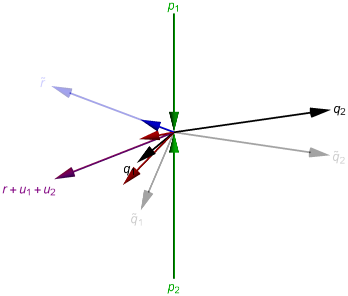

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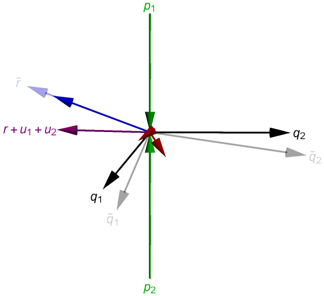

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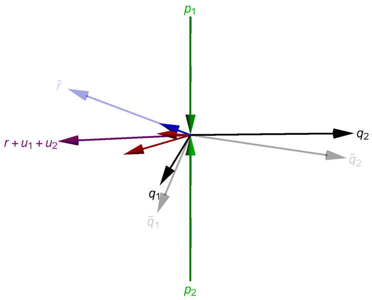

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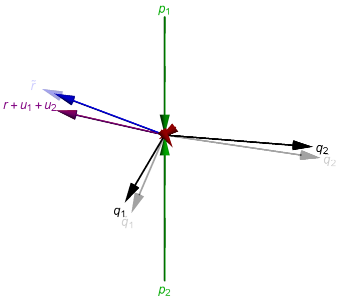

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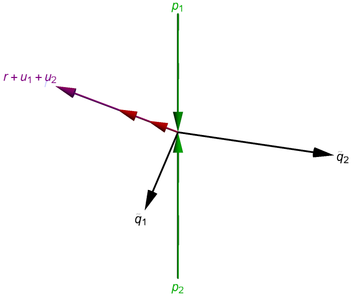
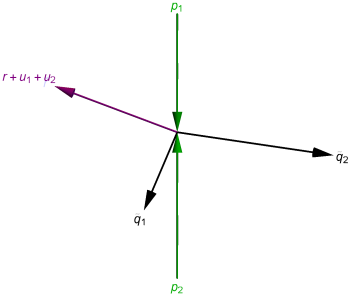


Double soft limit of u_1 & u_2

(sector 1, $\xi_1 \to 0$)



$\to$ Both double unresolved limits approach the Born configuration

Consequences

Features

- Minimal number of subtraction kinematics
- Only one DU configuration
  $\rightarrow$ pole cancellation for each Born phase space point
- Expected improved convergence of invariant mass distributions, since $\tilde{q}^2 = q^2$

Unintentional features

- Construction in lab frame
- Original construction of 't Hooft Veltman corrections [Czakon,Heymes'14] is spoiled

't Hooft Veltman scheme

Treat resolved particles in 4 dimensions (momenta and polarisations)

- Avoid unnecessary ϵ-orders of the matrix elements
- Avoid growth of dimensionality of phase space integrals

Make resolved phase space 4-dim. using measurement function, e.g.

$$F_n \to F_n \mathcal{N}^{-(n-1)\epsilon} \prod_{i=1}^{n-1} \delta^{(-2\epsilon)}(q_i)$$

Finite parts: σ_F^{RR} σ_F^{RV} σ_F^{VV}

Finite remainder parts: $\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}$

Single (SU) and double (DU) unresolved parts: $\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$ $\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$

$\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$ $\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$

't Hooft Veltman scheme

Goal: Make SU and DU separately finite

Idea: Move "divergent parts" of SU to DU before applying 'tHV scheme

- SU contribution: $\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1}$ with

$$\sigma_{SU}^{c} = \int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{c} F_{n+1} + I_{n}^{c} F_{n} \right)$$

- We know: NLO cross section is finite
  $\rightarrow$ F_{n+1} part of SU is finite: Poles cancel between RR, RV and C1 (with NLO measurement function)
- With NNLO measurement function: Additional poles arise
  $\rightarrow$ SU no longer finite by itself
- Non-cancelling ϵ poles are generated by terms with F_n
  $\rightarrow$ can be moved to DU

$\rightarrow$ Task: Identify non-cancelling parts of SU

't Hooft Veltman scheme

Task: Identify non-cancelling parts of SU

- Use parametrised measurement functions

$$F_{n+1}^{\alpha} = F_{n+1}\,\theta\left(\min_{i,j}\eta_{ij} - \alpha\right)\,\theta\left(\min_{i}\frac{u_i^0}{E_{\text{norm}}} - \alpha\right)$$

- Construct:

$$\sigma_{\text{SU}}^{c} - \mathcal{I}_c^{\alpha} = \int \mathrm{d}\Phi_{n+1}\left(I_{n+1}F_{n+1} + I_n F_n - [I_{n+1}]_{1/\epsilon^2,1/\epsilon}F_{n+1}^{\alpha}\right)$$

- Rearrangements allow to extract the non-cancelling part:

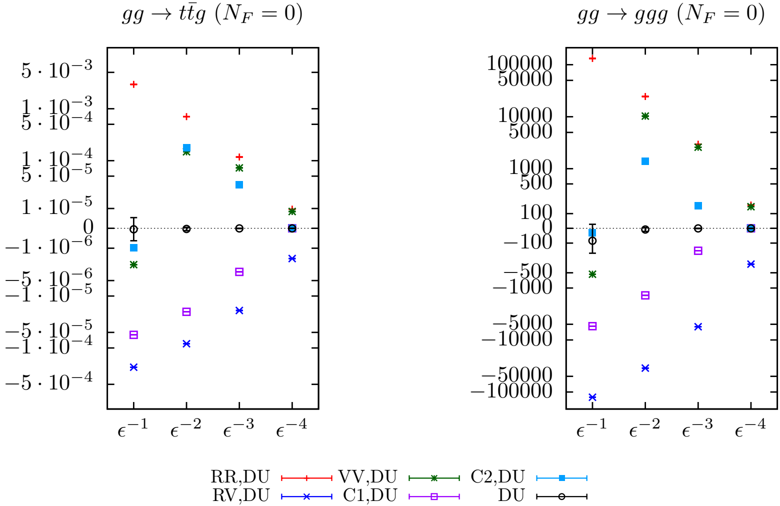
$$N^c(\alpha) = \int \mathrm{d}\Phi_{n+1}\,[I_n]_{1/\epsilon^2,1/\epsilon}F_n\,\theta_\alpha$$

- Analytically extract divergences ($\ln^k \alpha$) and cancel them exactly
- Take limit $\alpha \to 0$ to remove dependence on α
- Subtract from σ_{SU} and add to σ_{DU}
  $\to$ separately finite SU and DU contributions
  $\to$ ready for application of 'tHV scheme

Looks like slicing, but it is slicing *only* for divergences
$\to$ no actual slicing parameter in the result

Example: Pole cancellation at NNLO

Keep Born phase space point fixed – integrate double unresolved configurations



Conclusions

- Minimization of the STRIPPER scheme
- Fewer subsectors in triple collinear sectors
- Alternative phase space parametrisation
- New formulation of 't Hooft Veltman scheme
- Implementation of STRIPPER as a C++ library
- Currently performing tests for a variety of processes:
  $pp \to t\bar{t}$, $e^+e- \to 2/3j$, t decay, DIS, Drell-Yan, Higgs decay, dijets

Backup

Phase space

Common starting point for all phase spaces:

$$\mathrm{d}\Phi_n = \mathrm{d}Q^2 \left[\prod_{j=1}^{n_{fr}} \mu_0(r_j) \prod_{k=1}^{n_u} \mu_0(u_k) \delta_+ \left(\left(P - \sum_{j=1}^{n_{fr}} r_j - \sum_{k=1}^{n_u} u_k \right)^2 - Q^2 \right) \right]$$
$$\prod_{i=1}^{n_q} \mu_{m_i}(q_i) (2\pi)^d \delta^{(d)} \left(\sum_{i=1}^{n_q} q_i - q \right)$$

with

$$\text{with } \mu_m(k) \equiv \frac{\mathrm{d}^d k}{(2\pi)^d} 2\pi\delta\left(k^2 - m^2\right)\theta\left(k^0\right),$$

n: # final state particles, n_{fr}: # final state references, n_u: # additional partons

Factorization and subtraction terms

SU phase space

$$\iint_0^1 \mathrm{d}\eta\, \mathrm{d}\xi\, \eta^{a_1 - b_1\epsilon} \xi^{a_2 - b_2\epsilon}$$

DU phase space

$$\iiiint_0^1 \mathrm{d}\eta_1\, \mathrm{d}\xi_1\, \mathrm{d}\eta_2\, \mathrm{d}\xi_2\, \eta_1^{a_1 - b_1\epsilon} \xi_1^{a_2 - b_2\epsilon} \eta_2^{a_3 - b_3\epsilon} \xi_2^{a_4 - b_4\epsilon}$$

Factorized singular limits

$$\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}} \tilde{\mu}(\{x_i\})$$

$$\underbrace{\prod x_i^{a_i+1} \left\langle \mathcal{M}_{n+2} \middle| \mathcal{M}_{n+2} \right\rangle}_{\text{regular}} \mathrm{F}_{n+2}$$

Regularisation

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x \left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

The single unresolved (SU) contribution

- SU contribution

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \quad \text{with}$$

$$\sigma_{SU}^{c} = \int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{c}} F_{n+1} + I_{n}^{\mathrm{c}} F_{n} \right)$$

- NLO measurement function ($\alpha \neq 0$)

$$\int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{RR}} + I_{n+1}^{\mathrm{RV}} + I_{n+1}^{\mathrm{C1}} \right) F_{n+1}^{\alpha} = \text{finite in 4 dim.}$$

- All divergences cancel in d-dimensions

$$\sum_{c} \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^{2}} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1}^{\alpha} \equiv \sum_{c} \mathcal{I}^{c} = 0$$

SU finiteness for $\alpha = 0$

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{- \mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0}$$

$$\sigma_{SU}^{c} - \mathcal{I}^{c} = \int \mathrm{d}\Phi_{n+1} \left\{ \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} + I_{n}^{c,(0)} \right] F_{n} \right\}$$

$$- \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1} \theta_\alpha(\{\alpha_i\})$$

$$= \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)} F_{n+1} + I_{n}^{c,(-2)} F_{n}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)} F_{n+1} + I_{n}^{c,(-1)} F_{n}}{\epsilon} \right] (1 - \theta_\alpha(\{\alpha_i\}))$$

$$+ \int \mathrm{d}\Phi_{n+1} \left[I_{n+1}^{c,(0)} F_{n+1} + I_{n}^{c,(0)} F_{n} \right] + \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} \right] F_{n} \theta_\alpha(\{\alpha_i\})$$

$$=: \underbrace{Z^c(\alpha)}_{\text{integrable, zero volume for } \alpha \to 0} + \underbrace{C^c}_{\text{no divergences}} + \underbrace{N^c(\alpha)}_{\text{only } F_n \to \text{DU}}$$

The function $N^c(\alpha)$

Looks like slicing, but it is slicing *only* for divergences
$\rightarrow$ no actual slicing parameter in result

Power-log-expansion

$$N^c(\alpha) = \sum_{k=0}^{\ell_{\max}} \ln^k(\alpha) N_k^c(\alpha)$$

- all $N_k^c(\alpha)$ regular in α
- start expression independent of $\alpha \Rightarrow$ all logs cancel
- only $N_0^c(0)$ relevant

Putting parts together

$\sigma_{SU} - \sum_c N_0^c(0)$ and $\sigma_{DU} + \sum_c N_0^c(0)$
are finite in 4 dimension

$\Downarrow$

SU contribution

$\sigma_{SU} - \sum_c N_0^c = \sum_c C^c$
original expression σ_{SU} in 4-dim
without poles, no further ϵ pole cancellation

Calculation of $N_0^c(0)$

For each sector/contribution:

1. extraction of $\mathrm{d}\Phi_{n+1}$ from $\mathrm{d}\Phi_{n+2}\,|_{\text{SU pole}}$ (only for RR contribution)

$$\mathrm{d}\Phi_{n+2}\,|_{\text{SU pole}} = \Big(\underbrace{\mathrm{d}\Phi_n\,\mathrm{d}^d\mu(u_1)}_{\mathrm{d}\Phi_{n+1}}\,\mathrm{d}^d\mu(u_2)\Big)\Big|_{u_2\text{col/soft}}$$

2. expansion in ϵ up to ϵ^{-1} (except NLO parts): $\mathrm{d}^d\Phi_{n+1}\left(\frac{\cdots}{\epsilon^2}+\frac{\cdots}{\epsilon}\right)$
3. Identifying $\ln^k(\alpha)$'s from x_i integrations over θ function

$$\theta_\alpha(\hat{\eta}, u^0) = \theta(\hat{\eta}-\alpha)\theta(\hat{\xi}u_{max}/E_{norm}-\alpha)$$

   $\rightarrow$ discard them

4. perform integration over θ-functions of non-cancelling and non-vanishing (in $\alpha \rightarrow 0$ limit) terms