γ_5 in FDH

in collaboration with A. Signer
(arXiv:1710.09231)

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Loops and Legs in Quantum Field Theory
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To d, or not to d?

traditional dimensional schemes

- 't Hooft / Veltman (HV) '72
- conventional dim. reg. (CDR) '73
- dim. reduction (DRED) '79
- four-dim. helicity (FDH) '92

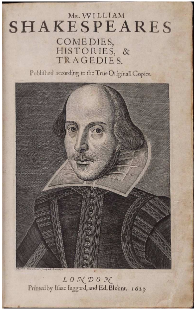
reformulations of dimensional schemes

- six-dim. formalism (SDF) '09
- four-dim. formalism (FDF) '14

non-dimensional schemes

- implicit reg. (IREG) '98
- loop regularization (LORE) '03
- four-dim. reg. / ren. (FDR) '12
- four-dim. unsubtraction (FDU) '16

→ comparison of the schemes: [arXiv:1705.01827]



- mathematical consistency
- unitarity, causality (equivalence to $\overline{\text{MS}}$ / BPHZ)
- symmetries (gauge invariance, SUSY, ...)
- computational efficiency (analytical/numerical automation)
- treatment of γ_5 (anomalies, ...)

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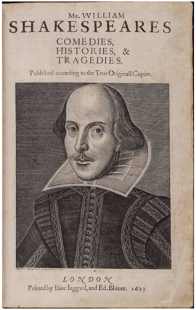

Dimensional regularization and γ_5

- (quasi) d-dim. phase-space integration:

$$\int \frac{d^4 k_{[4]}}{(2\pi)^4} \cdots \quad \to \quad \mu_{\text{DS}}^{4-d} \int \frac{d^d k_{[d]}}{(2\pi)^d} \cdots , \qquad\qquad d \equiv 4-2\epsilon$$

- special attention to be paid to treatment of γ_5
  $\to$ distinguish two classes:

(i) construction prescription:

['t Hooft/Veltman '72, Breitenlohner/Maison '77]

$$\gamma_5^{\text{BM}} \equiv \frac{i}{4!} \left(\varepsilon^{\mu\nu\rho\sigma} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \right)_{[4]} \equiv \frac{i}{4!} \varepsilon_{[4]}^{\mu\nu\rho\sigma} \left(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \right)_{[d]}$$

(ii) algebraic definition:

(same properties as for $d=4$)

$$\{\gamma_5^{\text{AC}}, \gamma_{[d]}^\mu\} \equiv 0$$

$d \neq 4$:

$$\{\gamma_5^{\text{BM}}, \gamma_{[d]}^\mu\} = 2\,\gamma_{[d-4]}^\mu \gamma_5^{\text{BM}} \neq \{\gamma_5^{\text{AC}}, \gamma_{[d]}^\mu\}$$

Both definitions yield different (intermediate) results.

Why not only using γ_5^{BM} ?

- complicated algebra (additional γ matrices, symmetrization,...)
- $d \neq 4$: symmetries of the unreg. theory broken explicitly
- additional counterterms to restore e.g. chiral and Lorentz invariance

Why not only using γ_5^{AC} ?

- demanding cyclic traces leads e.g. to $(d-4)\mathrm{Tr}\big[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\,\gamma_5^{\mathrm{AC}}\big]_{[d]} = 0$
- $d \neq 4$: gauge invariance broken explicitly through γ_5^{AC}-odd traces
- possible solution: redefinition of traces ('reading'/'cut' points)

Both definitions of γ_5 have disadvantages!

Why using dimensional regularization for γ_5 at all?

- finite-dim./implicit reg. schemes (IREG, FDR,. . .) face same difficulties
- contraction of Lorentz indices does not commute with renormalization

[Bruque,Cherchiglia,Pérez-Victoria '18]

This talk is about ...

...the formulation of *already existing* and *'implementation-friendly'* γ_5 schemes in FDH (i.e. γ_5 in the presence of evanescent d.o.f.).

(a) 't Hooft/Veltman, Breitenlohner/Maison:

$$\gamma_5^{\text{BM}} \equiv \frac{i}{4!}\,\varepsilon_{[4]}^{\mu\nu\rho\sigma}\,\big(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\big)_{[d]}$$

(b) anticommuting γ_5^{AC} with standard cyclic trace:

$$\{\gamma_5^{\text{AC}}, \gamma_{[d]}^{\mu}\} \equiv 0, \qquad \text{Tr}\big[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\,\gamma_5^{\text{AC}}\big]_{[d]} = 0$$

$\rightarrow$ hope/expectation (as FDH is somehow 'more' 4-dim.):
better features regarding treatment of γ_5

This talk is NOT about ...

...the definition of a *new* γ_5 scheme, e.g. with $\text{Tr}\big[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\,\gamma_5^{\text{AC}}\big]_{[d]} \neq 0$.

...a comprehensive study of *all* γ_5 schemes that do exist.

Outline

1. γ_5^{BM} and γ_5^{AC} in FDH

2. γ_5 in FDF

Dimensional schemes - a unified framework

$$\int \frac{d^4 k_{[4]}}{(2\pi)^4} \ldots \rightarrow \mu_{\text{DS}}^{4-d} \int \frac{d^d k_{[d]}}{(2\pi)^d} \ldots$$

[Stöckinger '05]

$S_{[4]}$	$\subset$	$QS_{[d]}$	$\subset$	$QS_{[d_s]}$	$\equiv$	$QS_{[d]}$	$\oplus$	$QS_{[n_\epsilon]}$
strictly four-dim.		quasi d-dim.		quasi d_s-dim.				'evanescent'
(unregularized)		(PS integration)		(FDH/DRED: $d_s = 4$)				space

	CDR	HV	FDH	DRED
singular vector fields (1PI; soft / collinear in initial / final state)	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[d_s]}$	$g^{\mu\nu}_{[d_s]}$
regular vector fields (all other VFs)	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[4]}$	$g^{\mu\nu}_{[4]}$	$g^{\mu\nu}_{[d_s]}$

FDH/DRED: $n_\epsilon = 2\epsilon$
$\rightarrow$ evanescent terms

CDR/HV: $n_\epsilon = 0$

γ_5 in FDH

- FDH: $g^{\mu\nu}_{[d_s]} = g^{\mu\nu}_{[d]} + g^{\mu\nu}_{[n_\epsilon]}, \quad \gamma^{\mu}_{[d_s]} = \gamma^{\mu}_{[d]} + \gamma^{\mu}_{[n_\epsilon]}, \quad \ldots$

- all (anti)commutators of $\gamma_5^{\rm BM}$ already fixed by definition:

$$\{\gamma_5^{\rm BM}, \gamma^{\mu}_{[d]}\} = 2\,\gamma^{\mu}_{[d-4]}\,\gamma_5^{\rm BM}$$

$$[\gamma_5^{\rm BM}, \gamma^{\mu}_{[n_\epsilon]}] = 0$$

  → *commutator* for $\gamma^{\mu}_{[n_\epsilon]}$ due to even number of γ matrices in the definition of $\gamma_5^{\rm BM}$

- same algebraic relations in d and d_s dimensions:

$$\{\gamma_5^{\rm BM}, \gamma^{\mu}_{[d_s]}\} = 2\,\gamma^{\mu}_{[d_s-4]}\,\gamma_5^{\rm BM}$$

- all (anti)commutators of $\gamma_5^{\rm AC}$ to be defined:

$$\{\gamma_5^{\rm AC}, \gamma^{\mu}_{[d]}\} \equiv 0$$

$$\{\gamma_5^{\rm AC}, \gamma^{\mu}_{[n_\epsilon]}\} \equiv 0$$

  → *anticommutator* for $\gamma^{\mu}_{[n_\epsilon]}$ (simpler algebra and UV ren., convenient choice for SUSY)

- same algebraic relations in d and d_s dimensions:

$$\{\gamma_5^{\rm AC}, \gamma^{\mu}_{[d_s]}\} = 0$$

γ_5 in FDH

- FDH: $g^{\mu\nu}_{[d_s]} = g^{\mu\nu}_{[d]} + g^{\mu\nu}_{[n_\epsilon]}, \quad \gamma^{\mu}_{[d_s]} = \gamma^{\mu}_{[d]} + \gamma^{\mu}_{[n_\epsilon]}, \quad \dots$

- all (anti)commutators of γ_5^{BM} already fixed by definition:

$$\{\, \gamma_5^{\mathrm{BM}}, \gamma^{\mu}_{[d]} \,\} = 2\, \gamma^{\mu}_{[d-4]}\, \gamma_5^{\mathrm{BM}}$$

$$[\, \gamma_5^{\mathrm{BM}}, \gamma^{\mu}_{[n_\epsilon]} \,] = 0$$

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- all (anti)commutators of γ_5^{AC} to be defined:

$$\{\, \gamma_5^{\mathrm{AC}}, \gamma^{\mu}_{[d]} \,\} \equiv 0$$

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  $\rightarrow$ *anticommutator* for $\gamma^{\mu}_{[n_\epsilon]}$ (simpler algebra and UV ren., convenient choice for SUSY)

- same algebraic relations in d and d_s dimensions:

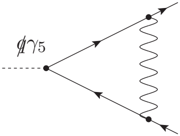
$$\{\, \gamma_5^{\mathrm{AC}}, \gamma^{\mu}_{[d_s]} \,\} = 0$$

γ_5^{BM}: different algebra compared to unreg. theory $\rightarrow$ symmetrization, e.g. [Larin '93]

$$\gamma^{\mu}_{[4]}\, \gamma_5 \;\rightarrow\; \frac{1}{2} \left(\gamma^{\mu}_{[d]}\, \gamma_5^{\mathrm{BM}} - \gamma_5^{\mathrm{BM}}\, \gamma^{\mu}_{[d]} \right)$$

Example 1: Pseudoscalar form factor in FDH

- one diagram at one-loop:



$$\sim g^{\mu\nu}_{[d_s]} = g^{\mu\nu}_{[d]} + g^{\mu\nu}_{[n_\epsilon]}$$

(unrenormalized) result for γ_5^{BM}:

$$\sim \varepsilon^{\mu\nu\rho\sigma}_{[4]} \int \dots \left(\not{q}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma - \gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\not{q}\right)_{[d]} \dots$$

$$\sim \frac{1 - \frac{n_\epsilon}{2}}{\epsilon} + \frac{9}{2} + \mathcal{O}(\epsilon)$$

(unrenormalized) result for γ_5^{AC}:

$$\sim \int \dots \left(\not{q}\gamma_5^{\text{AC}}\right)_{[d]} \dots$$

$$\sim \frac{1 + \frac{n_\epsilon}{2}}{\epsilon} + \frac{1}{2} + \mathcal{O}(\epsilon)$$

γ_5^{BM}: different sign for the n_ϵ-term compared to γ_5^{AC}, chiral and Lorentz invariance broken explicitly

$\rightarrow$ additional counterterm needed to restore symmetries

$$\delta Z^{\text{BM}} = \frac{n_\epsilon}{\epsilon} - 4$$

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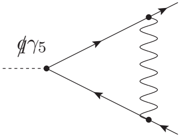

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(unrenormalized) result for γ_5^{AC}:

$$\sim \int \dots \left(\not{q}\gamma_5^{\text{AC}}\right)_{[d]} \dots$$

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γ_5^{BM}: different sign for the n_ϵ-term compared to γ_5^{AC}, chiral and Lorentz invariance broken explicitly

$\rightarrow$ additional counterterm needed to restore symmetries

$$\delta Z^{\text{BM}} = \frac{n_\epsilon}{\epsilon} - 4$$

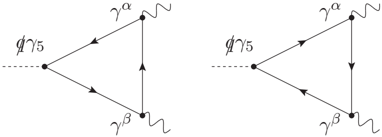
here γ_5^{BM} more complicated due to

- (strictly) four-dim. $\varepsilon^{\mu\nu\rho\sigma}_{[4]}$
- more γ matrices in numerator
- (anti)symmetrization 'by hand'
- additional counterterms

Example 2: AVV Correlator

- two diagrams at one-loop:

(same algebra as in CDR/HV, no n_ϵ-terms)



(unrenormalized) result for γ_5^{BM}:

$$\sim \varepsilon_{[4]}^{\mu\nu\rho\sigma} \int \mathrm{Tr}\big[(\not{q}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma - \gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\not{q})\ldots\big]_{[d]}$$

$$\sim \varepsilon_{[4]}^{\alpha\beta\mu\nu}(p_{1,\mu}p_{2,\nu})_{[d]} + \mathcal{O}(\epsilon)$$

(unrenormalized) result for γ_5^{AC}:

$$\sim \int \mathrm{Tr}\big[\not{q}\gamma_5^{\mathrm{AC}}\ldots\big]_{[d]} = 0$$

for standard cyclic trace

γ_5^{BM}: correct result for the (anomalous) axial Ward-identity ✓

$\rightarrow$ no additional counterterms needed to restore symmetry

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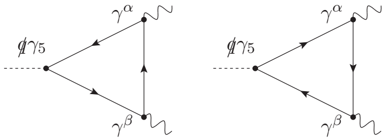

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for standard cyclic trace

γ_5^{BM}: correct result for the (anomalous) axial Ward-identity ✓

$\rightarrow$ no additional counterterms needed to restore symmetry

here γ_5^{AC} more complicated due to

- γ_5^{AC}-odd traces that vanish
- a breaking of the vector Ward-identity and therefore of gauge invariance

First conclusion

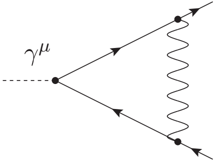
Even though FDH is somehow 'more 4-dimensional' than e.g. CDR and HV, it has the same features regarding γ_5.

Is this a general feature of dimensional regularization?
$\rightarrow$ Try to go more towards (strictly) 4 dimensions
within the dimensional framework.

FDF [Fazio,Mastrolia,Mirabella,Torres Bobadilla '14]

<u>basic idea</u>: *strictly* 4-dim. implementation of FDH algebra at NLO

- start from FDH-regularized quantities, e. g. $\not{k}_{[d]} = \not{k}_{[4]} + \not{k}_{[d-4]} \equiv \not{k}_{[4]} + i\,\mu\,\gamma_5$
- neglect *odd* powers of μ $\not{k}_{[d]}\not{k}_{[d]} = k^2_{[d]} \equiv k^2_{[4]} - \mu^2$



$$\sim \int d^d k_{[d]} \, \frac{\left[\gamma^\alpha\,(\not{k}_1 + i\,\mu\,\gamma_5)\,\gamma^\mu\,(\not{k}_2 + i\,\mu\,\gamma_5)\,\gamma_\alpha\right]_{[4]}}{\left[\,k_1^2\;k_2^2\;k^2\,\right]_{[d]}}$$

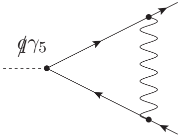
$$= \int d^d k_{[d]} \, \frac{f_1([4]) \;+\; f_2(\mu^2)}{g([d])} \qquad \equiv \qquad \mathcal{M}^{(1)}_{\text{FDH}}\Big|_{n_\epsilon = 2\epsilon}$$

FDH results recovered by using *strictly* 4-dim. numerator algebra.

- equivalence to FDH shown in many examples ($gg \to ggg$, $gg \to q\bar{q}$, $gg \to gH$,...)
- explicit representation of pol. vectors and spinors
  $\to$ enables use of generalized-unitarity methods
- compatible with color-kinematics duality

γ_5 in FDF

- FDF algebra realized in strictly four dimensions $\Rightarrow \gamma_5^{\text{BM}} = \gamma_5^{\text{AC}} \equiv \gamma_5$
- back to example 1:



FDF:

$$\sim \int \ldots \left[(\not{k}_1 + i\,\mu\,\gamma_5)\ \not{q}\gamma_5\ (\not{k}_2 + i\,\mu\,\gamma_5) \right]_{[4]} \cdots$$

$$\sim \frac{1}{\epsilon} + \frac{7}{2} + \mathcal{O}(\epsilon)$$

FDH + γ_5^{BM}:

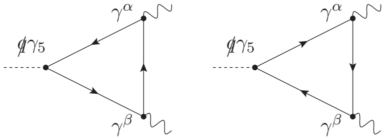
$$\sim \varepsilon^{\mu\nu\rho\sigma}_{[4]} \int \ldots \left(\not{q}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma - \gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\not{q} \right)_{[d]} \cdots$$

$$\sim \frac{1 - \frac{n_\epsilon}{2}}{\epsilon} + \frac{9}{2} + \mathcal{O}(\epsilon) \overset{n_\epsilon = 2\epsilon}{=} \frac{1}{\epsilon} + \frac{7}{2} + \mathcal{O}(\epsilon)$$

- FDF yields same results as FDH + γ_5^{BM} ✓
  $\rightarrow$ same counterterms needed to restore symmetries
- however: result obtained in a much simpler way ✓

γ_5 in FDF

- example 2: AVV correlator



FDF:

$$\sim \int \mathrm{Tr}\Big[(\not{k}_1 + i\,\mu\,\gamma_5)\;\not{q}\gamma_5\;(\not{k}_2 + i\,\mu\,\gamma_5)\ldots\Big]_{[4]}$$

$$\sim \varepsilon^{\alpha\beta\mu\nu}_{[4]}(p_{1,\mu}p_{2,\nu})_{[4]} + \mathcal{O}(\epsilon)$$

$\rightarrow$ result stems from μ^2 terms only, vanishes for $\mu = 0$ (unreg. theory)

FDH + γ_5^{BM}:

$$\sim \varepsilon^{\mu\nu\rho\sigma}_{[4]} \int \mathrm{Tr}\Big[(\not{q}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma - \gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\not{q})\ldots\Big]_{[d]}$$

$$\sim \varepsilon^{\alpha\beta\mu\nu}_{[4]}(p_{1,\mu}p_{2,\nu})_{[d]} + \mathcal{O}(\epsilon)$$

- FDF: correct result for the axial Ward-identity ✓
  (strictly 4-dim. algebra does not lead to breaking of gauge invariance)
- again: FDF algebra much simpler than FDH + γ_5^{BM} ✓

γ_5 in FDF

- FDF constitutes effective implementation of FDH + γ_5^{BM}.
  $\rightarrow$ conceptual reason:

$$
\begin{aligned}
\{\gamma_5, \not{k}_{[d]}\} &= \gamma_5 \big(\not{k}_{[4]} + i\,\mu\,\gamma_5\big) + \big(\not{k}_{[4]} + i\,\mu\,\gamma_5\big)\gamma_5 \\
&= \gamma_5 \big(\not{k}_{[4]} + i\,\mu\,\gamma_5\big) + \gamma_5 \big(-\not{k}_{[4]} + i\,\mu\,\gamma_5\big) \\
&= 2\,\big(i\,\mu\,\gamma_5\big)\,\gamma_5 = 2\,\not{k}_{[d-4]}\,\gamma_5
\end{aligned}
$$

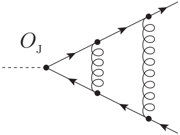
Although the algebra is realized in *strictly* 4 dimensions, the (anti)commutator is the same as for γ_5^{BM}!

however: FDF (so far) not formulated beyond one-loop

- multi-loop calculations: consider
  (i) FDH + γ_5^{BM}
  (ii) FDH + γ_5^{AC}

γ_5 in FDH beyond one loop

- pseudoscalar form factor at two loops in FDH
  (i) γ_5^{BM}, e.g.



$$O_{\text{J}} = \partial_\mu \left(\overline{\psi}\, \gamma^\mu \gamma_5\, \psi \right) \to \varepsilon^{\mu\nu\rho\sigma}_{[4]} \left\{ \partial_\mu \left(\overline{\psi}\, \gamma_\nu \gamma_\rho \gamma_\sigma\, \psi \right) \right\}_{[d]}$$

- as before: non-trivial operator renormalization: $O_{\text{J,ren.}} = \left(Z^{\text{BM}}_{\overline{\text{MS}}}\, Z^{\text{BM}}_5 \right) O_{\text{J,bare}}$
  → results in FDH [Signer,CG '17]

$$Z^{\text{BM}}_{\overline{\text{MS}}} = 1 + \left(\frac{\alpha_s}{4\pi}\right) C_F \frac{n_\epsilon}{\epsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left\{ C_A C_F \left[\frac{22}{3\epsilon} + n_\epsilon \left(-\frac{1}{\epsilon^2} + \frac{11}{3\epsilon} \right) + n_\epsilon^2 \left(\frac{1}{2\epsilon^2} + \frac{1}{4\epsilon} \right) \right] + C_F^2 \left[n_\epsilon \left(-\frac{1}{\epsilon^2} - \frac{4}{\epsilon} \right) - \frac{3 n_\epsilon^2}{4\epsilon} \right] + C_F N_F \left[\frac{5}{3\epsilon} + n_\epsilon \left(\frac{1}{2\epsilon^2} - \frac{1}{4\epsilon} \right) \right] \right\} + \mathcal{O}(\alpha_s^3)$$

$$Z^{\text{BM}}_5 = 1 + \left(\frac{\alpha_s}{4\pi}\right) \left\{ -4\, C_F \right\} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left\{ 22\, C_F^2 - \frac{107}{9}\, C_A + \frac{31}{18}\, C_F N_F \right\} + \mathcal{O}(\alpha_s^3)$$

- Z^{BM}_5 usually obtained by imposing

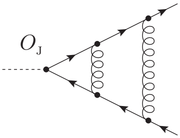
$$\partial_\mu\, j_5^\mu = 2m\, j_5 + \frac{\alpha_s}{4\pi}\, \varepsilon^{\mu\nu\rho\sigma}\, G^a_{\mu\nu} G^a_{\rho\sigma}$$

> anomalous term of $\mathcal{O}(\alpha_s) \Rightarrow (L{+}1)$-loop calculation needed to obtain L-loop value of Z^{BM}_5 (known up to two loops)

γ_5 in FDH beyond one loop

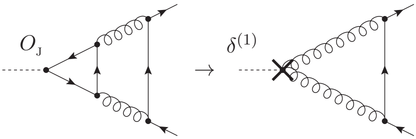
- pseudoscalar form factor at two loops in FDH
  (ii) $\gamma_5^{\mathrm{AC}} \to$ distinguish two classes of diagrams

<u>Type A</u>:
γ_5 attached to external quark line, e.g.



- no γ_5-odd traces $\to$ use γ_5^{AC}
- then: trivial op. ren. $O^{\mathrm{AC}}_{\mathsf{J,ren.}} = O^{\mathrm{AC}}_{\mathsf{J,bare}}$

<u>Type B</u>:
γ_5 attached to quark loop, e.g.



- vanishes for γ_5^{AC} and standard trace
- however: value of anomaly known from before (γ_5^{BM}, FDF, ...)
- op.ren. effectively reduced by one order

> Comparing this approach with γ_5^{BM}, the L-loop value of Z_5^{BM} can be obtained from a genuine L-loop calculation.

Conclusions

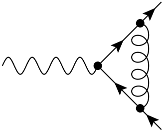
On the one hand: Even though FDH has some 4-dimensional features, it faces the same problems regarding the treatment of γ_5 as other 'traditional' dimensional schemes like CDR and HV.

This, however, is also true for finite-dim./implicit regularization schemes that stay in the physical dimension like IREG and FDR.

On the other hand: At least for one-loop computations, FDF constitutes an effective implementation of the γ_5 algebra that reduces the complexity of explicit computations.

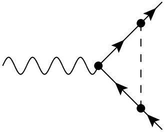
Supplement 1: Practical computations in FDH

- example: form factor in FDH / DRED
  → in principle two diagrams ($d_s = d + n_\epsilon$)



gauge field

$\sim\ \alpha_s\, g^{\mu\nu}_{[d]}$



ϵ-scalar with 'evanescent' coupling

$\sim\ \alpha_e\, g^{\mu\nu}_{[n_\epsilon]}$

→ can be simplified

$$\sim \int \frac{d^d k_{[d]}}{(2\pi)^d} \frac{\left[\gamma^\alpha \gamma^\rho \gamma^\mu \gamma^\sigma \gamma_\alpha\right]_{[d_s]}}{[\ldots][\ldots][\ldots]} \times \left[k_\rho k_\sigma\right]_{[d]}$$

$$\sim\ -2\left[\gamma^\sigma \gamma^\mu \gamma^\rho\right]_{[d_s]}\ +\ \underbrace{(4-d_s)}_{\equiv\ (2\epsilon - n_\epsilon)}\ \left[\gamma^\rho \gamma^\mu \gamma^\sigma\right]_{[d_s]}$$

- CDR / HV: $n_\epsilon = 0$
  → second term contributes

- FDH / DRED: $n_\epsilon = 2\epsilon$
  → second term vanishes

FDH / DRED:

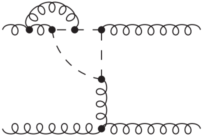
do algebra in $d_s = 4$ from the beginning
(even though nothing is (strictly) 4-dim.)

→ simplified algebra and integral reduction

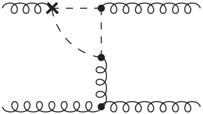
Supplement 2: ϵ-scalar couplings

- When is it mandatory to consider ϵ-scalars in FDH and DRED?

$gg \to gg$

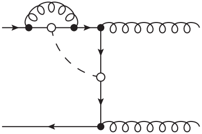


$\mathcal{O}(\alpha_s^2)$

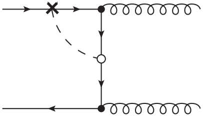


$\mathcal{O}(\alpha_s\, \delta Z_{\alpha_s})$

$q\bar{q} \to gg$



$\mathcal{O}(\alpha_e \alpha_s)$



$\mathcal{O}(\alpha_e\, \delta Z_{\alpha_e})$

- (ren.) couplings can be set equal, i. e. $\alpha_e \equiv \alpha_s$
- HOWEVER: in non-SUSY theories $\delta Z_{\alpha_e} \neq \delta Z_{\alpha_s}$

> L-loop calculation in FDH:
>
> α_e to be distinguished from α_s at $(L-1)$ loops
> (except you are lucky, e. g. $gg \to gg$)