Precise determination of the strong coupling from energy-energy correlation

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based on
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and
arXiv:1804.09146

Loops and Legs 2018
2018.05.01., St. Goar

Motivation

α_S is a fundamental parameter of the SM and must be determined precisely

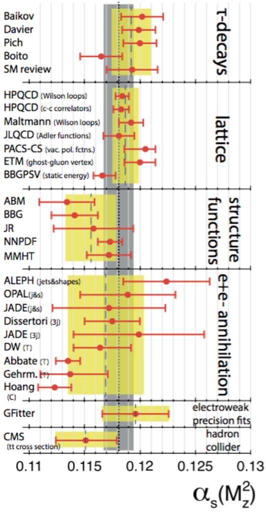
Obtained from fits to data

High precision measurements demand highly accurate theoretical predictions

One option: from 3-jet event shapes in e^+e^- collisions:

- extensively measured by multiple collaborations
- the Born contribution is proportional to α_S
- state-of-the-art theory: NNLO fixed-order and NNLL resummation (N^3LL for thrust and C-parameter)

α_S world average



[*S. Bethke, Nucl. Part. Phys. Proc.* **282-284** *(2017) 149*]

Determination from e^+e^- annihilation based on

- jet rates
- event shapes describing global topology (thrust, C-parameter, etc.)
- longstanding problem of low $\alpha_S(M_Z)$ from N^3LL+NNLO event shapes (with analytic hadronization models)

Can also consider observables based on particle correlations

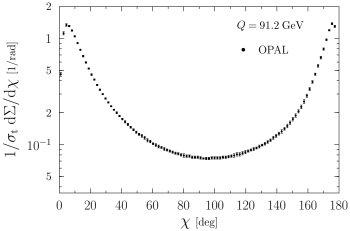
Energy-energy correlation (see also V. Shtabovenko's talk)

Energy-energy correlation in $e^+e^- \to$ jets:

$$\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi} = \frac{1}{\sigma_t}\int \sum_{i,j} \frac{E_i\, E_j}{Q^2} d\sigma_{e^+e^-\to ij+X}\delta(\cos\chi + \cos\theta_{ij})$$

E_i and E_j are particle energies, Q is the center-of-mass energy and $\theta_{ij} = \pi - \chi$ is the angle between the two particles

Was measured at LEP, PEP, PETRA, SLC and TRISTAN



[*OPAL Collaboration, P.D. Acton et al. Z. Phys.* **C59** *(1993)*]

Available data

Experiment	$\sqrt{s}$, GeV, data	$\sqrt{s}$, GeV, MC	Events
SLD	91.2(91.2)	91.2	60000
OPAL	91.2(91.2)	91.2	336247
OPAL	91.2(91.2)	91.2	128032
L3	91.2(91.2)	91.2	169700
DELPHI	91.2(91.2)	91.2	120600
TOPAZ	59.0 – 60.0(59.5)	59.5	540
TOPAZ	52.0 – 55.0(53.3)	53.3	745
TASSO	38.4 – 46.8(43.5)	43.5	6434
TASSO	32.0 – 35.2(34.0)	34.0	52118
PLUTO	34.6(34.6)	34.0	6964
JADE	29.0 – 36.0(34.0)	34.0	12719
CELLO	34.0(34.0)	34.0	2600
MARKII	29.0(29.0)	29.0	5024
MARKII	29.0(29.0)	29.0	13829
MAC	29.0(29.0)	29.0	65000
TASSO	21.0 – 23.0(22.0)	22.0	1913
JADE	22.0(22.0)	22.0	1399
CELLO	22.0(22.0)	22.0	2000
TASSO	12.4 – 14.4(14.0)	14.0	2704
JADE	14.0(14.0)	14.0	2112

Table: Data used in the extraction procedure. The average of $\sqrt{s}$ (used for MC generation) is given in the brackets.

Fixed-order calculation

The fixed-order expansion of EEC is

$$\left[\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi}\right]_{(\text{f.o.})} = \frac{\alpha_{\mathrm{S}}}{2\pi}\frac{d\bar{A}}{d\cos\chi} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2\frac{d\bar{B}}{d\cos\chi} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3\frac{d\bar{C}}{d\cos\chi} + \mathcal{O}(\alpha_{\mathrm{S}}^4)$$

We performed perturbative calculations up to NNLO using the CoLoRFulNNLO scheme [*V. Del Duca, G. Somogyi, Z. Trócsányi*]

The scheme was implemented in the `MCCSM` package [*A. Kardos*]

Has already been tested on $H \to b\bar{b}$ and $e^+e^- \to 3$ jets

CoLoRFulNNLO scheme (see also in A. Kardos's talk on Thursday)

Completely local subtraction for fully differential predictions at NNLO

The NNLO correction contains three separately divergent terms:

$$\sigma^{NNLO}[J] = \int_{m+2} d\sigma^{RR}_{m+2} J_{m+2} \ + \int_{m+1} d\sigma^{RV}_{m+1} J_{m+1} \ + \int_{m} d\sigma^{VV}_{m} J_{m}$$

In the $m+2$ parton term subtractions are needed to regularize 1- and 2-parton unresolved emissions:

$$\sigma^{NNLO}_{m+2} = \int_{m+2} \left\{ d\sigma^{RR}_{m+2} J_{m+2} - d\sigma^{RR,A_2}_{m+2} J_m - \left[d\sigma^{RR,A_1}_{m+2} J_{m+1} - d\sigma^{RR,A_{12}}_{m+2} J_m \right] \right\}_{d=4}$$

The $m+1$ parton term collects 1-parton emissions from the real-virtual term:

$$\sigma^{NNLO}_{m+1} = \int_{m+1} \left\{ \left(d\sigma^{RV}_{m+1} + \int_1 d\sigma^{RR,A_1}_{m+2} \right) J_{m+1} - \left[d\sigma^{RV,A_1}_{m+1} + \left(\int_1 d\sigma^{RR,A_1}_{m+2} \right)^{A_1} \right] J_m \right\}_{d=4}$$

The m parton term contains the double virtual term and integrated subtractions:

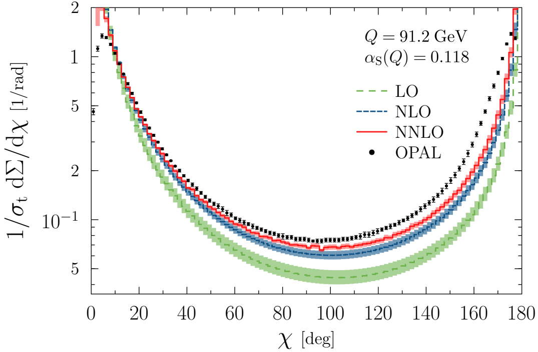
$$\sigma^{NNLO}_{m} = \int_{m} \left\{ d\sigma^{VV}_{m} + \int_2 \left[d\sigma^{RR,A_2}_{m+2} - d\sigma^{RR,A_{12}}_{m+2} \right] + \int_1 \left[d\sigma^{RV,A_1}_{m+1} + \left(d\sigma^{RR,A_1}_{m+2} \right)^{A_1} \right] \right\}_{d=4} J_m$$

CoLoRFulNNLO scheme

General features:

- fully local counterterms
  (mathematically well defined)
- fully differential predictions
  (with jet measurement functions defined in four dimensions)
- explicit expressions including flavor and color
  (using color space notation)
- analytic cancellation of ϵ poles in (double) virtual contribution
  (important analytic check)

EEC at NNLO



Higher order predictions improve agreement with data for medium angles

Sizable differences remain due to hadronization and resummation corrections

In the forward ($\chi = 180^\circ$) and back-to-back ($\chi = 0^\circ$) regions fixed-order calculations diverge due to multiple soft emissions

Resummation

EEC resummation is known in the back-to-back region up to NNLL, [*D. de Florian, M. Grazzini*, (2005)] (and partially to N^{3}LL [*I. Moult, H. X. Zhu*,(2018)])

$$\left[\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi}\right]_{\text{(res.)}} = \frac{Q^2}{8}H(\alpha_S)\int_0^\infty db\, J_0(b\,Q\sqrt{y})\; S(Q,b)$$

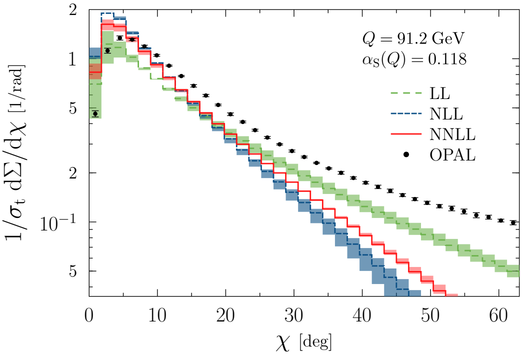
where $y = \sin^2\left(\frac{\chi}{2}\right)$ and the Sudakov form factor collects all log-enhanced terms

$$S(Q,b) = \exp\left\{-\int_{b_0^2/b^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_S(q^2))\,\ln\frac{Q^2}{q^2} \;+\; B(\alpha_S(q^2))\right]\right\}$$

The functions $A(\alpha_S)$, $B(\alpha_S)$ and $H(\alpha_S)$ can be computed perturbatively

$$A(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{4\pi}\right)^n A^{(n)}, \quad B(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{4\pi}\right)^n B^{(n)}, \quad H(\alpha_S) = 1+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{4\pi}\right)^n H^{(n)}$$

Resummation



The pure resummed results capture the general behavior of the data for small angles

Differences become sizable even for moderate values of χ

log-R matching

Resummed and fixed-order calculations are complementary to each other

In our case the fixed-order expansion of the EEC diverges for both small and large angles making the determination of a simple cumulant unreliable

We use a linear combination of moments to suppress the singularity at $\chi = \pi$

$$\frac{1}{\sigma_t}\tilde{\Sigma}(\chi) \equiv \frac{1}{\sigma_t}\int_0^{\chi} d\chi'(1+\cos\chi')\,\frac{d\Sigma}{d\chi'}$$

with fixed-order expansion

$$\left[\frac{1}{\sigma_t}\tilde{\Sigma}(\chi)\right]_{\text{(f.o.)}} = 1 + \frac{\alpha_S}{2\pi}\bar{\mathcal{A}}(\chi) + \left(\frac{\alpha_S}{2\pi}\right)^2\bar{\mathcal{B}}(\chi) + \left(\frac{\alpha_S}{2\pi}\right)^3\bar{\mathcal{C}}(\chi) + \mathcal{O}(\alpha_S^4)$$

The integration constants in $\bar{\mathcal{A}}$, $\bar{\mathcal{B}}$ and $\bar{\mathcal{C}}$ are fixed by

$$\frac{1}{\sigma_t}\tilde{\Sigma}(\pi) = \frac{1}{\sigma_t}\int\sum_{i,j}\frac{E_iE_j}{Q^2}(1-\cos\theta_{ij})\,d\sigma_{e^+e^-\to ij+X} = 1 \quad \text{(in massless QCD)}$$

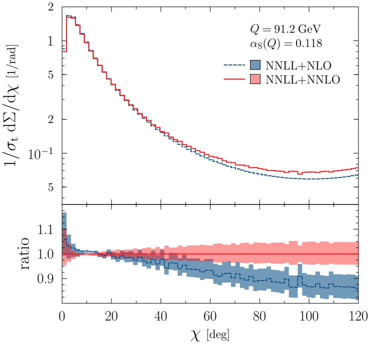
log-R matching

Non-logarithmically enhanced constant terms from $H(\alpha_S)$ must not be exponentiated and thus should not appear in the formula for the matched expression

At NNLL+NNLO we compute

$$\ln\left[\frac{1}{\sigma_t}\tilde{\Sigma}\right] = \ln\left\{\frac{1}{H(\alpha_S)}\left[\frac{1}{\sigma_t}\tilde{\Sigma}\right]_{(\text{res.})}\right\} - \ln\left\{\frac{1}{H(\alpha_S)}\left[\frac{1}{\sigma_t}\tilde{\Sigma}\right]_{(\text{res.})}\right\}\Bigg|_{\text{f.o.}}$$
$$+ \frac{\alpha_S}{2\pi}\bar{\mathcal{A}} + \left(\frac{\alpha_S}{2\pi}\right)^2\left(\bar{\mathcal{B}} - \frac{1}{2}\bar{\mathcal{A}}^2\right) + \left(\frac{\alpha_S}{2\pi}\right)^3\left(\bar{\mathcal{C}} - \bar{\mathcal{B}}\bar{\mathcal{A}} + \frac{1}{3}\bar{\mathcal{A}}^3\right)$$

NNLL+NLO vs NNLL+NNLO



Sizable difference between NNLL+NLO and NNLL+NNLO for $\chi > 40^\circ$

Reduced uncertainty band from scale variation at NNLL+NNLO (not apparent on plot due to normalization)

Finite b-quark mass corrections

At low energies the assumption of vanishing quark masses is not entirely justified

We include mass effects directly at the level of matched distributions

$$\frac{1}{\sigma_t}\frac{d\Sigma(\chi,Q)}{d\cos\chi} = (1-r_b(Q))\left[\frac{1}{\sigma_t}\frac{d\Sigma(\chi,Q)}{d\cos\chi}\right]_{massless} + r_b(Q)\left[\frac{1}{\sigma_t}\frac{d\Sigma(\chi,Q)}{d\cos\chi}\right]^{NNLO*}_{massive}$$

The complete NNLO correction to the massive distribution is unknown, hence we supplement the massive NLO prediction of `Zbb4` [*P. Nason, C. Oleari*, (1997)] with the NNLO coefficient of the massless calculation

We define the fraction of b-quark events as

$$r_b(Q) \equiv \frac{\sigma_{massive}(e^+e^- \to b\bar{b})}{\sigma_{massive}(e^+e^- \to \text{hadrons})}$$

Distributions were generated assuming a pole b-quark mass of $m_b = 4.75$ GeV for each Q separately

Non-perturbative corrections

NLO MC events were produced by particle level generators to extract point-by-point multiplicative correction factors

Simultaneously allows for the estimation of the missing statistical correlations of data points

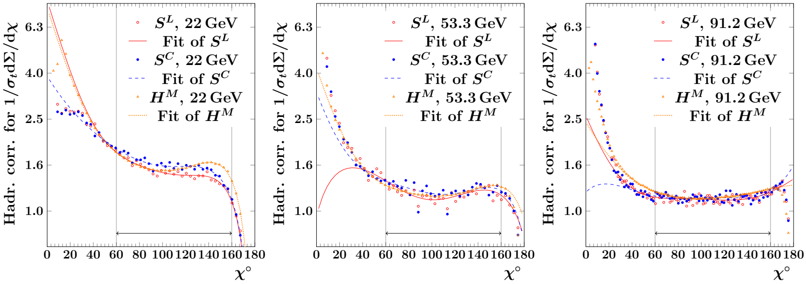
Four setups were used:

- S^L: default setup, `Sherpa2.2.4`, using Lund string model
- S^C: for hadronization systematics, `Sherpa2.2.4`, using cluster hadronization
- H^M: for MC cross check, `Herwig7.1.1`, using cluster hadronization
- DMW: for cross check with analytic hadronization model

Hadronization corrections are ratios of hadron to parton level distributions in the MCs

Simulated samples were reweighted to data at hadron level on an event-by-event basis to assure a better description of data

Non-perturbative corrections



Hadronization corrections are parametrized using smooth functions to tame statistical fluctuations

Parametrization is valid only in the fit range

Fit procedure and estimation of uncertainties

`MINUIT2` was used to minimize

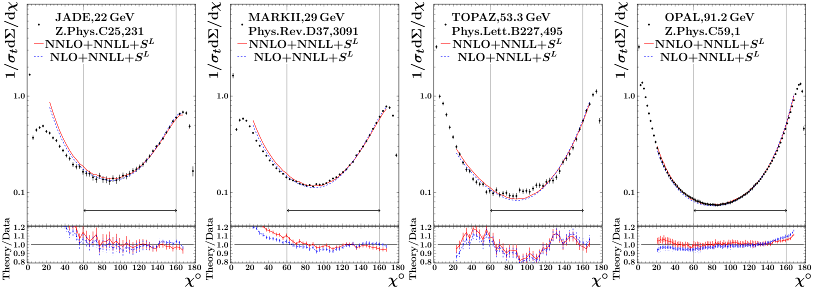
$$\chi^2(\alpha_S) = \sum_{\text{datasets}} \chi^2(\alpha_S)_{\text{dataset}}$$

Correlations between datapoints were taken into account

$$\chi^2(\alpha_S) = (\vec{D} - \vec{P}(\alpha_S))V^{-1}(\vec{D} - \vec{P}(\alpha_S))^T,$$

- $\vec{D}$: vector of data points
- $\vec{P}(\alpha_S)$: vector of calculated predictions
- V: covariance matrix for $\vec{D}$

Fit to data

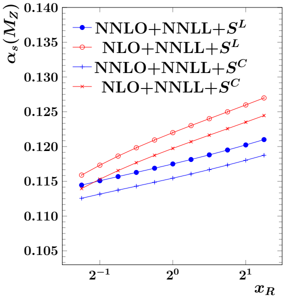


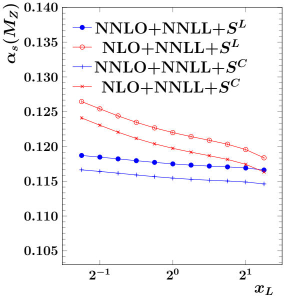
Fit range: $[60^\circ, 160^\circ]$ ($[117^\circ, 165^\circ]$ for DMW setup)

Fit range was chosen to avoid regions where the theoretical prediction or hadronization corrections become unreliable

The result is insensitive to a $\pm 5^\circ$ change in the fit range

Fit uncertainties



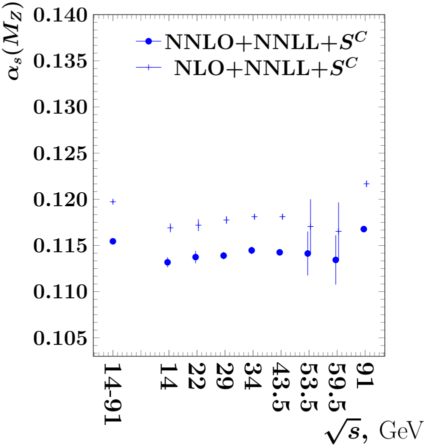


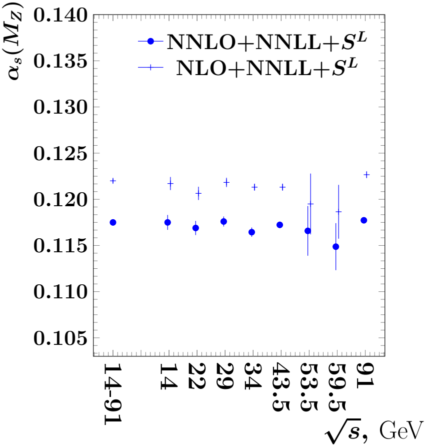
The uncertainties that were estimated:

- Variation of renormalization scale, $x_R \in [1/2, 2]$: (*ren.*)
- Variation of resummation scale, $x_L \in [1/2, 2]$: (*res.*)
- Variation of hadronization model S^L or S^C: (*hadr.*)
- Fit uncertainty is $\chi^2 + 1$ criterion from `MINUIT2`: (*exp.*)

Notice reduced slope at NNLL+NNLO

Fit dependence on Q





Fits performed for each energy separately

The obtained results are consistent for different energies

Final results

Global fit at NNLL+NLO:

$\alpha_S(M_Z) = 0.12200 \pm 0.00023(\mathit{exp.}) \pm 0.00113(\mathit{hadr.}) \pm 0.00433(\mathit{ren.}) \pm 0.00293(\mathit{res.})$

with combined uncertainty: $\alpha_S(M_Z) = 0.12200 \pm 0.00535$

Global fit at NNLL+NNLO:

$\alpha_S(M_Z) = 0.11750 \pm 0.00018(\mathit{exp.}) \pm 0.00102(\mathit{hadr.}) \pm 0.00257(\mathit{ren.}) \pm 0.00078(\mathit{res.})$

with combined uncertainty: $\alpha_S(M_Z) = 0.11750 \pm 0.00287$

The effect of NNLO on central value is moderate but not negligible, *ren.* uncertainty down by a factor of 2, *res.* uncertainty down by a factor of 3

The overall uncertainty is dominated by theoretical uncertainty (*ren.* and *res.*)

Cross check with analytic hadronization model

Hadronization effects modelled by a multiplicative correction to the Sudakov

$$S_{NP} = e^{-\frac{1}{2} a_1 b^2} (1 - 2\, a_2\, b)$$

(a_1 and a_2 are fit parameters)

Results strongly depend on fit range but are close to MC in the range $[117^\circ, 165^\circ]$

For NNLL+NLO: $\alpha_{\mathrm{S}}^{DMW}(M_Z) = 0.12154 \pm 0.00045(exp.)$

For NNLL+NNLO: $\alpha_{\mathrm{S}}^{DMW}(M_Z) = 0.11781 \pm 0.00037(exp.)$

Cannot describe hadronization away from back-to-back region

Conclusions

Presented a new measurement of $\alpha_S(M_Z)$ using global fit of EEC in e^+e^- collisions to NNLL+NNLO predictions

$$\alpha_S(M_Z) = 0.11750 \pm 0.00287$$

The value is compatible with the world average ($\alpha_S(M_Z) = 0.1181 \pm 0.0011$) and the estimated uncertainty is dominated by the uncertainty of the fixed-order prediction

Impact of NNLO corrections:

- better modelling of the shape of the distribution
- theoretical uncertainties are reduced
- non-negligible shift of extracted $\alpha_S(M_Z)$ towards lower values

Latest progress on the theoretical side:

- Everything but the two-loop jet function for N^3LL resummation is now known [*I. Moult, H. X. Zhu*,(2018)]
- Analytic NLO results are also available [*L.J. Dixon et al.*,(2018)]

Acknowledgements

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EMBERI ERŐFORRÁSOK
MINISZTÉRIUMA