Top pair production at NNLO

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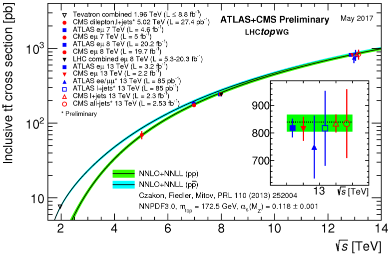
European Commission

National Science Centre Poland

Loops and Legs in Quantum Field Theory, St. Goar, Germany, 1 May 2018

Top pair production: the status of QCD calculations

- A single *complete* NNLO result for total and differential cross section obtained with STRIPPER methodology [Czakon, Fiedler, Mitov '13; Czakon, Heymes, Mitov '16]



- Flavour off-diagonal channels at NNLO from q_T subtraction [Bonciani, Catani, Grazzini, Sargsyan, Torre '15]
- Approximate NNLO [Broggio, Papanastasiou, Signer '14] and N^{3}LO [Kidonakis '14]
- Soft and small-mass resummation at NNLL [Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, Yang '18]
- Small-q_T resummation at NNLL [Li, Li, Shao, Yang, Zhu '13; Catani, Grazzini, Torre '14]

The q_T slicing method

[Catani, Grazzini '07, '15]

$$p + p \to F\,(q_T) + X$$

$$\sigma^{F}_{\mathrm{N}^m\mathrm{LO}} = \int_0^{q_{T,\mathrm{cut}}} dq_T \, \frac{d\sigma^{F}_{\mathrm{N}^m\mathrm{LO}}}{dq_T} + \int_{q_{T,\mathrm{cut}}}^{\infty} dq_T \, \frac{d\sigma^{F}_{\mathrm{N}^m\mathrm{LO}}}{dq_T}$$

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enough to know in small-q_T approximation

known

Soft Collinear Effective Theory (SCET)

$$\text{SCET} \simeq \text{QCD}\Big|_{\text{IR limit}}$$

- Hard degrees of freedom are integrated out into Wilson coefficients, which are then used to adjust new couplings of the (effective) theory.

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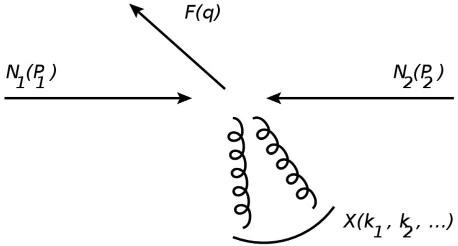
The new fields decouple in the Lagrangian

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_c + \mathcal{L}_{\bar{c}} + \mathcal{L}_s$$

- The separation of fields in the Lagrangian into collinear, anti-collinear and soft sectors, facilitates proofs of factorization theorems

Small-q_T factorization in SCET



where $F = H, Z, W, ZZ, WW, t\bar{t}, \ldots$

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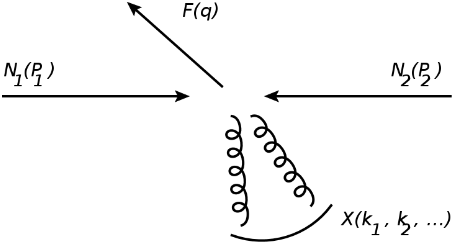

where $F = H, Z, W, ZZ, WW, t\bar{t}, \dots$

$$\frac{d\sigma^F}{d\Phi} = \mathcal{B}_1 \otimes \mathcal{B}_2 \otimes \mathcal{H} \otimes \mathcal{S} + \mathcal{O}\left(\frac{q_T^2}{q^2}\right)$$

Small-q_T factorization in SCET

Gluons' momenta in light-cone coordinates

$$k_i^\mu = \left(k_i^+, k_i^-, \boldsymbol{k}_i^\perp\right) \qquad \text{where} \qquad k^\pm = k^0 \pm k^3$$

Expansion parameter

$$\lambda = \frac{q_T^2}{q^2} \ll 1$$

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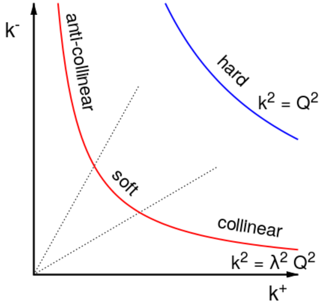
Regions

collinear	$k_i^\mu \sim (1, \lambda^2, \lambda)\, Q^2$	$\mathcal{B}_1$
anti-collinear	$k_i^\mu \sim (\lambda^2, 1, \lambda)\, Q^2$	$\mathcal{B}_2$
hard	$k_i^\mu \sim (1, 1, 1)\, Q^2$	$\mathcal{H}$
soft	$k_i^\mu \sim (\lambda, \lambda, \lambda)\, Q^2$	$\mathcal{S}$



Top pair production at small-q_T through NNLO

$$\frac{d\sigma^{\text{NNLO}}}{dq_T\, dy\, dM\, d\cos\theta} = \sum_{i,\bar{i}} \mathcal{B}_{i/N_1} \otimes \mathcal{B}_{\bar{i}/N_2} \otimes \text{Tr}\left[\mathcal{H}_{i\bar{i}} \otimes \mathcal{S}_{i\bar{i}}\right]$$

where

q_T, y, M	:	transverse momentum, rapidity, mass of top quark pair
θ	:	scattering angle of the top quark in $t\bar{t}$ rest frame

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- $\mathcal{B}$ - known up to NNLO [Gehrmann, Lübbert, Yang '12, '14]
- $\mathcal{H}$ - known up to NNLO [Czakon '08; Baernreuther, Czakon, Fiedler '13]
- $\mathcal{S}$ - known up to NLO in small-q_T limit [Li, Li, Shao, Yan, Zhu '13; Catani, Grazzini, Torre '14] (and up to NNLO in the threshold limit [Wang, Xu, Yang and Zhu '18])

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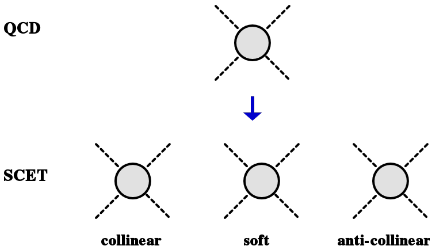
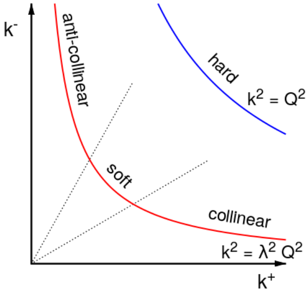
Calculating the missing NNLO correction to the soft function in the small-q_T limit, $\mathcal{S}$, is the aim of this phase of our work.

Rapidity divergences and analytic regulator





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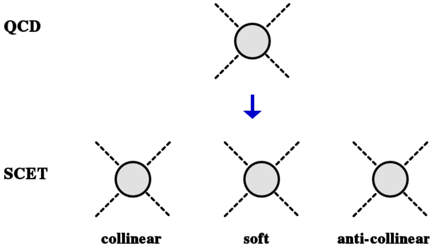

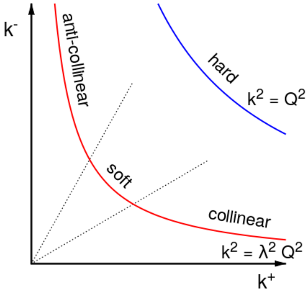

Modification of the measure [Becher, Bell '12]

$$\int d^d k\, \delta^+(k^2) \to \int d^d k \left(\frac{\nu}{k_+}\right)^{\alpha} \delta^+(k^2)$$

- The regulator is necessary at intermediate steps of the calculation.
- Rapidity divergences do not appear in QCD, hence, the complete SCET result has to stay finite in the limit $\alpha \to 0$.

Kinematics and notation

- Partonic process

$$q(p_1) + \bar{q}(p_2) \;\rightarrow\; t(p_3) + \bar{t}(p_4) + \textstyle\sum_i g(k_i)$$

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- Invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad\qquad M^2 = (p_3 + p_4)^2$$

$$t_1 = (p_1 - p_3)^2 - m_t^2 \qquad\qquad u_1 = (p_1 - p_4) - m_t^2$$

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- Small-q_T limit

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- Momenta

$$n = (1,0,0,1)\,, \qquad\qquad \bar{n} = (1,0,0,-1)$$

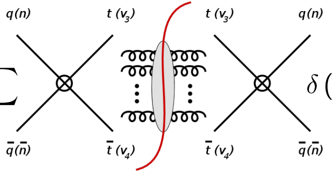
$$k_i^\mu = (n\cdot k_i)\,\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot k_i)\,\frac{n^\mu}{2} + k_{i\perp}^\mu$$

$$p_1^\mu = m_t n\,, \qquad p_2^\mu = m_t \bar{n}\,, \qquad p_{3,4}^\mu = m_t v_{3,4}^\mu + \lambda_{3,4}^\mu$$

Soft function

- Represents corrections coming from exchanges of real, soft gluons, whose transverse momenta sum up to a fixed value q_T.

$$S_{\text{bare}}(q_T, v_3, v_4) \propto \sum$$



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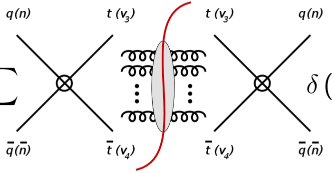

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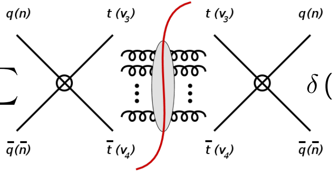
$$S_{\text{bare}}(q_T, v_3, v_4) \propto \sum$$  $$\delta\left(q_T - |\textstyle\sum_i k_{i\perp}|\right) \prod_i \delta^+(k_i^2)$$

- external momenta → Wilson Lines (Born kinematics)
- eikonal Feynman rules

$$\boldsymbol{S}_{i\bar{i}} = \sum_{n=0}^{\infty} \boldsymbol{S}_{i\bar{i}}^{(n)} \left(\frac{\alpha_s}{4\pi}\right)^n \qquad \boldsymbol{S}_{i\bar{i}}^{(n)} = \sum_{\{j\}} \boldsymbol{w}_{\{j\}}^{i\bar{i}} I_{\{j\}}$$

colour matrices ↑ ↑ phase space integrals

Renormalization

separately divergent

$$\frac{d\sigma}{d\Phi} = \mathcal{B}_1^{(\text{bare})} \otimes \mathcal{B}_2^{(\text{bare})} \otimes \text{Tr}\left[\mathcal{H}^{(\text{bare})} \otimes \mathcal{S}^{(\text{bare})}\right]$$

finite

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$$= Z_B \mathcal{B}_1^{(\text{bare})} \otimes Z_B \mathcal{B}_2^{(\text{bare})} \otimes \text{Tr}\left[\boldsymbol{Z}_H^\dagger \mathcal{H}^{(\text{bare})} \boldsymbol{Z}_H \otimes \boldsymbol{Z}_S^\dagger \mathcal{S}^{(\text{bare})} \boldsymbol{Z}_S\right]$$

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&= \mathcal{B}_1(\mu) \otimes \mathcal{B}_2(\mu) \otimes \mathrm{Tr}\left[\mathcal{H}(\mu) \otimes \mathcal{S}(\mu)\right]
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separately finite

$$\frac{d}{d\mu}\frac{d\sigma}{d\Phi} = 0 \quad \rightarrow \quad \text{Renormalization Group Equations for } \mathcal{B}, \mathcal{H} \text{ and } \mathcal{S}$$

Renormalization

- RG equation

$$\frac{d}{d\ln\mu}\boldsymbol{S}_{i\bar{i}}(\mu) = -\gamma^{s\dagger}_{i\bar{i}}\,\boldsymbol{S}_{i\bar{i}}(\mu) - \boldsymbol{S}_{i\bar{i}}(\mu)\,\gamma^{s}_{i\bar{i}}$$

- Soft anomalous dimension

$$\gamma^s = -\boldsymbol{Z}_s^{-1}\frac{d\boldsymbol{Z}_s}{d\ln\mu}$$

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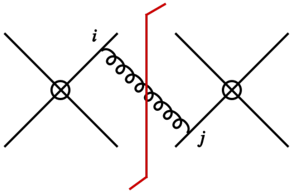
Specifically, at the order α_s^2, we get

$$\underbrace{\boldsymbol{S}^{(2)}}_{\text{finite part only}} = \overbrace{\boldsymbol{Z}_s^{\dagger(2)}\tilde{\boldsymbol{S}}^{(0)}_{\text{bare}} + \tilde{\boldsymbol{S}}^{(0)}_{\text{bare}}\boldsymbol{Z}_s^{(2)} + \boldsymbol{Z}_s^{\dagger(1)}\tilde{\boldsymbol{S}}^{(0)}_{\text{bare}}\boldsymbol{Z}_s^{(1)}}^{\text{pole part only}}$$

$$+ \underbrace{\boldsymbol{Z}_s^{\dagger(1)}\tilde{\boldsymbol{S}}^{(1)}_{\text{bare}} + \tilde{\boldsymbol{S}}^{(1)}_{\text{bare}}\boldsymbol{Z}_s^{(1)} + \tilde{\boldsymbol{S}}^{(2)}_{\text{bare}} - \frac{\beta_0}{\epsilon}\,\tilde{\boldsymbol{S}}^{(1)}_{\text{bare}}}_{\text{finite + pole part}}$$

Soft function at NLO



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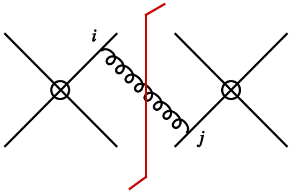


- Known in analytic form

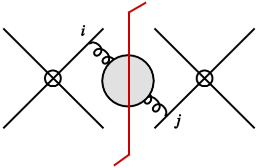
  [Li, Li, Shao, Yan, Zhu '13; Catani, Grazzini, Torre '13]

$$
\begin{aligned}
\boldsymbol{S}^{(1)}_{i\bar{i}} = {} & 4L_\perp \left(2\boldsymbol{w}^{13}_{i\bar{i}} \ln \frac{-t_1}{m_t M} + 2\boldsymbol{w}^{23}_{i\bar{i}} \ln \frac{-u_1}{m_t M} + \boldsymbol{w}^{33}_{i\bar{i}} \right) \\
& - 4 \left(\boldsymbol{w}^{13}_{i\bar{i}} + \boldsymbol{w}^{23}_{i\bar{i}} \right) \mathrm{Li}_2 \left(1 - \frac{t_1 u_1}{m_t^2 M^2} \right) + 4\boldsymbol{w}^{33}_{i\bar{i}} \ln \frac{t_1 u_1}{m_t^2 M^2} \\
& - 2\boldsymbol{w}^{34}_{i\bar{i}} \frac{1+\beta_t^2}{\beta_t} \Big[L_\perp \ln x_s - \mathrm{Li}_2 \left(-x_s \,\mathrm{tg}^2 \frac{\theta}{2} \right) + \mathrm{Li}_2 \left(-\frac{1}{x_s} \,\mathrm{tg}^2 \frac{\theta}{2} \right) \\
& + 4 \ln x_s \ln \cos \frac{\theta}{2} \Big] , \qquad \text{where} \qquad L_\perp = \ln \frac{x_T^2 \mu^2}{4 e^{-2\gamma_E}}
\end{aligned}
$$

Soft function at NNLO

Three distinct groups of diagrams:

- Bubble



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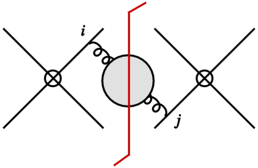
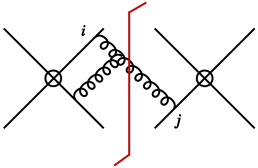


- Single-cut



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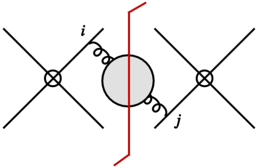

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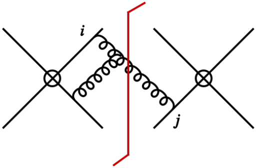

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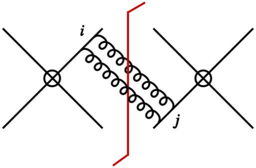

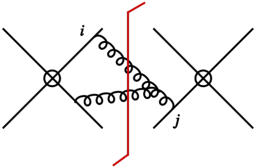

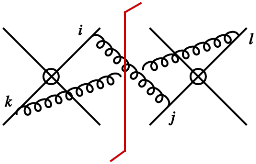

$+ \quad \ldots$

Double-cut NNLO integrals

Example:

$$\tilde{I}_{3gv,ij} = \int \frac{d^d k_1 \, d^d k_2 \; \delta^+(k_1^2)\, \delta^+(k_2^2)\, \delta((k_1+k_2)_T^2 - q_T^2)}{(n\cdot k_1)^\alpha \, (n\cdot k_2)^\alpha \, (n_i \cdot k_1)\, (n_j \cdot (k_1+k_2))\, (k_1+k_2)^2}$$

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To disentangle overlapping singularities and calculate regularized integrals we use the method of sector decomposition [Binoth, Heinrich, '00; Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke '17].

Sector decomposition

Two types of singularities

- Endpoint, *e.g.* soft:

$$\left(k_1^+, k_1^-, k_1^\perp\right) \to 0$$

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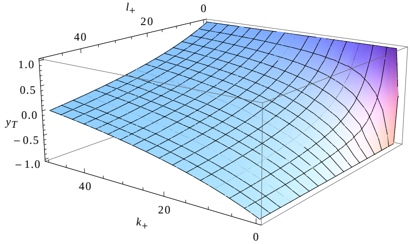
- Manifold, *e.g.* collinear

$$k_1 \cdot k_2 \to 0$$



The strategy

Given the integral: $$I_G = \int d^d k_1 d^d k_2 \; \mathcal{I}_G \times \mathcal{W}_G$$

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- Analytically integrate 3 out of $2d$ dimensions

- Map the remaining variables to a unit hypercube (split the original integral into a sum if necessary)

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$$= \sum_j \sum_{k \in \text{sectors}} \int_0^1 \prod_{i=1}^{2d-3} dx_i \, (\mathcal{I}_G \times \mathcal{W}_G)_{jk}$$

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$$= \sum_{j,m,n} \sum_{k \in \text{sectors}} \left(\int_0^1 a_{mn} \times \mathcal{W}_G \right) \alpha^m \epsilon^n$$

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- Numerically integrate series coefficients

$$= \sum_{j,m,n} \sum_{k \in \text{sectors}} c_{mn}^{jk} \, \alpha^m \epsilon^n$$

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single-cut	numeric integration of one-loop soft current
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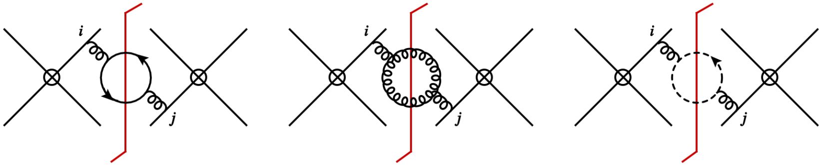
- Direct demonstration of the validity of small-q_T SCET factorization for top pair production at NNLO.
- Complete, small-q_T soft function for top pair production at NNLO.

Bubble



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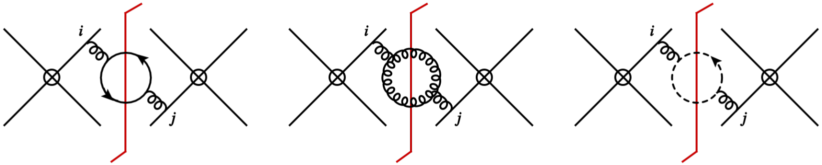
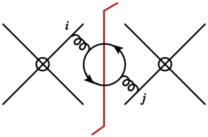
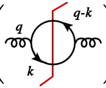


- Solvable analytically: direct cross check of our sector decomposition-based implementation
- Non-trivial tensor structure → challenging numerators
- Laboratory to stress-test sector decomposition-based methodology
- Comparable with n_f part of Renormalization Group prediction

Bubble part of the soft function from differential equations



$$\propto \int \frac{d^d q\, \delta(q_T - 1)\, \theta^+(q^2)\, n_i^\mu n_j^\nu}{q^4\, (n_i \cdot q)\, (n_j \cdot q)} \left(\quad \right)_{\mu\nu}$$



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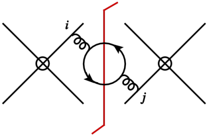

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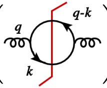
$\Bigg($  $\Bigg)_{\mu\nu}$

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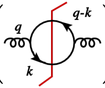
$\Bigg($  $\Bigg)_{\mu\nu}$

$$= \int \frac{d^d k\, N_{\mu\nu}\, \delta^+(k^2)\, \delta^+((q-k)^2)}{(n\cdot k)^\alpha\, (n\cdot(q-k))^\alpha\, k^2 (q-k)^2}$$

$$= T_{00}\, g^{\mu,\nu} + T_{qq}\, q^\mu q^\nu + T_{nn}\, n^\mu n^\nu + T_{qn}\, (n^\mu q^\nu + q^\mu n^\nu)$$

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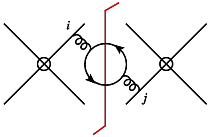

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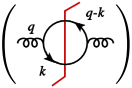

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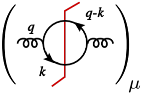

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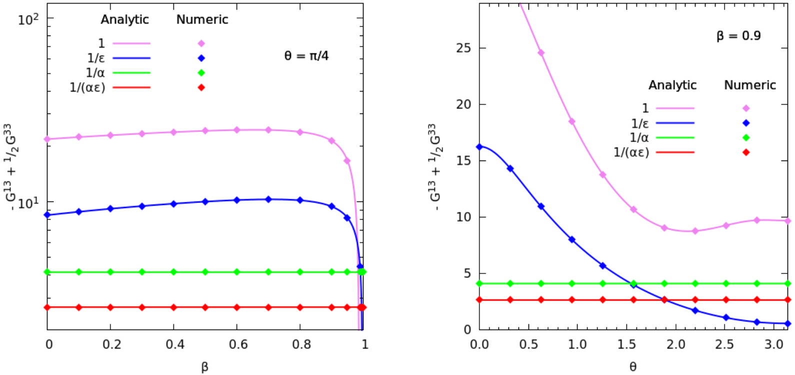
- Topology:

$$\int \frac{d^d k}{(n\cdot k)^{a_1+2\alpha}\, (\bar{n}\cdot k)^{a_2}\, (v_3\cdot k)^{a_3}\, (v_4\cdot k)^{a_4}\, (k^2 - m^2)^{a_5}\, ((n\cdot k)\,(\bar{n}\cdot k) - m^2 - 1)^{a_6}}$$

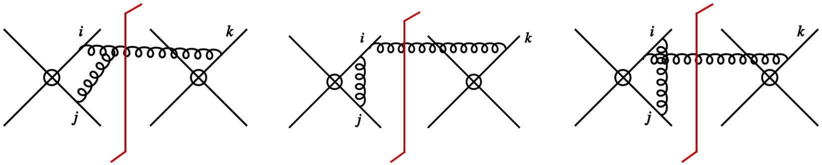
- IBPs $\rightarrow$ reduction $\rightarrow$ DE $\rightarrow$ solutions $\rightarrow \int dm^2 \rightarrow I_{jk}(\beta, \theta)$

Validation with the bubble: agreement with analytic result

$$-G^{13} + \frac{1}{2}G^{33}$$



Single-cut (real-virtual)



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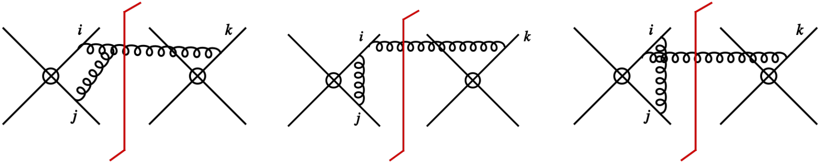

$$S^{(2)}_{\text{1-cut}} = \sum_{ijk} \int d^d l \, \frac{\delta^+(l^2)\,\delta(l_T - q_T)}{l_+^\alpha \, n_k \cdot l} \, n_k^\mu \, T_k^a \, J^\mu_{ij,a}(l)$$

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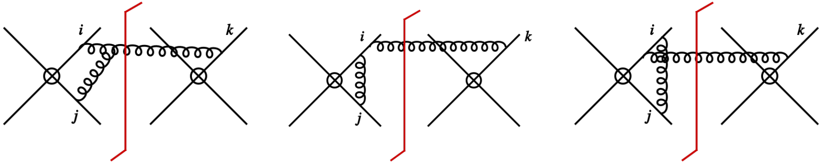

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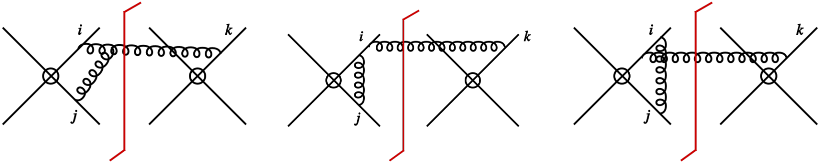

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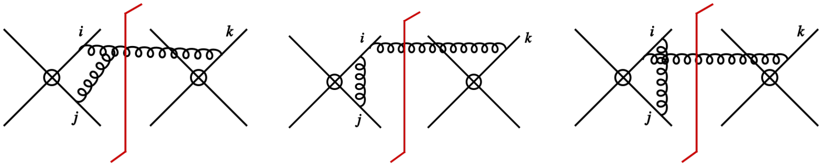

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- $S^{(2)}_{\text{1-cut}}$ can be obtained by a relatively simple integration over l^μ.
- Single-cut piece of the soft function exhibits both real and imaginary part. The latter when $i \neq j \neq k$, the former, otherwise.

Complete Soft Function at NNLO: structure of the result

- In momentum space

$$S^{(2,\text{bare})}(q_T,\beta,\theta) = \frac{1}{q_T^p}\left[S^{(2)}_{\text{bubble}}(\beta,\theta,\epsilon)+S^{(2)}_{\text{1-cut}}(\beta,\theta,\epsilon)+S^{(2)}_{\text{2-cut}}(\beta,\theta,\epsilon)\right]$$

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Fourier Transform

- In position space

$$S^{(2,\text{bare})}(L_\perp,\beta,\theta) = \left[\frac{1}{\epsilon}+L_\perp+L_\perp^2+\dots\right]$$

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$\hookrightarrow$ Momentum-space soft function has to be calculated up to order ϵ.

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can be cross-checked against RG; fixes all $L_\perp$-dependent terms in $S^{(2,0)}(L_\perp)$

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- The only term that has to be obtained through direct calculation is the $L_\perp$-independent part of $S^{(2,0)}(L_\perp)$.
- However, we calculate all terms and use the redundant ones for cross checks against Renormalization Group prediction.

Vanishing of higher order poles

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- $\frac{1}{\epsilon^3}$ pole cancels between 1-cut and 2-cut contributions

$$\frac{1}{\epsilon^3}\begin{pmatrix} 0.0004\,N_c^3 - 0.0007\,N_c + 0.0004\,N_c^{-1} & 0.0004\,N_c^2 - 0.0004\,N_c^{-2} - 7.\times 10^{-6} \\ 0.0004\,N_c^2 - 0.0004\,N_c^{-2} - 7.\times 10^{-6} & -0.0004\,N_c^3 - 0.00001\,N_c + 0.0003\,N_c^{-3} + 0.0002\,N_c^{-1} \end{pmatrix}$$

† We used $\beta = 0.4$, $\theta = 0.5$.

Comparison with Renormalization Group

- Double pole

$$\left[S^{(2,-2)}_{\text{direct}} + S^{(2,-2)}_{\text{RGE}}\right]_{\beta=0.4,\theta=0.5} =$$

$$\begin{pmatrix} -0.0003\,N_c^3 - 0.003\,N_c + 0.003\,N_c^{-1} & -0.001\,N_c^2 - 0.004\,N_c^{-2} + 0.005 \\ -0.001\,N_c^2 - 0.004\,N_c^{-2} + 0.005 & -0.0009\,N_c^3 + 0.002\,N_c + 0.003\,N_c^{-3} - 0.005\,N_c^{-1} \end{pmatrix}$$

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- Single pole

$$\left[S^{(2,-1)}_{\text{direct}} + S^{(2,-1)}_{\text{RGE}}\right]_{\beta=0.4,\theta=0.5} =$$

$$\begin{pmatrix} -0.0009\, N_c^3 - 0.0004\, N_c + 0.001\, N_c^{-1} & -0.0005\, N_c^2 - 0.001\, N_c^{-2} + 0.002 \\ -0.0005\, N_c^2 - 0.001\, N_c^{-2} + 0.002 & (1. \times 10^{-6})\, N_c^3 + 0.001\, N_c + 0.001\, N_c^{-3} - 0.002\, N_c^{-1} \end{pmatrix}$$

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Hence, all poles can be removed by the renormalization procedure. And that proves small-q_T factorization for top pair production up to NNLO

$$\frac{d\sigma^{\text{NNLO}}_{pp\to t\bar{t}}}{d\Phi} = \mathcal{B}_1 \otimes \mathcal{B}_2 \otimes \mathcal{H} \otimes \mathcal{S} + \mathcal{O}\left(\frac{q_T^2}{q^2}\right).$$

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PRELIMINARY

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- The soft function can now be used to obtain full $t\bar{t}$ cross section at NNLO as well for resummation up to NNLL'

Acknowledgements

The project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement NO. 665778.

European Commission

National Science Centre Poland