

Automated extraction of UV counterterms for arbitrary local operators

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UV counterterms

- UV counterterms are useful for renormalization
- For some BSM theories, higher-loop renormalization effects are unknown
- Local operators make computation difficult

Goal

We present a method to compute UV counterterms automatically for any Feynman diagram with local operators up to five loops

Results with the old program

- Computed five loop beta function for general colour group
 - Verified QCD result of [Baikov,Chetyrkin,Kühn '16]
 - Took 6 days on a pc with 32 cores
- Recomputed $H \rightarrow b\bar{b}$, R -ratio
 - Easy: took a few hours on one pc
- Computed $H \rightarrow gg$ [HRUVV '17]
 - Quartically divergent diagrams...
 - Hard: took two months
- Computed five-loop $N = 2$, $N = 3$ moments of non-singlet splitting function
 - Terrible IR divergences
 - Hard: also took two months

Five-loop Higgs decay to gluons

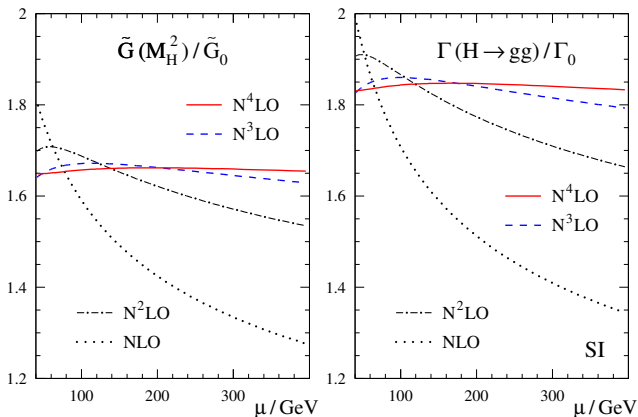


Figure: Left: the renormalization-scale dependence of $\tilde{G} = (\beta(a_S)/a_S)^2 G(M_H^2)$ at $n_f = 5$ in \overline{MS} . Right: decay width for $\alpha_S(M_Z^2) = 0.118$, $M_H = 125$ GeV and $\mu_t = 164$ GeV.

R^* operation

- The poles parts come from the divergent momentum configurations
- The recursive R^* operation takes care of combinatorics of subdivergences [Chetyrkin, Smirnov '83]
- We have extended R^* to Feynman integrals with arbitrary numerator structure [Herzog,Ruijl '17]

UV counterterm operation

$\Delta(G) =$ poles of G when all momenta go to ∞ with all contributions from subdivergences subtracted

Identifying divergent (sub)diagrams

$$\text{Diagram} = \int d^D p_1 \int d^D p_2 \frac{Q \cdot p_2 p_1^\nu}{p_1^2 p_2^2 p_3^2 p_4^4}$$

Identifying divergent (sub)diagrams

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- Get degree of divergence through power counting
- Each loop contributes +4 due to the measure
- All momenta $\rightarrow \infty$: $8 + 1 + 1 - 2 - 2 - 2 - 4 = 0$ (log)
- $p_2, p_3 \rightarrow \infty$: $4 + 1 - 2 - 2 = 1$ (linear)
- $p_4 \rightarrow 0$: $4 - 4 = 0$ (log IR)

R*-operation by example

$$K\left(\frac{2}{\epsilon^2} + 4 + 2\epsilon\right) \equiv \frac{2}{\epsilon^2}$$

$$K \left(\text{circle with nodes 1, 2} \right) = \Delta \left(\text{circle with nodes 1, 2} \right)$$

$$K \left(\text{circle with nodes 1, 2, 3, 4} \right) = \Delta \left(\text{circle with nodes 1, 2, 3, 4} \right) + \Delta \left(\text{circle with nodes 2, 3} \right) \cdot \text{circle with nodes 1, 4}$$

Consider all sets of non-overlapping divergent subdiagrams

R*-operation by example

$$K\left(\frac{2}{\epsilon^2} + 4 + 2\epsilon\right) \equiv \frac{2}{\epsilon^2}$$

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Consider all sets of non-overlapping divergent subdiagrams

$$\begin{aligned} K \text{ (circle with 1, 2, 3, 4, 5, 6) } &= \Delta \left(\text{circle with 1, 2, 3, 4, 5, 6} \right) \\ &+ \Delta \left(\text{circle with 5, 6} \right) \cdot \text{circle with 1, 2, 3, 4} + \Delta \left(\text{circle with 2, 3} \right) \cdot \text{circle with 4, 5, 6, 1} \\ &- \Delta \left(\text{circle with 5, 6} \right) \Delta \left(\text{circle with 2, 3} \right) \cdot \text{circle with 1, 4} \end{aligned}$$

Counterterm operation Δ

- For log diagrams, Δ does not depend on external momenta or masses!

$$\Delta \left(\text{Diagram 1} \right) = \Delta \left(\text{Diagram 2} \right) = \Delta \left(\text{Diagram 3} \right)$$

- Non-log diagrams have to be Taylor expanded first

Infrared rearrangement (IRR)

Rearrange diagrams to simpler ones we can compute (always possible)

How to compute Δ ?

- We use its definition:

$$\Delta(G) \stackrel{\text{IRR}}{=} \underbrace{\Delta(G')}_{\text{Simpler than } G} = K(G') - \underbrace{\text{subdivergences}(G')}_{\text{Lower-loop diagrams}}$$

How to compute Δ ?

- We use its definition:

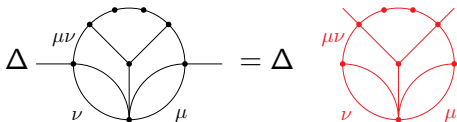
$$\Delta(G) \stackrel{\text{IRR}}{=} \underbrace{\Delta(G')}_{\text{Simpler than } G} = K(G') - \underbrace{\text{subdivergences}(G')}_{\text{Lower-loop diagrams}}$$

- Recursive application:

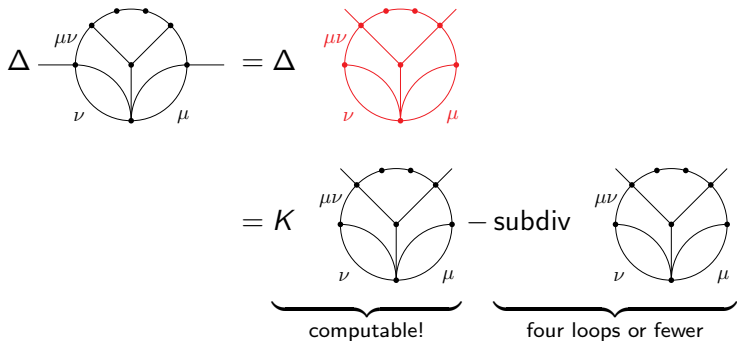
$$\begin{aligned} \Delta\left(\text{circle with two external lines and a loop}\right) &= K\left(\text{circle with two external lines and a loop}\right) \\ \Delta\left(\text{circle with two external lines and a loop with a subdivergence}\right) &= K\left(\text{circle with two external lines and a loop with a subdivergence}\right) - \Delta\left(\text{circle with two external lines and a loop}\right) \cdot \text{circle with two external lines} \end{aligned}$$

- FORCER can compute all rearranged diagrams [Ruijl,Ueda,Vermaseren '17]

Five-loop example



Five-loop example



Feynman rules and R^*

Is it ok to apply R^* after Feynman rule substitution?

$$\Delta \left(\text{diagram with nested loops} \right) \stackrel{?}{=} \Delta \left((D + \dots) \cdot \text{diagram with a bubble} + \dots \right)$$

- The D does not commute with (nested) Δ
- Feynman rules do not commute with R^* !

Feynman rules and R^*

- To get results in $\overline{\text{MS}}$, apply R^* before the Feynman rules:

$$\Delta \left(\text{Diagram 1} \right) = K \left(\text{Diagram 2} \right)$$

$$- \Delta \left(\text{Diagram 3} \right) * \text{Diagram 4}$$

$$+ \tilde{\Delta} \left(\text{Diagram 5} \right) * \Delta \left(\text{Diagram 6} \right) * \text{Diagram 7}$$

The diagrams are as follows:

- Diagram 1: A circle with two external wavy lines on the left and right. A wavy line enters from the top, loops around the top half of the circle, and exits from the top.
- Diagram 2: A circle with two external wavy lines on the left and right. A wavy line enters from the top, loops around the top half of the circle, and exits from the top. A small black dot is located on the top edge of the circle.
- Diagram 3: A semi-circle with two external wavy lines on the left and right. A wavy line enters from the top, loops around the top half of the semi-circle, and exits from the top.
- Diagram 4: A circle with two external wavy lines on the left and right. A wavy line enters from the top, loops around the top half of the circle, and exits from the top. A small black dot is located on the top edge of the circle.
- Diagram 5: Two separate wavy lines.
- Diagram 6: A semi-circle with two external wavy lines on the left and right. A wavy line enters from the top, loops around the top half of the semi-circle, and exits from the top.
- Diagram 7: A semi-circle with two external wavy lines on the left and right. A wavy line enters from the top, loops around the bottom half of the semi-circle, and exits from the top.

- Power-counting is still valid

Tricky bookkeeping (I)

- Tricky index matching between disconnected diagrams
- Need to remember original connectivity
- Indices only known after substituting the Feynman rules

$$\Delta \left(\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \end{array} \right) * \begin{array}{c} \delta^{\mu\rho} \delta^{\nu\sigma} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \end{array}$$

Tricky bookkeeping (II)

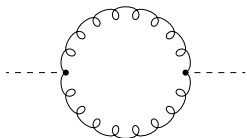
Taylor expansion in Q:

$$\Delta \left(\text{Diagram 1} \right) = \frac{1}{2} Q^\alpha Q^\beta \Delta \left(2 \text{Diagram 2} + \text{Diagram 3} + 2 \text{Diagram 4} \right)$$

- New derivative vertices where momenta are set to 0
- R^* only cares about the decrease in UV divergence
- 'Delay' the derivative computation until after R^*

Local operators

- Only dim. of vertices and propagators is important for R^*
- Local operators can be viewed as special vertices
- Use abstract vertices and propagators:



$$\begin{aligned}
 &vx(Q, p1, p2, vhg, 2) * \\
 &vx(-p1, -p2, -Q, vhg, 2) * \\
 &prop(p1, g1, -2) * prop(p2, g1, -2)
 \end{aligned}$$

Derivatives in abstract notation

- Derivate wrt Q through p_1 on the vertex

$$vx(Q, p_1, p_2, v_{hgg}, 2)$$

- becomes

$$Q(\mu) * vx(Q_0, p_1, p_2, v_{hgg}, 1, Q_0, p_1, \mu)$$

- The R^* implementation can parse this notation

Overview

Workflow:

- The user defines Feynman rules and provides abstract Feynman graph
- The R^* program generates all abstract counterterms
- All the Feynman rules are evaluated and derivatives are taken
- All resulting diagrams are computed with FORCER

Conclusions

- R^* can be used to automatically compute UV counterterms
- Application on abstract diagrams allows for local operators

Work in progress:

- Recomputed five-loop beta function
- Will be applied to compute splitting functions
- Will be applied to theories with dim. 6 operators