Automated extraction of UV counterterms for arbitrary local operators

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UV counterterms

- UV counterterms are useful for renormalization
- For some BSM theories, higher-loop renormalization effects are unknown
- Local operators make computation difficult

Goal

We present a method to compute UV counterterms automatically for any Feynman diagram with local operators up to five loops

Results with the old program

- Computed five loop beta function for general colour group
  - Verified QCD result of [Baikov,Chetyrkin,Kühn '16]
  - Took 6 days on a pc with 32 cores
- Recomputed $H \to b\bar{b}$, R-ratio
  - Easy: took a few hours on one pc
- Computed $H \to gg$ [HRUVV '17]
  - Quartically divergent diagrams...
  - Hard: took two months
- Computed five-loop $N = 2, N = 3$ moments of non-singlet splitting function
  - Terrible IR divergences
  - Hard: also took two months

Five-loop Higgs decay to gluons

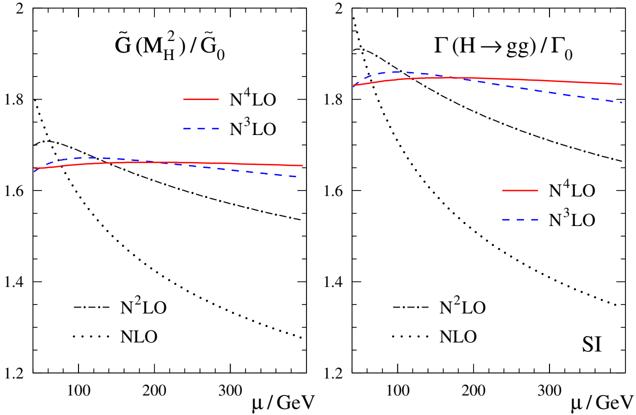


Figure: Left: the renormalization-scale dependence of $\widetilde{G} = (\beta(a_S)/a_S)^2 G(M_{\mathrm{H}}^2)$ at $n_f = 5$ in $\overline{\mathrm{MS}}$. Right: decay width for $\alpha_S(M_Z^2) = 0.118$, $M_{\mathrm{H}} = 125$ GeV and $\mu_t = 164$ GeV.

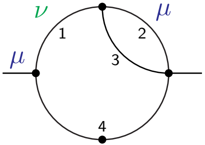
R^* operation

- The poles parts come from the divergent momentum configurations
- The recursive R^* operation takes care of combinatorics of subdivergences [Chetyrkin, Smirnov '83]
- We have extended R^* to Feynman integrals with arbitrary numerator structure [Herzog,Ruijl '17]

UV counterterm operation

$\Delta(G) =$ poles of G when all momenta go to ∞ with all contributions from subdivergences subtracted

Identifying divergent (sub)diagrams



$$= \int d^D p_1 \int d^D p_2 \frac{Q \cdot p_2 p_1^\nu}{{p_1}^2 {p_2}^2 {p_3}^2 {p_4}^4}$$

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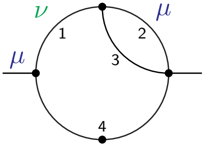

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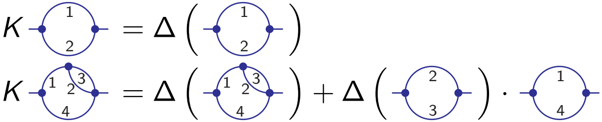
- Get degree of divergence through power counting
- Each loop contributes $+4$ due to the measure
- All momenta $\to \infty$: $8 + 1 + 1 - 2 - 2 - 2 - 4 = 0$ (log)
- $p_2, p_3 \to \infty$: $4 + 1 - 2 - 2 = 1$ (linear)
- $p_4 \to 0 : 4 - 4 = 0$ (log IR)

R^*-operation by example

$$K(\frac{2}{\epsilon^2} + 4 + 2\varepsilon) \equiv \frac{2}{\epsilon^2}$$



Consider all sets of non-overlapping divergent subdiagrams

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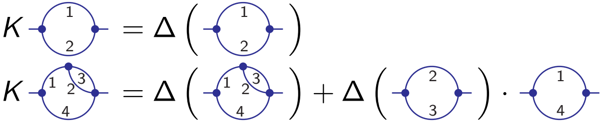

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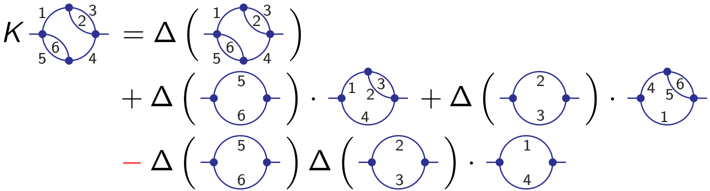
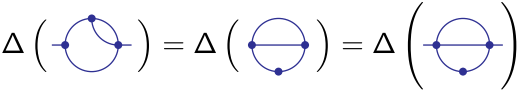


Counterterm operation Δ

- For log diagrams, Δ does not depend on external momenta or masses!



- Non-log diagrams have to be Taylor expanded first

Infrared rearrangement (IRR)

Rearrange diagrams to simpler ones we can compute (always possible)

How to compute Δ?

- We use its definition:

$$\Delta(G) \stackrel{\text{IRR}}{=} \underbrace{\Delta(G')}_{\text{Simpler than G}} = K(G') - \underbrace{\text{subdivergences}(G')}_{\text{Lower-loop diagrams}}$$

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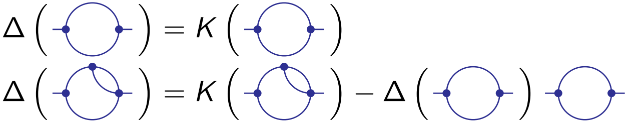
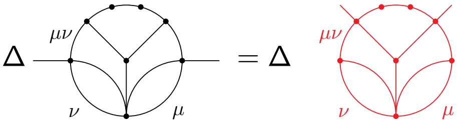
- Recursive application:

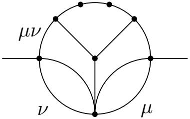


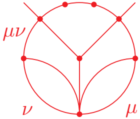
- FORCER can compute all rearranged diagrams [Ruijl,Ueda,Vermaseren '17]

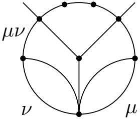
Five-loop example

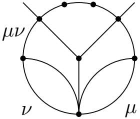


Five-loop example

Δ 

$= \Delta$ 

$= K$ 

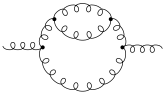
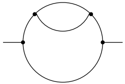
$-$ subdiv 

computable! (the K term)

four loops or fewer (the subdiv term)

Feynman rules and R^*

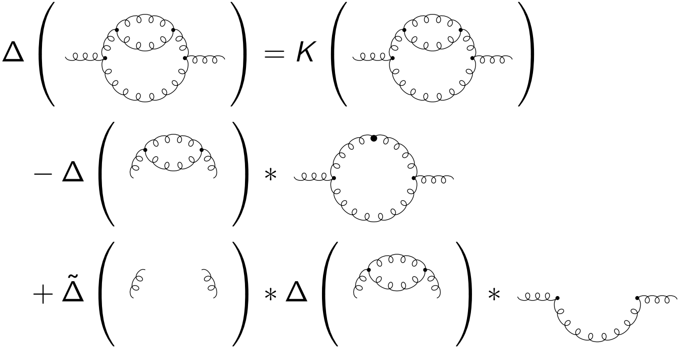
Is it ok to apply R^* after Feynman rule substitution?

$\Delta \Big($  $\Big) \stackrel{?}{=} \Delta \Big((D + \ldots) \cdot$  $+ \ldots \Big)$

- The D does not commute with (nested) Δ
- Feynman rules do not commute with R^*!

Feynman rules and R^*

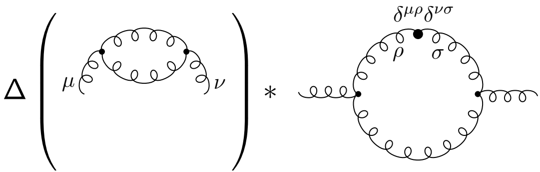
- To get results in $\overline{\text{MS}}$, apply R^* before the Feynman rules:



- Power-counting is still valid

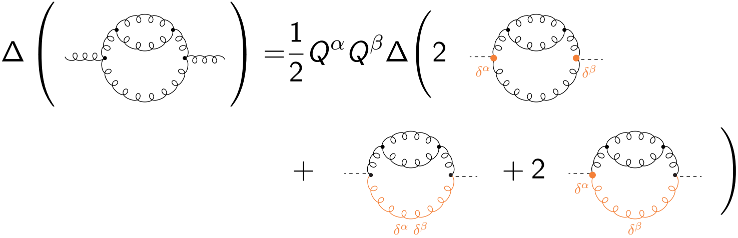
Tricky bookkeeping (I)

- Tricky index matching between disconnected diagrams
- Need to remember original connectivity
- Indices only known after substituting the Feynman rules



Tricky bookkeeping (II)

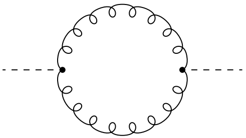
Taylor expansion in Q:



- New derivative vertices where momenta are set to 0
- R^* only cares about the decrease in UV divergence
- 'Delay' the derivative computation until after R^*

Local operators

- Only dim. of vertices and propagators is important for R^*
- Local operators can be viewed as special vertices
- Use abstract vertices and propagators:



```
vx(Q,p1,p2,vhgg,2)*
vx(-p1,-p2,-Q,vhgg,2)*
prop(p1,gl,-2)*prop(p2,gl,-2)
```

Derivatives in abstract notation

- Derivate wrt Q through p_1 on the vertex

```
vx(Q,p1,p2,vhgg,2)
```

- becomes

```
Q(mu)*vx(Q0,p1,p2,vhgg,1,Q0,p1,mu)
```

- The R^* implementation can parse this notation

Overview

Workflow:

- The user defines Feynman rules and provides abstract Feynman graph
- The R^* program generates all abstract counterterms
- All the Feynman rules are evaluated and derivatives are taken
- All resulting diagrams are computed with FORCER

Conclusions

- R^* can be used to automatically compute UV counterterms
- Application on abstract diagrams allows for local operators

Work in progress:

- Recomputed five-loop beta function
- Will be applied to compute splitting functions
- Will be applied to theories with dim. 6 operators