The massive three loop form factor in the planar limit

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in collaboration with
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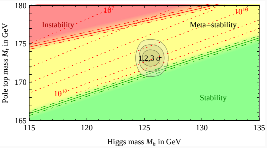
Plan of this talk

1. Preliminary
2. Computational details
3. Renormalization
4. Infrared structure
5. Results
6. Conclusion

The 'top' sector

✓ The heaviest SM particle
- probes the Higgs sector most
- plays unique role in understanding
  the EW symmetry breaking



Buttazzo et. al. Jul'13

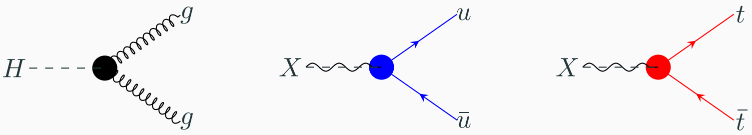
✓ New physics potential :
perfect place to manifest it

✓ Does not hadronize - opportunity to study it as a single particle -
Spin properties, Interaction vertices, Precise description of mass

✓ High precision will be achieved at the future electron-positron colliders
– In order to match the experimental accuracy, precise predictions are required on the theoretical side as well

Form factors

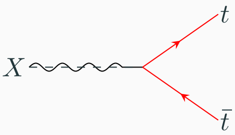
- ✓ The form factors are basic building blocks for many physical quantities
- ✓ They exhibit a universal infrared behavior - leads to information on anomalous dimensions
- ✓ The massive cusp anomalous dimension controls the infrared structure of massive form factors - studying the form factors helps in better understanding of the massive cusp
- ✓ Another important motive is to study high energy behavior of the massive form factors



Preliminary

The process

We consider the decay of a color neutral massive particle to a pair of heavy quark of mass m.



Notation

$$X(q) \to t(q_1) + \bar{t}(q_2)$$

$$X = V, A, S, P$$

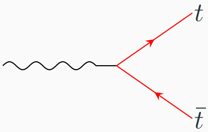
$$s = \frac{q^2}{m^2} = -\frac{(1-x)^2}{x}$$

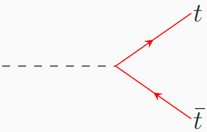
The general structure

Vector and Axial Vector

V: $-i\delta_{ij}v_Q\left(\gamma^\mu\, F_{V,1} + \frac{i}{2m}\sigma^{\mu\nu}q_\nu\, F_{V,2}\right)$

A: $-i\delta_{ij}a_Q\left(\gamma^\mu\gamma_5\, F_{A,1} + \frac{1}{2m}q^\mu\gamma_5\, F_{A,2}\right)$





Scalar and Pseudo Scalar

$-\frac{m}{v}\delta_{ij}\left[s_Q F_S + i p_Q \gamma_5 F_P\right]$

The form factors are expanded in the strong coupling constant as

$$F_I = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n F_I^{(n)}$$

To obtain $F_I^{(n)}$ $\Rightarrow$ appropriate projector on the amplitudes

$$P_{V,i} = \frac{i}{v_Q} \frac{\not{q}_2 - m}{m} \left(\gamma_\mu g^1_{V,i} + \frac{1}{2m}(q_{2\mu} - q_{1\mu}) g^2_{V,i}\right) \frac{\not{q}_1 + m}{m} \,,$$

$$P_{A,i} = \frac{i}{a_Q} \frac{\not{q}_2 - m}{m} \left(\gamma_\mu \gamma_5 g^1_{A,i} + \frac{1}{2m}(q_{1\mu} + q_{2\mu}) \gamma_5 g^2_{A,i}\right) \frac{\not{q}_1 + m}{m} \,,$$

$$P_S = \frac{v}{2m s_Q} \frac{\not{q}_2 - m}{m} \left(g_S\right) \frac{\not{q}_1 + m}{m} \,, \; P_P = \frac{v}{2m p_Q} \frac{\not{q}_2 - m}{m} \left(i\gamma_5 g_P\right) \frac{\not{q}_1 + m}{m} \,,$$

$g_I \equiv g_I(s, d)$ and are determined by applying the projectors on the generic Lorentz structure.

A study in literature

NLO and beyond NLO

$F_{V,i}^{(1)}, F_{A,i}^{(1)}$ [Arbuzov, Bardin, Leike '92; Djouadi, Lampe, Zerwas '95]

$F_{S}^{(1)}, F_{P}^{(1)}$ [Braaten, Leveille '80; Sakai '80; Drees, Hikasa '90]

$F_{V,i}^{(2)}, F_{A,i}^{(2)}$ [Altarelli, Lampe '93; Ravindran, van Neerven '98; Catani, Seymour '99]

$F_{S}^{(2)}, F_{P}^{(2)}$ [Gorishnii *et. al.* '91; Chetyrkin, Kwiatkowski '95; Harlander, Steinhauser '97]

NNLO

$F_{I}^{(2)}$ [Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi '04,'05]

$F_{V,i}^{(2)}(\mathcal{O}(\epsilon))$ [Gluza, Mitov, Moch, Riemann '09]

$F_{I}^{(2)}(\mathcal{O}(\epsilon^2))$ [Ablinger, Behring, Blümlein, Falcioni, Freitas, Marquard, Rana, Schneider '17]

NNNLO

$F_{V,i}^{(3)}|_{\text{large N}}$ (talk by M. Steinhauser) [Henn, Smirnov, Smirnov, Steinhauser '16]

$F_{V,i}^{(3)}|_{n_l\, contributions}$ [Lee, Smirnov, Smirnov, Steinhauser '18]

$F_{V,i}^{(4)}|_{partial\, poles\, in\, large\, Q^2}$ [Ahmed, Henn, Steinhauser '17]

Goal

In this talk, we present

- *Automatizing the technique to compute the first order factorizable system of differential equations.*
- *Results for the master integrals which contribute to color-planar diagrams and full light quark dependence.*
- *Color-planar (N_C^3) and complete light quark (n_l) contributions for $F_{V,i}^{(3)}$, $F_{A,i}^{(3)}$, $F_S^{(3)}$ and $F_P^{(3)}$.*

J. Ablinger, J. Blümlein, P. Marquard, N. Rana and C. Schneider,
`Heavy Quark Form Factors at Three Loops in the Planar Limit`,
arXiv:1804.07313 [hep-ph].

J. Ablinger, J. Blümlein, P. Marquard, N. Rana and C. Schneider,
DESY 18-053.

* A parallel computation in arXiv:1804.07310 [hep-ph] (talk by M. Steinhauser)

Computational details

The generic procedure

$d = 4 - 2\epsilon$

- Diagrammatic approach -> QGRAF [Nogueira] to generate diagrams
- FORM [Vermaseren] for algebraic manipulation :
  Lorentz, Dirac and Color [Ritbergen, Schellekens, Vermaseren] *algebra*
- Decomposition of the dot products to obtain scalar integrals

$$\frac{2l.p}{l^2(l-p)^2} = \frac{l^2-(l-p)^2+p^2}{l^2(l-p)^2} = \frac{1}{(l-p)^2} - \frac{1}{l^2} + \frac{p^2}{l^2(l-p)^2}$$

- Identity relations among scalar integrals : *IBPs & SRs*
- Algebraic linear system of equations relating the integrals

$$\Downarrow$$

Master integrals (MIs)

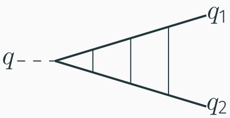
- Crusher [Marquard, Seidel] for reduction to master integrals
- Computation of MIs : *Differential eqns.*

Computing the master integrals

A scalar integral can be expressed as

$$J(\nu_1, \ldots, \nu_n) = \left((4\pi)^{2-\epsilon} e^{\epsilon\gamma_E}\right)^3 \int \frac{d^d l_1}{(2\pi)^d} \frac{d^d l_2}{(2\pi)^d} \frac{d^d l_3}{(2\pi)^d} \frac{1}{D_1^{\nu_1} \ldots D_n^{\nu_n}}$$

For example



$$\begin{aligned} &l_1^2 - m^2, (l_1 - q)^2 - m^2, (l_1 - l_2)^2, \\ &l_2^2 - m^2, (l_2 - q)^2 - m^2, (l_2 - l_3)^2, \\ &l_3^2 - m^2, (l_3 - q)^2 - m^2, (l_1 - q_1)^2 \end{aligned}$$

To evaluate the integral $\rightarrow$ Feynman parametrization, Mellin-Barnes ...

We use the method of differential equations!

Using differential equations

$$\frac{q^2}{m^2} = -\frac{(1-x)^2}{x}$$

The integral is a function of d, q^2 and m^2.

$$J(1,1,1,1,1,1,1,1,1) \equiv f(d,\, q^2,\, m^2) \equiv f(d,\, x)$$

The idea is to obtain a differential eqn. for the integral $w.r.t.$ x and solve it.

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$$\frac{d}{dx} J_i = \text{some combinations of integrals}$$

$$\Downarrow \text{ IBP identities}$$

$$= \sum_j c_{ij} J_j$$

c_{ij}'s are rational function of x.

Using differential equations

$$\frac{q^2}{m^2} = -\frac{(1-x)^2}{x}$$

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The idea is to obtain a differential eqn. for the integral *w.r.t.* x and solve it.

$$d_x \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ \vdots \\ J_n \end{pmatrix} = \begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \end{bmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ \vdots \\ J_n \end{pmatrix}$$

Using differential equations

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To solve such a system, it would be best to organize it in such a way that it diagonalizes, or at least it takes a block-triangular form.

$$d_x \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ \vdots \\ J_n \end{pmatrix} = \begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ 0 & \bullet & \bullet & \bullet & \cdots & \bullet \\ 0 & \bullet & \bullet & \bullet & \cdots & \bullet \\ 0 & 0 & 0 & \bullet & \cdots & \bullet \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \bullet \end{bmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ \vdots \\ J_n \end{pmatrix}$$

Let's consider the 12^{th} blob from below

$$\frac{d}{dx}\begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}\begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} + \begin{pmatrix} R_1(\epsilon, x) \\ R_2(\epsilon, x) \\ R_3(\epsilon, x) \end{pmatrix},$$

$$c_{11} = \frac{(7 + 6x + 7x^2 - 2d(1 + x + x^2))}{x(1 + x)^2},$$

$$c_{12} = \frac{(-4 + d)(-10 + 3d)}{2(-3 + d)^2(1 + x)^2},$$

$$c_{13} = \frac{(d^2(15 + 8x + 15x^2) + 8(20 + 9x + 20x^2) - 2d(49 + 24x + 49x^2))}{4(-3 + d)^2 x(1 + x)^2}, \ldots$$

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For these topologies, the integrals can have, at max, a cubic pole in ϵ.

$$J_i = \frac{1}{\epsilon^3}J_i^{-3} + \frac{1}{\epsilon^2}J_i^{-2} + \frac{1}{\epsilon}J_i^{-1} + J_i^0 + \epsilon J_i^1 + \cdots$$

Series expansion and compare each order of ϵ!

Let's consider the 12^{th} blob from below

$$\frac{d}{dx}\begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}\begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} + \begin{pmatrix} R_1(\epsilon, x) \\ R_2(\epsilon, x) \\ R_3(\epsilon, x) \end{pmatrix},$$

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Each order in ϵ-expansion gives a much simpler form

$$\frac{d}{dx}\begin{pmatrix} J_1^{-3} \\ J_2^{-3} \\ J_3^{-3} \end{pmatrix} = \begin{bmatrix} \frac{1}{x} + \frac{2}{1-x} & 0 & \frac{1}{1+x} - \frac{2}{x} - \frac{3}{1-x} \\ -\frac{1}{x} + \frac{2}{1+x} & \frac{1}{1+x} - \frac{1}{x} - \frac{1}{1-x} & \frac{1}{x} - \frac{2}{1+x} \\ \frac{1}{x} + \frac{2}{1-x} & 0 & \frac{1}{1+x} - \frac{2}{x} - \frac{3}{1-x} \end{bmatrix}\begin{pmatrix} J_1^{-3} \\ J_2^{-3} \\ J_3^{-3} \end{pmatrix} + \begin{pmatrix} R_1^{-3}(x) \\ R_2^{-3}(x) \\ R_3^{-3}(x) \end{pmatrix},$$

Algorithm - I : the homogeneous part

- It boils down to solving a system of linear first order diff. eqns.
- A natural first step is to reduce the system to a higher order equation in a single unknown
  `Note that, the inverse operation is trivial!`
- The classical/naive method to achieve this uncoupling is the cyclic vector algorithm
- But, it gives a complicated decoupled equation.
- Few smarter uncoupling algorithms

`Zürcher`
`Incomplete Zürcher`
`Abramov and Zima`

Implemented in `OreSys` [Gerhold, Schneider]

The 12^{th} blob from below

Applying one of the algorithms to ϵ^{-3} part

$$\left[\frac{d^2}{dx^2} - \frac{2}{1-x}\frac{d}{dx} + \left(\frac{2}{x} - \frac{2}{1+x} - \frac{2}{(1+x)^2}\right)\right] I_1^{-3}(x) = r_1^{-3}(x)$$

$$\left[\frac{d}{dx} - \left(\frac{1}{1-x} + \frac{1}{x} - \frac{1}{1+x}\right)\right] I_2^{-3}(x) = r_2^{-3}(x)$$

Solving for homogeneous part of each diff. eqns.

$$y_1(x) = \frac{x}{1-x^2}\,, \quad y_2(x) = 1 - \frac{2x}{1-x^2} H(0,x)\,; \qquad \mu(x) = \frac{1}{x} - x\,;$$

Next, use variation of constant to obtain the solution

$$I_1^{-3}(x) = y_1(x)\left[C_1 - \int dx \frac{r_1^{-3}(x) y_2(x)}{W(y_1,y_2)}\right] + y_2(x)\left[C_2 + \int dx \frac{r_1^{-3}(x) y_1(x)}{W(y_1,y_2)}\right]$$

$$I_2^{-3}(x) = \frac{1}{\mu(x)}\left(C_3 + \int dx \mu(x) r_2^{-3}(x)\right)$$

$W(y_1, y_2)$ is the Wronskian.

Finally $J_i^{-3}(x) = f_i(\{I_1^{-3}(x), I_2^{-3}(x)\})$!

Algorithm - II : the nonhomogeneous part

- Structure of homogeneous part is same at each order in ϵ-expansion
- Hence the homogeneous solutions and uncoupling procedure are similar for each order
- Start with the ϵ^{-3} part
- Find the best uncoupling for the sub-system and solve for the corresponding homogeneous solutions
- Now at each order in ϵ, find the nonhomogeneous parts keeping the uncoupling structure fixed
- Solve order by order in ϵ using variation of constant

All of this have been automatized using

`Sigma` [Schneider] , `OreSys` [Gerhold, Schneider]
and `HarmonicSums` [Ablinger, Blümlein, Schneider]

The results are obtained in terms of HPLs and Cyclotomic HPLs.

Boundary conditions

Boundary conditions are fixed by imposing regularity of the integrals in the limit of vanishing space-like momentum $q^2 \to 0$ $i.e.$ $x \to 1$.

- In the planar limit, the integrals are regular in $y = 1 - x$, and hence can be expanded as follows

$$J_i(y) = \sum_{n=0}^{\infty} \sum_{j=-3}^{r} \epsilon^j C_{i,j}(n) y^n$$

- $q^2 \to 0$ reduces them to known two-point integrals, providing $C_{i,j}(0)$
- The differential equations now can be solved to obtain $C_{i,j}(n)$ for sufficiently high order (n) in y

Iterated integrals and Harmonic polylogarithms (HPLs)

Given a set of integration kernels $K_i(t)$, one can define

$$\mathcal{I}(i_n, \dots i_1, x) = \int_{x_0}^{x} K_{i_n}(t)\mathcal{I}(i_{n-1}, \dots i_1, t)dt$$

Classic example is $\mathrm{Li}_n(x)$. We have five kernels K_m

$$\Big\{0, 1, -1, \{6, 0\}, \{6, 1\}\Big\} \equiv \Big\{\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}, \frac{1}{1-x+x^2}, \frac{x}{1-x+x^2}\Big\}$$

and correspondingly we define the HPLs as

$$H(m_n, \dots, m_1, x) = \int_0^x K_{m_n}(t)H(m_{n-1}, \dots, m_1, t)dt$$

Some important properties :

Shuffle algebra, Scaling invariance and integration-by-parts identities

Iterated integrals and Harmonic polylogarithms (HPLs)

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New!

$$\Big\{0, 1, -1, \{6, 0\}, \{6, 1\}\Big\} \equiv \Big\{\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}, \frac{1}{1-x+x^2}, \frac{x}{1-x+x^2}\Big\}$$

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Renormalization

We consider a hybrid scheme for UV renormalization.

Heavy quark mass and wave function $(Z_{m,\mathrm{OS}}, Z_{2,\mathrm{OS}})$:	On-shell
QCD strong coupling constant (Z_{a_s}) :	$\overline{MS}$

The renormalization of F_I for these topologies, is straightforward

$$F_{V,i} = Z_{2,\mathrm{OS}} \hat{F}_{V,i} \qquad F_S = Z_{m,\mathrm{OS}} Z_{2,\mathrm{OS}} \hat{F}_S$$

$$F_{V,i} = Z_{2,\mathrm{OS}} \hat{F}_{V,i} \qquad F_P = Z_{m,\mathrm{OS}} Z_{2,\mathrm{OS}} \hat{F}_P$$

where $\hat{F}_I$ contains the counterterms from mass renormalization.

For Axial-Vector and Pseudo-Scalar currents, for these topologies *i.e.* the non-singlet case, both the γ_5-matrices appear in the same chain of Dirac matrices, which allows us to use anti-commuting γ_5 in D space-time dimensions.

Infrared structure

The infrared singularities factorize as a multiplicative factor

[Becher, Neubert '09]

$$F_I(\epsilon, x) = Z(\epsilon, x, \mu) F_I^{fin}(x, \mu)$$

$Z(\epsilon, x, \mu)$ is universal/independent of current
$F_I^{fin}(x, \mu)$ is finite as $\epsilon \to 0$

Renormalization group evolution of $Z(\epsilon, x, \mu)$ provides

$$Z(\epsilon, x, \mu) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\left[\frac{\Gamma_0}{2\epsilon}\right] + \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\frac{1}{\epsilon^2}\left(\frac{\Gamma_0^2}{8} - \frac{\beta_0\Gamma_0}{4}\right) + \frac{1}{\epsilon}\left(\frac{\Gamma_1}{4}\right)\right]$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^3 \left[\frac{1}{\epsilon^3}\left(\frac{\Gamma_0^3}{48} - \frac{\beta_0\Gamma_0^2}{8} + \frac{\beta_0^2\Gamma_0}{6}\right) + \frac{1}{\epsilon^2}\left(\frac{\Gamma_0\Gamma_1}{8} - \frac{\beta_1\Gamma_0}{6}\right) + \frac{1}{\epsilon}\left(\frac{\Gamma_2}{6}\right)\right]$$

Γ_n is the n^{th} order massive cusp anomalous dimension.

[Korchemsky, Radyushkin '87, '92; Grozin, Henn, Korchemsky, Marquard '14, '15]

Results

Results & Checks

- We have computed the master integrals to obtain color-planar and full light quark contributions of massive form factors.
- We have obtained corresponding results for aforementioned currents

$$F_{V,1}^{(3)}, F_{V,2}^{(3)}, F_{A,1}^{(3)}, F_{A,2}^{(3)}, F_{S}^{(3)}, F_{P}^{(3)}$$

✓ $F_{V,i}^{(3)}$ match with the results from Henn $et\,al.$ (color-planar limit)

✓ Complete light quark contributions of $F_{V,i}^{(3)}$ match with the results from Lee $et\,al.$

✓ We agree the results for all other currents with Lee $et\,al.$

✓ The results reproduce the universal infrared structure

✓ Chiral Ward identity is satisfied between $F_{A,i}^{(3)}$ and $F_{P}^{(3)}$

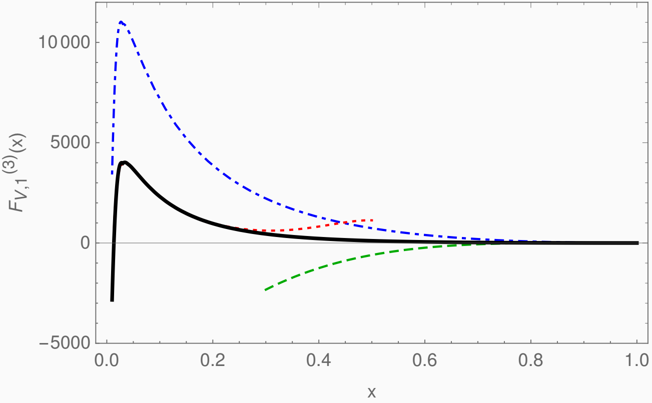
Form factors at various kinematical regions

Low energy region	$q^2 \ll m^2$	$x \to 1$
High energy region	$q^2 \gg m^2$	$x \to 0$

$\textit{Dotted}$: *large q*, $\textit{Dashed}$: *small q*

$N_C = 3, n_l = 5$

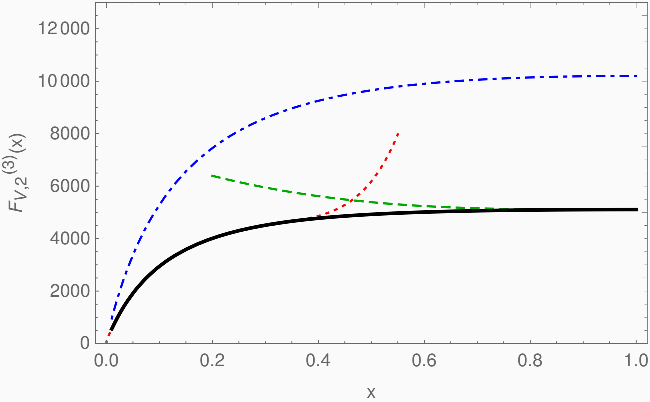
$\mathcal{O}(\epsilon^0)$ part of $F_{V,1}^{(3)}$



Dotted : *large* q, *Dashed* : *small* q

$N_C = 3, n_l = 5$

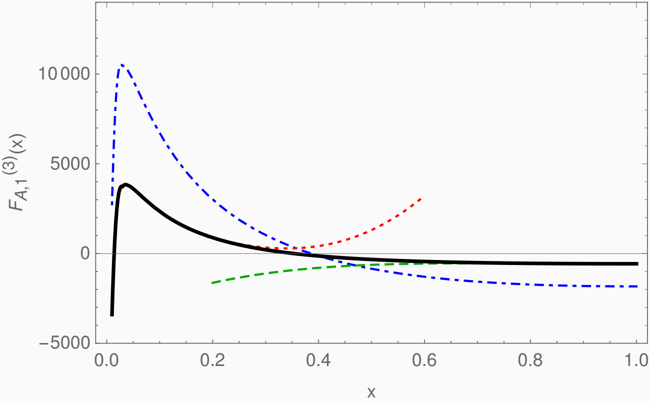
$\mathcal{O}(\epsilon^0)$ part of $F_{V,2}^{(3)}$



Dotted : *large q*, *Dashed* : *small q*

$N_C = 3, n_l = 5$

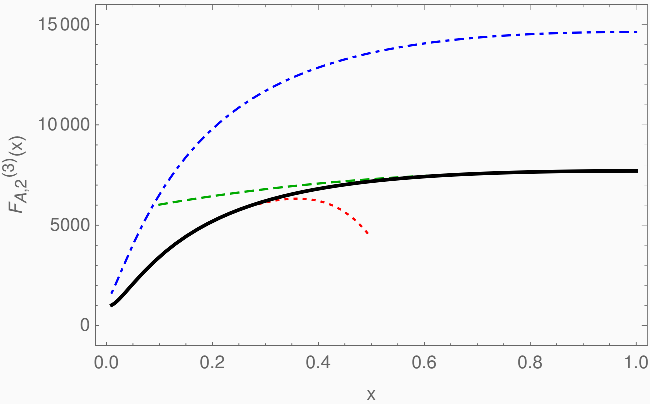
$\mathcal{O}(\epsilon^0)$ part of $F_{A,1}^{(3)}$



$\color{red}{Dotted}: large\ q$, $\color{green}{Dashed}: small\ q$

$N_C = 3, n_l = 5$

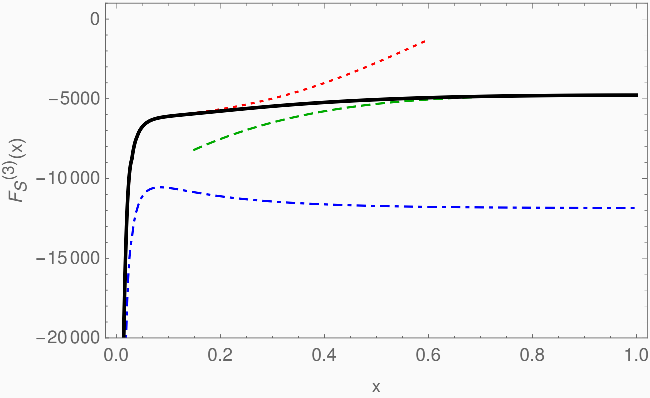
$\mathcal{O}(\epsilon^0)$ part of $F_{A,2}^{(3)}$



Dotted : *large q*, *Dashed* : *small q*

$N_C = 3, n_l = 5$

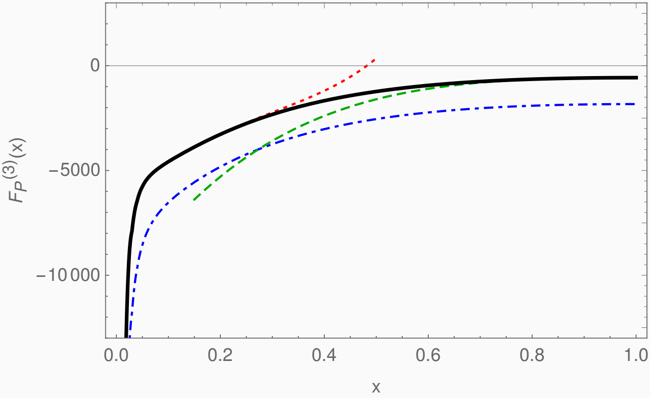
$\mathcal{O}(\epsilon^0)$ part of $F_S^{(3)}$



$\color{red}{Dotted}$: *large q*, $\color{green}{Dashed}$: *small q*

$N_C = 3, n_l = 5$

$\mathcal{O}(\epsilon^0)$ part of $F_P^{(3)}$



Conclusion

Summary

- We have computed the master integrals which appear in the three-loop massive form factors in the color-planar limit and for full light quark (n_l) contributions.
- We have solved the first order factorizable system of differential equations by a new method.
- The use of `OreSys` to uncouple the differential equations has made it possible to automatize the procedure.
- The method applies to all first order factorizable systems in any basis.
- The alphabet contains sixth root of unity letters in the real representation.
- Finally, we have obtained the color-planar and complete light quark three-loop contributions to the heavy quark form factors for vector, axial-vector, scalar and pseudo-scalar currents.

Thank You!