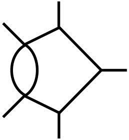
Multi-Loop Numerical Unitarity

A Framework for Computing Two-Loop MEs



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Loops & Legs, St. Goar, May 2018

ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

Alexander von Humboldt Stiftung/Foundation

References

Samuel Abreu, FFC, Harald Ita, Matthieu Jaquier and Ben Page (Freiburg)

arXiv:1703.05255

Samuel Abreu, FFC, Harald Ita, Matthieu Jaquier, Ben Page (Freiburg) and Mao Zeng (UCLA)

arXiv:1703.05273

Samuel Abreu, FFC, Harald Ita, Ben Page (Freiburg) and Mao Zeng (UCLA)

arXiv:1712.03946

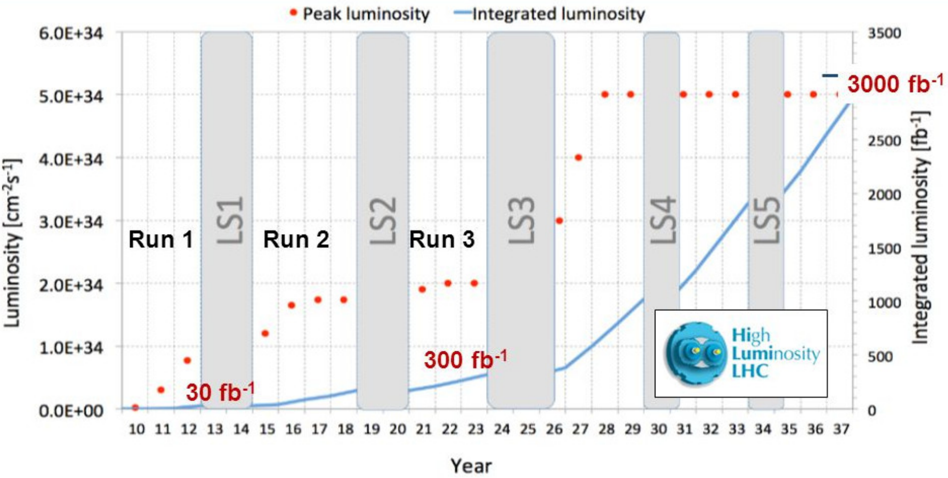
Outline

CHALLENGES FOR QCD PRECISION

TWO-LOOP NUMERICAL UNITARITY

RESULTS AND OUTLOOK

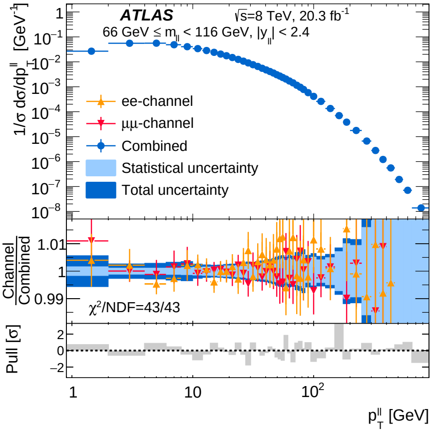
The *atto*barn Era



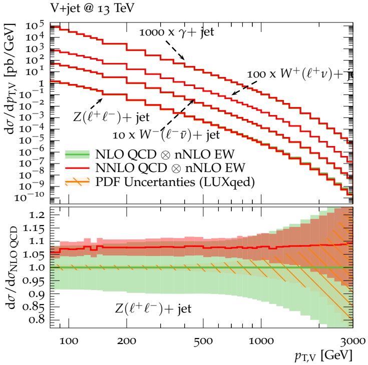
Few % Frontier at the LHC

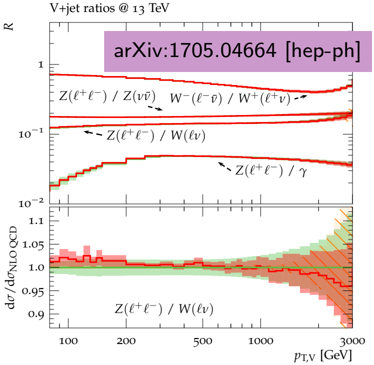
- p_T^{ll} in Drell-Yan, an impressive example of precise differential measurements by ATLAS (8 TeV)
- By normalizing to inclusive Z cross section, improvement in uncertainties
- Total uncertainties below 1% for $p_T^{ll} < 200$ GeV

arXiv:1512.02192 [hep-ex]



Few % Frontier in Theory





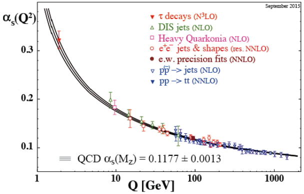
arXiv:1705.04664 [hep-ph]

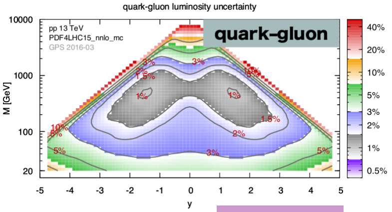
- $p_T^{ll'}$, an impressive example of precise differential predictions
- Uncertainty estimates from NNLO QCD, NLO EW including higher orders Sudakov logs and PDF uncertainties

Lindert, Pozzorini, Boughezal, Campbell, Denner, Dittmaier, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Kallweit, Maierhöfer, Mangano, Morgan, Mück, Petriello, Salam, Schönherr, Williams

Parametric Dependence of QCD Predictions

In order to compute quantum QCD corrections two fundamental inputs are required: the strong coupling α_s and the *Parton Distribution Functions*





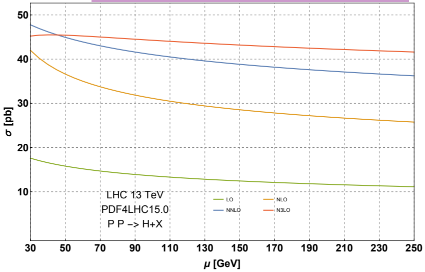
From G. Salam

- Perturbative calculations are also required for the partonic cross sections associated to the signal studied
- Naively at the LHC ($\alpha_s \sim 0.1$) one is to expect NLO QCD corrections to be of order $\sim 10\%$ and NNLO QCD at $\sim 1\%$

Perturbative Improvements for Predictions

- The smallness of α_s and α allows systematic improvements for SM predictions
- In particular hard-processes can be described ever more precisely by systematic additions of higher-order QCD corrections
- As an example Higgs production up to N3LO QCD

B. Mistlberger, arXiv:1802.00833 [hep-ph]



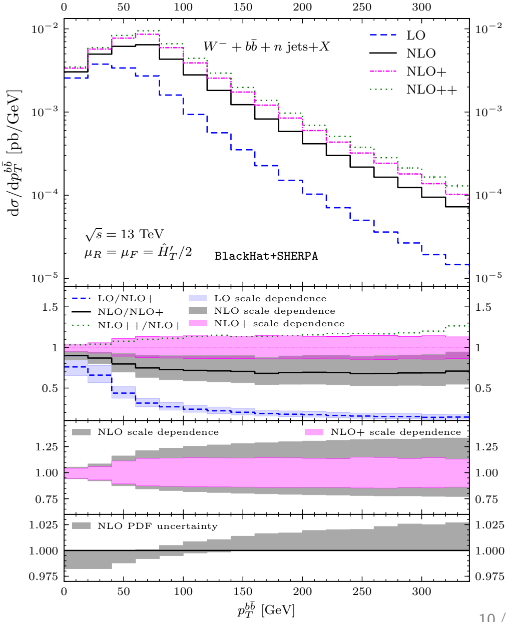
- But computations at NNLO QCD and beyond are challenging in particular for processes with many scales and colored partons
- Inherent need for automation to tackle these problems, even though judicious choices for studies will be mandatory (computationally intensive calculations)

When to push for higher orders...

A Higgs Boson Background

- A key irreducible background to $H(\to b\bar{b})W$ measurement are QCD production of $Wb\bar{b}$+jets
- This signature gives access to y_b
- NLO+ a exclusive sum: adds NLO corrections to hard contributions
- From NLO++ and $\sim 10\%$ $p_T^{\rm veto}$ sensitivity deduce need for NNLO QCD

Anger, FFC, Ita, Sotnikov
arXiv:1712.05721 [hep-ph]



NNLO QCD for Multi-Scale processes

- Great advances over the last few years on NNLO QCD studies for $2 \to 2$ processes, with up to four scales (notice VBF studies by a scheme that exploits DIS calculations, see Cruz-Martinez's talk)
- Physics cases make precision studies for more complex processes necessary, like $H + 2j$, $V + 2j$, $3j$, $t\bar{t} + j$, $VV'j$, among other (more than five scales!)
- About a decade ago, $2 \to 3$ was the frontier for NLO QCD (one-loop) calculations, and the work beyond relied mainly on efficient numerical algorithms (now available through many powerful tools, e.g. *BlackHat, GoSam, HELAC-1Loop/CutTools, Madgraph, NJet, OpenLoops, Recola,* $\cdots$)

Key Building Blocks for NNLO QCD Corrections

- Strategy to handle and cancel IR divergences
- Two-loop matrix elements

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Regarding IR structure

$\rightarrow$ real *hard*

$\rightarrow$ virtual *easy*

From L. Magnea's talk

Key Building Blocks for NNLO QCD Corrections

Full $\mathcal{O}(\epsilon^0)$ structure

- Strategy to handle and cancel IR divergences
- Two-loop matrix elements

$\rightarrow$ real *hard*

$\rightarrow$ virtual *hard*

Key Building Blocks for NNLO QCD Corrections

- Strategy to handle and cancel IR divergences
- Two-loop matrix elements

- Many recent advances and complete calculations (e.g. $t\bar{t}$, $2j$, VV', Vj, HH, etc)
- Several well-developed approaches
  - Antenna subtraction
  - ColorfulNNLO
  - Nested soft-collinear subtractions
  - N-Jettiness slicing
  - Projection to born
  - q_T slicing
  - SecToR Improved Phase sPacE for real Radiation
  - $\ldots$
- Different degrees of automation, we might have public tools in the near future

Key Building Blocks for NNLO QCD Corrections

- Strategy to handle and cancel IR divergences
- Two-loop matrix elements

- Great steps towards understanding mechanisms to compute multi-scale master Feynman integrals, including insights into functional forms over the last few years (see many talks on the topic during this workshop)
- Also new efficient tools developed for multi-loop integral reduction
- Integrand reduction techniques have shown a lot of power to tackle complicated amplitudes (see Badger and Torres-Bobadilla's talks and also related work by Boels et al). Here we focus on the numerical unitarity method (see Ben Page's talk)

Outline

CHALLENGES FOR QCD PRECISION

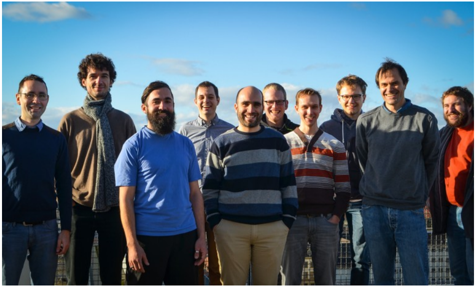
TWO-LOOP NUMERICAL UNITARITY

RESULTS AND OUTLOOK

A Two-Loop Numerical Unitarity Framework

Freiburg: Samuel Abreu, Jerry Dormans, FFC, Harald Ita, Matthieu Jaquier, Ben Page, Evgenij Pascual, Vasily Sotnikov

UCLA: Mao Zeng



Amplitudes through Generalized Unitarity

Write down $\mathcal{A}$ in terms of *master* integrals:

$$\mathcal{A} = \int \prod_j \left[\frac{d\ell_j}{(2\pi)^d}\right] \sum_i \frac{\mathcal{N}_i(\ell)}{\rho_1 \cdots \rho_{n_i}} = \sum_i c_i \int \prod_j \left[\frac{d\ell_j}{(2\pi)^d}\right] \frac{m_i^{\text{master}}(\ell)}{\rho_1 \cdots \rho_{n_i}}$$

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Drop the integral symbol, by constructing sets of *surface* terms:

$$\sum_i \frac{\mathcal{N}_i(\ell)}{\rho_1 \cdots \rho_{n_i}} = \sum_i c_i \frac{m_i^{\text{master}}(\ell)}{\rho_1 \cdots \rho_{n_i}} + \sum_j c_j \frac{m_j^{\text{surface}}(\ell)}{\rho_1 \cdots \rho_{n_j}}$$

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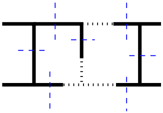
Factorization gives towards the on-shell surfaces $\{\rho_1, \cdots, \rho_{n_l}\} = 0$:

$$\sum_i \frac{\mathcal{N}_i(\ell)}{\rho_1 \cdots \rho_{n_i}} \quad \longrightarrow$$



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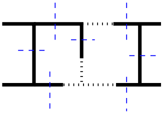
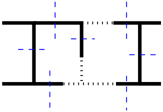


And so we get access to the set of (maximal) *cut equations*, like:

$$N(\Gamma, \ell_\Gamma) \equiv \sum_k c_k m_k (\Gamma, \ell_\Gamma) = \quad [\text{diagram}] \quad \equiv R(\Gamma, \ell_\Gamma)$$



Challenges in Multi-Loop Numerical Unitarity

See Ben Page's talk for more details!

- Efficient algorithm to color decompose the amplitude's integrand [Ochirov, Page '16]
- Systematic master/surface decomposition of all propagator structures [Ita '15], [Abreu et al. (FFC) 17']
- On-the-fly reconstruction of functional dependence on regulators [Giele, Kunszt, Melnikov '08], [Peraro '16]
- Algorithm to handle subleading poles over on-shell phase spaces [Abreu et al. (FFC) 17']
- Fast implementation of multi-dimensional cuts and on-shell parameterizations (through Berends-Giele off-shell recursions, and van Neerven-Vermaseren basis and [Ita '15])
- Ensure numerical stability of calculation (through high-precision arithmetics and exploiting exact kinematics [von Manteuffel, Schabinger '14], [Peraro '16])
- Availability of master integrals (analytic expressions, for 5-pt examples see [Papadopoulos, Tommasini, Wever '15], [Gehrmann, Henn, lo Presti '15], or employing numerical tools like SecDec and Fiesta)

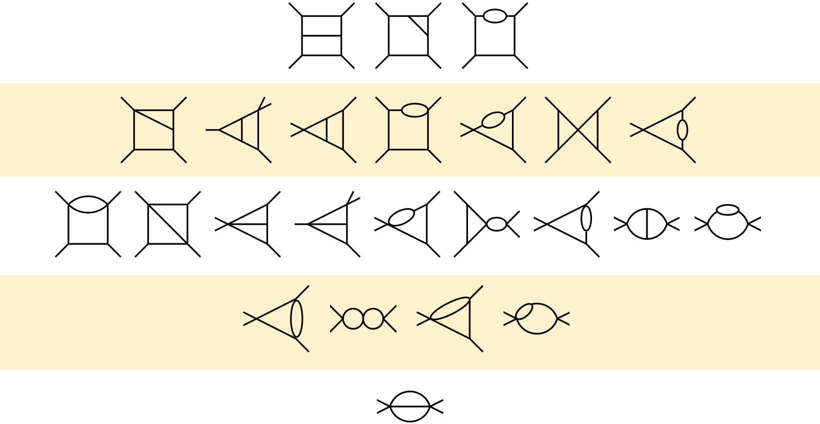
Modular Library

We are constructing a C++ framework for D-dimensional multi-loop numerical unitarity, with a highly modular structure

- Hierarchical relations between propagator structures
- Decompositions of numerator functions into master/surface terms
- Color handling with interaction to algebraic libraries
- Automated construction of cut equations handling subleading poles
- Engine to solve off-shell recursions to compute trees and multi-loop cuts
- Toolkit to handle kinematic structures using high-precision and exact arithmetics
- D-dimensional on-shell phase spaces generator
- Machinery for univariate functional reconstruction
- Integral library

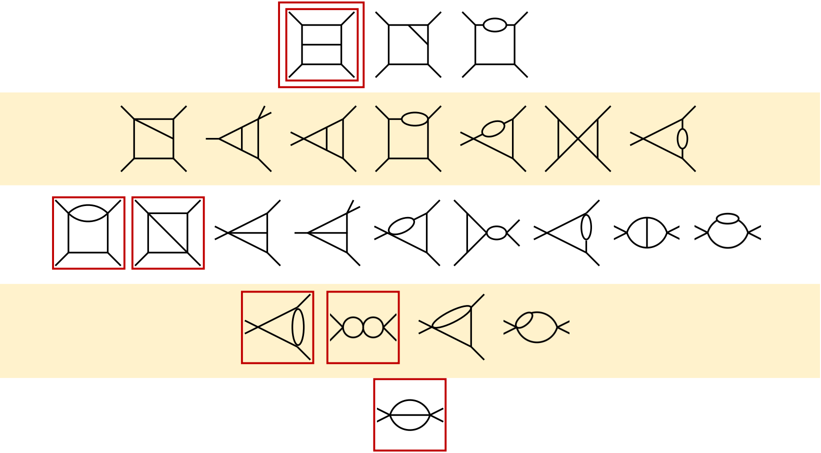
Some dependencies like `Givaro, GMP, Lapack, MPACK, QD`

The Planar two-loop four-point Hierarchy



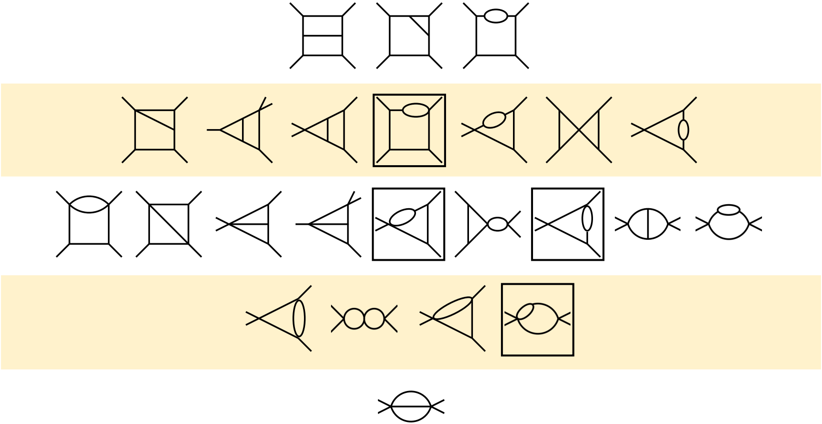
The full hierarchy of diagrams for the planar 2-loop 4-gluon amplitudes

The Planar two-loop four-point Hierarchy



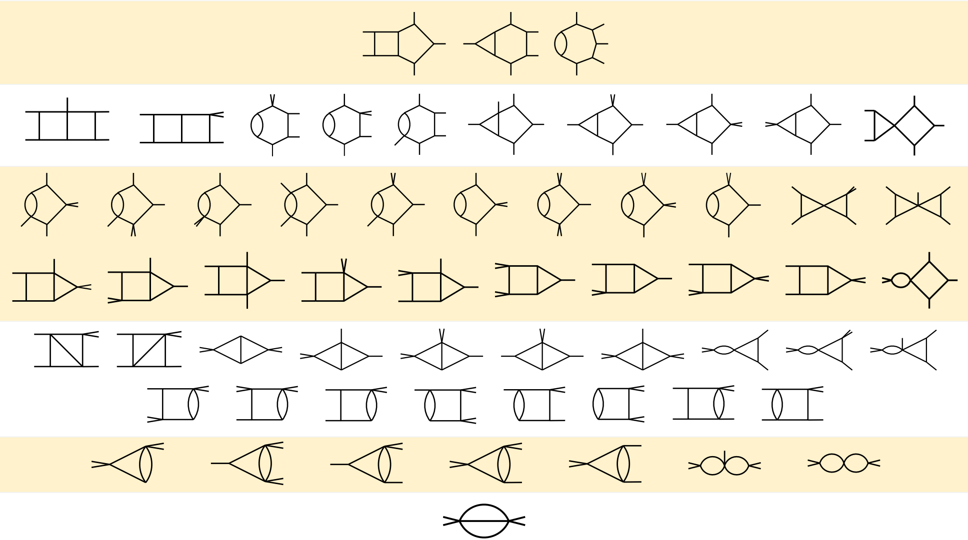
The *master* integrals

The Planar two-loop four-point Hierarchy



The set of subleading pole diagrams

The Two-Loop Five Gluon Amplitudes



The full hierarchy of diagrams for the planar 2-loop 5-gluon amplitudes

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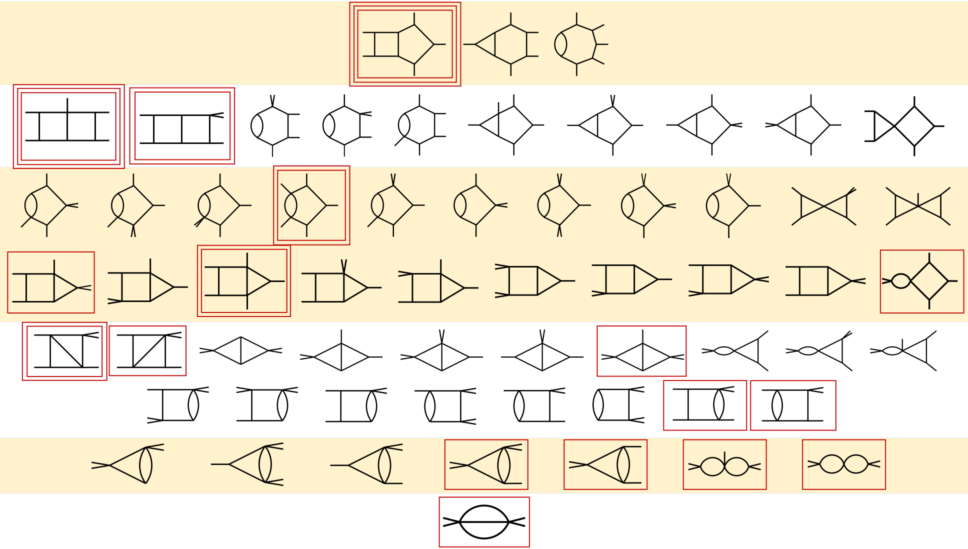
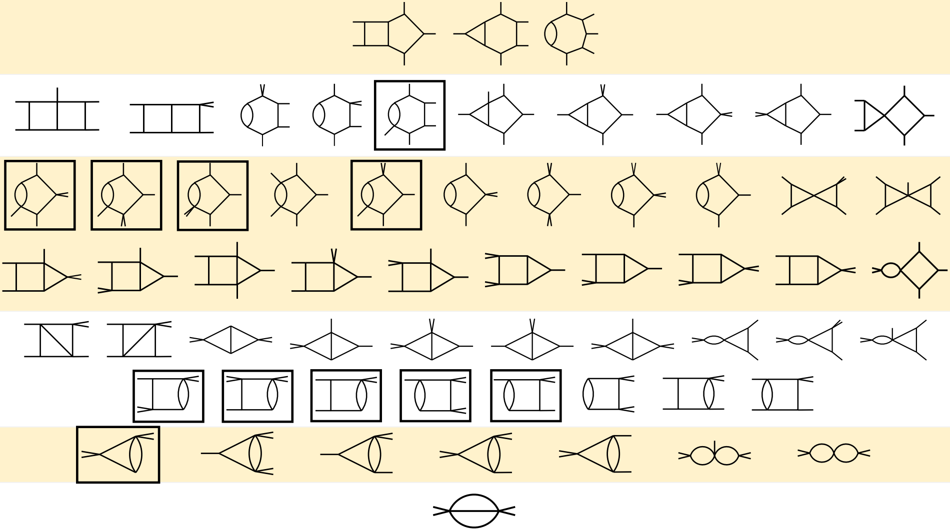


The master integrals

The Two-Loop Five Gluon Amplitudes



The set of subleading pole diagrams

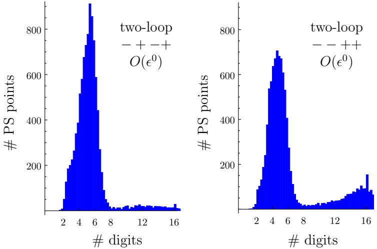
CHALLENGES FOR QCD PRECISION

TWO-LOOP NUMERICAL UNITARITY

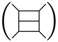
RESULTS AND OUTLOOK

Four- and Five-Gluon Amplitudes

- We have computed 4-gluon and 5-gluon planar amplitudes
- Floating point 4-pt calculation shows large cancellations
- Univariate reconstruction exploited to extract (known) analytic results
- Exact coefficient results extracted for 5-pt amps



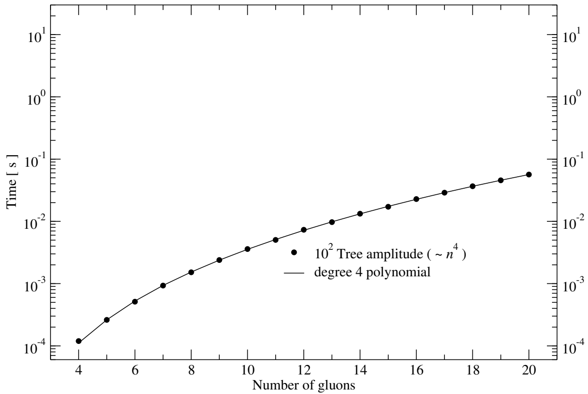
$$c_1 \left(\right) = \frac{9x + \frac{\epsilon\left(-x^3 - \frac{32x^2}{11} - \frac{97x}{44} - \frac{5}{22}\right)}{\frac{x^2}{33} + \frac{2x}{33} + \frac{1}{33}} + \cdots}{27 - 81(4-2\epsilon) + 90(4-2\epsilon)^2 + \cdots}$$



$\mathcal{A}^{(2)}/\mathcal{A}^{(0)}$	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$(1^-, 2^-, 3^+, 4^+, 5^+)$	12.5000000	25.46246919	-1152.843107	-4072.938337	-3637.249566
$(1^-, 2^+, 3^-, 4^+, 5^+)$	12.5000000	25.46246919	-6.121629624	-90.22184214	-115.7836685

See Ben Page's talk

Scaling Properties of Gluon Amplitudes



- Polynomial complexity to compute color-ordered amplitudes
- Asymptotic regime only for very large n at 1 and 2 loops
- Initial benchmarking for two-loop five-point amplitudes promising

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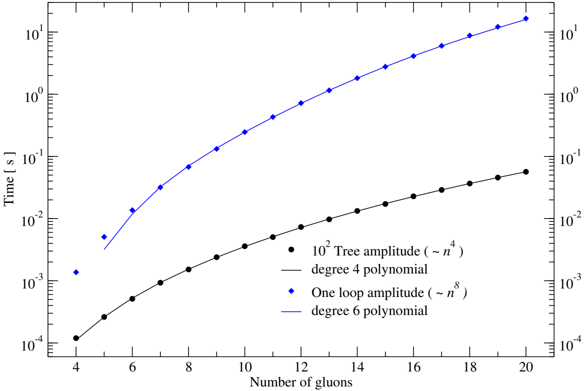

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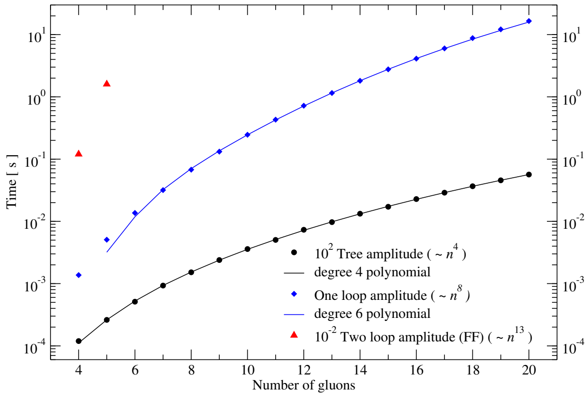

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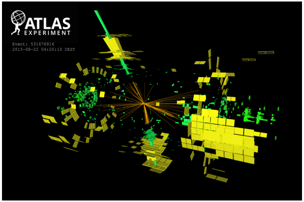
Outlook

- Exploiting the physics potential of the LHC's experiments will require very precise predictions from the SM
- Important advances in IR subtraction, integral and full-amplitude computations open the path to multi-scale NNLO QCD predictions
- Multi-loop numerical unitarity appears as a robust method to tackle two-loop calculations relevant for phenomenology and formal studies
- We expect an initial release in the near future



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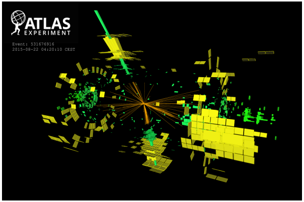
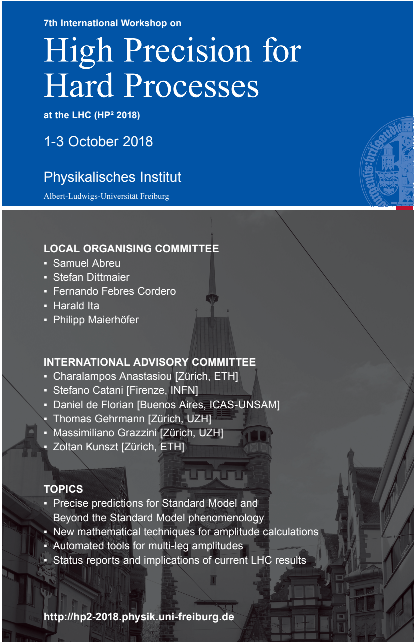


Thanks!

HP2 2018 in Freiburg



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