Non-resonant and electroweak corrections to top pair production

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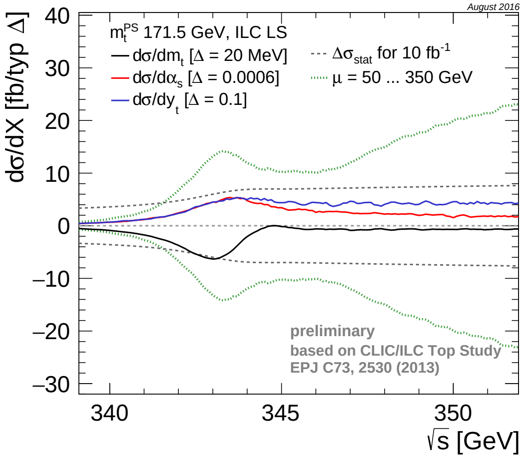
M. Beneke, AM, T. Rauh, P. Ruiz-Femenía arXiv:1711.10429

M. Beneke, AM, J. Piclum, T. Rauh arXiv:1506.06865

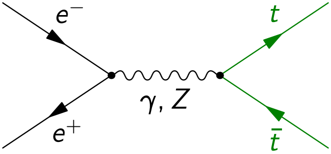
$\sigma(e^+e^- \to t\bar{t})$ near threshold

- Allows clean and precise determination of top-quark mass, $\Delta m_t^{\overline{\text{MS}}} \approx 50\,\text{MeV}$
- Sensitive to top-quark width, Yukawa coupling, α_s

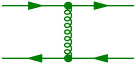
Limited by theory uncertainty: [Simon 2016]



$\sigma(e^+e^- \to t\bar{t})$ near threshold



- Kinematics: $v \ll 1, \quad E_{\mathrm{kin}} \sim m_t v^2, \quad |\mathbf{p}| \sim m_t v$
- Dominant interaction:



$\Rightarrow$ Colour Coulomb potential $-\frac{C_F \alpha_s}{r}$

- $t\bar{t}$ "decays during bound state formation":

$$v \sim \alpha_s \Rightarrow E_{\mathrm{kin}} \sim m_t \alpha_s^2 \sim -E_1$$

$$\alpha \sim \alpha_s^2 \Rightarrow \Gamma_{t\bar{t}} \sim m_t \alpha \sim -E_1$$

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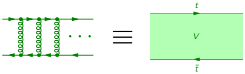
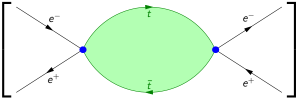
Potential non-relativistic effective field theory

[Pineda, Soto 97; Beneke, Signer, Smirnov 99; Brambilla et al. 99]

$$
\begin{aligned}
\mathcal{L}_{\text{PNREFT}} = \psi^\dagger \left(i\partial_0 + \frac{\partial^2 + i m_t \Gamma_t}{2m_t} \right) \psi + \mathcal{L}_{\text{anti-quark}} \\
- \int d^3\mathbf{r} \left[\psi^\dagger \psi\right](x+\mathbf{r}) \frac{C_F \alpha_s}{r} \left[\chi^\dagger \chi\right](x) \\
+ \{\text{NLO}\}
\end{aligned}
$$

- Propagator: Coulomb Green Function 

- $\sigma_{\text{LO}} \propto \text{Im}$ 

- Higher orders suppressed by powers of $v \sim \alpha_s \sim \sqrt{\alpha} \sim y_t$

PNREFT at higher orders

Scales: $m_t, m_W, m_Z, m_H \gg m_t v \gg m_t v^2 \gg \Lambda_{\text{QCD}}$

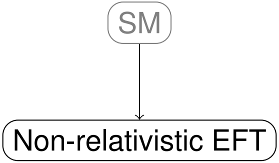
- hard modes: $k \sim m_t$
- soft modes: $k \sim m_t v$
- potential modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$

SM

PNREFT at higher orders

Scales: $m_t, m_W, m_Z, m_H \gg m_t v \gg m_t v^2 \gg \Lambda_{\text{QCD}}$

- hard modes: $k \sim m_t \to$ (local) effective vertices
- soft modes: $k \sim m_t v$
- potential modes: $k_0 \sim m_t v^2$, $\vec{k} \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$



PNREFT at higher orders

Scales: $m_t, m_W, m_Z, m_H \gg m_t v \gg m_t v^2 \gg \Lambda_{QCD}$

- hard modes: $k \sim m_t \to$ (local) effective vertices
- soft modes: $k \sim m_t v \to$ (non-local) potentials
- potential light particle modes $\to$ (non-local) potentials
- potential top quark modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$

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↓

Non-relativistic EFT

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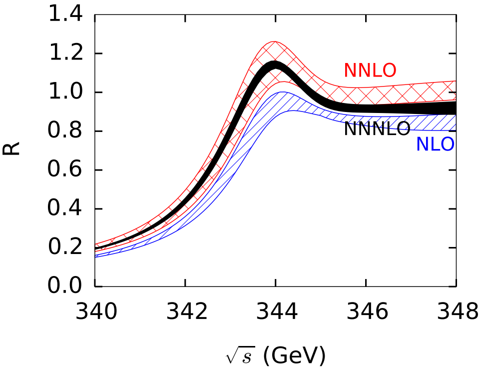
Potential non-relativistic EFT

NNNLO QCD corrections

[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]



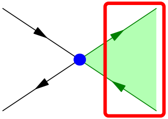
- Apparent convergence at NNNLO, 3% scale uncertainty
- Similar convergence at NNLO + NNLL [Hoang, Stahlhofen 2013]

Why go beyond QCD?

- Required for consistency,
  finite-width divergences $\propto \frac{\Gamma_t}{\epsilon}$ at NNLO
- Effects can be sizeable $\sim 10\%$
- Sensitivity to Yukawa coupling

$\Rightarrow$ Electroweak and non-resonant corrections to NNLO,
Yukawa corrections up to NNNLO

Electroweak corrections to $t\bar{t}$ resonance

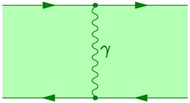


- Expand $p_t^2 = m_t^2 - i m_t \Gamma_t$ in non-relativistic limit:

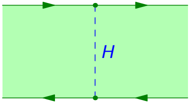
$$\mathcal{L}_{\psi,\text{kinetic}} = \psi^\dagger \left(i\partial_0 + \frac{\partial^2 + i m_t \Gamma_t}{2 m_t} + \frac{\left(\partial^2 + i m_t \Gamma_t\right)^2}{8 m_t^3} + \dots \right) \psi$$

Time dilatation: $\Gamma_{t\bar{t}} < 2\Gamma_t$

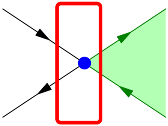
- Potential corrections:



QED Coulomb potential (NLO)

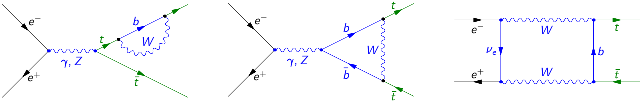


Higgs potential (NNNLO)



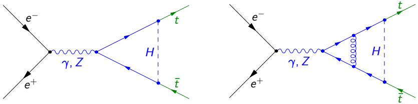
- NNLO Electroweak corrections:

[Grzadkowski, Kühn, Krawczyk, Stuart 1986; Guth, Kühn 1991; Hoang, Reißer 2004 & 2006]



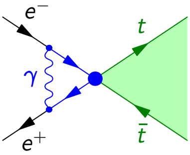
- NNLO + NNNLO Yukawa corrections:

[Eiras, Steinhauser 2006]

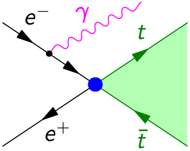


Initial-state radiation

Photon corrections to initial state:



γ hard, hard-collinear



γ ultrasoft, (ultra)soft-collinear

$\hookrightarrow$ large logarithms $\log^2 \frac{m_t}{m_e}$, resummed into structure functions

[Fadin, Kuraev 1985; Fadin, Khoze 1987]

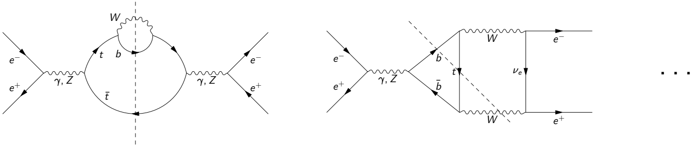
$$\sigma(s) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, \Gamma_{ee}(x_1)\Gamma_{ee}(x_2)\hat{\sigma}(x_1 x_2 s) + \sigma^{\mathrm{ISR}}_{\mathrm{const}}(s)$$

Non-resonant production

Unstable particle effective field theory [Beneke, Chapovsky, Signer, Zanderighi 2003-2004]

$$\sigma = \sigma_{\text{res}} + \sigma_{\text{non-res}}$$

$\sigma_{\text{non-res}}$: produce decay products (W, b, gluons) *without* intermediate $t\bar{t}$ resonance:



starting at NLO (α/v) [Beneke, Jantzen, Ruiz-Femenía 2010]

Formally $p_t^2 - m_t^2 \sim m_t^2 \gg m_t\Gamma_t$ in top-quark propagators
$\hookrightarrow$ width not resummed
$\hookrightarrow$ endpoint divergences as $p_t^2 - m_t^2 \to 0$
cancel against $\frac{\Gamma_t}{\epsilon}$ finite-width divergences in resonant part

dimensional regularisation required

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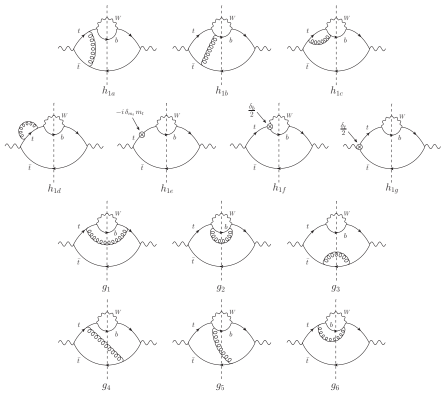
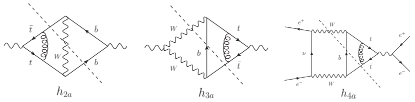
NNLO non-resonant production





“Interference” contribution

UV divergent & endpoint divergent

$\mathcal{O}(100)$ diagrams calculated with MadGraph5_aMC@NLO

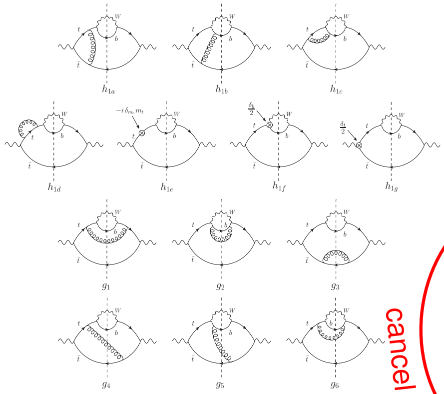
“Squared” contribution

UV finite & endpoint divergent

“Automated” contribution

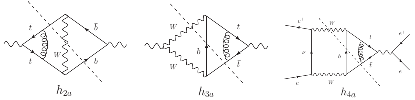
UV divergent & endpoint finite

NNLO non-resonant production



"Squared" contribution

UV finite & endpoint divergent



"Interference" contribution

UV divergent & endpoint divergent

$\mathcal{O}(100)$ diagrams
calculated with
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"Automated" contribution

UV divergent & endpoint finite

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NNLO non-resonant production

$$\sigma = \sigma_{\text{res}} + \sigma_{\text{sq}} + \sigma_{\text{int}} + \sigma_{\text{aut}}$$

d dim. phase space → σ_{res}

4 dim. phase space → σ_{aut}

Split into separately finite pieces:

$$\sigma = \left[\sigma_{\text{res}} + \sigma_{\text{sq}} + \sigma_{\text{int}}^{\text{(EP div)}}\right] + \left[\sigma_{\text{int}}^{\text{(EP fin)}} + \sigma_{\text{aut}}\right]$$

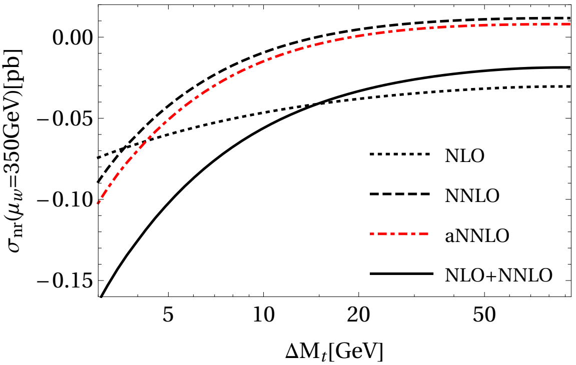
$$\int_y^1 dt\, g_{ia}(t) = \sum_n \frac{\hat{g}_{ia}^{(1,n)}(1-y)^{-n\epsilon}}{-n\epsilon} + \int_y^1 dt \left[g_{ia}(t) - \sum_n \frac{\hat{g}_{ia}^{(1,n)}}{(1-t)^{1+n\epsilon}}\right]$$

$t = \frac{p_t^2}{m_t^2}$, endpoint divergence for $t \to 1$

y: cut on invariant mass

NNLO non-resonant production

Effect of invariant mass cut



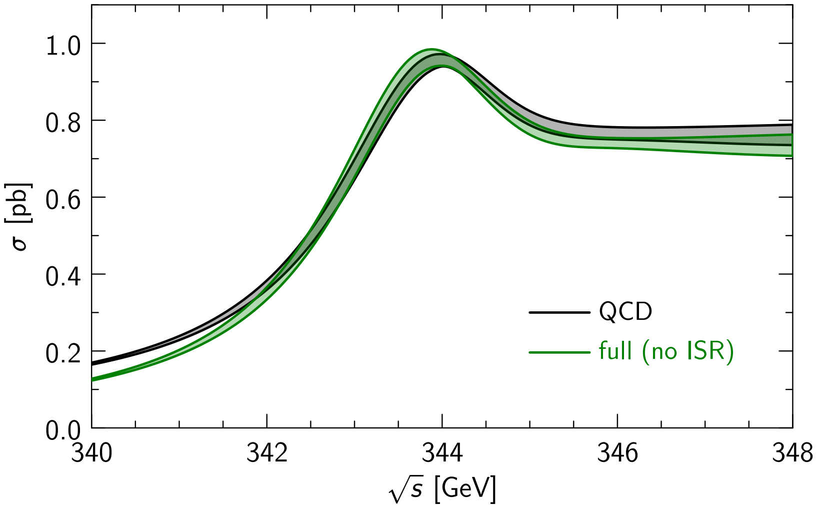
$$(m_t - \Delta M_t)^2 \leq p_t^2 \leq (m_t + \Delta M_t)^2$$

NLO: [Beneke, Jantzen, Ruiz-Femenía 2010]

aNNLO: $\Gamma_t \ll \Delta M_t \ll m_t$

[Jantzen,Ruiz-Femenía 2013; see also Hoang, Reißer, Ruiz-Femenía 2010]

Top-pair production cross section

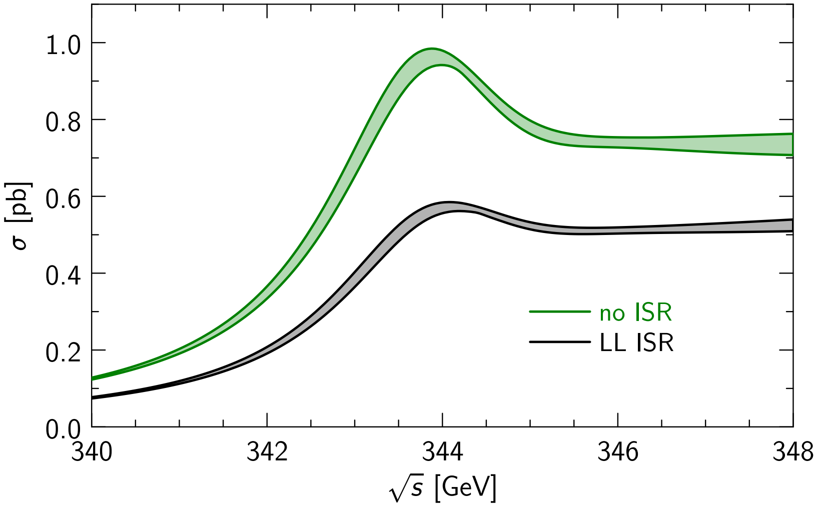


$m_t^{\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}\,,\qquad \Gamma_t = 1.33\,\mathrm{GeV}\,,\qquad m_H = 125\,\mathrm{GeV}\,,$

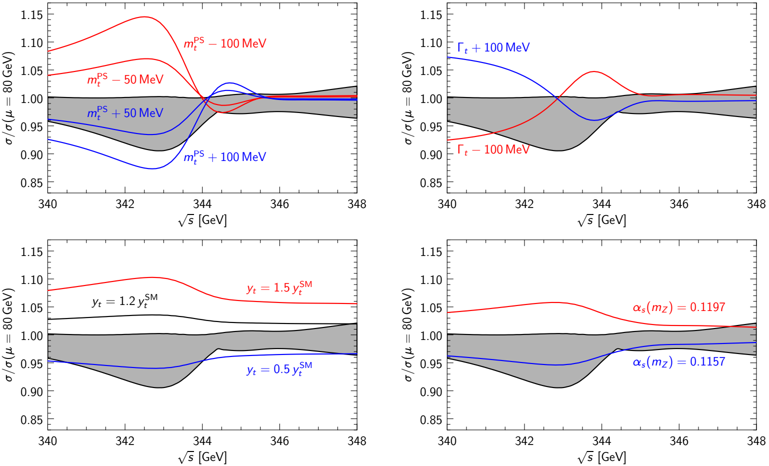
$\alpha_s(m_Z) = 0.1177\,,\qquad \alpha(m_Z) = 1/128.944\,,\qquad m_W, m_Z$

Top-pair production cross section

Initial-state radiation



Determination of top-quark properties



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Conclusions

- $\sigma(e^+e^- \to t\bar{t})$ sensitive to top-quark mass, width, Yukawa coupling
- NNNLO QCD + Higgs and NNLO electroweak corrections
  $\sim 3\%$ QCD scale uncertainty
- Electroweak effects are important,
  exceed scale uncertainty below peak
- All corrections included in version 2 of `QQbar_threshold`
  `http://qqbarthreshold.hepforge.org/`
- Future improvements:
  - Resonant NNNLO electroweak corrections
  - NNLL resummation [Hoang, Stahlhofen 2013]
  - NNNNLO Higgs potential
  - NLL initial-state radiation

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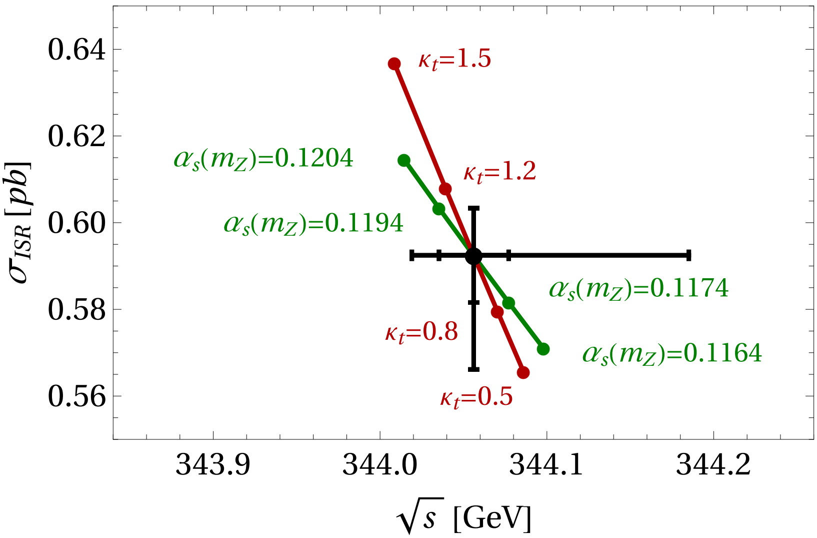
Backup



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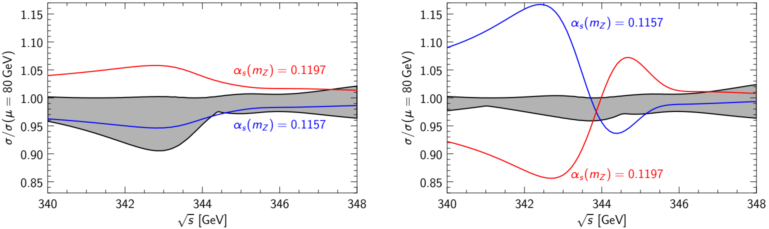
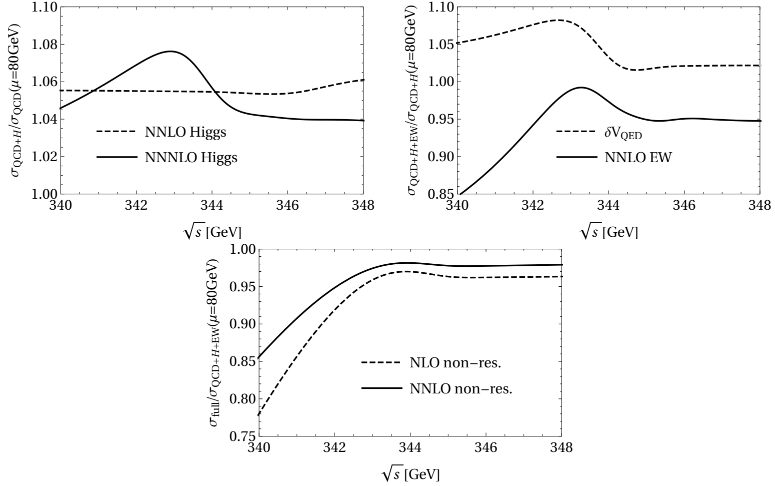
PS vs. $\overline{\text{MS}}$ scheme

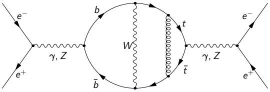


$m_t^{\text{PS}}(20\,\text{GeV}) = 171.5\,\text{GeV}$ $\bar{m}_t(\bar{m}_t) = 163.4\,\text{GeV}$

Size of single corrections



Cancellation of endpoint divergences



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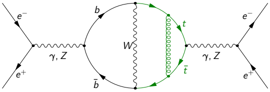
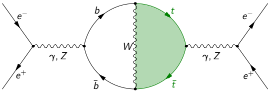


$\Cap$ $\Cap$

$\sigma_{\text{non-res}}$



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