NLO mixed QCD-EW corrections to Higgs gluon fusion

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Loops & Legs in Quantum Field Theory 2018

Karlsruher Institut für Technologie

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in collaboration with

K. Melnikov (TTP, KIT) & L. Tancredi (CERN)

Topics

1. Motivations
2. Amplitude
3. Differential Equations and UT functions
4. Virtual NLO QCD-EW evaluation
5. NLO cross-section
6. Conclusions

Higgs at the LHC [Anastasiou...,2009][LHC H CSWG,2017][Mistlberger,2018]

- Existence of physics beyond the Standard Model
- Lack of direct evidence of new physics (at the LHC)

Higgs boson: good candidate

- Effects of new physics: $\delta g/g \sim 5\%$ at $\Lambda = 1\,\text{TeV}$
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Uncertainty on theoretical predictions for Higgs physics lower than $O(1\%)$

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- Gluon fusion: main H prodction at the LHC, $\mathcal{O}(5\%)$ from QCD-EW

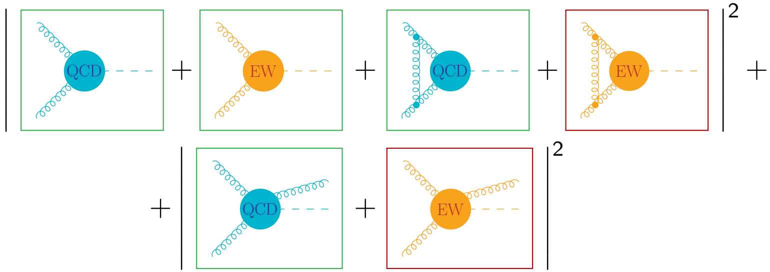
Theoretical uncertainties

δ_{scale}	$\delta_{\text{PDF-TH}}$	$\delta_{\text{QCD-EW}}$	$\delta_{t,\ b,\ c}$	δ_{1/m_t}
$\sim 2\%$	1.16%	1%	0.83%	1%

Mixed QCD-EW contributions

[Aglietti. . . ,2006][Degrassi. . . ,2004]

$\overline{\sigma}_{\text{NLO}} \propto$



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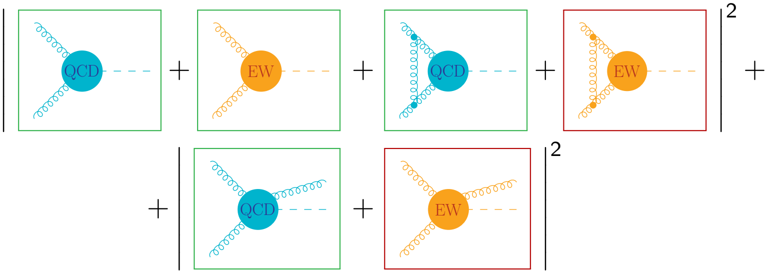

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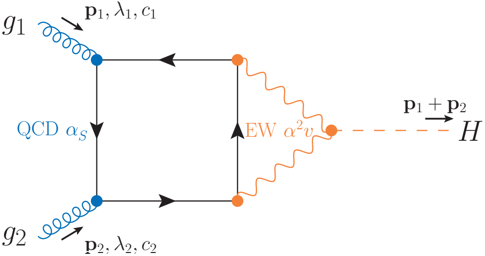
Compute σ_{NLO} for physical values of $m_{W,Z}$ and m_H

$gg \to H$ NLO QCD-EW amplitude

$gg \to H$: Leading Order

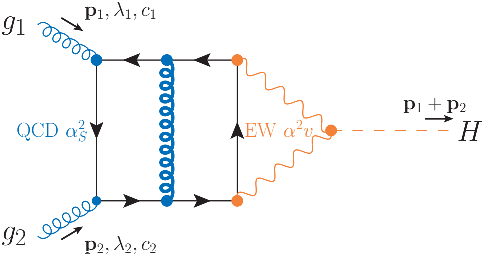


$$\mathcal{M}_{2\,\lambda_1\lambda_2}^{\,c_1c_2} = \delta^{c_1c_2}\epsilon_{\lambda_1}(\mathbf{p}_1)\cdot\epsilon_{\lambda_2}(\mathbf{p}_2)\mathcal{F}_2(s, m_W, m_Z)$$

- H couples to EW vector bosons: t suppressed (large mass)
- Light quarks taken to be massless (W: $\{u, d, s, c\}$; Z: $\{u, d, s, c, b\}$)
- W and Z never in the same diagram: two-scale problem (s & m^2)

$gg \to H$ NLO QCD-EW amplitude

$gg \to H$: virtual NLO



$$\mathcal{M}_3{}^{c_1 c_2}_{\lambda_1 \lambda_2} = \delta^{c_1 c_2} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2) \mathcal{F}_3(s, m_W, m_Z)$$

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Analysis of the amplitude

- $\mathcal{M}_3{}^{c_1 c_2}_{\lambda_1 \lambda_2}$: 47 3-loop Feynman Diagrams for both W and Z EW bosons

$$\mathcal{F}(s, m_W, m_Z) = -\mathrm{i}\frac{\alpha^2 \alpha_S(\mu) v}{64\pi \sin^4\theta_W} \sum_{V=W,Z} C_V\, A(m_V^2/s, \mu^2/s)$$

- $C_W = 4$
- $C_Z = \dfrac{2}{\cos^4\theta_W}\left(\dfrac{5}{4} - \dfrac{7}{3}\sin^2\theta_W + \dfrac{22}{9}\sin^4\theta_W\right)$

$$A(m^2/s, \mu^2/s) = A_{2\mathrm{L}}(m^2/s) + \frac{\alpha_S(\mu)}{2\pi} A_{3\mathrm{L}}(m^2/s, \mu^2/s) + \mathcal{O}(\alpha_S^2)$$

- $A_{3\mathrm{L}}(m^2/s, \mu^2/s)$: linear combination of 95 Master Integrals $\mathcal{I}(s, m, \epsilon)$

Evaluation of Master Integrals

1. Change of variables: only one dimensionful parameter

$$s \qquad\qquad y := \frac{\sqrt{1-4m^2/s}-1}{\sqrt{1-4m^2/s}+1}$$

By dimensional analysis

$$\mathcal{I}(s, y, \epsilon) = (-s - \mathrm{i}0)^{a-3\epsilon} \mathcal{J}(y, \epsilon)$$

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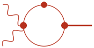
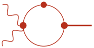
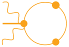
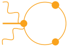
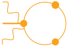
2. Evaluation of $\mathcal{J}(y,\epsilon)$ via Differential Equations

System of Linear Partial Differential Equations

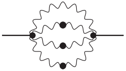
$\frac{\partial}{\partial y}\left[\frac{1+y}{1-y}\epsilon\right.$  $\left.\right] = \frac{-4\epsilon}{1-y^2}\left[\frac{1+y}{1-y}\epsilon\right.$  $\left.\right] + \frac{2\epsilon}{y}\left[\frac{y}{1-y}\right.$  $\left.\right]$

$\frac{\partial}{\partial y}\left[\frac{y}{1-y}\right.$  $\left.\right] = -\frac{1+y}{1-y}\epsilon\left[\frac{y}{1-y}\right.$  $\left.\right]$

A simple form for the MIs

[Henn,2013][Argeri. . . ,2014]

In many cases of interest, the MIs can be expresed using UT functions

$$\epsilon^3 (-s)$$

$$= 1 - 3\,\epsilon \log(-s) + \epsilon^2 \frac{9 \log^2(-s) - \pi^2}{2} + O\left(\epsilon^3\right)$$

UT function

Function having an ϵ-expansion with weight n coefficients at order ϵ^n

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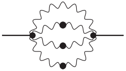

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Weight W

Number of nested integrations over $\mathrm{d}\log R(\xi)$, $R(\xi)$ rational functions

$$F_n(y) = \int_0^y \dots \int_0^{\xi_n} \mathrm{d}\log R_n(\xi) \dots \mathrm{d}\log R_1(\xi) \quad \Rightarrow \quad W(F_n) := n$$

- Weight w functions in rational points give weight w constants
- $W(F_a \, F_b) = W(F_a) + W(F_b)$

The UT Cauchy problem

[Remiddi...,1999][Henn,2013][Lee,2014]

A UT function $\mathbf{F}(y,\epsilon)$ satisfies

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Differential Equations

$$\frac{\mathrm{d}}{\mathrm{d}y}\mathbf{F}(y,\epsilon) = \epsilon \sum_{a=1}^{A} B_a \frac{\mathrm{d}\log R_a(y)}{\mathrm{d}y}\mathbf{F}(y,\epsilon)$$

Canonical form — homogeneous dependence over ϵ

Fuchsian system — only simple poles in y (partial fractioning on $\mathrm{d}\log$s)

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Boundary Conditions

Integration constants: the solution of the DE is compared in $y \to y_0$ to a boundary function

$$\lim_{y\to y_0}[\mathbf{F}(y,\epsilon) - \mathbf{L}(y,\epsilon)] = 0$$

If y_0 is a rational point, $\mathbf{L}(y \to y_0,\epsilon)$ is also a UT expression

Form of the solution

[Goncharov,1994][Remiddi. . . ,1999][Henn,2013][Argeri. . . ,2014][Lee,2014]

Tuning the MIs to satisfy the UT Cauchy problem:

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Tuning the MIs to satisfy the UT Cauchy problem:

- Canonical form: solution as a Dyson series in ϵ

$$\mathbf{F}(y,\epsilon) = \mathcal{P}_y \mathrm{e}^{\epsilon\int A(\xi)\,\mathrm{d}\xi}\,\mathbf{F}_0(\epsilon) = \mathbf{F}_0^{(0)} + \epsilon\left[\int_y A(\xi_1)\mathbf{F}_0^{(0)}\,\mathrm{d}\xi_1 + \mathbf{F}_0^{(1)}\right] +$$

$$+\epsilon^2\left[\int_y A(\xi_1)\int_{\xi_1} A(\xi_2)\mathbf{F}_0^{(0)}\,\mathrm{d}\xi_2\mathrm{d}\xi_1 + \int_y A(\xi_1)\mathbf{F}_0^{(1)}\,\mathrm{d}\xi_1 + \mathbf{F}_0^{(2)}\right] + O\left(\epsilon^3\right)$$

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- Fuchsian system: iterated integrations as Goncharov Polylogarithms

$$G(\mathbf{a}_w; y) := \int_0^y \frac{1}{\xi - a_w} G(\mathbf{a}_{w-1}; \xi)\,\mathrm{d}\xi$$

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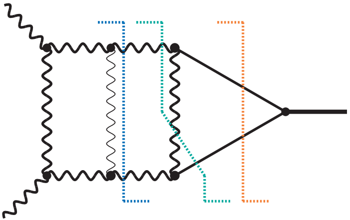
Canonical Fuchsian DEs for NLO $gg \to H$

$$\mathrm{d}\mathbf{F}(y,\epsilon) = \epsilon \left[B_+ \,\mathrm{d}\log(1-y) + B_r \,\mathrm{d}\log(y^2 - y + 1) + B_- \,\mathrm{d}\log(y+1) + B_0 \,\mathrm{d}\log y \right] \mathbf{F}(y,\epsilon)$$



s	0	m^2	$4m^2$	$[\infty]$
y	$+1$	$e^{i\pi/3}$	-1	$[0]$
Kernel	$\frac{1}{\xi-1}$	$\frac{2\xi-1}{\xi^2-\xi+1}$	$\frac{1}{\xi+1}$	$\left[\frac{1}{\xi}\right]$

Boundary Conditions

[Smirnov,2002]

UT functions: efficient numerical matching using PSLQ algorithm, at

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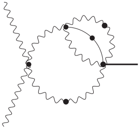
- Taylor expansion in $\sim \sqrt{s}$ to generate the large-mass integrals

A 3-loop example



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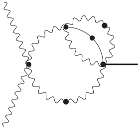
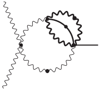
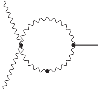
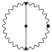
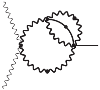
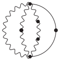
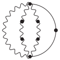


-  $\rightarrow$  $\times$ 
-  $\rightarrow$  $+\; s\dfrac{2(1+\epsilon)}{2-\epsilon}$  $+\; \mathcal{O}\left(\dfrac{(-s)^2}{(m^2)^4}\right)$

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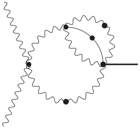
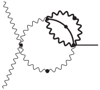
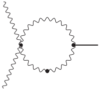
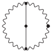
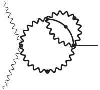
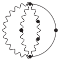
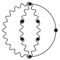


-  $\rightarrow$  $\times$ 
-  $\rightarrow$  $+ \; s\,\frac{2(1+\epsilon)}{2-\epsilon}$  $+ \; \mathcal{O}\left(\frac{(-s)^2}{(m^2)^4}\right)$

w	0	1	2	3	4	5	6
Values	1		π^2	$\zeta(3)$	π^4	$\pi^2\zeta(3)$ $\zeta(5)$	π^6 $\zeta^2(3)$

All integrals have been checked against (Py)SecDec

Expression of the Form Factor

[Catani,1998]

The virtual NLO contribution is divergent

UV div. removed by α_S renormalization

IR div. described by Catani's formula, removed by real corrections

$$A_{3L} = \mathbf{I}_g^{(1)} A_{2L} + A_{3L}^{\text{fin}}$$

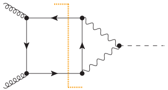
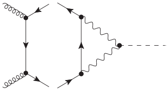
$$\mathbf{I}_g^{(1)} = \left(-\frac{s}{\mu^2}\right)^{-\epsilon} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \left[-\frac{C_A}{\epsilon^2} - \frac{\beta_0}{\epsilon}\right]$$

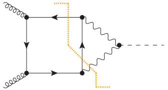
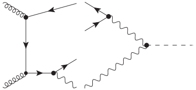
$s = \mu = m_H = 125.09\,\text{GeV},\ m_W = 80.385\,\text{GeV},\ m_Z = 91.1876\,\text{GeV},\ N_C = 3,\ N_f = 5$

$$\begin{aligned}
A_{2L}(m_Z^2/m_H^2, 1) &= -6.880846 \; -i\,0.5784119 \\
A_{2L}(m_W^2/m_H^2, 1) &= -10.71693 \; -i\,2.302953 \\
A_{3L}^{\text{fin}}(m_Z^2/m_H^2, 1) &= -2.975801 \; -i\,41.19509 \\
A_{3L}^{\text{fin}}(m_W^2/m_H^2, 1) &= -11.31557 \; -i\,54.02989
\end{aligned}$$

Analysis of the imaginary part

Diagram level Imaginary parts: on-shell intermediate particles

 $\Rightarrow$  $\Rightarrow$ $s > 0$

 $\Rightarrow$  $\Rightarrow$ $s > m^2$

Analysis of the imaginary part

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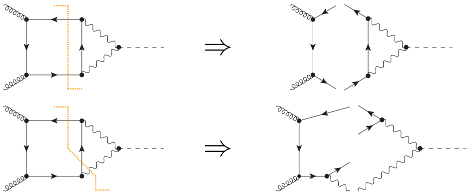
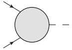


$\Rightarrow \quad s > 0$

$\Rightarrow \quad s > m^2$

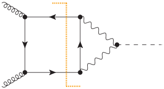
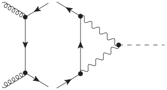
Amplitude level Not all intermediate configurations are allowed

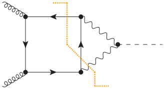
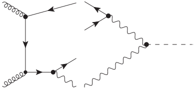
H cannot couple to massless fermions

 $= 0$

Analysis of the imaginary part

Diagram level Imaginary parts: on-shell intermediate particles

 $\Rightarrow$  $\Rightarrow$ $s > 0$

 $\Rightarrow$  $\Rightarrow$ $s > m^2$

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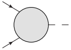
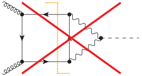
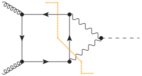
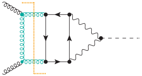
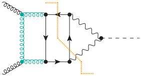
H cannot couple to massless fermions  $= 0$

$s = 0$ $s = m^2$

2 loops   $\Rightarrow$ $-\mathrm{i}\,2.302953$

3 loops   $\Rightarrow$ $-\mathrm{i}\,54.02989$

Soft-gluon limit

[Catani. . . ,2001][de Florian. . . ,2012][Forte. . . ,2013]

Real emissions: challenging problem

Soft-gluon limit

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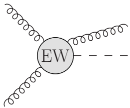
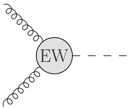
Real emissions: challenging problem

$gg \to H$ PDFs suppress extra gluon with large momentum

Soft limit

$\left|$  $\right|^2$

$$\underbrace{=}_{E_g \to 0} \frac{\alpha_S}{4\pi} C_A \frac{2\, p_1 \cdot p_2}{p_1 \cdot p_4 \; p_2 \cdot p_4}$$

$\left|$  $\right|^2 + O\left(p_4^{-1}\right)$

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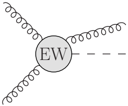
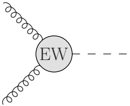


$$\left| \cdots \right|^2 \underbrace{=}_{E_g \to 0} \frac{\alpha_S}{4\pi} C_A \frac{2\, p_1 \cdot p_2}{p_1 \cdot p_4\, p_2 \cdot p_4} \left| \cdots \right|^2 + O\left(p_4^{-1}\right)$$



$$\sigma = \int_0^1 \int_0^1 f(x_1,\mu)\, f(x_2,\mu)\, \sigma_{\mathrm{LO}}\, z\, G(z,\mu,\alpha_S)\, \mathrm{d}x_2 \mathrm{d}x_1$$

- $z := m_H^2/(S_{\mathrm{h}} x_1 x_2)$, $\quad gg \to H$ energy
- $G = \delta(1-z) + \frac{\alpha_S}{2\pi}\left[8C_A\left(\mathcal{D}_1 + \frac{\mathcal{D}_0}{2}\log\frac{m_H^2}{\mu^2}\right) + \left(\frac{2\pi^2}{3}C_A + \frac{\sigma_{\mathbf{NLO}}^{\mathbf{fin}}}{\sigma_{\mathbf{LO}}}\right)\delta(1-z)\right]$
- $\mathcal{D}_0 = \left[\frac{1}{1-z}\right]_+$, $\quad \mathcal{D}_1 = \left[\frac{\log(1-z)}{1-z}\right]_+ + (2-3z+2z^2)\frac{\log[(1-z)/\sqrt{z}]}{1-z} - \frac{\log(1-z)}{1-z}$

Numerical values for the cross-section

QCD vs. QCD-EW

$$\sigma_{\text{LO}}^{\text{QCD}} = 20.6\,\text{pb} \qquad \sigma_{\text{LO}}^{\text{QCD-EW}} = 21.7\,\text{pb} \quad \Rightarrow \quad +5.3\% \text{ at LO}$$

$$\sigma_{\text{NLO}}^{\text{QCD}} = 32.7\,\text{pb} \qquad \sigma_{\text{NLO}}^{\text{QCD-EW}} = 34.4\,\text{pb} \quad \Rightarrow \quad +5.2\% \text{ at NLO}$$

NNPDF30 for PDFs and running of $\alpha_S(\mu)$

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NNPDF30 for PDFs and running of $\alpha_S(\mu)$

Robust result

- PDFs suppress large energy for the extra gluon
- Internal consistency check: standard and improved $\mathcal{D}_1$ give the same increase between σ_{LO} and σ_{NLO}

Conclusions

- NLO virtual corrections to QCD-EW $gg \to H$ have been evaluated
- In soft-gluon limit

$$\sigma^{\text{QCD-EW}}_{\text{NLO}} = \sigma^{\text{QCD}}_{\text{NLO}} \left(1 + 5.2\%\right)$$

Theoretical uncertainties now

δ_{scale}	$\delta_{\text{PDF-TH}}$	$\delta_{\text{QCD-EW}}$		$\delta_{t,\ b,\ c}$	δ_{1/m_t}
$\sim 2\%$	1.16%	0.7%	$\mu \in [m_H/4, m_H]$	0.83%	1%

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- NLO virtual corrections to QCD-EW $gg \to H$ have been evaluated
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- Next step: determination of the real corrections

$$gg \to Hg \qquad qg \to Hq \qquad \bar{q}g \to H\bar{q} \qquad q\bar{q} \to Hg$$

  - Necessary ingredient for further improvements (no soft-gluon approximation)
  - Interesting problem both for physics and mathematics

Thank you for your attention

Karlsruher Institut für Technologie

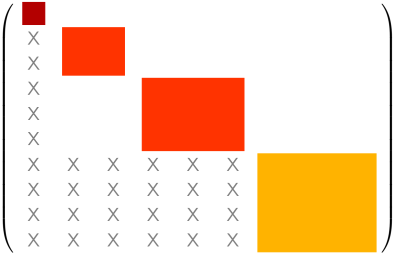
BACKUP SLIDES

The plan for 3-loop NLO QCD-EW

DEs have a natural block-triangular form

- Off-diagonal terms related to subtopologies
- MIs with fewer denominators have simpler equations
- Blocks correspond to topologies requiring more than 1 MIs (up to 4)
- Different topologies with same number of denominators are not related

$$\frac{\mathrm{d}\mathbf{J}(y,\epsilon)}{\mathrm{d}y} =$$



$$\mathbf{J}(y,\epsilon)$$

The plan for 3-loop NLO QCD-EW

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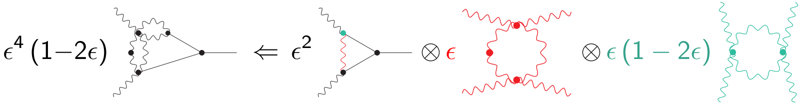
Bottom-up approach

Put into a UT form MIs having few denominators and off-diagonal terms, then modify one by one higher topologies

A two-steps approach

[Argeri. . . ,2014][Gehrmann. . . ,2014] [Lee,2014][Primo. . . ,2016][Gituliar. . . ,2017][Frellesvig. . . ,2017][Meyer,2017]

1. Study of $\mathbf{J}(y,\epsilon)$ $\qquad A(y,\epsilon) \rightsquigarrow A_0(y) + \epsilon A_1(y)\,[+\ldots]$
   - Building blocks



   - Maximal cut
     - All possible propagators are put on-shell
     - DEs: all terms not featuring cut propagators are put to 0
     - Requiring MIs with $\mathrm{d}\log$-form in all remaining integration variables

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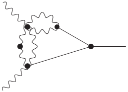
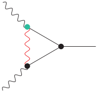
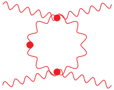
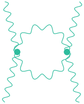
$\epsilon^4\,(1-2\epsilon)$  $\Leftarrow \epsilon^2$  $\otimes\, \epsilon$  $\otimes\, \epsilon\,(1-2\epsilon)$ 

   - Maximal cut
     - All possible propagators are put on-shell
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     - Requiring MIs with $\mathrm{d}\log$-form in all remaining integration variables
2. Study of $A(y,\epsilon)$ $\quad A_0(y) + \epsilon A_1(y)\,[+\ldots] \rightsquigarrow \epsilon\, B_a\, \mathrm{d}_y \log R_a(y)$
   - Integrating away of $A_0(y)$

     Fuchsian structure can be spoiled: logs in $A_1(y)$
   - Algebraic techniques: `Fuchsia` & `CANONICA`

     BC can become non-UT: rescaling of lower UT MIs by ϵ-polynomials