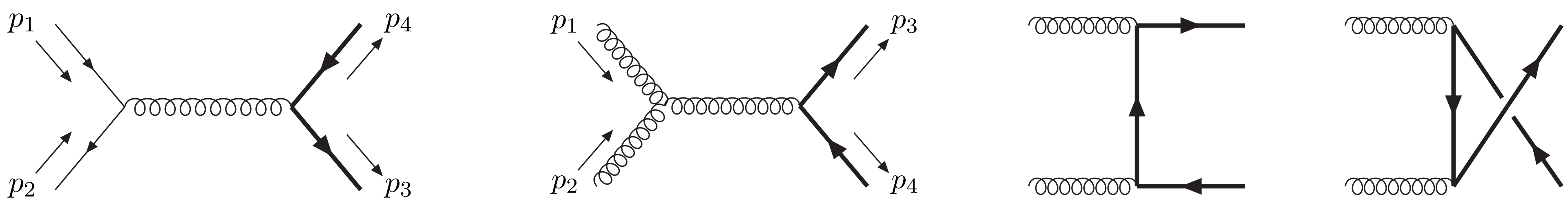
Soft gluons and top quark pair production

Li Lin Yang
Peking University

Based on: 1804.05218 (with Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang)
1803.07623 (with Wang, Xu, Zhu)

LOOPS AND LEGS IN QUANTUM FIELD THEORY
14th DESY Workshop on Elementary Particle Theory,
St. Goar, Germany, April 29 – May 04, 2018

Top quark pair production



A standard candle for the LHC and future colliders

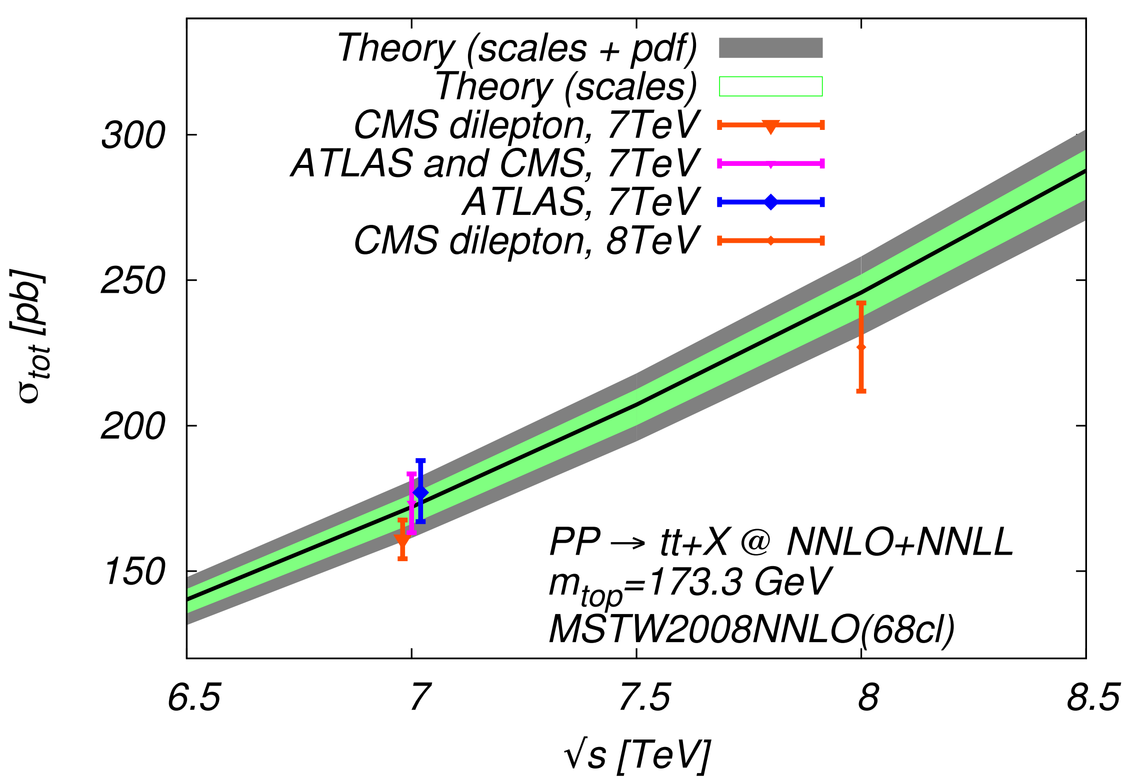
* Top quark related to many important issues: hierarchy problem, vacuum stability, origin of fermion masses, ...
* Major background to many searches

NNLO QCD for top pair

A milestone in precision calculations

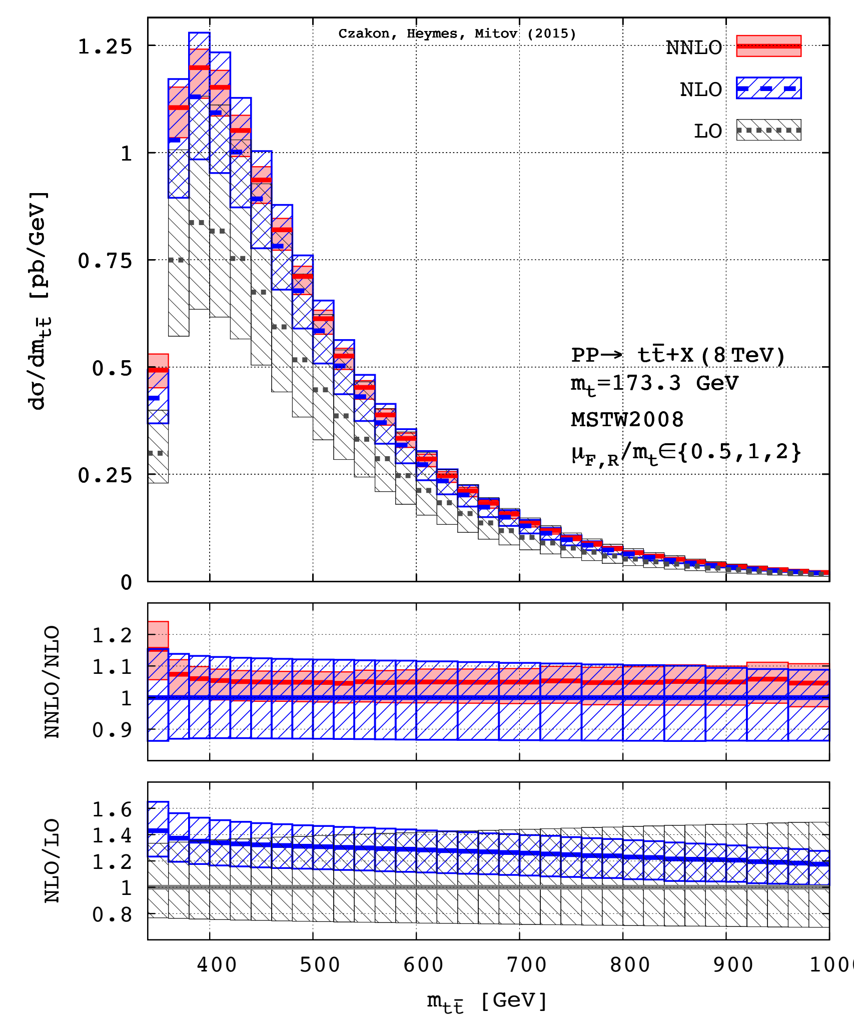
Total cross section

Baernreuther, Czakon, Mitov: 1204.5201;
Czakon, Fiedler, Mitov: 1303.6254



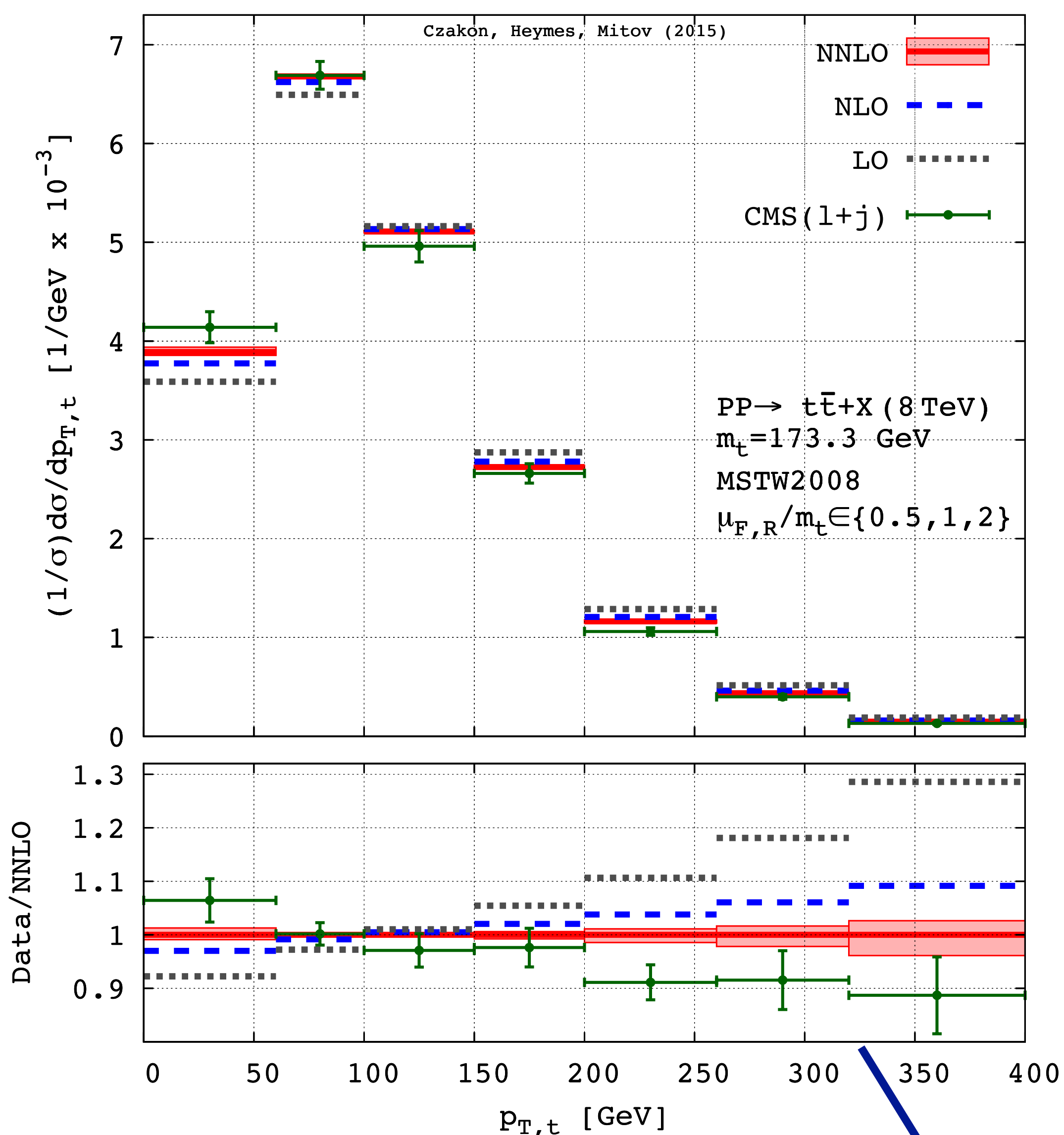
Differential distributions

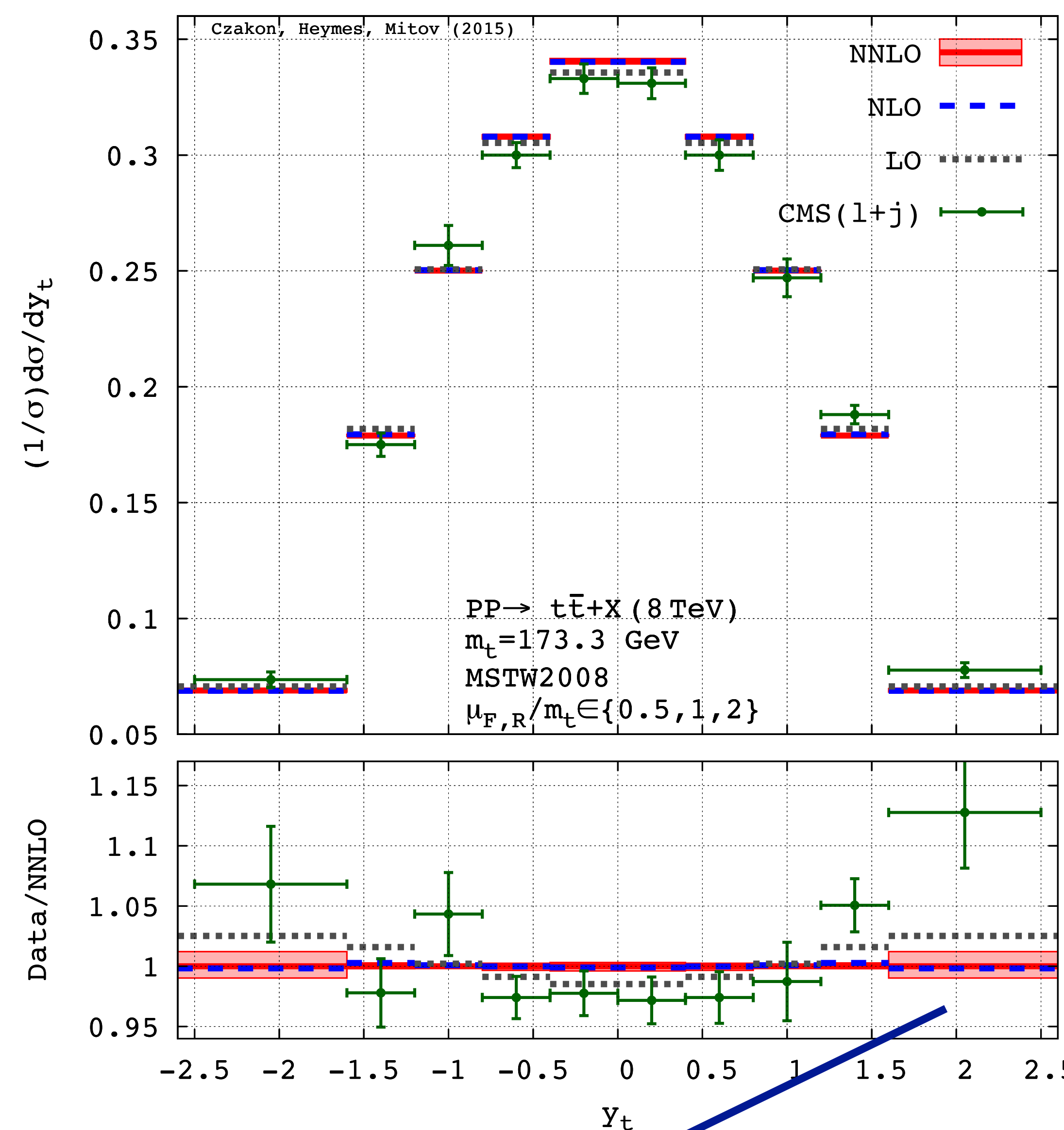
Czakon, Heymes, Mitov: 1511.00549



Differential distributions

Czakon, Heymes, Mitov: 1511.00549





Some tension at high energy (boosted kinematics)

NNLO with dynamic scale

Czakon, Heymes, Mitov: 1606.03350

The problem here: it's a multiple-scale process with complicated kinematics!

$$\mu_0 \sim m_t \,,$$

$$\mu_0 \sim m_T = \sqrt{m_t^2 + p_T^2} \,,$$

$$\mu_0 \sim H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} \,,$$

$$\mu_0 \sim H_T' = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} + \sum_i p_{T,i} \,,$$

$$\mu_0 \sim E_T = \sqrt{\sqrt{m_t^2 + p_{T,t}^2}\sqrt{m_t^2 + p_{T,\bar{t}}^2}} \,,$$

$$\mu_0 \sim H_{T,\text{int}} = \sqrt{(m_t/2)^2 + p_{T,t}^2} + \sqrt{(m_t/2)^2 + p_{T,\bar{t}}^2} \,,$$

$$\mu_0 \sim m_{t\bar{t}} \,,$$

NNLO with dynamic scale

Czakon, Heymes, Mitov: 1606.03350

The problem here: it's a multiple-scale process with complicated kinematics!

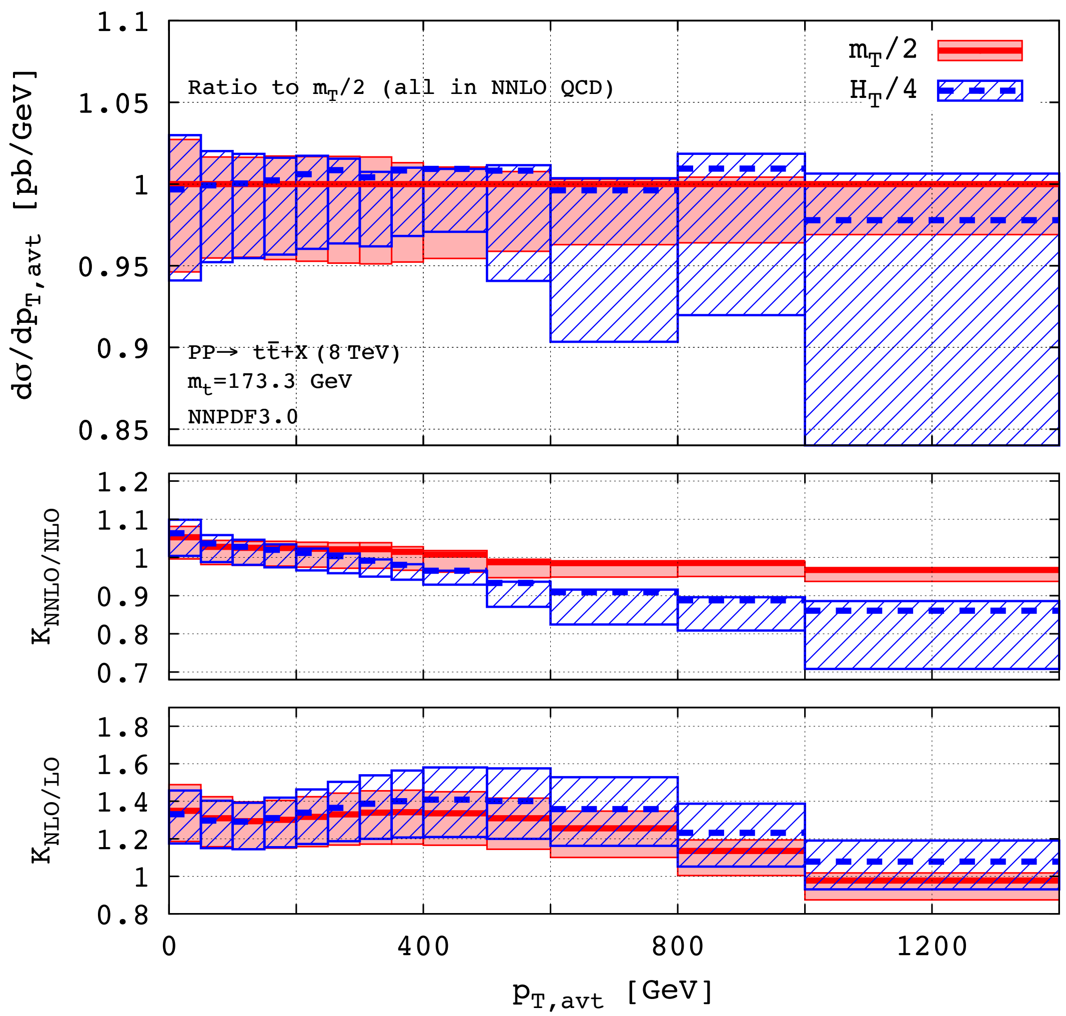
$$\mu_0 \sim m_t \,,$$
$$\mu_0 \sim m_T = \sqrt{m_t^2 + p_T^2} \,,$$
$$\mu_0 \sim H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} \,,$$
$$\mu_0 \sim H_T' = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} + \sum_i p_{T,i} \,,$$
$$\mu_0 \sim E_T = \sqrt{\sqrt{m_t^2 + p_{T,t}^2}\sqrt{m_t^2 + p_{T,\bar{t}}^2}} \,,$$
$$\mu_0 \sim H_{T,\mathrm{int}} = \sqrt{(m_t/2)^2 + p_{T,t}^2} + \sqrt{(m_t/2)^2 + p_{T,\bar{t}}^2} \,,$$
$$\mu_0 \sim m_{t\bar{t}} \,,$$

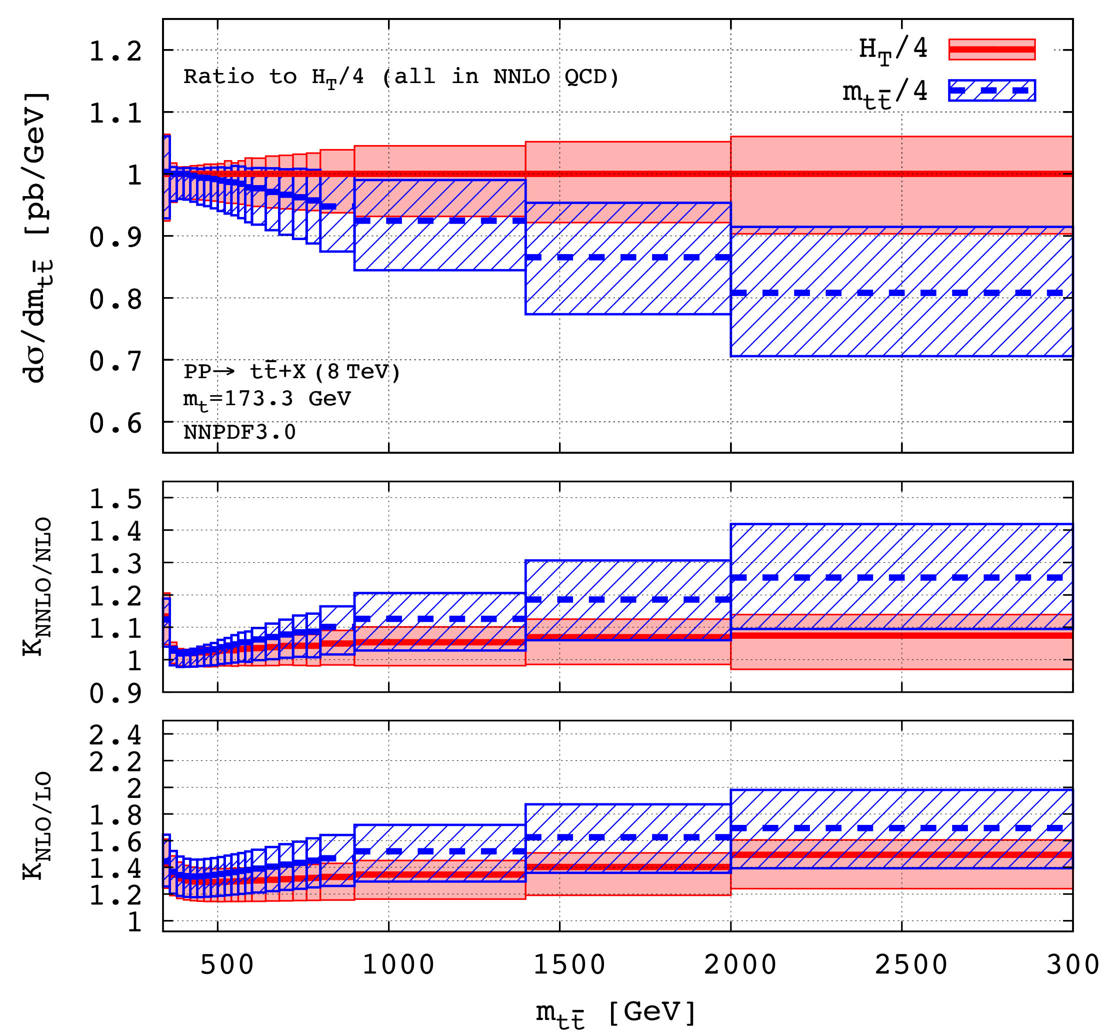
$$\mu_0 = \begin{cases} \frac{m_T}{2} & \text{for}: \quad p_{T,t},\ p_{T,\bar{t}} \text{ and } p_{T,t/\bar{t}}\,, \\ \frac{H_T}{4} & \text{for}: \quad \text{all other distributions}\,. \end{cases}$$

Determine optimal "scale scheme" by minimizing higher order corrections

NNLO with dynamic scale

Czakon, Heymes, Mitov: 1606.03350

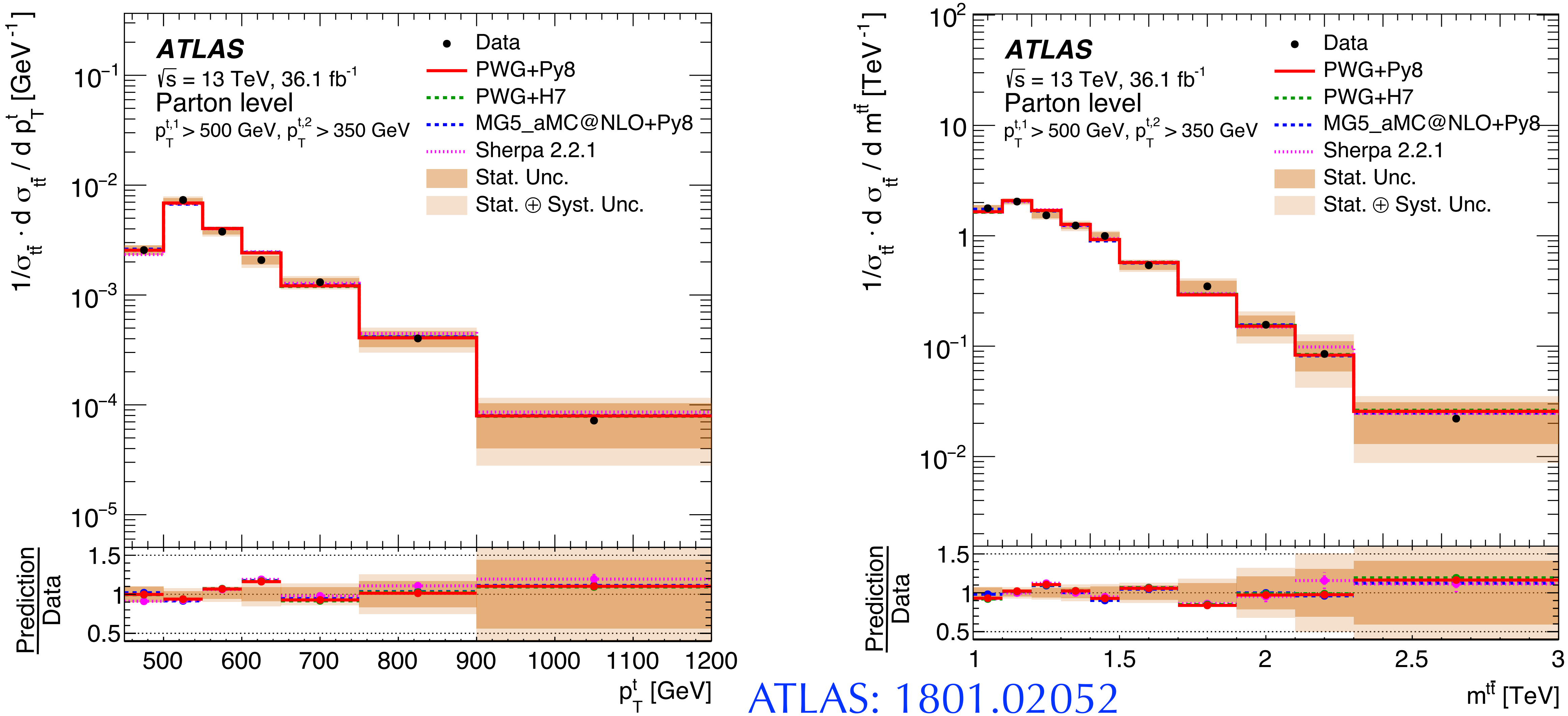




Vastly different behaviors with different scheme choices (especially in the boosted region)

Boosted top quarks

Sensitive to new physics, interesting in its own right!



ATLAS: 1801.02052

Actively being probed by LHC experiments

Producing boosted tops

Hard extra emissions suppressed

↓

soft gluons

↓

$$\ln \frac{\hat{s} - M_{t\bar{t}}^2}{M_{t\bar{t}}^2}$$

Producing boosted tops

Hard extra emissions suppressed

↓

soft gluons

↓

$$\ln \frac{\hat{s} - M_{t\bar{t}}^2}{M_{t\bar{t}}^2}$$

Top quark nearly massless

↓

quasi-collinear gluons

↓

$$\ln \frac{m_t^2}{M_{t\bar{t}}^2}$$

Producing boosted tops

Hard extra emissions suppressed

↓

soft gluons

↓

$$\ln \frac{\hat{s} - M_{t\bar{t}}^2}{M_{t\bar{t}}^2}$$

Top quark nearly massless

↓

quasi-collinear gluons

↓

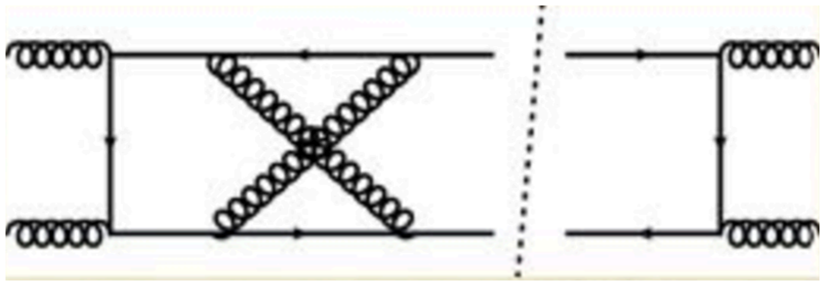
$$\ln \frac{m_t^2}{M_{t\bar{t}}^2}$$

Need to resum both!

Ferroglia, Pecjak, **LLY**: 1205.3662

Soft gluon resummation

Hard function



Kidonakis, Sterman: hep-ph/9705234

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

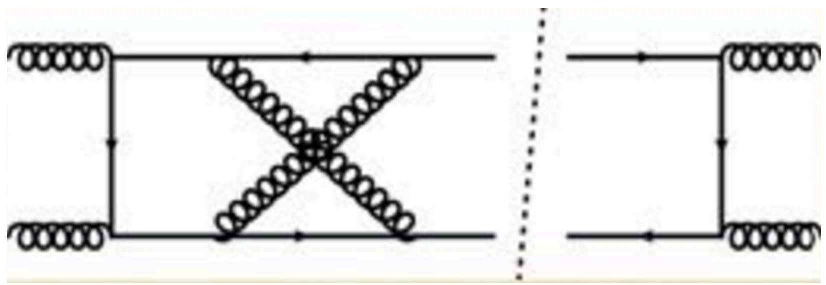
Evolving from the scale of hard scatterings

Soft gluon resummation

Hard function

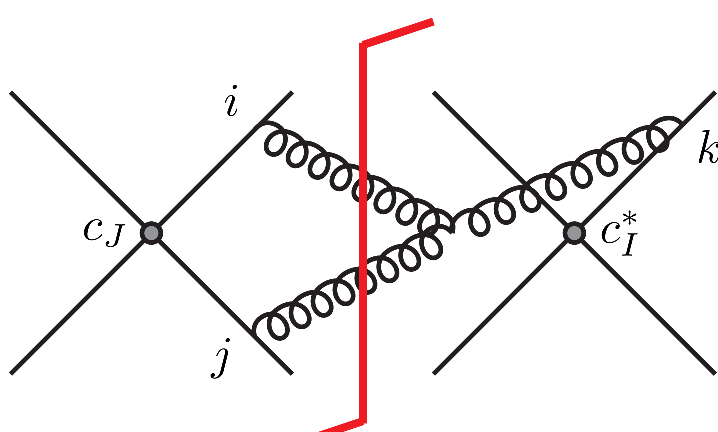
Kidonakis, Sterman: hep-ph/9705234

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827



Evolving from the scale of hard scatterings

to the scale of soft interactions



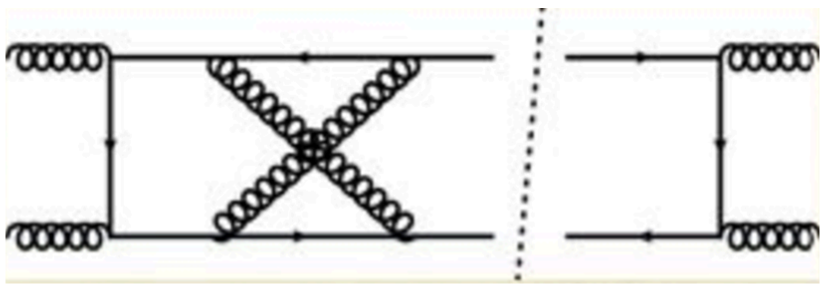
Soft function

Soft gluon resummation

Hard function

Kidonakis, Sterman: hep-ph/9705234

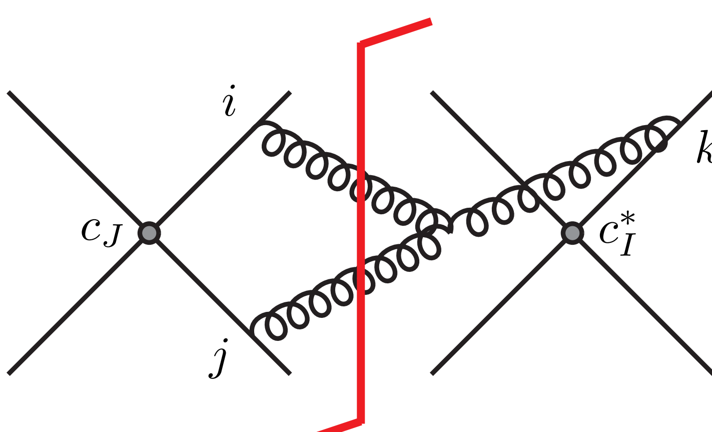
Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827



Evolving from the scale of hard scatterings

Governed by IR structure

to the scale of soft interactions



Soft function

IR anomalous dimension

Becher, Neubert: 0904.1021

Ferroglia, Neubert, Pecjak, **LLY**: 0907.4791; 0908.3676

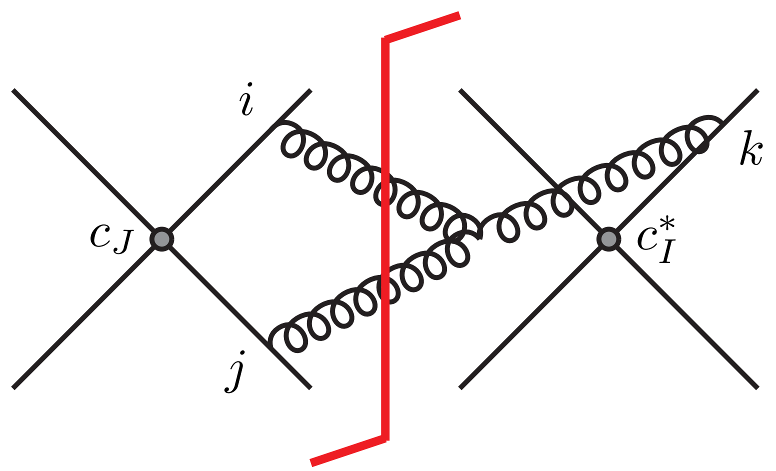
$$\begin{aligned}\boldsymbol{\Gamma} = &\sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) \\ &- \sum_{(I,J)} \frac{\boldsymbol{T}_I \cdot \boldsymbol{T}_J}{2}\, \gamma_{\rm cusp}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s) \\ &+ \sum_{I,j} \boldsymbol{T}_I \cdot \boldsymbol{T}_j\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{m_I \mu}{-s_{Ij}} \qquad (2 \\ &+ \sum_{(I,J,K)} i f^{abc}\, \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_K^c\, F_1(\beta_{IJ},\beta_{JK},\beta_{KI}) \\ &+ \sum_{(I,J)} \sum_k i f^{abc}\, \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_k^c\, f_2\Big(\beta_{IJ}, \ln\frac{-\sigma_{Jk}\, v_J \cdot p_k}{-\sigma_{Ik}\, v_I \cdot p_k}\Big)\end{aligned}$$

$$F_1(\beta_{12},\beta_{23},\beta_{31}) = \frac{\alpha_s^2}{12\pi^2} \sum_{i,j,k} \epsilon_{ijk}\, g(\beta_{ij})\, r(\beta_{ki})$$

$$\begin{aligned} r(\beta) &= \beta \coth\beta\,, \\ g(\beta) &= \coth\beta \left[\beta^2 + 2\beta \ln(1-e^{-2\beta}) - \mathrm{Li}_2(e^{-2\beta}) + \frac{\pi^2}{6}\right] \\ &\quad - \beta^2 - \frac{\pi^2}{6}\,. \qquad (5) \end{aligned}$$

3-parton correlations

The soft function



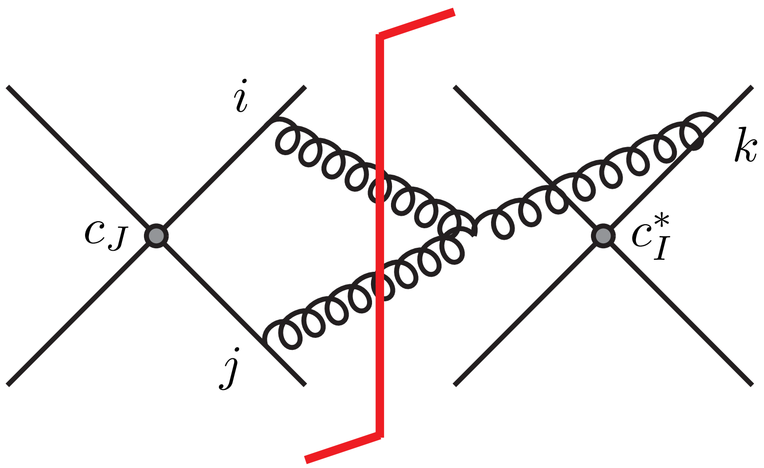
Known at NLO

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Known at NNLO in the massless limit (except an off-diagonal 3-parton piece)

Ferroglia, Pecjak, **LLY**: 1207.4798

The soft function



Known at NLO

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

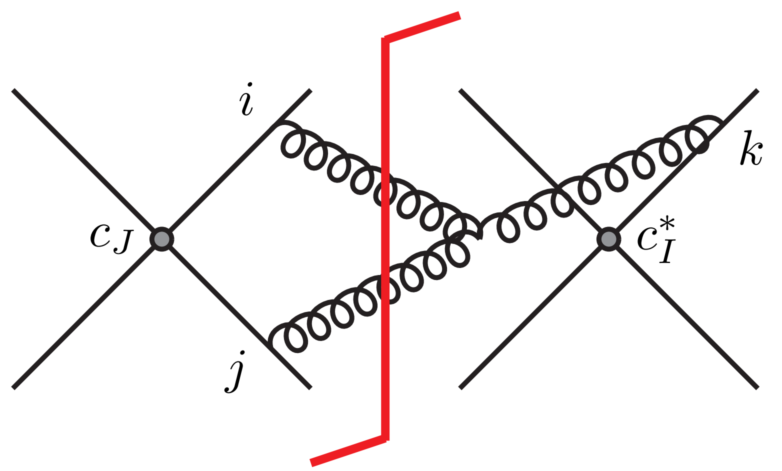
Known at NNLO in the massless limit (except an off-diagonal 3-parton piece)

Ferroglia, Pecjak, **LLY**: 1207.4798

Recent calculation at NNLO with massive tops

Wang, Xu, **LLY**, Zhu: 1804.05218

The soft function



Known at NLO — Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Known at NNLO in the massless limit (except an off-diagonal 3-parton piece)

Ferroglia, Pecjak, **LLY**: 1207.4798

Recent calculation at NNLO with massive tops

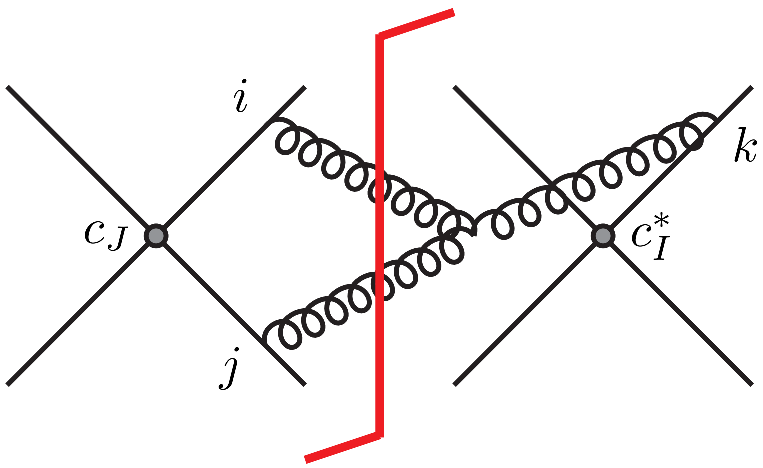
Wang, Xu, **LLY**, Zhu: 1804.05218

Standard techniques → Inverse unitarity

Standard techniques → IBP identities

Standard techniques → Differential equations

The soft function



Known at NLO

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Known at NNLO in the massless limit (except an off-diagonal 3-parton piece)

Ferroglia, Pecjak, **LLY**: 1207.4798

Recent calculation at NNLO with massive tops

Wang, Xu, **LLY**, Zhu: 1804.05218

Standard techniques → Inverse unitarity

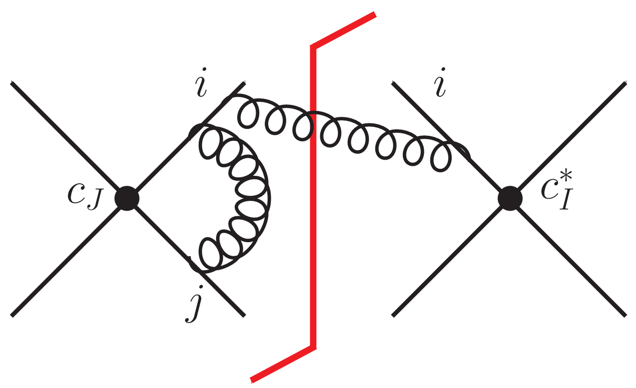
Standard techniques → IBP identities

Standard techniques → Differential equations

Non-trivial: boundary conditions

The boundary conditions

We choose the boundary to be $\beta \equiv \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}}} \to 0$



Some virtual-real integrals develop Coulomb/Glauber-type singularities in this limit

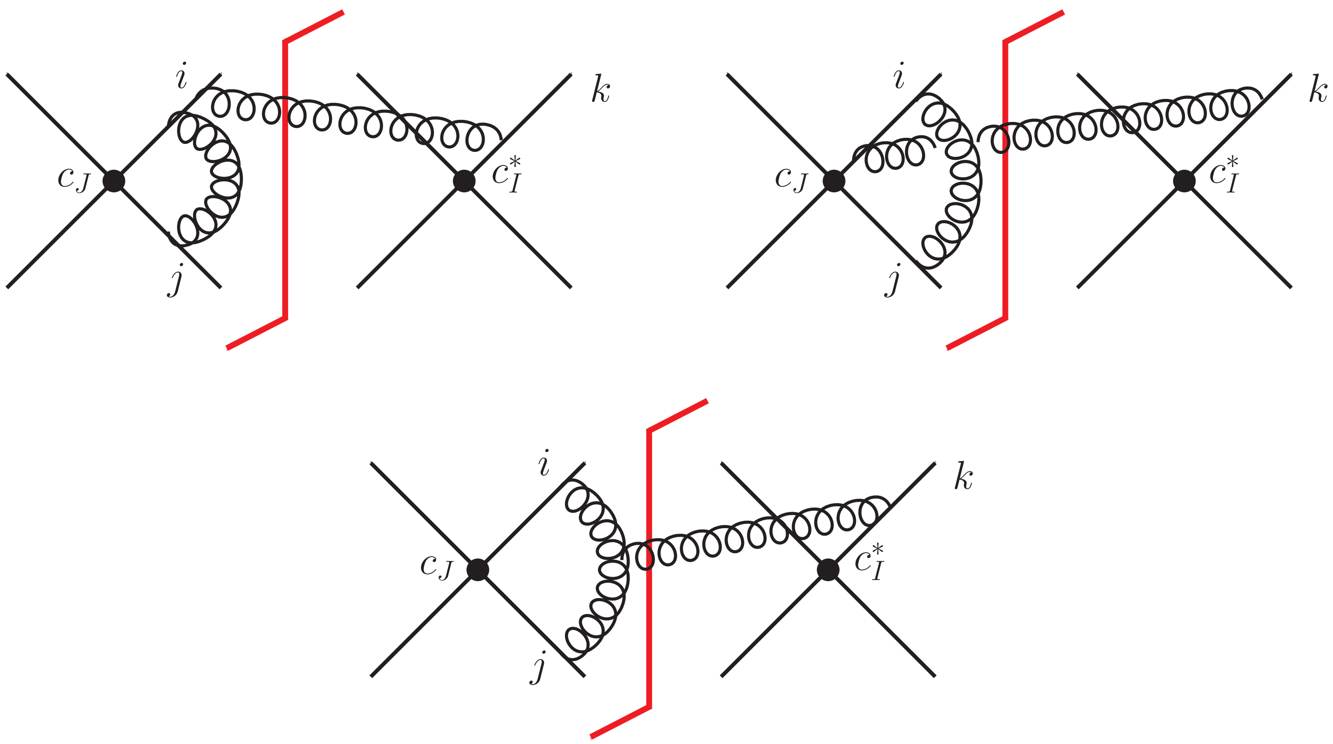
Carefully extract the asymptotic behavior, e.g.

$$g_6^{(4)}(\epsilon, \beta \to 0, y) \approx \frac{(e^{-2i\pi\epsilon} - 1)\, \beta^{2\epsilon}\, \Gamma(1-2\epsilon)\, \Gamma(1+\epsilon)}{4^{1-2\epsilon}\, \Gamma(1-\epsilon)}$$

Note: virtual integrals available in

Bierenbaum, Czakon, Mitov: 1107.4384
Czakon, Mitov: 1804.02069

The 3-parton terms



$$i f^{abc} \boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c$$

- Off-diagonal purely-imaginary contributions
- Do not enter the NNLO cross section
- Not calculated in the massless case

Ferroglia, Pecjak, **LLY**: 1207.4798

A piece of final result

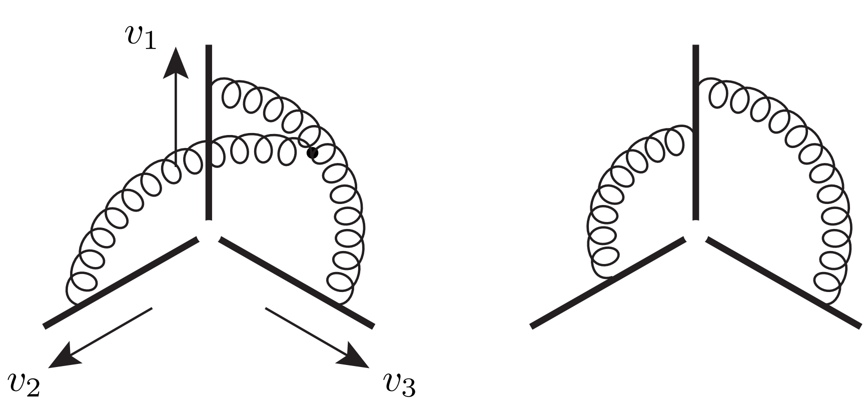
$$
\begin{aligned}
\tilde{\boldsymbol{s}}_{22}^{q\bar{q},(2)}(0,\beta,y)\Big|_{T_F N_l} &= \frac{16(7\beta^2-126\beta+127)}{243\beta}G_1 + \frac{8(5\beta^2+90\beta+53)}{81\beta}\big(G_{-1,-1}-G_{-1,1}-2G_{0,-1}\big) \\
&- \frac{16(7\beta^2+126\beta+127)}{243\beta}G_{-1} + \frac{8(5\beta^2-90\beta+53)}{81\beta}\big(G_{1,-1}-G_{1,1}+2G_{0,1}\big) \\
&+ \frac{8(\beta^2+18\beta+1)}{27\beta}\big(-G_{-1,-1,-1}+G_{-1,-1,1}+2G_{-1,0,-1}-2G_{-1,0,1}-G_{-1,1,-1}+G_{-1,1,1} \\
&+ 2G_{0,-1,-1}-2G_{0,-1,1}-4G_{0,0,-1}\big) + \frac{8(\beta^2-18\beta+1)}{27\beta}\big(4G_{0,0,1}+2G_{0,1,-1}-2G_{0,1,1} \\
&- G_{1,-1,-1}+G_{1,-1,1}+2G_{1,0,-1}-2G_{1,0,1}-G_{1,1,-1}+G_{1,1,1}\big) \\
&+ \frac{32}{243}\Big[28G_{-1/y}+98G_{1/y}+30\big(2G_{0,-1/y}+G_{-1/y,-1}+G_{-1/y,1}-2G_{-1/y,-1/y}\big) \\
&+ 105\big(2G_{0,1/y}+G_{1/y,-1}+G_{1/y,1}-2G_{1/y,1/y}\big)+18\big(4G_{0,0,-1/y}+2G_{0,-1/y,-1}+2G_{0,-1/y,1} \\
&- 4G_{0,-1/y,-1/y}-G_{-1/y,-1,-1}+G_{-1/y,-1,1}+2G_{-1/y,0,-1}+2G_{-1/y,0,1}-4G_{-1/y,0,-1/y} \\
&+ G_{-1/y,1,-1}-G_{-1/y,1,1}-2G_{-1/y,-1/y,-1}-2G_{-1/y,-1/y,1}+4G_{-1/y,-1/y,-1/y}\big) \\
&+ 63\big(4G_{0,0,1/y}+2G_{0,1/y,-1}+2G_{0,1/y,1}-4G_{0,1/y,1/y}-G_{1/y,-1,-1}+G_{1/y,-1,1}+2G_{1/y,0,-1} \\
&+ 2G_{1/y,0,1}-4G_{1/y,0,1/y}+G_{1/y,1,-1}-G_{1/y,1,1}-2G_{1/y,1/y,-1}-2G_{1/y,1/y,1}+4G_{1/y,1/y,1/y}\big) \\
&- \frac{332}{3}-\frac{5\pi^2}{2}+6\zeta_3\Big], \qquad (84)
\end{aligned}
$$

It is remarkable that all the results can be written in terms of multiple polylogarithms

Validation: IR structure

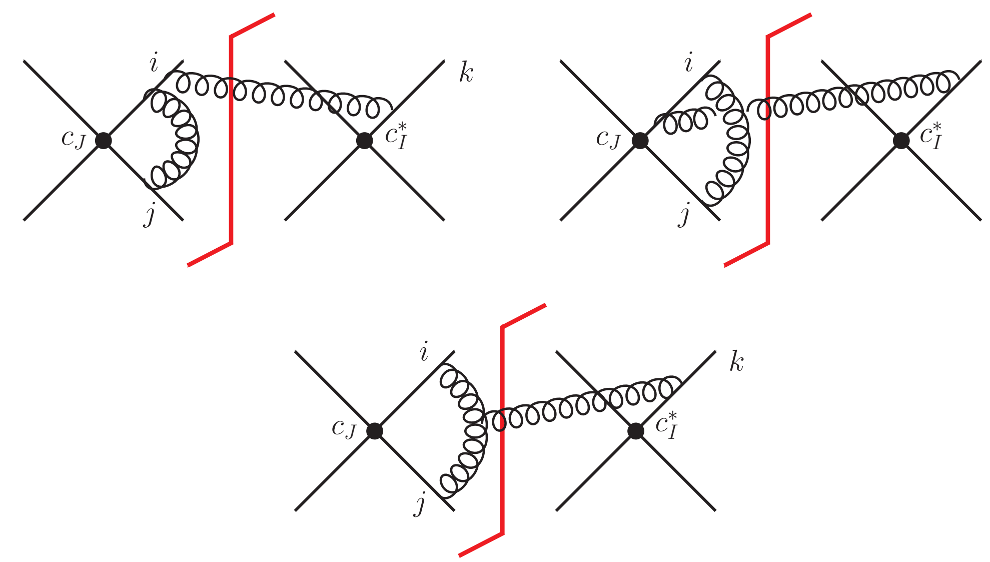
The soft divergence generated from real emissions should be the same as the virtual amplitude! (required by KLN theorem)

virtual



$$+\sum_{(I,J)}\sum_{k} i f^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_k^c\,f_2\Big(\beta_{IJ},\ln\frac{-\sigma_{Jk}\,v_J\cdot p_k}{-\sigma_{Ik}\,v_I\cdot p_k}\Big)$$

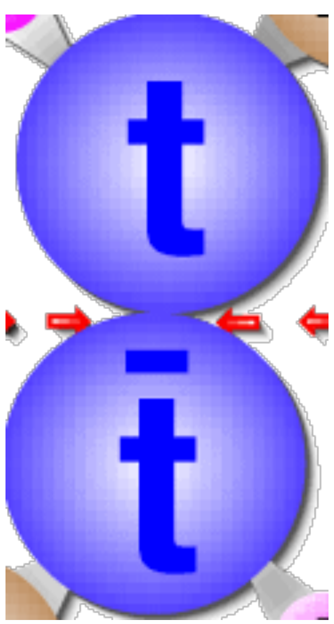
real



3-parton correlations: non-trivial cross-check!

Validation: threshold limit

It is interesting to check the threshold limit where the top quarks are produced at rest



Color singlet: same as Drell-Yan and Higgs production

Belitsky: hep-ph/9808389

Color octet Czakon, Fiedler: 1311.2541

Note: singlet-octet mixing terms do NOT vanish in the threshold limit!

Validation: boosted limit

In the limit where the top quarks are highly boosted

↓

Factorization Ferroglia, Pecjak, **LLY**: 1205.3662

$$\boldsymbol{S}_{\rm massive}(s,t,m_t,N) \to \boldsymbol{S}_{\rm massless}(s,t,N)\, S_D^2(m_t/N)$$

Validation: boosted limit

In the limit where the top quarks are highly boosted

Factorization Ferroglia, Pecjak, **LLY**: 1205.3662

$$\boldsymbol{S}_{\mathrm{massive}}(s,t,m_t,N) \to \boldsymbol{S}_{\mathrm{massless}}(s,t,N)\, S_D^2(m_t/N)$$

Ferroglia, Pecjak, **LLY**: 1207.4798

Also obtain the missing 3-parton piece for free

Validation: boosted limit

In the limit where the top quarks are highly boosted

Factorization Ferroglia, Pecjak, **LLY**: 1205.3662

$$\boldsymbol{S}_{\rm massive}(s,t,m_t,N) \to \boldsymbol{S}_{\rm massless}(s,t,N)\, S_D^2(m_t/N)$$

Ferroglia, Pecjak, **LLY**: 1207.4798

Also obtain the missing 3-parton piece for free

Allows to extract the soft fragmentation function

Soft and small-mass factorization

Ferroglia, Pecjak, **LLY**: 1205.3662

In Mellin space: $Q \sim \sqrt{s}, \sqrt{-t} \gg Q/N \gg m_t \gg m_t/N$

$$\hat{\sigma}(N, \mu_f) \sim \mathrm{Tr}\big[\boldsymbol{H}(L_h, \mu_f)\, \boldsymbol{S}(L_s, \mu_f)\big] C_D^2(L_c, \mu_f)\, S_D^2(L_{sc}, \mu_f)$$

Soft and small-mass factorization

Ferroglia, Pecjak, **LLY**: 1205.3662

In Mellin space: $Q \sim \sqrt{s}, \sqrt{-t} \gg Q/N \gg m_t \gg m_t/N$

$$\hat{\sigma}(N,\mu_f) \sim \mathrm{Tr}\big[\boldsymbol{H}(L_h,\mu_f)\,\boldsymbol{S}(L_s,\mu_f)\big]C_D^2(L_c,\mu_f)\,S_D^2(L_{sc},\mu_f)$$

$$\ln \frac{Q^2}{\mu_f^2}$$

hard log

Soft and small-mass factorization

Ferroglia, Pecjak, **LLY**: 1205.3662

In Mellin space: $Q \sim \sqrt{s}, \sqrt{-t} \gg Q/N \gg m_t \gg m_t/N$

$$\hat{\sigma}(N, \mu_f) \sim \mathrm{Tr}\big[\boldsymbol{H}(L_h, \mu_f)\, \boldsymbol{S}(L_s, \mu_f)\big] C_D^2(L_c, \mu_f)\, S_D^2(L_{sc}, \mu_f)$$

$$\ln \frac{Q^2}{\mu_f^2}$$

hard log

$$\ln \frac{Q^2}{\bar{N}^2 \mu_f^2}$$

soft log

Soft and small-mass factorization

Ferroglia, Pecjak, **LLY**: 1205.3662

In Mellin space: $Q \sim \sqrt{s}, \sqrt{-t} \gg Q/N \gg m_t \gg m_t/N$

$$\hat{\sigma}(N, \mu_f) \sim \mathrm{Tr}\big[\boldsymbol{H}(L_h, \mu_f)\, \boldsymbol{S}(L_s, \mu_f)\big] C_D^2(L_c, \mu_f)\, S_D^2(L_{sc}, \mu_f)$$

$\ln \frac{Q^2}{\mu_f^2}$

hard log

$\ln \frac{Q^2}{\bar{N}^2 \mu_f^2}$

soft log

$\ln \frac{m_t^2}{\mu_f^2}$

collinear log
(small-mass)

Soft and small-mass factorization

Ferroglia, Pecjak, **LLY**: 1205.3662

In Mellin space: $Q \sim \sqrt{s}, \sqrt{-t} \gg Q/N \gg m_t \gg m_t/N$

$$\hat{\sigma}(N, \mu_f) \sim \mathrm{Tr}\big[\boldsymbol{H}(L_h, \mu_f)\, \boldsymbol{S}(L_s, \mu_f)\big] C_D^2(L_c, \mu_f)\, S_D^2(L_{sc}, \mu_f)$$

$\ln \frac{Q^2}{\mu_f^2}$

hard log

$\ln \frac{Q^2}{\bar{N}^2 \mu_f^2}$

soft log

$\ln \frac{m_t^2}{\mu_f^2}$

collinear log (small-mass)

$\ln \frac{m_t^2}{\bar{N}^2 \mu_f^2}$

soft-collinear log (emergent)

Consistency of factorizations

There exist 3 factorization formulas involving C_D an S_D

Consistency of factorizations

There exist 3 factorization formulas involving C_D an S_D

$$\boldsymbol{H}_{\rm massive}(s,t,m_t) \to \boldsymbol{H}_{\rm massless}(s,t)\, C_D^2(m_t)$$

Mitov, Moch: hep-ph/0612149

Consistency of factorizations

There exist 3 factorization formulas involving C_D an S_D

$$\boldsymbol{H}_{\text{massive}}(s,t,m_t) \to \boldsymbol{H}_{\text{massless}}(s,t)\, C_D^2(m_t)$$

Mitov, Moch: hep-ph/0612149

$$\boldsymbol{S}_{\text{massive}}(s,t,m_t,N) \to \boldsymbol{S}_{\text{massless}}(s,t,N)\, S_D^2(m_t/N)$$

Ferroglia, Pecjak, **LLY**: 1205.3662
Wang, Xu, **LLY**, Zhu: 1804.05218

Consistency of factorizations

There exist 3 factorization formulas involving C_D an S_D

$$\boldsymbol{H}_{\text{massive}}(s,t,m_t) \to \boldsymbol{H}_{\text{massless}}(s,t)\, C_D^2(m_t)$$

Mitov, Moch: hep-ph/0612149

$$\boldsymbol{S}_{\text{massive}}(s,t,m_t,N) \to \boldsymbol{S}_{\text{massless}}(s,t,N)\, S_D^2(m_t/N)$$

Ferroglia, Pecjak, **LLY**: 1205.3662
Wang, Xu, **LLY**, Zhu: 1804.05218

$$D_{t/t}(m_t,N) \to C_D(m_t)\, S_D(m_t/N)$$

Fragmentation function

Korchemsky, Marchesini: hep-ph/9210281
Cacciari, Catani: hep-ph/0107138
Gardi: hep-ph/0501257
Neubert: 0706.2136

Consistency of factorizations

There exist 3 factorization formulas involving C_D an S_D

$$\boldsymbol{H}_{\text{massive}}(s,t,m_t) \to \boldsymbol{H}_{\text{massless}}(s,t)\, C_D^2(m_t)$$

Mitov, Moch: hep-ph/0612149

$$\boldsymbol{S}_{\text{massive}}(s,t,m_t,N) \to \boldsymbol{S}_{\text{massless}}(s,t,N)\, S_D^2(m_t/N)$$

Ferroglia, Pecjak, **LLY**: 1205.3662
Wang, Xu, **LLY**, Zhu: 1804.05218

$$D_{t/t}(m_t,N) \to C_D(m_t)\, S_D(m_t/N)$$

Fragmentation function

Korchemsky, Marchesini: hep-ph/9210281
Cacciari, Catani: hep-ph/0107138
Gardi: hep-ph/0501257
Neubert: 0706.2136

All consistent at NNLO!

Soft and small-mass resummation

Massless hard function

$$\boldsymbol{H}(L_h, \mu_h \sim Q)$$

Massless soft function

$$\boldsymbol{S}(L_s, \mu_s \sim Q/\bar{N})$$

$$\mu_f$$

RG flow

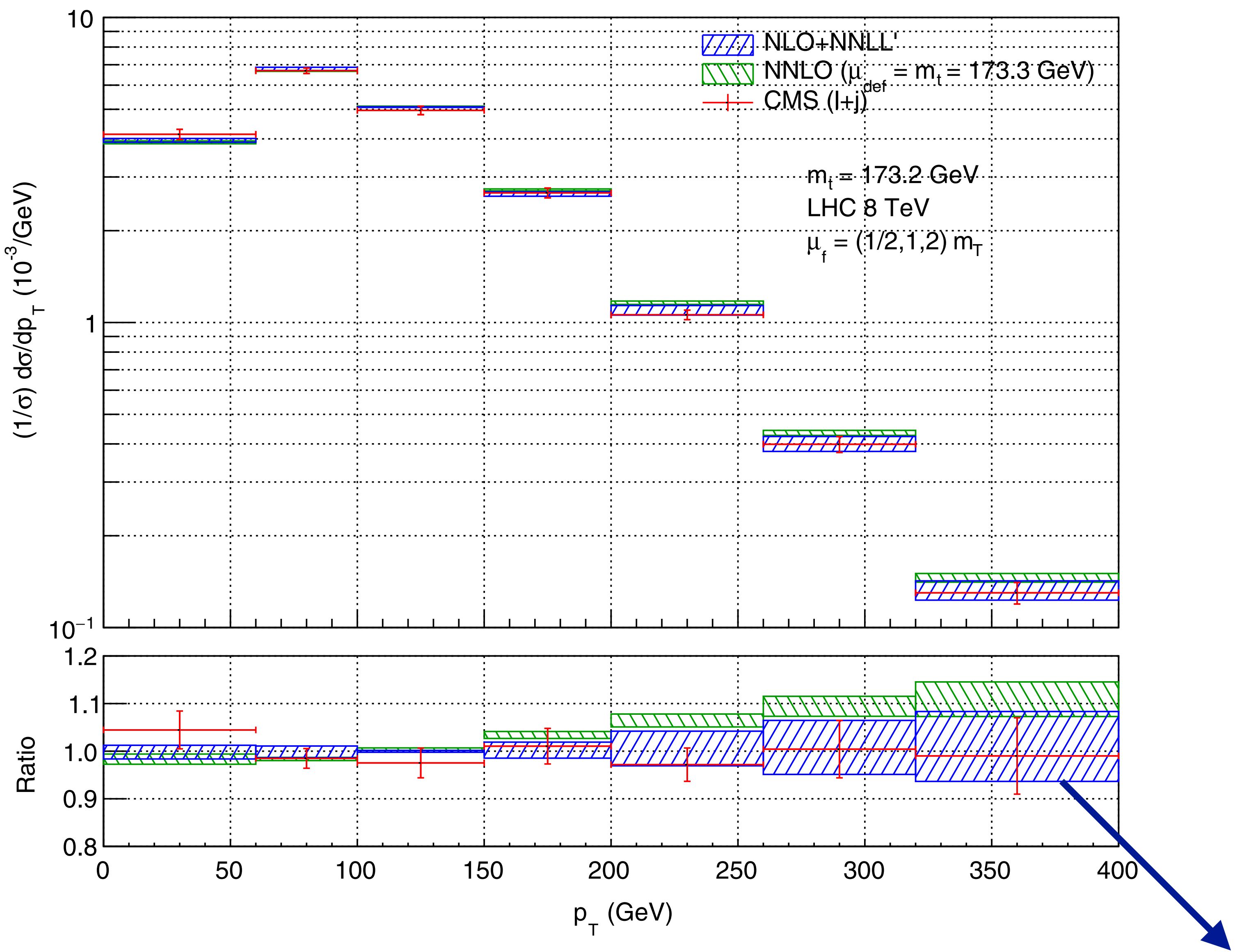
$$C_D(L_c, \mu_c \sim m_t)$$

$$S_D(L_{sc}, \mu_{sc} \sim m_t/\bar{N})$$

All ingredients known at NNLO (for NNLL′ resummation)

NLO+NNLL′

Pecjak, Scott, Wang, **LLY**: 1601.07020



Resummation soften the spectrum

NNLO+NNLL′

A joint effort of the NNLO group and the resummation group

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

$$d\sigma^{\mathrm{(N)NLO+NNLL}'} = d\sigma^{\mathrm{NNLL}'_{b+m}} + \left(d\sigma^{\mathrm{(N)NLO}} - d\sigma^{\mathrm{NNLL}'_{b+m}} \Big|_{\substack{\mathrm{(N)NLO} \\ \mathrm{expansion}}} \right)$$

NNLO+NNLL′

A joint effort of the NNLO group and the resummation group

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

$$d\sigma^{\text{(N)NLO+NNLL}'} = d\sigma^{\text{NNLL}'_{b+m}} + \left(d\sigma^{\text{(N)NLO}} - d\sigma^{\text{NNLL}'_{b+m}} \Big|_{\substack{\text{(N)NLO} \\ \text{expansion}}} \right)$$

$$d\sigma^{\text{NNLL}'_b} + \left(d\sigma^{\text{NNLL}_m} - d\sigma^{\text{NNLL}_m} \Big|_{m_t \to 0} \right)$$

soft & small mass resummation

match to soft resummation

NNLO+NNLL′

A joint effort of the NNLO group and the resummation group

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

$$d\sigma^{\text{(N)NLO+NNLL}'} = d\sigma^{\text{NNLL}'_{b+m}} + \left(d\sigma^{\text{(N)NLO}} - d\sigma^{\text{NNLL}'_{b+m}} \Big|_{\substack{\text{(N)NLO} \\ \text{expansion}}} \right)$$

$$d\sigma^{\text{NNLL}'_b} + \left(d\sigma^{\text{NNLL}_m} - d\sigma^{\text{NNLL}_m} \Big|_{m_t \to 0} \right)$$

match to NNLO

soft & small mass resummation

match to soft resummation

NNLO+NNLL′

A joint effort of the NNLO group and the resummation group

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

$$d\sigma^{\mathrm{(N)NLO+NNLL}'} = d\sigma^{\mathrm{NNLL}'_{b+m}} + \left(d\sigma^{\mathrm{(N)NLO}} - d\sigma^{\mathrm{NNLL}'_{b+m}} \Big|_{\substack{\mathrm{(N)NLO} \\ \mathrm{expansion}}} \right)$$

$$d\sigma^{\mathrm{NNLL}'_b} + \left(d\sigma^{\mathrm{NNLL}_m} - d\sigma^{\mathrm{NNLL}_m} \Big|_{m_t \to 0} \right)$$

match to NNLO

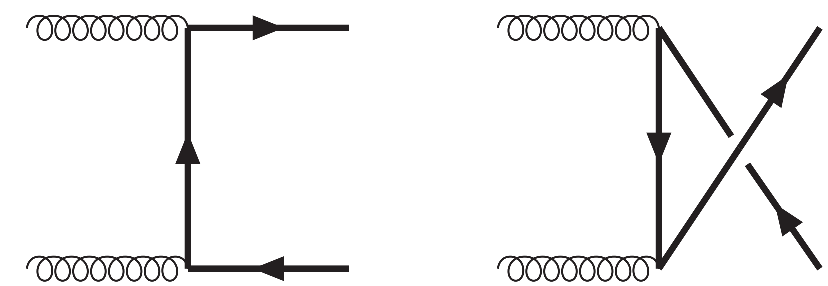
soft & small mass resummation

match to soft resummation

Careful to avoid double-counting!

Scale choices

In the boosted limit, t- and u-channel propagators push the effective hard scale to



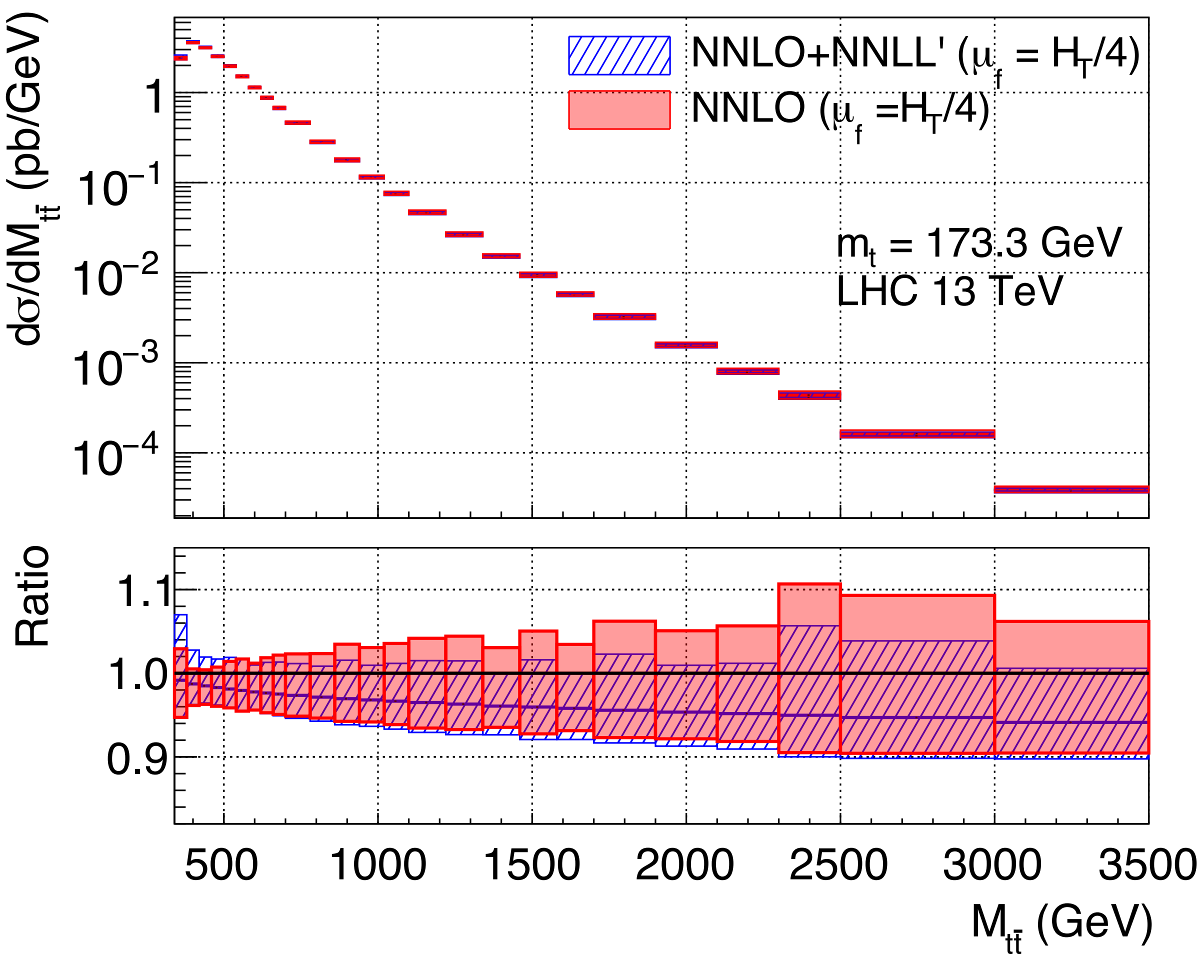
$$-t_1\big|_{m_t\to 0} \approx \frac{M_{t\bar{t}}^2}{2}(1-\cos\theta) + m_t^2\cos\theta \xrightarrow{\cos\theta\to 1} p_T^2 + m_t^2 \equiv m_T^2 = H_T^2/4\,,$$

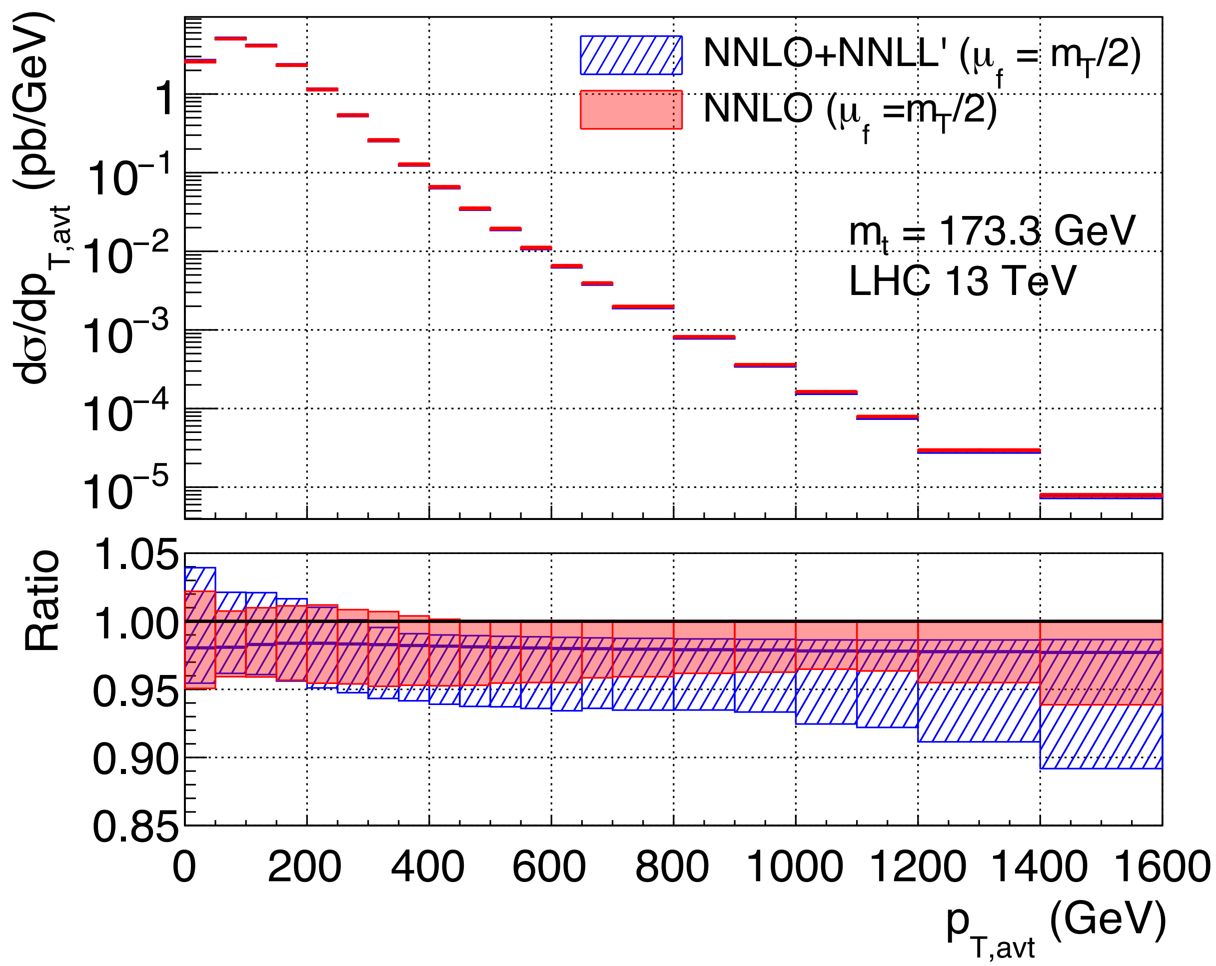
$$-u_1\big|_{m_t\to 0} \approx \frac{M_{t\bar{t}}^2}{2}(1+\cos\theta) - m_t^2\cos\theta \xrightarrow{\cos\theta\to -1} m_T^2 = H_T^2/4\,.$$

$$\frac{\mathcal{H}_{gg}^{\rm NLO}(\mu_h)}{\mathcal{H}_{gg}^{\rm LO}(\mu_h)}\bigg|_{t_1\to 0} = 1+\frac{\alpha_s(\mu_h)}{36\pi}\left[-78\ln^2\left(\frac{-t_1}{\mu_h^2}\right)+24\ln\left(\frac{-t_1}{\mu_h^2}\right)(3+2\ln x_t)+37\pi^2-168\right]$$

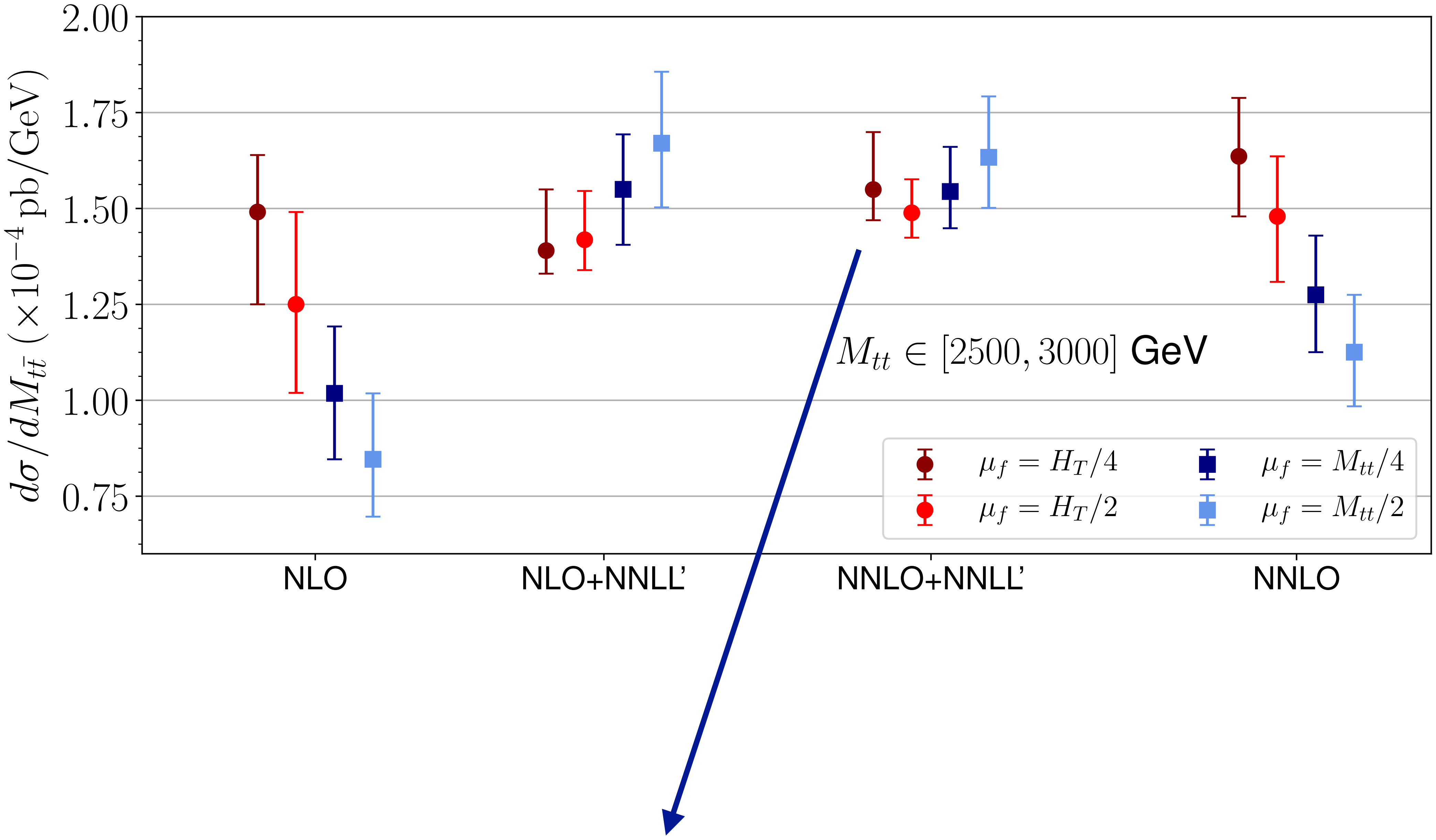
Support the findings of Czakon, Heymes, Mitov: 1606.03350

NNLO+NNLL′


NNLO+NNLL' (μ_f = H_T/4)
NNLO (μ_f = H_T/4)
m_t = 173.3 GeV
LHC 13 TeV
$d\sigma/dM_{t\bar{t}}$ (pb/GeV)
Ratio
$M_{t\bar{t}}$ (GeV)


NNLO+NNLL' (μ_f = m_T/2)
NNLO (μ_f = m_T/2)
m_t = 173.3 GeV
LHC 13 TeV
$d\sigma/dp_{T,avt}$ (pb/GeV)
Ratio
$p_{T,avt}$ (GeV)

NNLO+NNLL′



Matched result insensitive to scale scheme choices

Summary and outlook

- Soft gluons are important in top quark pair production
- We have studied their impacts at NNLO and beyond
  - Two-loop IR divergences
  - NNLO soft real emissions
  - Resummation of soft logarithms (+small-mass logarithms)
- Future prospects
  - Numerical results for rapidity distributions
  - From on-shell top quarks to top jets

Thank you!