

# Gross-Neveu-Yukawa Models at Four Loops

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Loops & Legs, St. Goat 2018

[[arXiv:1605.09423](https://arxiv.org/abs/1605.09423)] & [[arXiv:1703.08801](https://arxiv.org/abs/1703.08801)] & [[arXiv:1709.05057](https://arxiv.org/abs/1709.05057)]

# Overview

1 Motivation

2 Summary & Outlook

# Motivation

- 🕒 Obtain reliable & precise predictions
- 🕒 In HEP the SM (QFT with  $\mathcal{L}_{SM}$ ) provides collider observables
- 🕒 New physics  $\mathcal{L}^{\text{“real”}} = \mathcal{L}_{SM} + \mathcal{L}_X$  ?
- 🕒 New physics includes realization of *SUSY*? (MSSM, nMSSM, ...)
- 🕒 Origin of  $\mathcal{L}_{SM}$ ?
- 🕒 Origin of QFTs?

## How to proceed?

- ➡ Build and run LHC to find new physics
- ➡ Calculate observables at higher order in PT
- ➡ Investigate on various QFTs (may not have HEP relevancy\*)

# Work in collaboration with...

(from west to east)

➡ *Simon Fraser University (Burnaby)*

Igor Herbut

➡ *University of Alberta (Edmonton)*

Joseph Maciejko

Chien-Hung Lin

➡ *University of Heidelberg*

Michael Scherer (→ Köln)

Luminita Mihaila

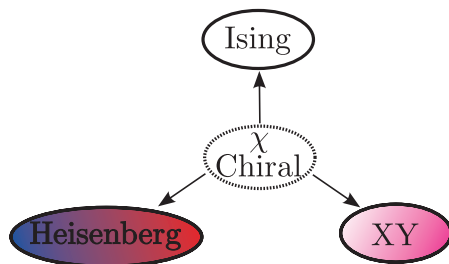
Bernhard Ihrig (→ Köln)

➡ *DESY Zeuthen*

Peter Marquard

# Content

- Perturbativ RGE analysis at 4-loops around  $D = 4 - \epsilon$
- Gross-Neveu-Yukawa Models



# Models

- Emerge as LEET in condensed matter physics



Strongly coupled systems in  $D = 2 + 1$

- Dirac systems with  $N$   $\psi$ 's coupled to a bosonic order parameter  $\phi$
- Lorenzian Symmetry
- Fermion Kinetic Term in  $\mathcal{L}$  (no gap)

$$\mathcal{L}_\psi = \bar{\psi}(x) \not{\partial} \psi(x).$$

$$\not{\partial} = \gamma^\mu \cdot \partial_\mu.$$

+Clifford algebra:  $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ .

(Ex.Rep.:  $\gamma^0 = \sigma_3, \gamma^1 = \sigma_1, \gamma^2 = \sigma_2$ )

- Global Symmetry/ DoF[ $\phi$ ]  $\rightarrow$  {Ising, Heisenberg, XY}

# Models

- $\phi \in \mathbb{R}$  (1 DoF) & 'Global  $Z_2$ ' (no cubics in pot.)  $\rightarrow$   $\chi$ -Ising

$$\mathcal{L}_{\chi I} = \mathcal{L}_\psi + g\phi\bar{\psi}\psi + \frac{1}{2}\phi(m^2 - \partial_\mu^2)\phi + \lambda\phi^4.$$

- $\phi \in \mathbb{C}$  (2 DoF) & Global  $U_1 \rightarrow \chi$ -XY

$$\mathcal{L}_{\chi XY} = \mathcal{L}_\psi + g\phi\bar{\psi}P_+\psi + g\phi^*\bar{\psi}P_-\psi + |\partial_\mu\phi|^2 + m^2|\phi|^2 + \lambda|\phi|^4.$$

- $\vec{\phi} \in \mathbb{R}^3$  (3 DoF) & Global  $SU_2 \rightarrow \chi$ -Heisenberg

$$\mathcal{L}_{\chi H} = \mathcal{L}_\psi + g\bar{\psi}(\vec{\phi} \cdot \vec{\sigma})\psi + \frac{1}{2}\vec{\phi} \cdot (m^2 - \partial_\mu^2)\vec{\phi} + \lambda(\vec{\phi} \cdot \vec{\phi})^2.$$

 Renormalizability around  $D = 4$

# Renormalization

➡ All UV divergences absorbable via  $\overline{\text{MS}}$  field and coupling redefinitions

$$\begin{aligned}\phi &\rightarrow \phi^0 = \sqrt{Z_\phi} \phi, & \psi &\rightarrow \psi^0 = \sqrt{Z_\psi} \psi, \\ \lambda &\rightarrow \lambda^0 = \mu^\epsilon Z_\lambda \lambda, & g &\rightarrow g^0 = \mu^{\epsilon/2} Z_g g.\end{aligned}$$

➡ With

$$Z_{\phi^4} = Z_\lambda Z_\phi^2, \quad Z_{\psi\psi\phi} = Z_g \sqrt{Z_\phi} Z_\psi, \quad Z_{\phi^2} = Z_\phi Z_{m^2},$$

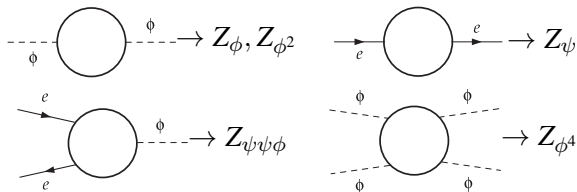
renormalized Lagrangians read

$$\begin{aligned}\mathcal{L}_{\chi I} &= Z_\psi \mathcal{L}_\psi + Z_{\psi\psi\phi} g \phi \bar{\psi} \psi + \frac{1}{2} \phi (Z_{\phi^2} m^2 - Z_\phi \partial_\mu^2) \phi + Z_{\phi^4} \lambda \phi^4. \\ \mathcal{L}_{\chi XY} &= Z_\psi \mathcal{L}_\psi + Z_{\psi\psi\phi} g (\phi \bar{\psi} P_+ \psi + \phi^* \bar{\psi} P_- \psi) \\ &\quad + Z_\phi |\partial_\mu \phi|^2 + Z_{\phi^2} m^2 |\phi|^2 + Z_{\phi^4} \lambda |\phi|^4. \\ \mathcal{L}_{\chi H} &= \mathcal{L}_\psi + Z_{\psi\psi\phi} g \bar{\psi} (\vec{\phi} \cdot \vec{\sigma}) \psi \\ &\quad + \frac{1}{2} \vec{\phi} \cdot (Z_{\phi^2} m^2 - Z_\phi \partial_\mu^2) \vec{\phi} + Z_{\phi^4} \lambda (\vec{\phi} \cdot \vec{\phi})^2.\end{aligned}$$

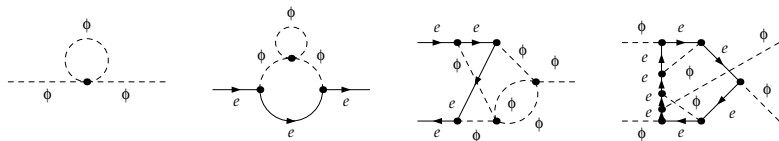


# How to obtain Zs?

- Using DREG get poles in small  $\epsilon = 4 - d$  of 1-PI  $n$ -point functions at  $L = 1, 2, 3, 4$  loops for “any” kinematics

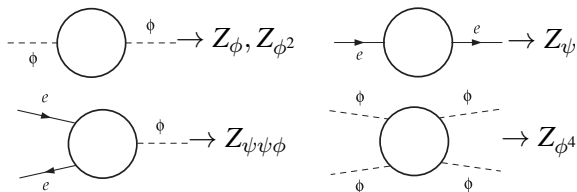


- Example Diagrams (Ising)



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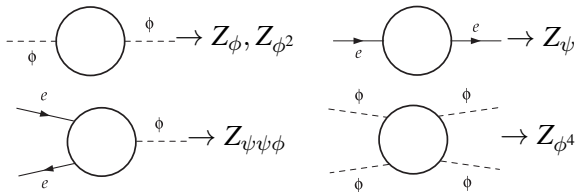


- Number of Diagrams

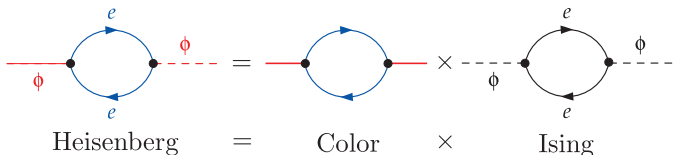
	$\chi^{Ising}$ & $\chi^{Heisenberg}$				$\chi^{XY}$			
Loops	1	2	3	4	1	2	3	4
$Z_\psi$	1	4	31	323	2	14	200	4014
$Z_\phi, Z_{\phi^2}$	2	6	36	358	2	9	112	2198
$Z_{\psi\psi\phi}$	1	11	145	2199	2	41	1002	28701
$Z_{\phi^4}$	9	93	1476	26976	9	173	5029	147023

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- $\chi$ Heisenberg diagrams factorize  $(T_F^a = \frac{1}{2}\sigma^a \in su_2)$



# How to obtain Zs?

## ➡ Choosing “massiv tadpole kinematics”

- ▶ Use single IR regulator mass  $M$  in every denominator
- ▶ Expand in small external momenta  $|p_i|/M \ll 1$

➡ All integrals are 1-scale tadpole integrals

➡ Use **infrared rearrangement** [Chetyrkin,Misiak,Munz]  
to consistently subtract artificial terms  $\sim M^2$

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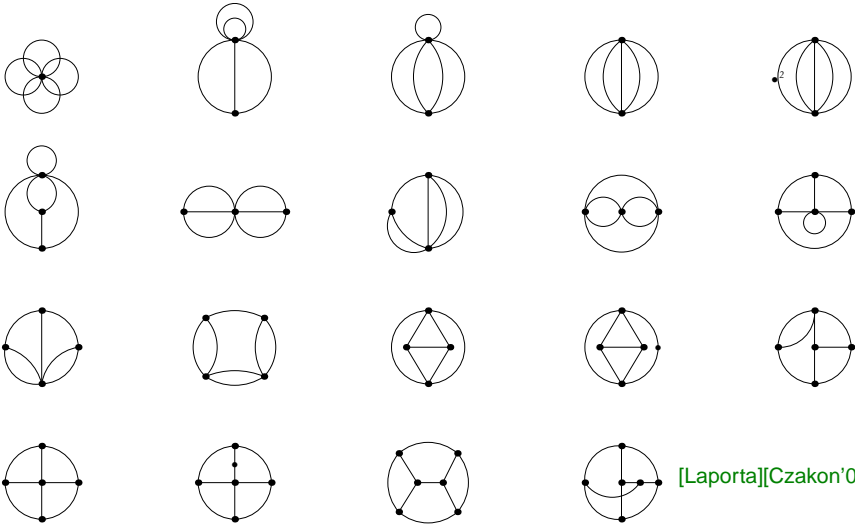
## ➡ All integrals are 1-scale tadpole integrals

- ## ➡ Use infrared rearrangement [Chetyrkin,Misiak,Munz]
- to consistently subtract artificial terms  $\sim M^2$

## ➡ Fully automatized setup

- ▶ *QGRAF* [Nogueira]
- ▶ *q2e* [Seidensticker]
- ▶ *exp* [Seidensticker]
- ▶ *FORM* [Vermaseren & Co]
- ▶ *MATAD* [Steinhauser] OR *CRUSHER* [Marquard,Seidel]
- ▶ *COLOR* [Ritbergen,Schellekens,Vermaseren'98]
  
- ▶ modified *majoranas.pl* [Harlander]
- ▶ *RecalcPrefac* [NZ]

# 19 Fully Massive Master Integrals



[Laporta][Czakon'04]

# $\beta$ - & $\gamma$ - Functions

- $\beta_y = \frac{dy}{d \ln \mu} \quad \beta_\lambda = \frac{d\lambda}{d \ln \mu} \quad \left( \frac{g^2}{8\pi} \rightarrow y, \frac{\lambda}{8\pi} \rightarrow \lambda \right)$

$$\beta_y = -\epsilon y + \beta_y^{(1L)} + \beta_y^{(2L)} + \beta_y^{(3L)} + \beta_y^{(4L)},$$

$$\beta_\lambda = -\epsilon \lambda + \beta_\lambda^{(1L)} + \beta_\lambda^{(2L)} + \beta_\lambda^{(3L)} + \beta_\lambda^{(4L)},$$

- $\gamma_x = d \ln Z_x / d \ln \mu \quad \forall x \in \{\psi, \phi, \phi^2\}$

$$\gamma_\psi = \gamma_\psi^{(1L)} + \gamma_\psi^{(2L)} + \gamma_\psi^{(3L)} + \gamma_\psi^{(4L)},$$

$$\gamma_\phi = \gamma_\phi^{(1L)} + \gamma_\phi^{(2L)} + \gamma_\phi^{(3L)} + \gamma_\phi^{(4L)},$$

$$\gamma_{\phi^2} = \gamma_{\phi^2}^{(1L)} + \gamma_{\phi^2}^{(2L)} + \gamma_{\phi^2}^{(3L)} + \gamma_{\phi^2}^{(4L)}.$$

# $\beta$ - & $\gamma$ - Functions for $\chi$ Ising

Example Results @ 1 & 2 Loops:

$$\begin{aligned}\beta_{y,\chi^1}^{(1L)} &= (3 + 2N)y^2, & \beta_{\lambda,\chi^1}^{(1L)} &= 36\lambda^2 + 4Ny\lambda - Ny^2, \\ \beta_{y,\chi^1}^{(2L)} &= 24y\lambda(\lambda - y) - \left(\frac{9}{8} + 6N\right)y^3, & \beta_{\lambda,\chi^1}^{(2L)} &= 4Ny^3 + 7Ny^2\lambda - 72Ny\lambda^2 - 816\lambda^3, \\ \gamma_{\psi,\chi^1}^{(1L)} &= \frac{y}{2}, & \gamma_{\phi,\chi^1}^{(1L)} &= 2Ny, & \gamma_{\phi^2,\chi^1}^{(1L)} &= -12\lambda, \\ \gamma_{\psi,\chi^1}^{(2L)} &= -\frac{y^2}{16}(12N + 1), & \gamma_{\phi,\chi^1}^{(2L)} &= 24\lambda^2 - \frac{5Ny^2}{2}, & \gamma_{\phi^2,\chi^1}^{(2L)} &= 144\lambda^2 - 2Ny(y - 12\lambda).\end{aligned}$$

✓ 2-loop results [Rosenstein,Kovner,Yu'93]



# $\beta$ - & $\gamma$ - Functions for $\chi$ Ising

Example Result @ 4 Loops:

$$\begin{aligned} \beta_{\lambda, \chi^1}^{(4L)} = & 41472 \left( -39\zeta_3 - 60\zeta_5 + \frac{\pi^4}{10} - \frac{3499}{96} \right) \lambda^5 + \frac{1}{240} \lambda N y^4 \left( -60\zeta_3 (912N^2 - 4156N - 4677) \right. \\ & \left. + 1200\zeta_5 (157 - 168N) - 4\pi^4 (450N + 41) + 25(4N(337N + 3461) + 5847) \right) \\ & + \frac{N y^5 \left( 480\zeta_3 (12N(14N - 15) + 277) + 2400\zeta_5 (128N + 65) + 8\pi^4 (64N - 77) + 160N(1289 - 386N) - 67095 \right)}{1920} \\ & + \frac{1}{80} \lambda^2 N y^3 \left( 835200\zeta_5 + 1920\zeta_3 (3N(4N - 61) + 19) + 72\pi^4 (24N + 31) - 40N(288N + 15649) + 1057825 \right) \\ & + \frac{4}{5} \lambda^3 N y^2 \left( -86400\zeta_5 + 540\zeta_3 (4N - 69) + 7890N - 288\pi^4 - 72605 \right) + \frac{36}{5} \left( -17280\zeta_3 + 96\pi^4 - 6775 \right) \lambda^4 N y. \end{aligned}$$

✓  $y = 0 \rightarrow Z_2 \phi^4$  text book results [Kleinert, Schulte-Frohlinde'01]

⚠ 6-loop  $O(N) \phi^4$  [Batkovich, Chetyrkin, Kompaniets'17], 7-loop ...

✓ large  $N$  limit for critical exponents  $\rightarrow$  talk by John

[Derkachov, Gracey, Kivel, Manashov, Strohmaier, Stepanenko, Vasiliev][Gracey'17,18]

# $\beta$ - & $\gamma$ - Functions for $\chi^{XY}$

$N = 1/2, y \rightarrow 2h^2, \lambda \rightarrow \lambda^2/4$  [S.Thomas'05@KITP]

$$\beta_{h^2} = -\epsilon \bar{h}^2 + 12\bar{h}^4 \quad \bar{h}^2 = h^2/(4\pi)^2, \bar{\lambda}^2 = \lambda^2/(4\pi)^2$$
$$+ 8\bar{h}^2 \bar{\lambda}^4 - 64\bar{h}^4 \bar{\lambda}^2 + 8\bar{h}^6$$
$$- 40\bar{h}^2 \bar{\lambda}^6 + 300\bar{h}^4 \bar{\lambda}^4 + 1632\bar{h}^6 \bar{\lambda}^2 - [1652 - 576\zeta_3] \bar{h}^8,$$

$$\beta_{\lambda^2} = -\epsilon \bar{\lambda}^2 + 20\bar{\lambda}^4 + 8\bar{h}^2 \bar{\lambda}^2 - 16\bar{h}^4$$
$$- 80\bar{h}^2 \bar{\lambda}^4 + 16\bar{h}^4 \bar{\lambda}^2 + 256\bar{h}^6 - 240\bar{\lambda}^6$$
$$+ 904\bar{h}^2 \bar{\lambda}^6 + [3832 + 3264\zeta_3] \bar{h}^4 \bar{\lambda}^4 - [8664 + 2688\zeta_3] \bar{h}^6 \bar{\lambda}^2$$
$$- [768 + 3072\zeta_3] \bar{h}^8 + [4936 + 3072\zeta_3] \bar{\lambda}^8,$$

$$\gamma_\phi = 4\bar{h}^2 - 24\bar{h}^4 + 8\bar{\lambda}^4$$
$$- 40\bar{\lambda}^6 - 60\bar{h}^2 \bar{\lambda}^4 + 160\bar{h}^4 \bar{\lambda}^2 + [20 + 192\zeta_3] \bar{h}^6,$$

$$\gamma_\psi = 4\bar{h}^2 - 16\bar{h}^4$$
$$- 44\bar{h}^2 \bar{\lambda}^4 + 128\bar{h}^4 \bar{\lambda}^2 + [192\zeta_3 - 4] \bar{h}^6.$$

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$$\gamma_\phi = 4\bar{h}^2 - 16\bar{h}^4 + 16\bar{h}^6(5 + 12\zeta_3),$$
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✓  $h^2 = \lambda^2$  limit reproduces Wess-Zumino model [Avdeev,Goroshni'82]

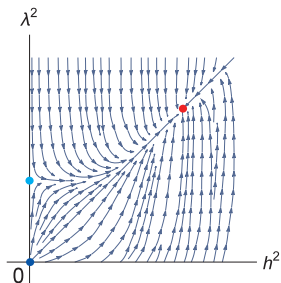
# RG Fixed Points @ 3 Loops $\chi^{XY}$

➡ Searching for IR fixed points ( $\mu \rightarrow 0$ ) at  $d = 3$ :

$$(\bar{h}_*^2, \bar{\lambda}_*^2) = (0, 0), \quad (\text{unstable})$$

$$(\bar{h}_*^2, \bar{\lambda}_*^2) = \left( 0, \frac{\epsilon}{20} + \frac{3\epsilon^2}{100} - \frac{384\zeta_3 - 103}{20000}\epsilon^3 \right), \quad (\text{unstable})$$

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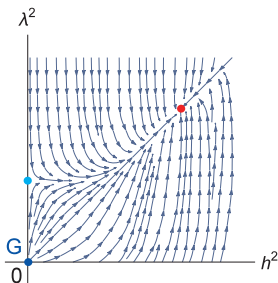
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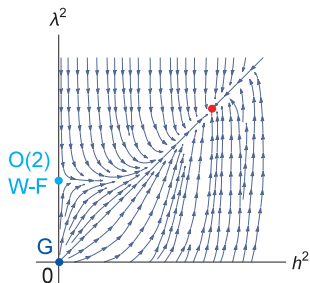
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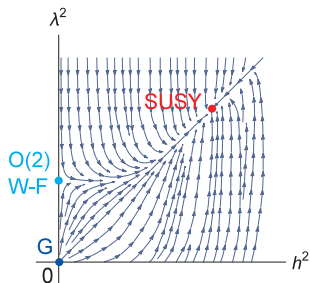
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$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} \Phi^\dagger \Phi + \left( \int d^2\theta \frac{h}{3} \Phi^3 + \text{h.c.} \right),$$

$$\Phi(y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y),$$

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🚫 SUSY non-renormalization theorems

[Strassler'03@TASI]

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- 🚫 SUSY non-renormalization theorems [Strassler'03@TASI]

- ➔ At the SUSY fixed point we check:

$$\gamma_\phi^* = \gamma_\psi^* = \frac{\epsilon}{3} + \mathcal{O}(\epsilon^5).$$

# Determination of Critical Exponents

## Phenomenology of Quantum Phasetransitions

$$x = (X - X_c)/X_c$$

$$\xi \sim |x|^{-\nu}(1 + C|x|^\omega + \dots)$$

- ▶ correlation length around Quantum Critical Point (QCP)  $\xi$
- ▶ correlation length exponent  $\nu$
- ▶ subleading/stability exponent  $\omega$

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## Which phase transition?

$$\mathcal{L}_{\chi XY} = \bar{\psi} \not{\partial} \psi + |\partial_\mu \phi|^2 + m^2 |\phi|^2 + \lambda^2 |\phi|^4 + h(\phi^* \psi^T i \sigma_2 \psi + h.c.).$$

- ▶ **Semimetal phase** Global  $U(1)$  symmetry for  $m^2 > 0$

$$\psi \rightarrow e^{i\theta} \psi, \quad \phi \rightarrow e^{2i\theta} \phi, \quad \langle 0 | \phi | 0 \rangle = \phi_0 = 0.$$

- ▶ **Phasetransition** at QCP for  $m^2 = 0$
- ▶ **Superconducting phase**

Spontaneously broken global  $U(1)$  symmetry for  $m^2 < 0$

$$\langle 0 | \phi | 0 \rangle = \phi_0 \neq 0.$$

# Critical Exponents $\chi^{XY}$

⚠ Stability exponent ( $\omega \geq 0$  IR stable/unstable \*)

$$\omega = \left. \frac{d\beta_{h^2}(\lambda^2 = h^2)}{dh^2} \right|_{h=h_*} = \epsilon - \frac{1}{3}\epsilon^2 + \left( \frac{1}{18} + \frac{2}{3}\zeta_3 \right) \epsilon^3 + a\epsilon^4 + \mathcal{O}(\epsilon^5).$$



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✓ OR: Using “Grassmannian continuation” of the SUSY Lagrangian

$$\gamma_{m^2}^* = - \left. \frac{d\beta_{h^2}(\lambda^2 = h^2)}{dh^2} \right|_{h=h_*} = -3h_*^2 \left. \frac{d\gamma_\phi(\lambda^2 = h^2)}{dh^2} \right|_{h=h_*}.$$

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➡ Naive extrapolation to  $d = 2 + 1$  ( $\epsilon = 1$ ):

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➡ Use Padé approximations



# Emergent $\mathcal{N} = 1$ SUSY in $\chi$ Ising

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💡  $D = 3$  (using Pauli-matrices)

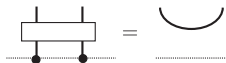
$$\text{tr}\{\gamma^\mu \gamma^\nu \gamma^\rho\} \sim \varepsilon^{\mu\nu\rho}.$$

# Clifford for Pedestrians

$$\begin{array}{|c} \hline \\ \hline \end{array} = \frac{1}{2} \left[ \begin{array}{|c} \hline \\ \hline \end{array} + \begin{array}{|c} \diagup \\ \diagdown \end{array} \right] \quad \begin{array}{|c} \hline \\ \hline \end{array} = \frac{1}{2} \left[ \begin{array}{|c} \hline \\ \hline \end{array} - \begin{array}{|c} \diagup \\ \diagdown \end{array} \right]$$



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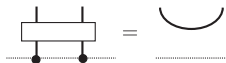


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



## Irreducible Bilinears in $D$ Dimensions




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




**Dimension of corresponding  $SO(D)$  representation**

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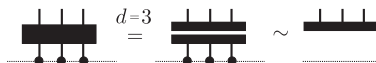

**Irreducible Bilinears in  $D$  Dimensions**




**Dimension of corresponding  $SO(D)$  representation**

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**In  $D = 3$  ( $\varepsilon_{ijk}$  has 1 lin. indep. component!)**





# Saving $\mathcal{N} = 1$ SUSY “with Trump”

➡ Modify DREG & demand in  $D$  dimensions:

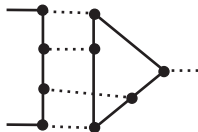


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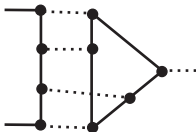


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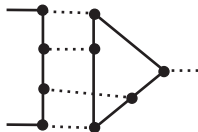
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✓ SUSY relations restored

$$\beta_y^{(4L)*} = \beta_\lambda^{(4L)*} \rightarrow \lambda^* = y^*,$$

$$\nu^{-1} = \frac{D - \gamma^*}{2}.$$

# Comparing Results $\chi^{XY}$

$N = 1/2$	$1/\nu$	$\gamma_\phi^*$	$\gamma_\psi^*$	$\omega$
<i>this work</i> , $P_{[2/2]}$	1.128	1/3	1/3	0.872
<i>this work</i> , $P_{[3/1]}$	1.130	1/3	1/3	0.870
conf. bootstrap <sup>1</sup>	1.090	1/3	1/3	0.910
$N = 2$	$1/\nu$	$\gamma_\phi^*$	$\gamma_\psi^*$	$\omega$
<i>this work</i> , $P_{[2/2]}$	0.840	0.810	0.117	0.796
<i>this work</i> , $P_{[3/1]}$	0.841	0.788	0.108	0.780
functional RG <sup>2</sup>	0.862	0.88	0.062	0.878
Monte Carlo <sup>3</sup>	1.06(5)	0.71(3)		

<sup>1</sup>[Bobev,El-Showk,Mazac,Paulos'15]

<sup>2</sup>[Classen, Herbut, Scherer'17]

<sup>3</sup>[Li,Jiang,Jian,Yao,'17]

# Comparing Results $\chi$ Ising

$N = 1/4$	$1/\nu$	$\gamma_\phi^*$	$\gamma_\psi^*$	$\omega$
<i>this work, <math>P_{[2/2]}</math></i>	1.415	0.171	0.171	0.843
<i>this work, <math>P_{[3/1]}</math></i>	1.415	0.170	0.170	0.838
FRG <sup>4</sup> (Regulator 1)	1.385	0.174	0.174	0.765
FRG <sup>4</sup> (Regulator 2)	1.395	0.167	0.167	0.782
conf. bootstrap <sup>5</sup>		0.164	0.164	

<sup>4</sup>[Gies,Hellwig,Wipf,Zanusso'17]

<sup>5</sup>[Iliesiu,Kos,Poland,Pufu,Simmons-Duffin,Yacoby'16]

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$N = 2$	$1/\nu$	$\gamma_\phi^*$	$\gamma_\psi^*$	$\omega$
<i>this work</i> , $P_{[2/2]}$	0.931	0.7079	0.0539	0.794
<i>this work</i> , $P_{[3/1]}$	0.945	0.6906	0.0506	0.777
$(2 + \epsilon)$ , $(\epsilon^4, \text{Padé})^6$	0.931	0.745	0.082	
FRG <sup>7</sup>	0.994(2)	0.7765	0.0276	
conformal bootstrap <sup>8</sup>	0.88	0.742	0.044	
Monte Carlo <sup>9</sup>	1.20(1)	0.62(1)	0.38(1)	

<sup>6</sup>[Gracey,Luthe,Schroder'16]

<sup>7</sup>[Knorr'16]

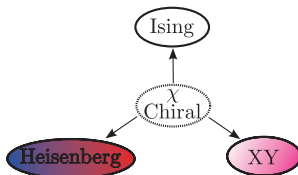
<sup>8</sup>[Iliesiu,Kos,Poland,Pufu,Simmons-Duffin'17]

<sup>9</sup>[Chandrasekharan,Li'13]

# Summary & Outlook

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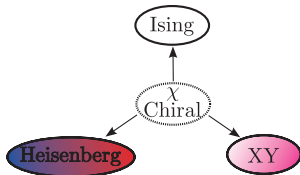
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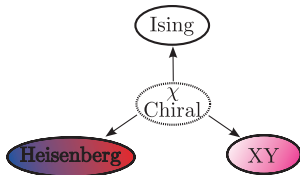
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## Exercise

🧐 Is the QFT with

$$\mathcal{L} = \bar{\psi}_\alpha (\not{\partial} + g[T_R^a]_\beta^\alpha \phi_a) \psi^\beta - \frac{1}{2} \phi_a \partial_\mu^2 \phi_a + \lambda (\phi_a \phi_a)^2,$$

a renormalizable theory in  $d = 4$  when  $T_R$  obey a finite dimensional simple Lie algebra?


To backup slides →

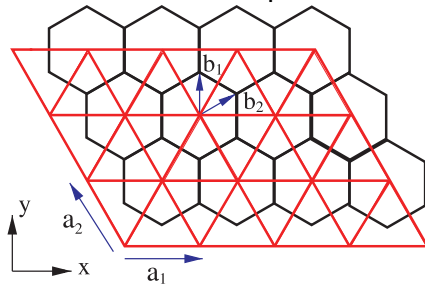
# Where to find SUSY?

 @ LHC

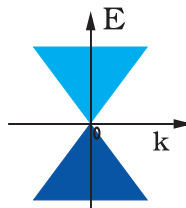
# Where to find SUSY?

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 On the surface of topological insulator  
@ the semimetal-superconductor QCP



[S.-S.Lee'07]



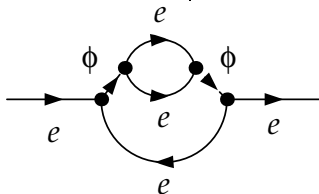
# $\chi^{XY}$ in smart

$$\mathcal{L}_{\chi^{XY}} = \bar{\psi} \not{\partial} \psi + |\partial_\mu \phi|^2 + m^2 |\phi|^2 + \lambda^2 |\phi|^4 + h(\phi^* \psi^T i \sigma_2 \psi + h.c.).$$

VS.

$$\mathcal{L}_{\chi^{XY}} = \bar{\psi} \not{\partial} \psi + |\partial_\mu \phi|^2 + m^2 |\phi|^2 + \lambda^2 |\phi|^4 + g \phi \bar{\psi} P_+ \psi + g \phi^* \bar{\psi} P_- \psi.$$

	$\chi^{XY}$ v1				$\chi^{XY}$ v2			
Loops	1	2	3	4	1	2	3	4
$Z_\psi$	1	3	16	116	2	14	200	4014
$Z_\phi, Z_{\phi^2}$	2	4	22	148	2	9	112	2198
$Z_{\psi\psi\phi}$	0	2	25	296	2	41	1002	28701
$Z_{\phi^4}$	4	35	369	4388	9	173	5029	147023



# Renormalizability

 Is the QFT with

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Lie algebra	Betti numbers
$A_r \sim SU(r+1)$	$2, 3, \dots, r+1$
$B_r \sim SO(2r+1)$	$2, 4, 6, \dots, 2r$
$C_r \sim Sp(2r)$	$2, 4, 6, \dots, 2r$
$D_r \sim SO(2r)$	$2, 4, 6, \dots, 2r-2, r$
$G_2$	$2, 6$
$F_4$	$2, 6, 8, 12$
$E_6$	$2, 5, 6, 8, 9, 12$
$E_7$	$2, 6, 8, 10, 12, 14, 18$
$E_8$	$2, 8, 12, 14, 18, 20, 24, 30$





# Superspace

$$d^2\theta \equiv -\frac{1}{4}d\theta^\alpha d\theta^\beta \varepsilon_{\alpha\beta},$$

$$d^2\bar{\theta} \equiv -\frac{1}{4}d\bar{\theta}^\alpha d\bar{\theta}^\beta \varepsilon_{\alpha\beta},$$

$$\theta^2 \equiv \theta^\alpha \theta_\alpha = \theta^\alpha \varepsilon_{\alpha\beta} \theta^\beta,$$

$$\int d^2\theta d^2\bar{\theta} \Phi^\dagger \Phi = i\bar{\psi}\not{\partial}\psi + |\partial_\mu\phi|^2 + |F|^2. \quad \textit{kinetic terms}$$

$$\int d^2\theta \frac{h}{3}\Phi^3 = h\phi^2 F + h\phi\psi^T i\sigma_2\psi. \quad \textit{super potential terms}$$

Auxiliary fields are eliminated using EQM:  $F = -h\phi^{*2}$ ,  $F^* = -h\phi^2$ .