Universität Zürich[UZH]

Two-loop integrals for μe-scattering

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St.Goar , 1st May 2018 – Loops&Legs in Quantum Field Theory

Outline

- Motivation: the muon (g-2) in the Standard Model
- Leading hadronic contribution from μe-scattering
- Status of μe-scattering at NNLO in QED: virtual amplitude
- Outline and Conclusions

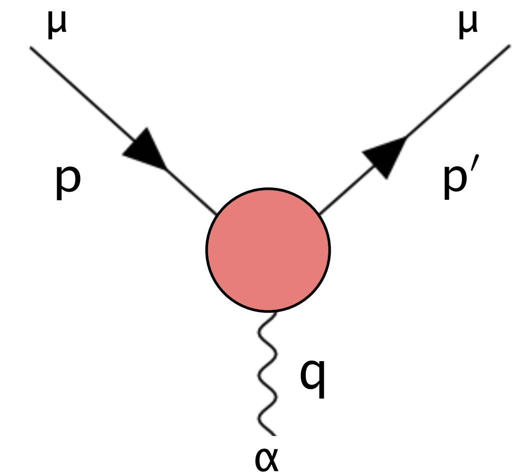
References: arXiv:1709.07435

In collaboration with: S. Di Vita, S.Laporta, P.Mastrolia, M.Passera, U.Schubert and W.J.Torres Bobadilla

The muon g-2

- Muon anomalous magnetic moment

$$\vec{m} = 2(1 + a_\mu)\frac{Qe}{2m_\mu}\vec{s}$$



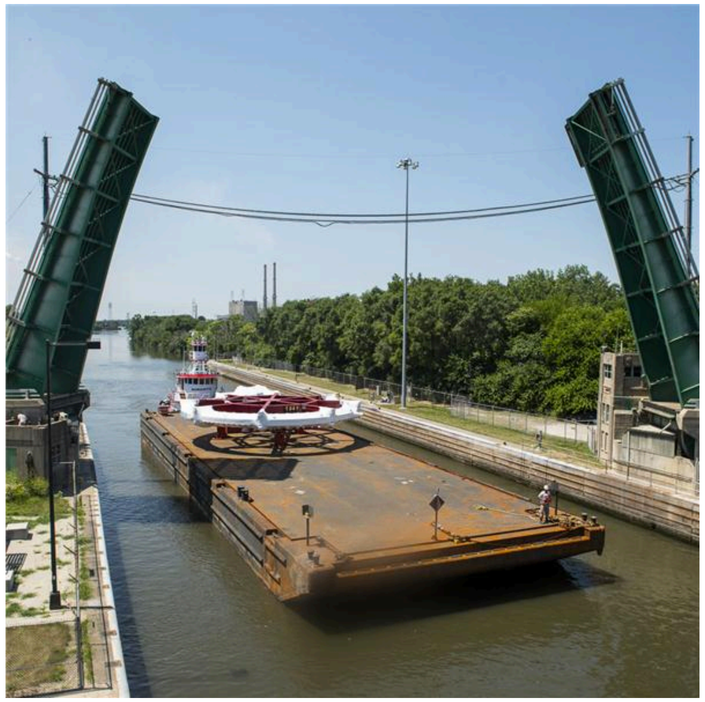
- Contributions from quantum effects, $a_\mu = F_2(0)$

$$\bar{u}(p')\Gamma_\alpha u(p) = \bar{u}(p')\left[\gamma_\alpha F_1(q^2) + \frac{i\sigma_{\alpha\beta}q^\beta}{2m_\mu^2}F_2(q^2) + \ldots\right]u(p)$$

- Experimental measure by BNL-E821, 0.5ppm accuracy

$$a_\mu^{exp} = 116\,592\,089\,(63) \times 10^{-11}$$

[E821 06]



- Upcoming validation with higher precision
  - FNAL-E989 aims at $\pm 16 \times 10^{-11}$ (0.14ppm)
  - later confirmation from J-Parc E34

SM vs experiment

- (g−2) in the Standard Model

$$a_\mu = a_\mu^{QED} + a_\mu^{Weak} + a_\mu^{Had}$$

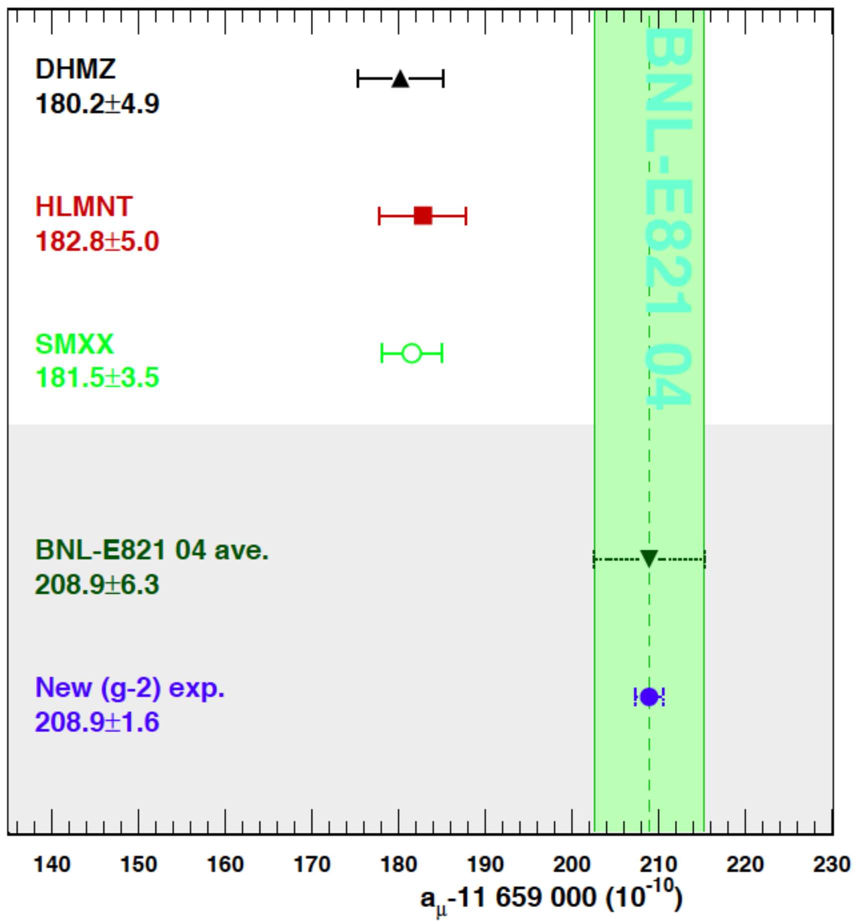
  - theory prediction at 0.48 ppm accuracy

- Longest standing deviation from the SM

$a_\mu^{SM} \times 10^{11}$	$\Delta a_\mu \times 10^{11}$	σ
116 591 761 (57)	330 (85)	3.9
116 591 818 (51)	273 (81)	3.4
116 591 841 (58)	250 (86)	2.9

[Jegerlehner 15, Davier 16, Hagiwara et al 11]

  - new measurements can push σ above 5
  - theoretical error will dominate

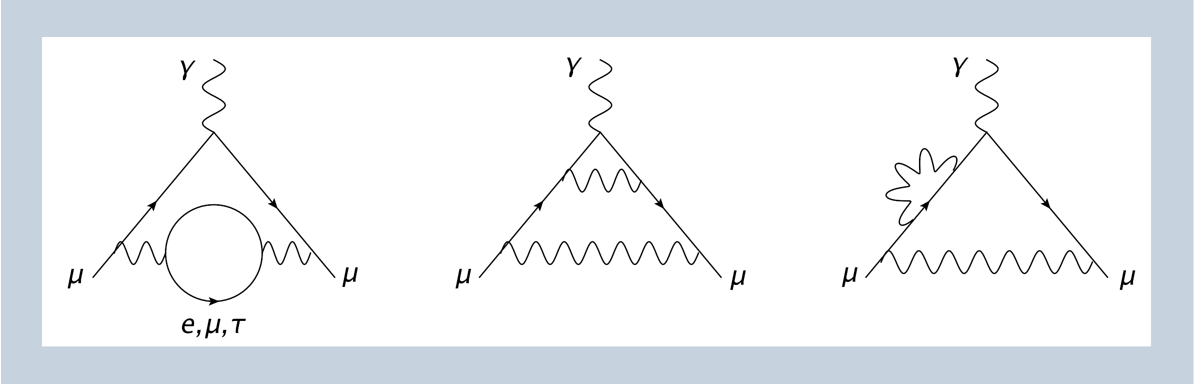


[Blum, Denig, Lovashenko et al 13]

Electro–weak contributions

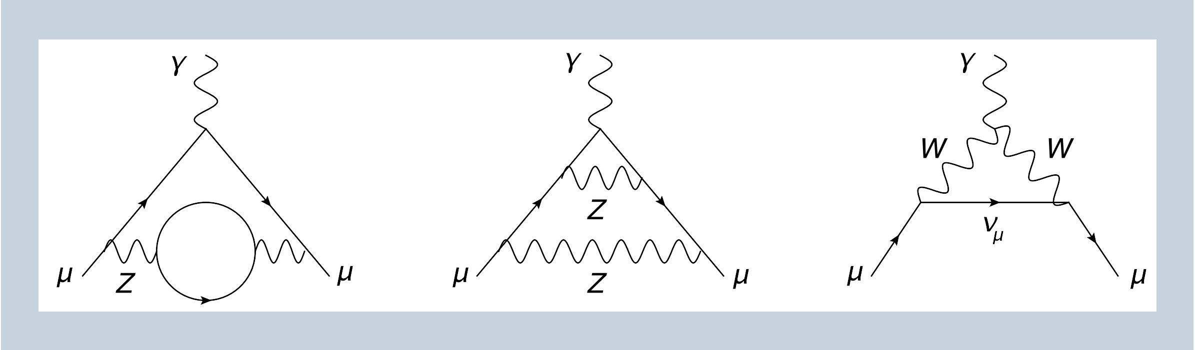
- EW sector is under complete control

- $$a_\mu^{\text{QED}} = 116\,584\,718.944(21)(77) \times 10^{-11}$$
  - 99.99% of the total
  - known up to five loops
  - uncertainty far below Δa_μ



[Schwinger 48,Sommerfield; Petermann; Suura andWichmann 57 Elend 66, Kinoshita and Lindquist 81, Kinoshita et al. 90, Remiddi, Laporta, Barbieri et al; Czarnecki and Skrzypek,Passera 04 Friot, Greynat and de Rafael 05, Mohr, Kinoshita & Nio 04–05, Aoyama, Hayakawa, Kinoshita et al 07, Taylor and Newell 12, Kinoshita et al. 12–15, Steinhauser et a 13–15–16, Yelkhovsky, Milstein, Starshenko, Laporta, Aoyama, Hayakawa, Kinoshita, Nio 12–15,Laporta 17,...]

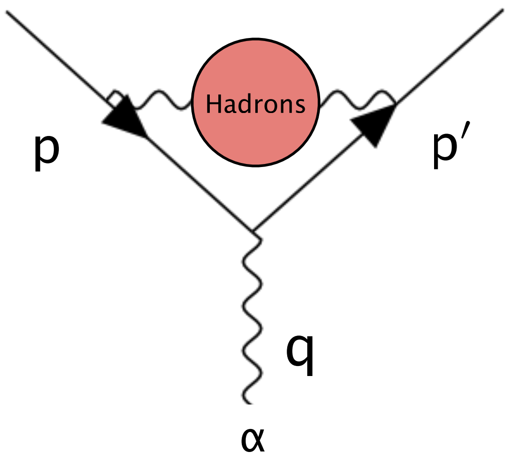
- $$a_\mu^{\text{Weak}} = (153.6 \pm 1) \times 10^{-11}$$
  - contributes to 1.5 ppm
  - known up to two loops
  - uncertainty from hadronic loop



[Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda 71, Kukhto et al. 92, Czarnecki, Krause, Marciano 95, Knecht, Peris, Perrottet, de Rafael 02, Czarnecki, Marciano and Vainshtein 02, Degrassi and Giudice 98; Heinemeyer, Stockinger, Weiglein (04), Gribouk and Czarnecki 05, Vainshtein 03, Gnendiger, Stockinger, Stockinger-Kim 13,...]

Hadronic contribution

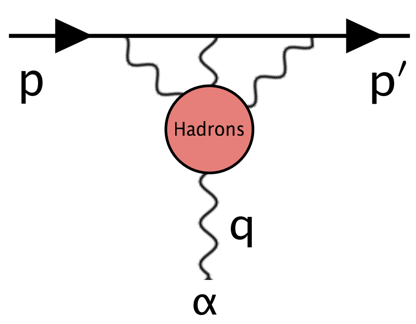
- Hadronic contribution: 60 ppm of the total value
  - non–perturbative, large uncertainties
- $\Pi_{\mathrm{Had}}(q^2)$ largest contribution to central value



$$a_\mu^{\mathrm{HLO}} = 6870\,(42) \times 10^{-11}$$
$$= 6926(33) \times 10^{-11}$$
$$= 6949(37)(21) \times 10^{-11}$$

[Jegerlehner 15, Davier 16, Hagiwara et al 11]

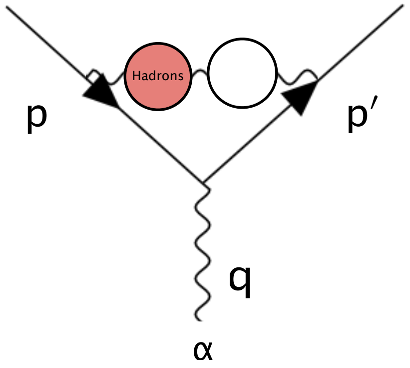
- Light–by–Light



$$a_\mu^{\mathrm{LBL}} = 102\,(39) \times 10^{-11}$$

[Knecht, Nyffeler 02, Melnikov, Vainshtein 03...., Jegerlehner 15]

- Hadronic NLO



$$a_\mu^{\mathrm{NHLO}} = -98\,(1) \times 10^{-11}$$

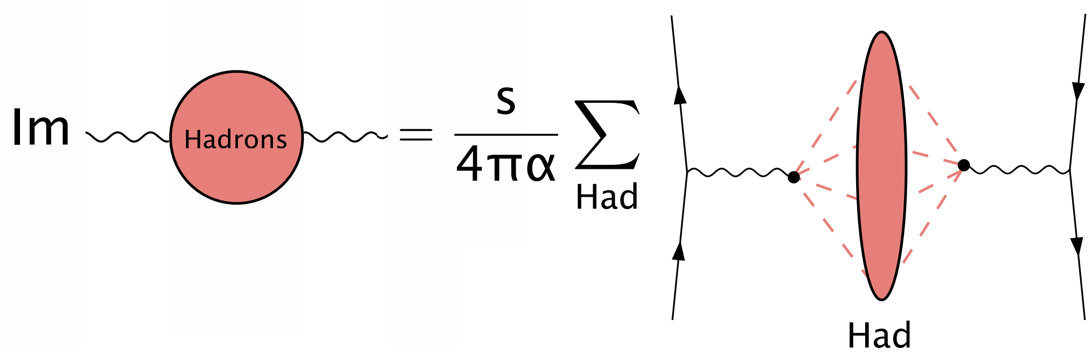
[Krause 96, Alemany et al 98,..., Hagiwara et al 11]

Dispersive approach to a_μ^{HLO}

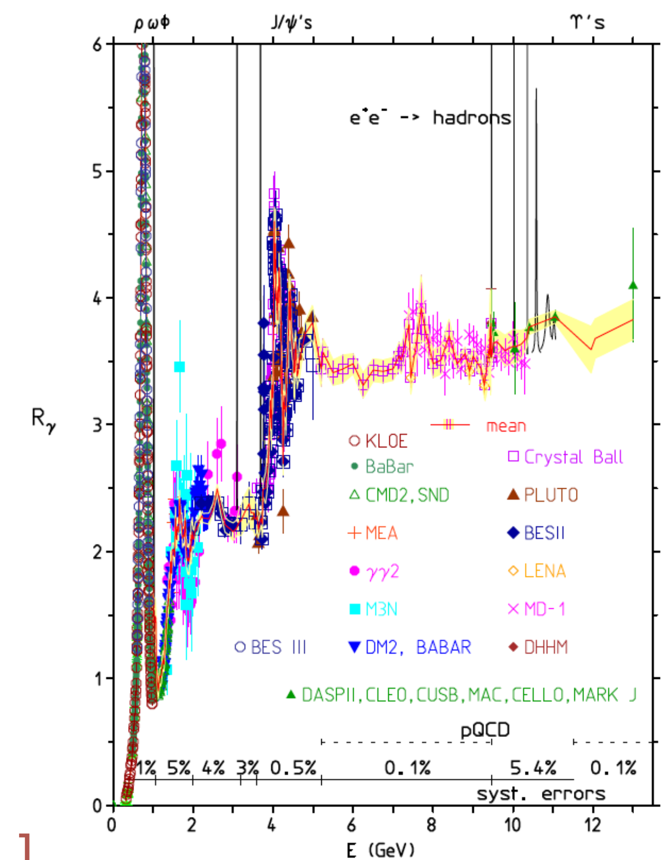
- a_μ^{HLO} computed from dispersion relations

$$a_\mu^{HLO} = \frac{\alpha}{\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}\int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)s/m^2}\,\mathrm{Im}\Pi(s)_{\mathrm{Had}}$$

  - Unitarity relates $\mathrm{Im}\,\Pi_{\mathrm{had}}(s)$ to $e^+e^- \to$ Had cross-section



[Jegerlehner 15]



- Extract a_μ^{HLO} from experimental data

$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)s/m^2}\,\sigma_{e^+e^-\to\mathrm{Had}}(s)$$

[Bouchiat, Michiel 61, Durand 62, Gourdin, de Rafael 69,...]

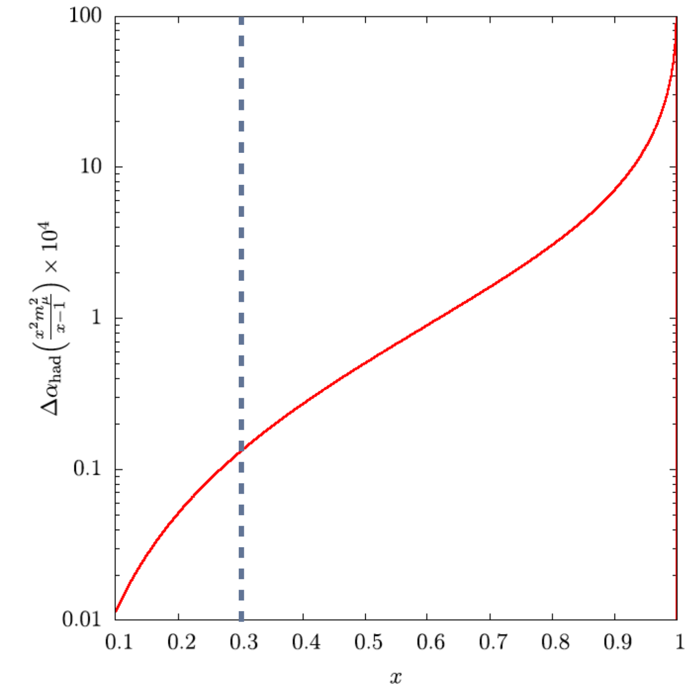
  - enhanced region $s \lesssim 2\,\mathrm{GeV}$
  - Improve accuracy to 0.22 ppm requires 0.4% error on $\sigma_{e^+e^-\to\mathrm{Had}}$

a_μ^{HLO} from muon–electron scattering

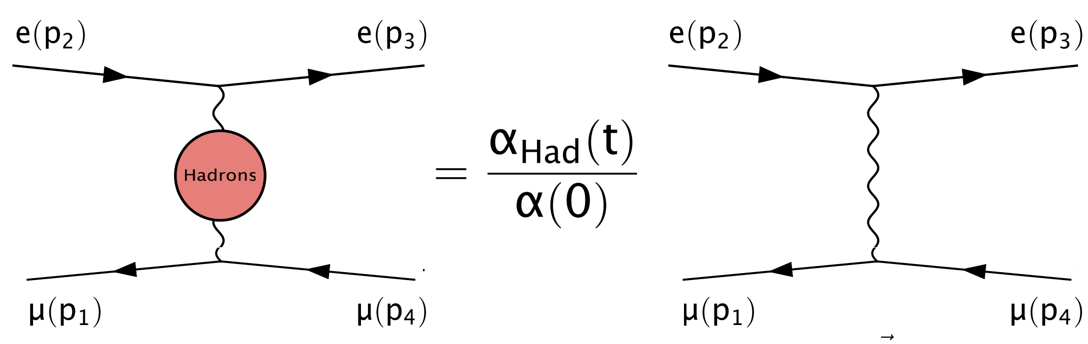
- Alternatively, compute a_μ^{HLO} from space-like data

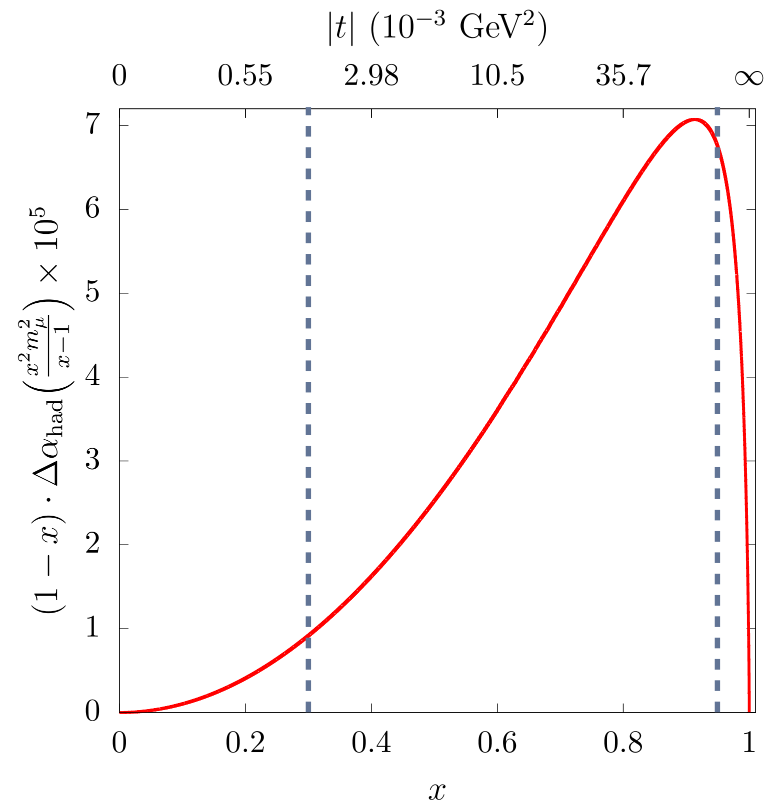
$$a_\mu^{HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\Delta\alpha_{Had}[t(x)] \qquad t(x) = \frac{x^2 m^2}{x-1} \leq 0$$

[Lautrup, Peterman, de Rafael 72]

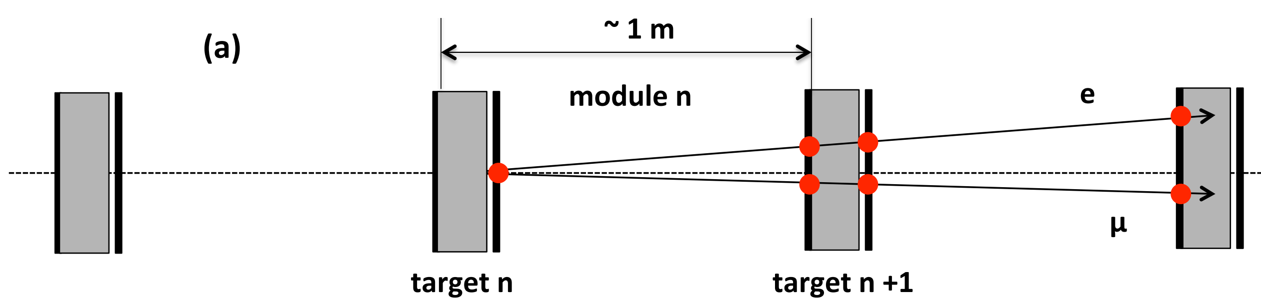


- Extract $\Delta\alpha_{Had}[t(x)] = -\overline{\Pi}_{Had}[t(x)]$ from μe scattering





- MUonE proposal: 150 GeV μ-beam on atomic e

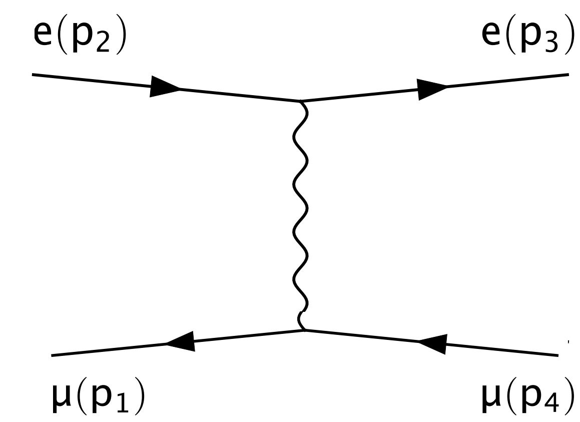


[Carloni Calame, Passera et al 15, Abbiendi, Carloni Calame, Marconi et al 16]

a_μ^{HLO} from muon–electron scattering

- Running coupling from μe-scattering

$$\frac{d\sigma^{HLO}}{dt} = \left|\frac{\alpha_{Had}(t)}{\alpha(0)}\right|^2 \frac{d\sigma^{LO}}{dt}$$



  - LO contribution from QED

$$\frac{d\sigma_{LO}}{dt} = 4\pi\alpha^2 \frac{(m^2 + m_e^2) - su - t^2/s}{t^2 \lambda(s, m^2, m_e^2)}$$

  - Kinematics $s = (p_1 + p_2)^2, \quad t = (p_2 - p_3)^2, \quad u = 2m^2 + 2m_e^2 - s - t$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

- Measure the cross section, subtract everything but the hadronic vac. pol.

  - 20×10^{-11} estimated statistical uncertainty on a_μ^{HLO} (0.3%)
  - systematics (exp. and th.) must be below 10 ppm

- Theory goal: Monte Carlo for QED μe at NNLO

μe–scattering at NNLO

- Needed fixed order corrections to μe–scattering

$$\sigma_{NLO} = \underbrace{\int dLIPS_2 \left(2\mathrm{Re}\mathcal{M}^{(0)\,*}\mathcal{M}^{(1)}\right)}_{\text{Virtual}} + \underbrace{\int dLIPS_3 |\mathcal{M}^{(0)}_{\gamma}|^2}_{\text{Real}}$$

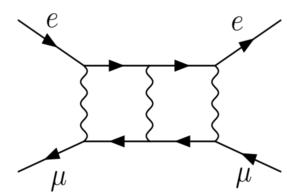
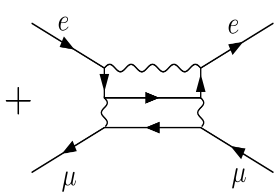
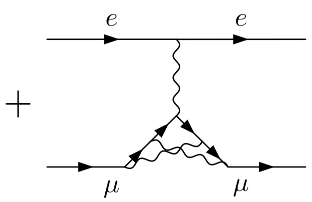
[Nikishov 61, Eriksson 61, ...]

$$\sigma_{NNLO} = \underbrace{\int dLIPS_2\left(2\mathrm{Re}\,\mathcal{M}^{(0)*}\mathcal{M}^{(2)} + |\mathcal{M}^{(1)}|^2\right)}_{\text{Double Virtual}} + \underbrace{\int dLIPS_3 2\mathrm{Re}\,\mathcal{M}^{(0)*}_{\gamma}\mathcal{M}^{(1)}_{\gamma}}_{\text{Real–Virtual}} + \underbrace{\int dLIPS_4|\mathcal{M}^{(0)}_{\gamma\gamma}|^2}_{\text{Double Real}}$$

- $\mathcal{M}^{(2)}(m_e^2, m^2)$ unknown (out of reach?)
    - given $m_e^2/m^2 \approx 2 \cdot 10^{-5}$, consider massless e
    - $\mathcal{M}^{(2)}(0, m^2)$ can be computed, log–dependence on m_e^2 to be retrieved

Workflow

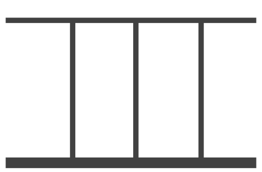
Amplitude generation

- $\mathcal{M}^{(2)}(e\mu \to e\mu) =$  $+$  $+$  $+ \dots$

Algebraic decomposition

- $$\mathcal{M}^{(2)}(e\mu \to e\mu) = \sum_k c_k(s,t,m^2,\varepsilon) I_k^{(2)}(s,t,m^2,\varepsilon)$$

Loop integral evaluation

- 
$$= -\frac{4}{3t^2\varepsilon^4\left(m^2-s\right)} + \frac{14G\left(1;\frac{s}{m^2}\right) - 9G\left(0;\frac{2m^2-t-\sqrt{t\left(t-4m^2\right)}}{2m^2}\right) + 18G\left(1;\frac{2m^2-t-\sqrt{t\left(t-4m^2\right)}}{2m^2}\right)}{6t^2\varepsilon^3\left(m^2-s\right)} + \mathcal{O}(\varepsilon^{-2})$$

Renormalisation, subtractions, ps-integration

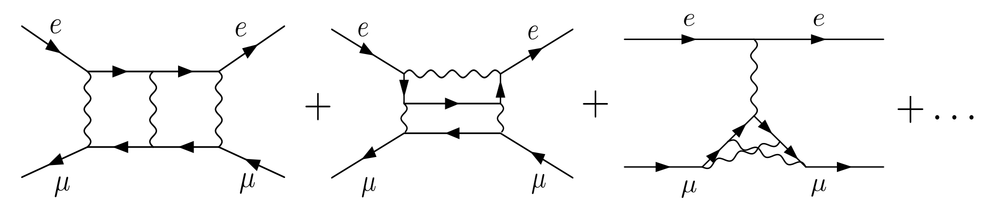
Workflow

Amplitude generation

Algebraic decomposition

Loop integral evaluation

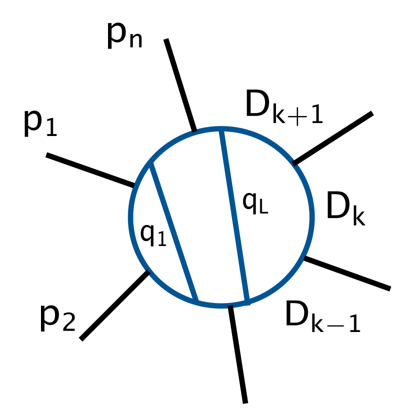
Renormalisation, subtractions, ps-integration

- $\mathcal{M}^{(2)}(e\mu \to e\mu) =$ 

  - 69 diagrams+ 1 Loop renormalisation

- $\mathcal{M}^{(2)}(e\mu \to e\mu) = \sum_k c_k(s,t,m^2,\varepsilon) I_k^{(2)}(s,t,m^2,\varepsilon)$

  - work at the integrand-level [Ossola, Papadopoulos, Pittau 06]



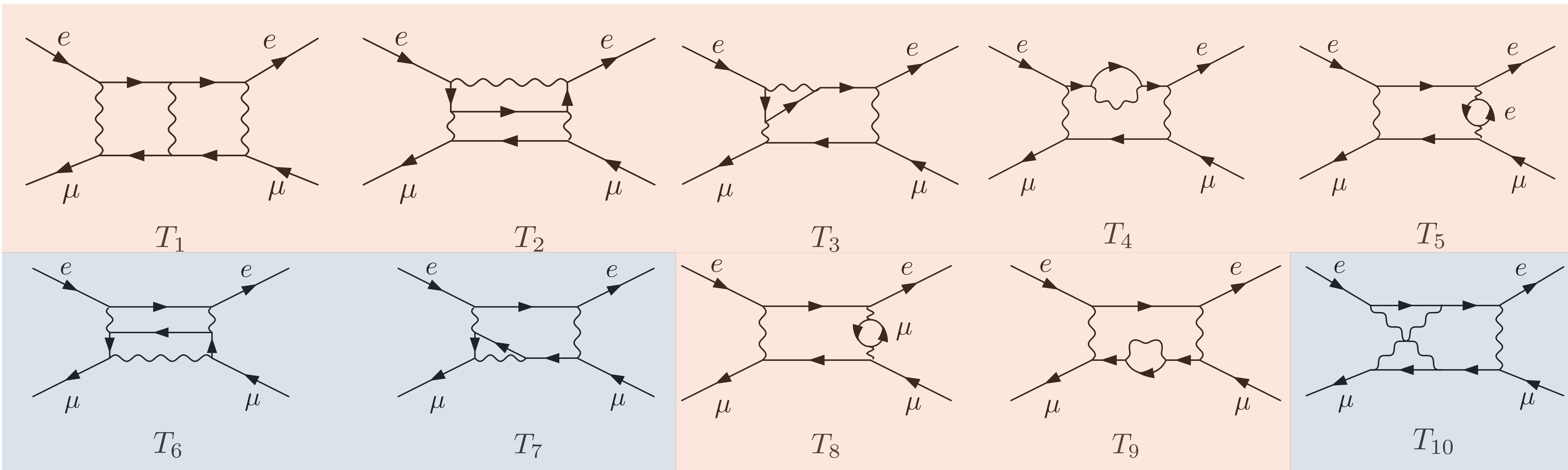
$$= \frac{N_{i_1 \cdots i_k \cdots i_m}(q_j)}{D_1 \cdots D_k \ldots D_m}$$

  - adaptive integrand decomposition [Mastrolia, Peraro, **AP** 16]
  - **Torres**' talk this afternoon
  - reduction to master integrals via integration-by-parts

[Chetyrkin, Tkachov 81, ..., Laporta 01]

Master integrals for μe–scattering

- Four-point topologies for μe-scattering at two loops:



- Most planar integrals known analytically from different processes
  - $t\bar{t}$ production in QCD [Gehrmann Remiddi 01, Bonciani Mastrolia Remiddi 04, ...]
  - BhaBha scattering in QED [Bonciani, Ferroglia 08, Asatrian, Greub, Pecjak 08, ...]
  - heavy-to-light quark decay in QCD [Bonciani, Ferroglia, Gehrmann 08, ...]

- Unknown integrals with more massive lines

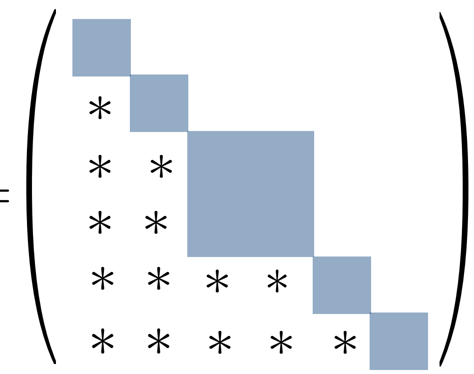
Differential equations method

- Master integrals $\vec{I} = (I_1, I_2, \ldots, I_N)$ fulfil coupled 1st order PDEs in the kinematics

$$\frac{\partial}{\partial x_i}\vec{I}(\vec{x},\varepsilon) = \mathbf{A}_i(\vec{x},\varepsilon)\vec{I}(\vec{x},\varepsilon)$$

[Kotikov 91, Remiddi 97, Gehrmann, Remiddi 00, …]

- Computation of the master integrals: solve PDEs + boundary conditions
  - $\mathbf{A}_i(\vec{x},\varepsilon)$ are block-triangular
  - $\mathbf{A}_i(\vec{x},\varepsilon)$ are rational in $\vec{x}$ and ε

$\mathbf{A}_i(\vec{x},\varepsilon) =$ 

- Master integrals determined by series expansion for $\varepsilon \approx 0$

$$\vec{I}(\vec{x},\varepsilon) = \sum_{k=0}^{\infty} I^{(k)}(\vec{x})\varepsilon^k$$

  - PDEs for Taylor coefficients $\vec{I}^{(k)}(\vec{x})$ (mostly) triangular

Canonical differential equations

- Systems of PDEs are not unique
  - change of variables: $\vec{x} \to \vec{y}(\vec{x})$ $$\frac{\partial}{\partial y_i}\vec{I} = \left[\frac{\partial x_j}{\partial y_i}\mathbf{A}_j(\vec{y},\varepsilon)\right]\vec{I}$$
  - change of basis: $\vec{I}(\vec{x},\varepsilon) = \mathbf{B}(\vec{x},\varepsilon)\vec{J}(\vec{x},\varepsilon)$ $$\frac{\partial}{\partial x_i}\vec{J} = \mathbf{B}^{-1}\left[\mathbf{A}_i\mathbf{B} - \frac{\partial}{\partial x_i}\mathbf{B}\right]\vec{J}$$
- Cast PDEs to canonical (=simplest) form

$$d\vec{I}(\vec{x},\varepsilon) = \varepsilon\left[\sum_{i=1}^{m}\mathbf{M}_i\,d\log\eta_i(\vec{x})\right]\vec{I}(\vec{x},\varepsilon)$$ [Henn 13]

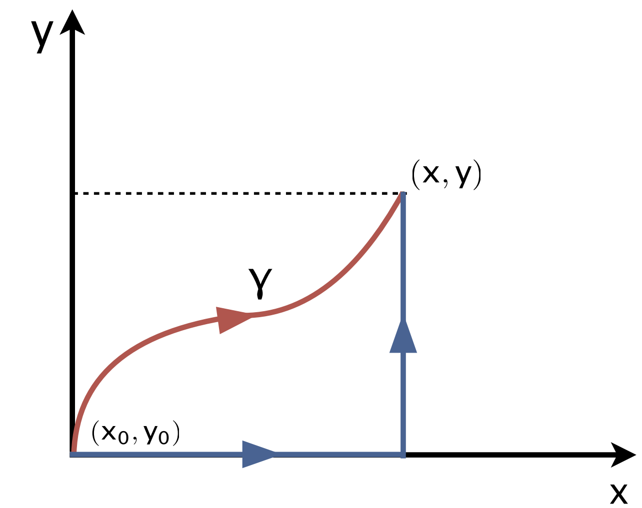
  - order-by-order decoupling

$$d\vec{I}^{(k)}(\vec{x}) = \sum_{i=1}^{m} d\log\eta_i(\vec{x})\,\vec{I}^{(k-1)}(\vec{x})$$

  - known classes of iterated integrals

Canonical differential equations

- Algorithmic solution in canonical form



$$\vec{I}(\vec{x}, \varepsilon) = \left[1 + \sum_{k=1}^{\infty} \int_{\gamma} d\mathbf{A} \dots d\mathbf{A} \right] \vec{I}(\vec{x}_0, \varepsilon)$$

  - algebraic $\eta_i(\vec{x})$: Chen iterated integrals [Chen 77]

  - rational $\eta_i(\vec{x})$: Generalised polylogarithms (GPLs)

$$\boxed{A_i(\vec{x}) = \sum_{j=1}^{m} \frac{\mathbf{M}_j}{x_i - \omega_j}}$$

$$G(\vec{0}_n; x) = \frac{1}{n!} \text{dlog}^n x \qquad G(\vec{\omega}_n; x) = \int_0^x \frac{dt}{t - \omega_1} G(\vec{\omega}_{n-1}; t)$$

[Goncharov 98, Remiddi, Vermaseren 99, Gehrmann, Remiddi 00, ...]

- canonical form: easy to solve but hard to find

Finding the canonical form

- Many strategies
  - unit leading singularity [Henn 13]
  - magnus exponential [Argeri, Di Vita, Mastrolia et al 14]
  - rational Ansätze for basis change [Gehrmann, Von Manteuffel, Tancredi et al 14,…]
  - reduction to fuchsian form and eigenvalue normalisation [Lee 15, Lee, Smirnov 16]
  - factorisation of the Picard-Fuchs operator [Adams, Chaubery, Weinzierl 17]
- Some public tools: **Canonica, Fuchsia, Epsilon**
- Go-condition: at $\varepsilon = 0$ decoupled 1st PDEs with algebraic homogeneous solutions
- Beyond one-loop: irreducible higher order PDEs
  - transcendental homogeneous solutions
  - iterated integrals over elliptic curves (many talks during L&L!)

Magnus method

- If canonical form exists, find the $\varepsilon = 0$ solution at once
  - ansatz: ε–linear PDEs

$$\frac{\partial}{\partial x_i}\vec{I} = \left[\mathbf{A}_i^{(0)}(\vec{x}) + \varepsilon\,\mathbf{A}_i^{(1)}(\vec{x})\right]\vec{I}$$

  - solve PDEs for $\varepsilon = 0$

$$\frac{\partial}{\partial x_i}\mathbf{B}(\vec{x}) = \mathbf{A}_i^{(0)}\mathbf{B}(\vec{x})$$

  - rotate to canonical form $\vec{I} = \mathbf{B}\vec{J}$

$$\frac{\partial}{\partial x_i}\vec{J} = \varepsilon\left[\mathbf{B}^{-1}\mathbf{A}_i^{(1)}\mathbf{B}\right]\vec{J}$$

- Formal solution of matrix differential equation by the Magnus exponential

[Magnus 54]

Magnus exponential

- One-variable case

$$\frac{d}{dx}\mathbf{B}(x) = \mathbf{A}^{(0)}(x)\mathbf{B}(x)$$

  - Magnus exponential

$$\mathbf{B}(x) = \exp\left(\sum_{k=1}^{\infty}\Omega_k[\mathbf{A}^{(0)}](x)\right)$$

$$\Omega_k[\mathbf{A}](t) = \begin{cases} \Omega_1[\mathbf{A}](t) = & \int dt_1 \mathbf{A}(t_1) \\ \Omega_2[\mathbf{A}](t) = & \int dt_1 dt_2 [\mathbf{A}(t_1), \mathbf{A}(t_2)] \\ \Omega_3[\mathbf{A}](t) = & \int dt_1 dt_2 dt_3 [\mathbf{A}(t_1), [\mathbf{A}(t_2), \mathbf{A}(t_3)]]_{(1,3)} \\ & \dots \end{cases}$$

- For Feynman integrals expressible in GPLs

$$\Omega_k[\mathbf{A}_i^{(0)}](\vec{x}) = 0 \qquad k > k_{max}$$

  - analytic change of basis from Magnus exponential

[Argeri, Di Vita, Mastrolia et al 14]

Magnus exponential

- Multivariate generalisation:

$$\frac{\partial}{\partial x_i}\mathbf{B}(\vec{x}) = \mathbf{A}_i^{(0)}\mathbf{B}(\vec{x}) \qquad i = 1, \dots, n$$

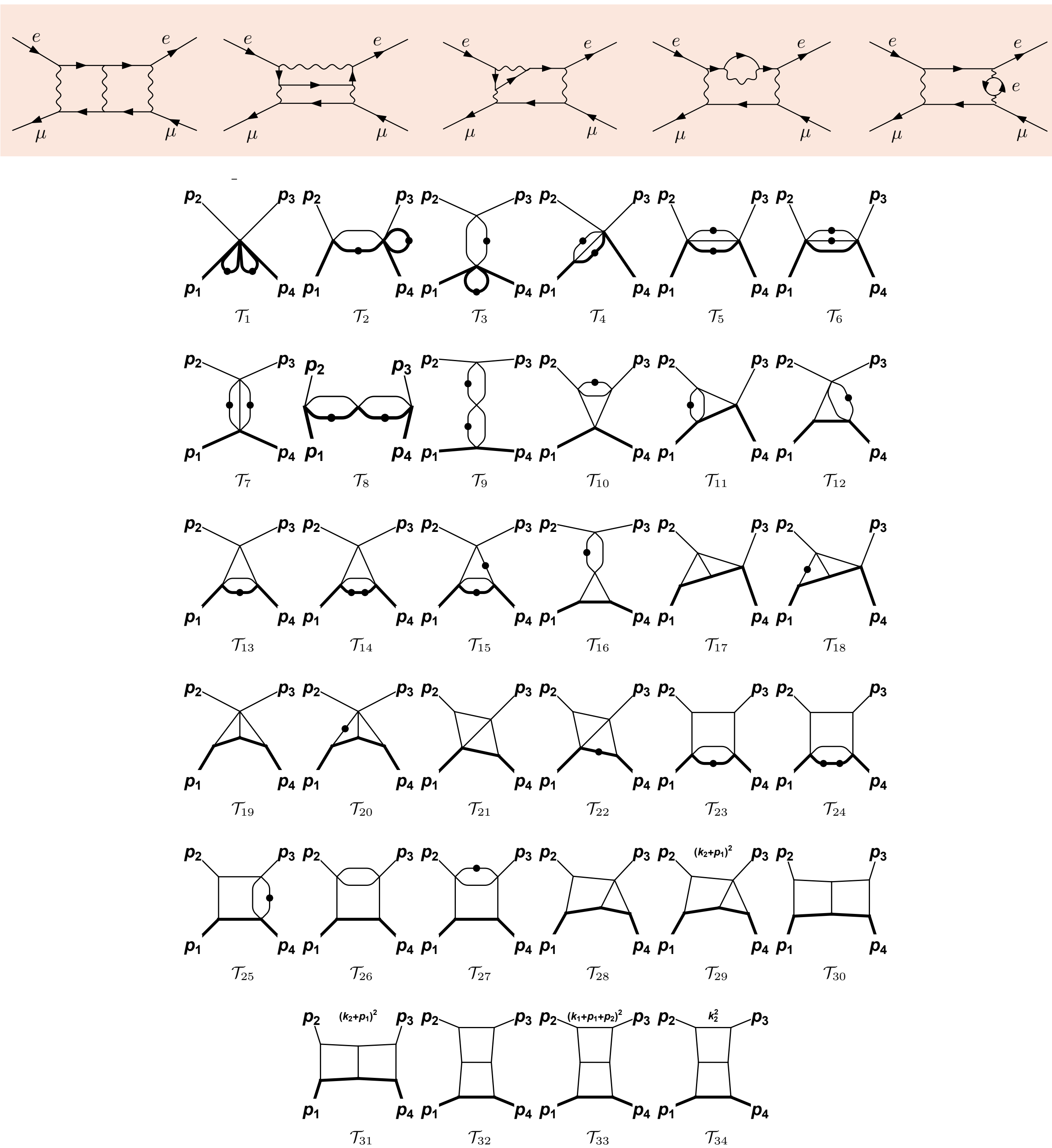
  - chained Magnus rotations

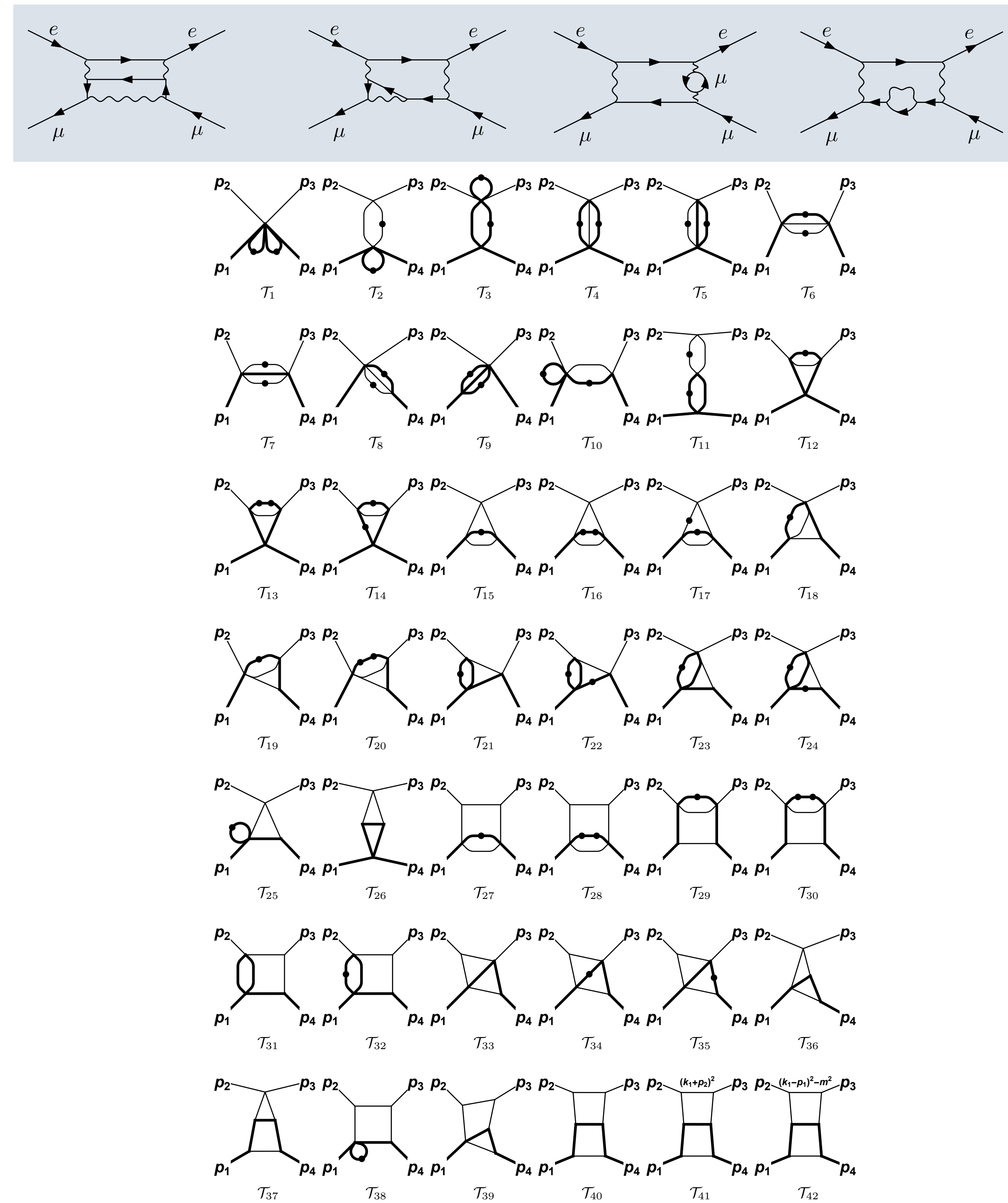
$$\mathbf{B}(\vec{x}) = e^{\Omega[\hat{A}_n^{(0)}]}.e^{\Omega[\hat{A}_{n-1}^{(0)}]}\dots.e^{\Omega[A_1^{(0)}]}$$

- Method applied to several multiscale problems
  - two-loop QED form factor [Argeri, Di Vita, Mastrolia et al 14]
  - three-loop H+j [Di Vita, Mastrolia, Schubert, Yundin 14]
  - mixed EW-QCD corrections to Drell-Yan [Bonciani, Di Vita, Mastrolia, Schubert 16]
  - mixed EW-QCD corrections to WWH, WWZ(γ^*) [Di Vita, Mastrolia, **AP**, Schubert 17]
- Used to compute μe-scattering integrals [Mastrolia, Passera, **AP**, Schubert 17]

Planar integrals for μe−scattering

- 65 distinct master integrals identified with **Reduze**





Canonical form

- ε-linear PDEs in two variables $s/m^2\,, t/m^2$

$$\frac{\partial}{\partial x_i}\vec{I} = \left[\mathbf{A}_i^{(0)}(\vec{x}) + \varepsilon\,\mathbf{A}_i^{(1)}(\vec{x})\right]\vec{I} \qquad i = 1, 2$$

- Magnus exponential:

$$\mathbf{B}(\vec{x}) = \exp\left(\sum_k \Omega_k[\hat{A}_2^{(0)}](\vec{x})\right).\exp\left(\sum_j \Omega_j[A_1^{(0)}](\vec{x})\right)$$

  - change of basis: $\vec{I} = \mathbf{B}\,\vec{J}$
  - change of variables: $s = -m^2\,x \qquad t = -m^2\,\frac{(1-y)^2}{y}$

- Canonical form: $d\vec{J}(x, y, \varepsilon) = \varepsilon\left[\sum_{i=1}^{9} \mathbf{M}_i \,\mathrm{dlog}\,\eta_i(x, y)\right]\vec{J}(x, y, \varepsilon)$

$$\begin{array}{lll} \eta_1 = x & \eta_4 = y & \eta_7 = x + y \\ \eta_2 = 1 + x & \eta_5 = 1 + y & \eta_8 = 1 + xy \\ \eta_3 = 1 - x & \eta_6 = 1 - y & \eta_9 = 1 - y(1 - x - y) \end{array}$$

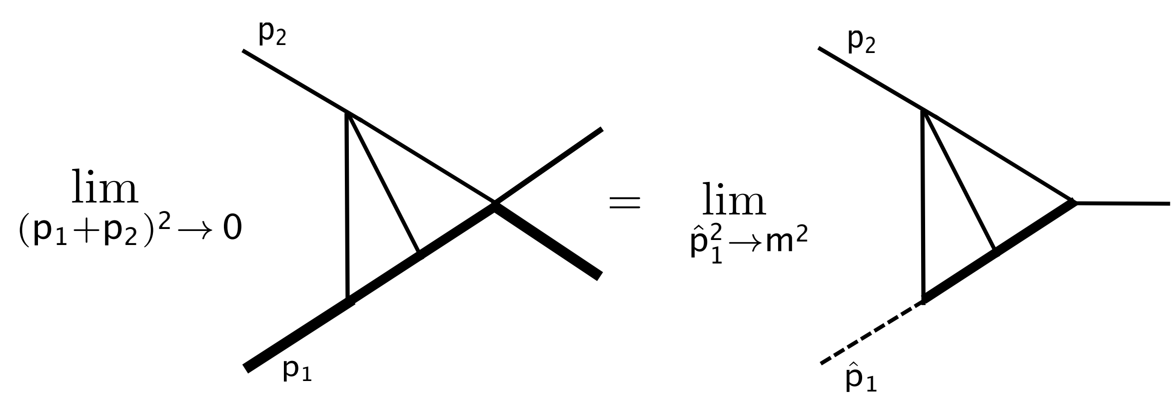
- Solution in terms of GPLs

Boundary conditions

- Boundary conditions from physical information
  - external input
  - regularity at pseudo-thresholds η_k of the PDEs

$$\lim_{\eta_k \to 0} \mathrm{M}_k \vec{\mathrm{I}}(\vec{x}, \epsilon) = 0$$

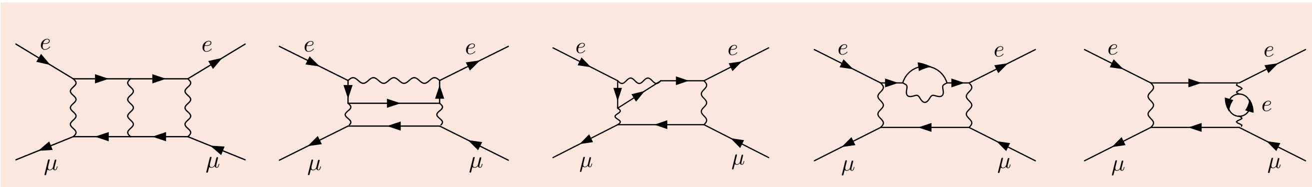
  - kinematic limits from auxiliary integrals

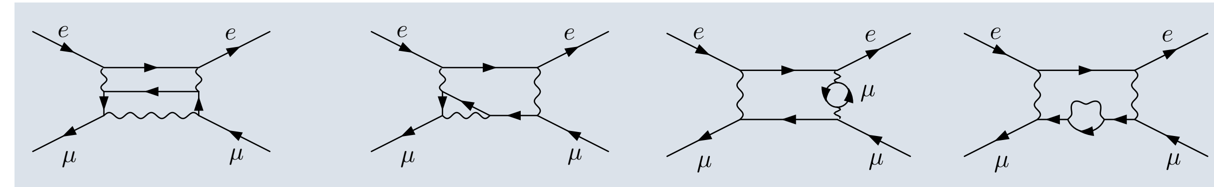


- Uniform combinations of constant GPLs fitted to ζ_k

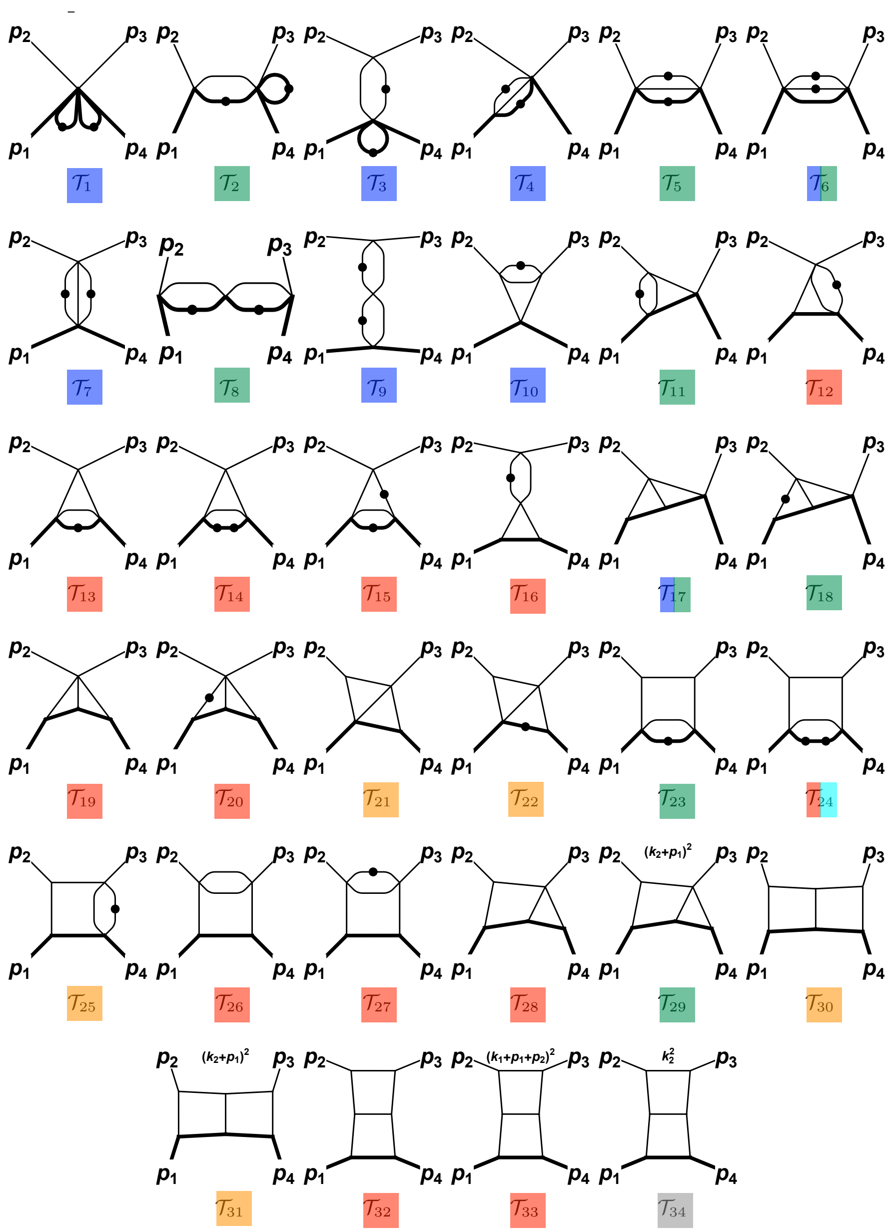
$$-59\zeta_4 = \pi^2 \left(G_{-1}^2 - 2\,G_{0,-(-1)^{\frac{1}{3}}} - 2\,G_{0,(-1)^{\frac{2}{3}}} \right) - 21\,\zeta_3\,G_{-1} - G_{-1}^4 - 18\,G_{0,0,0,-(-1)^{\frac{1}{3}}} - 18\,G_{0,0,0,(-1)^{\frac{2}{3}}}$$
$$+ 12\,G_{0,0,-(-1)^{\frac{1}{3}},-1} + 12\,G_{0,0,(-1)^{\frac{2}{3}},-1} + 12\,G_{0,-(-1)^{\frac{1}{3}},-1,-1} + 12\,G_{0,(-1)^{\frac{2}{3}},-1,-1} + 24\,G_{0,0,0,2}$$

Planar integrals

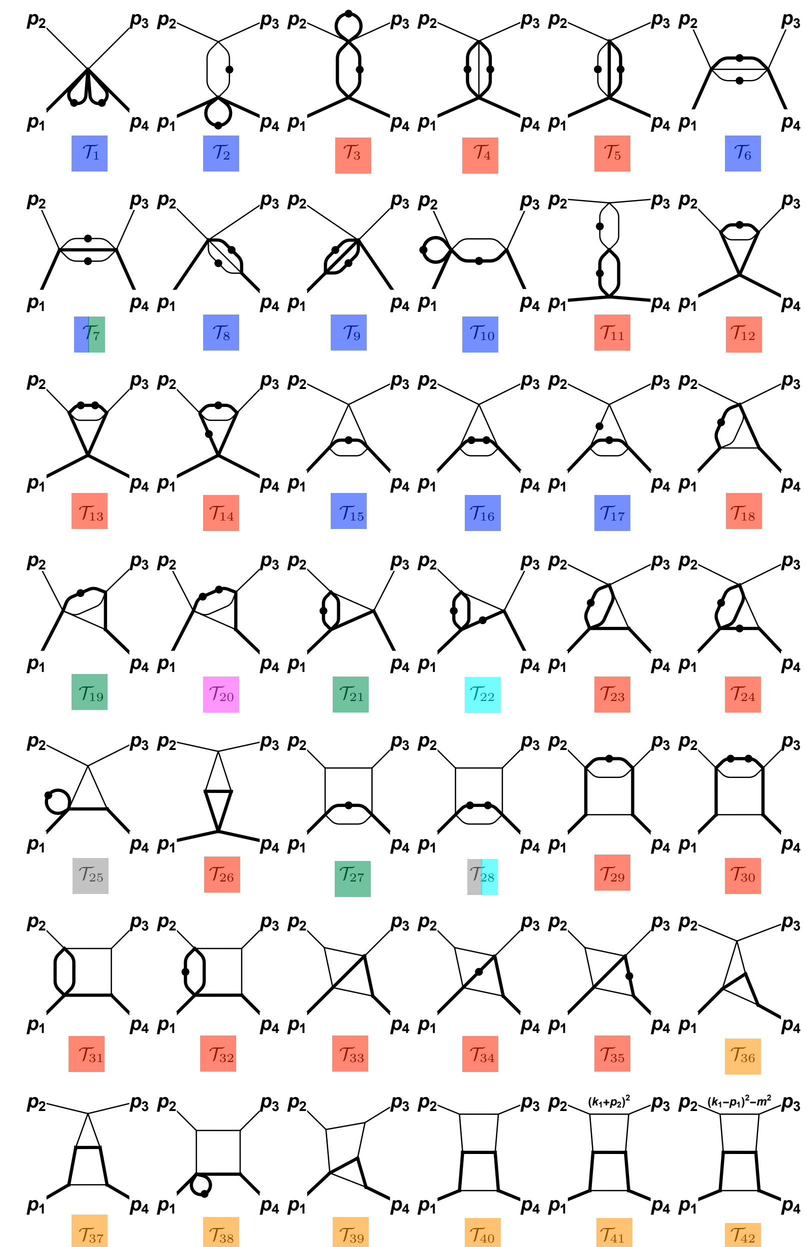




- Input
- $s \to 0$
- $t \to 4m^2$
- $u \to 2m^2$
- $s \to -m^2$
- $s/t \to 0$



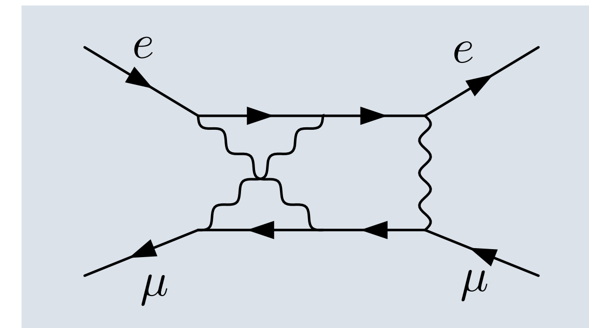
- Input
- $s \to 0$
- $t \to 0$
- $u \to m^2/2$
- $s \to -m^2$
- $t \to 4m^2$
- $s \to 2t - m^2 - \lambda_t$

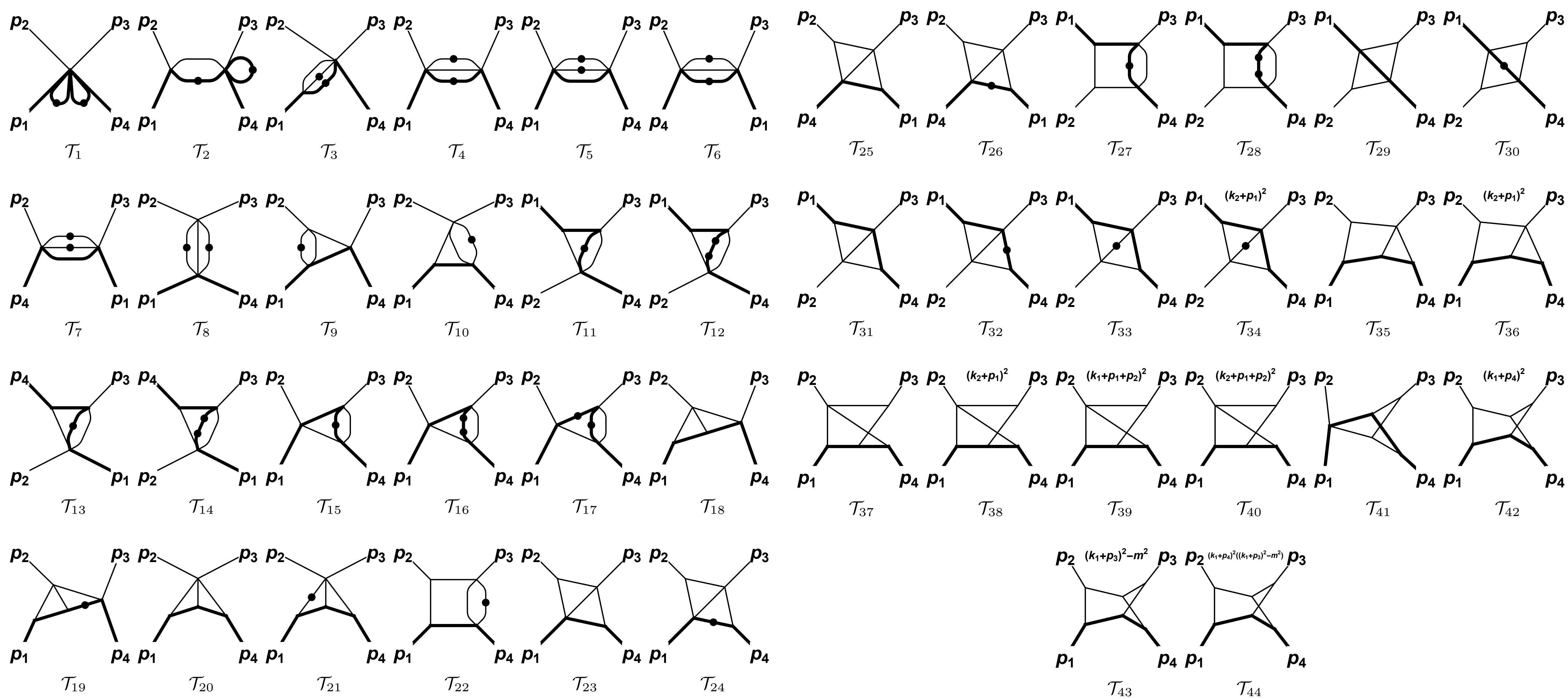


- Numerical evaluation (**GiNac**) validated against **SecDec**

Non-planar integrals for μe-scattering

- One single non-planar integral family is missing
  - 44 distinct master integrals identified with **Reduze**





[Di Vita, Laporta, Mastrolia, **AP**, Schubert in progress]

Canonical form

- Magnus exponential:

$$\mathbf{B}(\vec{x}) = \exp\left(\sum_k \Omega_k[\hat{A}_2^{(0)}](\vec{x})\right) . \exp\left(\sum_j \Omega_j[A_1^{(0)}](\vec{x})\right)$$

  - change of basis: $\vec{I} = \mathbf{B}\vec{J}$
  - change of variables: $t = -m^2 \frac{(1-w)^2}{w} \qquad s = m^2\left(1 + \frac{(1-w)^2}{w-z^2}\right)$

- Canonical form:

$$d\vec{J}(w,z,\varepsilon) = \varepsilon \left[\sum_{i=1}^{14} \mathbf{M}_i \mathrm{dlog}\eta_i(w,z)\right] \vec{J}(w,z,\varepsilon)$$

$$\begin{aligned}
\eta_1 &= w & \eta_2 &= 1+w & \eta_3 &= 1-w \\
\eta_4 &= z & \eta_5 &= 1+z & \eta_6 &= 1-z \\
\eta_7 &= w+z & \eta_8 &= w-z & \eta_9 &= w-z^2 \\
\eta_{10} &= 1-w+w^2-z^2 & \eta_{11} &= 1-3w+w^2+z^2 & \eta_{12} &= w^2-z^2+yz^2-y^2z^2
\end{aligned}$$

- Solution in terms of GPLs

Outlook and conclusions

- Experimental results for a_μ will improve soon
  - Theory prediction of a_μ^{Had} main source of uncertainty
  - Proposal for independent determination from μe-scattering
- Unknown QED prediction at NNLO are required
  - Virtual amplitude decomposed to master integrals
  - All planar master integrals are now available in the $m_e = 0$ limit
  - Non-planar integrals are on the way
  - $|\mathcal{M}_\gamma^{(1)}|^2$ can be computed with available tools
- Still a lot work need to get a NNLO generator

Outlook and conclusions

- Padova, 4th–5th September 2017

  µe scattering:
  Theory kickoff workshop

  Università degli Studi di Padova — INFN Istituto Nazionale di Fisica Nucleare

  - https://agenda.infn.it/conferenceDisplay.py?confId=13774

- Mainz, 19th–23th February 2018

  The Evaluation of the Leading Hadronic Contribution to the Muon Anomalous Magnetic Moment

  JG|U Johannes Gutenberg Universität Mainz — mitp Mainz Institute for Theoretical Physics

  - https://indico.mitp.uni-mainz.de/event/128/

- Next: **Zürich** February 2019