CP-odd invariants for multi-Higgs models and applications with discrete symmetry: explicit, spontaneous, and geometric violation of CP

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(Based on work with Christoph Luhn, Ivo de Medeiros Varzielas, Steve F. King:
arxiv:1603.06942 and 1704.xxxxx)

Outline

- CP Violation: Introduction
- CPV in multi-Higgs models
- Spontaneous CPV — the sales pitch
- CP-odd basis invariants
- Construction of new invariants
- Applications and Examples for explicit violation of CP
- Invariants for spontaneous CPV
- Minimizing complicated multi-Higgs potentials
- Summary of Results, Conclusions and Outlook

CP Violation: Introduction

- CP is a combined symmetry transformation of charge conjugation and space inversion, e.g. $\varphi(x) \mapsto X\varphi^*(x^P)$ with a unitary matrix X.
- Violation of CP seems to be simply necessary for Baryogenesis[1] and thus all our existence.
  - (Together with Baryon number violation.)
  - As measured by Planck: $n_B/n_\gamma = (6.10 \pm 0.04) \times 10^{-10}$
    In cosmic rays:[2] $\frac{\bar{p}}{p} \approx 10^{-4}$
- Overlap between flavour and CP: Only currently confirmed violation of CP happens in the quark Yukawa sector **but is not sufficient for succesful baryogenesis.**[3]
- Additional sources of CP violation are needed and can be introduced:
  - In an extended scalar sector [4] → later
  - Or in the lepton sector (As global fits are hinting at with $\delta_{CP} \simeq -\pi/2$.) → Leptogenesis [5] → Lepton-flavour-models
  - The latter also leads to an extended scalar sector: Flavour models normally need scalar particles that break extended symmetries.
- Via CPT theorem: Is a time direction built into the laws of nature?

[1] Sakharov, 1967

[2] Cline 2006, arxiv:0609145

[3] Kuzmin, Rubakov, Shaposhnikov 1985

[4] Lee, 1973

[5] Yanagida, Fukugita 1986

CPV in Multi-Higgs-Models

- In the most general 2HDM, additional CP violation is introduced in complex couplings[6].
- The most general 2HDM (when coupled to fermions) has FCNC.
- Forbidding the FCNC via symmetry automatically forbids CP violation in 2HDM $\rightarrow$ 3HDM, or more
- In flavour models, fermions transform under 3-dim irrep which needs to be broken $\rightarrow$ introduce scalars that transform under this irrep
- Extended scalar models can introduce two kinds of new CP violation:
- **Explicit** CPV: Parameters of potential violate CP in unbroken phase
- **Spontaneous** CPV: The Lagrangian is CP-invariant, vacuum violates CP
- Multi-Higgs potentials can have natural discrete symmetries, which can lead to
- **Geometric** CPV: The CP-violating phase is fixed by the symmetry. For a long time, only one example was known.
- Last but not least: A first Higgs particle has been discovered, there might be more! (Even if the new particle's mass is not going to be 750 GeV.)

[6] For a review of 2HDM: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva, 2011

Spontaneous CP Violation — the Sales Pitch

- Spontaneous violation of CP arises when the Lagrangian has a (generalised) CP symmetry, but the vacuum expectation value breaks this CP symmetry.
- CP is a symmetry which acts on known degrees of freedom in a known way.
- Models where CP is initially conserved have a greatly reduced number of parameters.
- Due to the controlled breaking of CP, low-energy observables are functions of possibly extremely-high-energy parameters.
- In models with a broken CP symmetry, there is no strong CP problem as initially the θ-term is forbidden.[7]
- For special cases, so-called geometric violation of CP can arise, in which the spontaneously CP violating phases are fixed by symmetry of the potential, and additionally are protected against renormalization corrections by a residual symmetry.[8]

[7] e.g. Branco, Lavoura, Silva, 1999

[8] Branco, Gerard, Grimus, 1983; Branco, Ivanov, 2015

CP-odd Basis invariants

- Extended scalar potentials are often too complicated to trivially recognize if CP is violated or not.
- Additionally basis changes obscure the situation.
- **CP-odd basis invariants**[9] solve both of these problems: eliminate basis changes and isolate CP-violating quantities.
- Example of CP-odd basis invariant in the fermion sector: Jarlskog invariant[10]
- Invariants exist that can indicate both explicit and spontaneous CPV and distinguish between the two.
- **However,** only for 2HDM a basis of invariants was known. For 3HDM with additional symmetries, these known invariants vanish trivially without indicating CP conservation.
- Observables of CP violation have to be functions of CP-odd invariants, but relation often non-trivial.
- CP is conserved $\Rightarrow$ All CP-odd invariants are zero
- At least one CP-odd invariant non-zero $\Rightarrow$ CP is violated

[9] Mendez, Pomarol, 1991; Lavoura, Silva, 1994; Botella, Silva, 1995; Branco, Rebelo, Silva-Marcos, 2005; Davidson, Haber, 2005; Gunion, Haber, 2005

[10] Jarlskog, 1985

Construction of new invariants

- Any even scalar potential can be written in a standard form:

$$V = \phi^{*a} Y_a^b \phi_b + \phi^{*a} \phi^{*c} Z_{ac}^{bd} \phi_b \phi_d$$

- Y and Z transform under symmetry and basis transformations.
- Any product of Ys and Zs where indices are correctly contracted form an invariant:

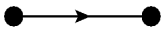
$$I_\sigma^{(n_Z, m_Y)} \equiv Y_{\sigma(a_1)}^{a_1} \ldots Y_{\sigma(a_{m_Y})}^{a_{m_Y}} Z_{\sigma(b_1)\sigma(b_2)}^{b_1 b_2} \ldots Z_{\sigma(b_{2n_Z-1})\sigma(b_{2n_Z})}^{b_{2n_Z-1} b_{2n_Z}} \text{ with } \sigma \in S_{m_Y+2n_Z}$$

- By construction, complex conjugation interchanges upper and lower indices
- The CP-odd part of any invariant can then be extracted:

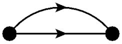
$$\mathcal{I} = I - I^*$$

Diagrams for invariants

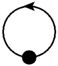
- Invariants can be expressed via diagrams[11] using the rule

$$X^{a\cdot}_{\cdot\cdot} X^{\cdot\cdot}_{a\cdot} =$$ 

- Additionally, because of the symmetry of Z^{ab}_{cd} under $a \leftrightarrow b$ and/or $c \leftrightarrow d$, one does not need to distinguish the two outgoing arrows:

$$Z^{ab}_{\cdot\cdot} Z^{\cdot\cdot}_{ab} =$$ 

- Contracting a tensor with itself forms a loop

$$X^{a\cdot}_{a\cdot} =$$ 

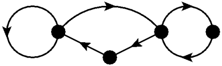
- As a CP transformation interchanges upper and lower indices,
the CP conjugate of a diagram is the identical diagram with inverted arrows.

[11] Davidson, Haber, 2005

The smallest CP-odd invariant (for explicit CP violation)

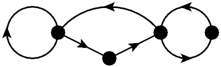
The smallest CP-odd invariant[12] (for explicit CP violation) is the difference $\mathcal{I}_1 = I_1 - I_1^*$ between

$$I_1 \equiv Z_{ae}^{ab} Z_{bf}^{cd} Y_c^e Y_d^f =$$



and its CP conjugate

$$I_1^* \equiv Z_{ab}^{ae} Z_{cd}^{bf} Y_e^c Y_f^d =$$



.

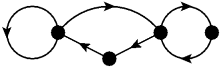
In whatever ways one tries to interchange the positions of vertices and arrows, it is impossible to make the diagrams equivalent.

[12] Branco, Rebelo, Silva-Marcos 2005

Beyond diagrams

- Each diagram can be expressed via a matrix m where m_{ij} denotes the number of arrows from the i-th tensor to the j-th tensor.
- Example:

$$I_1 \equiv Y^e_c Y^f_d Z^{ab}_{ae} Z^{cd}_{bf} =$$



$$= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

- For an arbitrary invariant, this matrix can be obtained from the permutation σ that defines the invariant
- A CP transformation interchanges upper and lower indices ⇔ inverts the direction of arrows ⇔ **transposes the matrix** m

Symmetries of diagrams in terms of matrices

- Diagrams are symmetric under permutations of the positions of vertices of the same type
- For an invariant with m_Y Y tensors and n_Z Z tensors, this means that for all $\sigma^Y \in S_{m_Y}$ and $\sigma^Z \in S_{n_Z}$, the matrices

$$\begin{pmatrix} \sigma_Y & 0 \\ 0 & \sigma_Z \end{pmatrix} m \begin{pmatrix} \sigma_Y & 0 \\ 0 & \sigma_Z \end{pmatrix}^T$$

  define identical invariants.
- From this follows

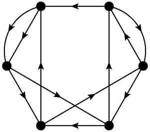
$$\text{Invariant CP-even} \Leftrightarrow m^T = \begin{pmatrix} \sigma_Y & 0 \\ 0 & \sigma_Z \end{pmatrix} m \begin{pmatrix} \sigma_Y & 0 \\ 0 & \sigma_Z \end{pmatrix}^T ,$$

- (Above expressions are for explicit CP violation. For spont. CPV, c.f. the paper.)
- The search for invariants then went as follows: Generate all matrices m, reduce to classes, test for CP-oddness.

Applications and Examples for explicit violation of CP

- A systematic search was performed and it was found there are for $m_Y = 0$ (for $m_Y \neq 0$ also a variety of invariants exist)
- no CP-odd invariants with n_Z up to 4
- only 3 "independent" CP-odd invariants with $n_Z = 5$
- only 5 "independent" CP-odd invariants with $n_Z = 6$.
- One of the most useful invariants is

$$I_2^{(6)} = Z^{a_1 a_2}_{a_7 a_{10}} Z^{a_3 a_4}_{a_{11} a_6} Z^{a_5 a_6}_{a_9 a_8} Z^{a_7 a_8}_{a_3 a_{12}} Z^{a_9 a_{10}}_{a_5 a_4} Z^{a_{11} a_{12}}_{a_1 a_2} =$$



- As example potentials we considered the general 2HDM; 3HDM and 6HDM invariant under A_4, S_4, $\Delta(27)$, $\Delta(54)$, or all larger $\Delta(3n^2)$ and $\Delta(6n^2)$ groups.
- We analysed the CP situation in many of those models for the first time, many new sources of (explict) CPV, full results see table in paper.

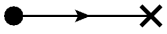
Results for explicit CP-violation

	$\mathcal{I}_2^{(6)}$	$\mathcal{I}_3^{(6)}$	$\mathcal{I}_4^{(6)}$	$\mathcal{I}_5^{(6)}$	CP
$(\mathbf{3}_{A_4}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	Eq. (4.5)
$(\mathbf{3}_{A_4}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	Eq. (4.10)
$2 \times (\mathbf{3}_{A_4}, \mathbf{1}_{SU(2)_L})$	*	*	*	*	NA
$2 \times (\mathbf{3}_{A_4}, \mathbf{2}_{SU(2)_L})$	*	*	*	*	NA
$(\mathbf{3}_{\Delta(27)}, \mathbf{1}_{SU(2)_L})$	0	0	Eq. (5.3)	Eq. (5.3)	NA
$(\mathbf{3}_{\Delta(27)}, \mathbf{2}_{SU(2)_L})$	0	0	Eq. (5.13)	Eq. (5.13)	NA
$2 \times (\mathbf{3}_{\Delta(27)}, \mathbf{1}_{SU(2)_L})$	*	*	*	*	NA
$2 \times (\mathbf{3}_{\Delta(27)}, \mathbf{2}_{SU(2)_L})$	*	*	*	*	NA
$(\mathbf{3}_{\Delta(3n^2)}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	Eq. (4.5)
$(\mathbf{3}_{\Delta(3n^2)}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	Eq. (4.10)
$2 \times (\mathbf{3}_{\Delta(3n^2)}, \mathbf{1}_{SU(2)_L})$	Eq. (6.5)	*	*	*	NA
$2 \times (\mathbf{3}_{\Delta(3n^2)}, \mathbf{2}_{SU(2)_L})$	*	*	*	*	NA
$(\mathbf{3}_{S_4}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.5)
$(\mathbf{3}_{S_4}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.10)
$2 \times (\mathbf{3}_{S_4}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.15)
$2 \times (\mathbf{3}_{S_4}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.20)
$(\mathbf{3}_{\Delta(54)}, \mathbf{1}_{SU(2)_L})$	0	0	*	*	NA
$(\mathbf{3}_{\Delta(54)}, \mathbf{2}_{SU(2)_L})$	0	0	*	*	NA
$2 \times (\mathbf{3}_{\Delta(54)}, \mathbf{1}_{SU(2)_L})$	0	*	*	*	NA
$2 \times (\mathbf{3}_{\Delta(54)}, \mathbf{2}_{SU(2)_L})$	*	*	*	*	NA
$(\mathbf{3}_{\Delta(6n^2)}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.5)
$(\mathbf{3}_{\Delta(6n^2)}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.10)
$2 \times (\mathbf{3}_{\Delta(6n^2)}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.15)
$2 \times (\mathbf{3}_{\Delta(6n^2)}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.20)

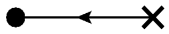
Table 1: Summary of CPIs and (if applicable) CP symmetry transformations for scalar potentials with discrete symmetry.

Invariants for spontaneous CP violation

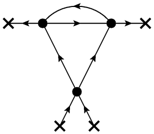
- Additional diagrammatic rules:

$$X^{a\cdot\cdot}_{\cdot\cdot} v_a =$$ 

and

$$X^{\cdot\cdot}_{a\cdot\cdot} v^{*a} =$$ 

- Again, a systematic search for invariants using the symmetries of the diagrams was performed and various new useful invariants were found.
- Example of a useful invariant

$$J_1^{(3,2)} \equiv Z^{a_1 a_2}_{a_4 a_5} Z^{a_3 a_4}_{a_2 a_6} Z^{a_5 a_6}_{a_7 a_8} v_{a_1} v_{a_3} v^{*a_7} v^{*a_8} =$$ 

- Again, we analyzed 24 example models with 3 or 6 Higgs fields, many for the first time, and found new sources of spontaneous CPV and for $N = 6$ new sources of **geometric** CPV.
- But, finding global minima of potentials with more than two non-zero vev components can be non-trivial.

Minimizing complicated Multi-Higgs-Potentials (1)

- Previously, only for the 3HDM with discrete symmetries, global minima had been obtained systematically.[13]
- We employed a symmetry-based method: Consider a potential

$$V = V_G + V_H \text{ with } H \subset G$$

- All minima of potentials generally fall into so-called orbits

$$[v_i]_H = \{H \cdot v_i\}$$

- Now consider the sub-potential V_G separately, it has minima in orbits

$$[v_i]_G = \{G \cdot v_i\}$$

- Consider V_H as terms that explicitly break G to H.
- By this, the of orbits of V_G are now split into several orbits organized by H.

[13] Ivanov, Nishi, 2014

Minimizing complicated Multi-Higgs-Potentials (2)

- In other words, by H degrees of freedom that were unphysical under G, are made physical.
- Now only the part of the potential that depends on those new dof has to be minimized, which is normally analytically possible.
- The minima so obtained are non-trivial, guaranteed to be global minima, and are often stable against parameter perturbations (=geometric).
- One would assume that the list of new minima so obtained is incomplete, but surprisingly, this method reproduces analytically the results for 3HDM identically.
- These are the first more systematic and analytical results for 6HDM.
- Conjecture that there is a symmetry reason for the completeness of the list of minima found because of special relations between groups considered.

	$J^{(3,2)}$	$J^{(3,3)}$	CP	SCPV
$\mathbf{3}_{\Delta(3n^2)=\Delta(6n^2)}$	0	0	X_0,X_{23}	No
$\mathbf{3}_{A_4}$	0	*	X_{23}	No
$\mathbf{3}_{S_4}$	0	*	$X_0,\ X_{23}$	No
$\mathbf{3}_{\Delta(27)=\Delta(54)}$	*	*	None	NA
$\mathbf{3}_{\Delta(27)=\Delta(54)},\ X_0$	*	*	X_0	S(G)CPV
$\mathbf{3}_{\Delta(27)=\Delta(54)},\ X_4$	*	*	X_4	S(G)CPV

	$J^{(3,2)}$	$J^{(3,3)}$	CP	SCPV
$2\times\mathbf{3}_{\Delta(3n^2)}$	*	*	None	NA
$2\times\mathbf{3}_{\Delta(3n^2)},CP_0$	*	*	X_0	S(G)CPV
$2\times\mathbf{3}_{\Delta(3n^2)},\ CP_{23}$	*	*	X_{23}	S(G)CPV
$2\times\mathbf{3}_{\Delta(3n^2)},\ CP_X$	*	*	X'	S(G)CPV
$2\times\mathbf{3}_{\Delta(6n^2)}$	0	*	X_0,X_{23}	No
$2\times\mathbf{3}_{A_4}$	*	*	None	NA
$2\times\mathbf{3}_{A_4},\ CP_0$	*	*	X_0	S(G)CPV
$2\times\mathbf{3}_{A_4},\ CP_{23}$	*	*	X_{23}	S(G)CPV
$2\times\mathbf{3}_{S_4}$	0	*	$X_0,\ X_{23}$	No
$2\times\mathbf{3}_{\Delta(27)}$	*	*	None	NA
$2\times\mathbf{3}_{\Delta(27)},\ CP_0$	*	*	X_0	S(G)CPV
$2\times\mathbf{3}_{\Delta(54)}$	*	*	None	NA
$2\times\mathbf{3}_{\Delta(54)},\ CP_0$	*	*	X_0	S(G)CPV

Summary of Results, Conclusions and Outlook (1)

- Additional Sources of CP violation beyond the SM are needed.
- A straightforward source can be an extended scalar sector.
- Besides that, most SM-extensions require an extended scalar sector.
- It is non-trivial to determine if CP is violated in such potentials.
- CP-odd basis invariants provide a model-independent tool for determining the CP properties of scalar potentials.
- Diagrammatic methods are helpful in finding CP-odd invariants.
- We further developed these methods and provide an algorithm to enable a systematic search for **all** invariants up to a given order by exploiting symmetries of diagrams.
- We performed such a search up to a specific order and found new non-zero CP-odd invariants for all potentials considered.
- Some of these invariants immediately provided new information about the CP situation in new and known potentials.

Summary of Results, Conclusions and Outlook (2)

- Furthermore, we analyse 24 example potentials for spontaneous breaking of CP
- For minimizing those potentials with up to 6 Higgs fields, we conjecture a method based on the symmetries of vev orbits.
- We find or exclude spontaneous CPV for all those models and
- Find new instances of geometric CPV.
- We also conjecture a relation between instances of geometric CPV and certain preserved CP-type symmetries and positively test it for all models considered.

Outlook

- Symmetries of diagrams as theoretical tool: Basis of invariants; Relation between invariance group and CPV; Relation between explicit and spontaneous CPV; New methods for minimizing potentials; Systematically express Observables by CPIs; etc; Eventually re-write physics in terms of invariants of internal symmetries/basis transformations.
- Could the CP violation found explain the baryon asymmetry?

BACKUP

Multi-Higgs-Potentials: Standard form

- Any even scalar potential can be written in a standard form:

$$V = \phi^{*a}\, Y_a^b\, \phi_b + \phi^{*a}\phi^{*c}\, Z_{ac}^{bd}\, \phi_b\phi_d$$

- For N SM-singlets:

$$\phi = (\varphi_1, \ldots, \varphi_N) \text{ and } \phi^* = (\varphi_1^*, \ldots, \varphi_N^*)$$

- For n Higgs-doublets:

$$\phi = (\varphi_1, \varphi_2, \ldots, \varphi_{2n-1}, \varphi_{2n}) = (h_{1,1}, h_{1,2}, \ldots, h_{n,1}, h_{n,2})$$

- By construction, Z_{ac}^{bd} is symmetric under $b \leftrightarrow d$ and/or $a \leftrightarrow c$.

Multi-Higgs-Potentials: Symmetry and basis transformations

- The potential is invariant under **symmetry transformations**. With

$$\phi_a \mapsto [\rho(g)]_a^{a'} \phi_{a'},$$

$$Y_a^b = \rho_a^{a'} \, Y_{a'}^{b'} \, \rho^\dagger{}_{b'}^{b}$$

$$Z_{ac}^{bd} = \rho_a^{a'} \, \rho_c^{c'} \, Z_{a'c'}^{b'd'} \rho^\dagger{}_{b'}^{b} \, \rho^\dagger{}_{d'}^{d}$$

- **Basis transformations** on the other hand change the potential but should not change the physics described by it. With

$$\phi_a \mapsto V_a^{a'} \phi_{a'} \ ,$$

the couplings of the potential transform as

$$Y_a^b \mapsto V_a^{a'} \, Y_{a'}^{b'} \, V^\dagger{}_{b'}^{b} \text{ and}$$

$$Z_{ac}^{bd} \mapsto V_a^{a'} \, V_c^{c'} \, Z_{a'c'}^{b'd'} \, V^\dagger{}_{b'}^{b} \, V^\dagger{}_{d'}^{d}$$

Multi-Higgs-Potentials: CP transformations

- **Complex conjugation** interchanges positions of upper and lower indices

$$\phi_a \mapsto (\phi_a)^* \equiv \phi^{*\,a}$$

- Reality of the potential, $V = V^*$, results in

$$(Y^a_b)^* = Y^b_a \text{ and } (Z^{ac}_{bd})^* = Z^{bd}_{ac}$$

  (These equations are not between tensors but between tensor components.)

- **(General) CP transformations** combine basis transformations and complex conjugation:

$$\phi_a \mapsto \phi^{*\,a'} U^a_{a'}$$

  Under which the couplings transform as

$$Y^b_a \mapsto U^{a'}_a \, (Y^{b'}_{a'})^* \, {U^\dagger}^b_{b'}$$

$$Z^{bd}_{ac} \mapsto U^{a'}_a U^{c'}_c \, (Z^{b'd'}_{a'c'})^* \, {U^\dagger}^b_{b'} {U^\dagger}^d_{d'}$$

Basis invariants: examples with Y

- Any product of Y and Z tensors where all indices are correctly contracted forms a basis invariant.
- Simplest example: Y_a^a
- With two Y tensors: $Y_a^a Y_b^b = (Y_a^a)^2$ and $Y_b^a Y_a^b$
- These correspond to possible permutations of indices:

$$Y_a^a Y_b^b \Leftrightarrow a \mapsto a \text{ and } b \mapsto b$$

  and

$$Y_b^a Y_a^b \Leftrightarrow a \mapsto b \text{ and } b \mapsto a$$

- Formally:

$$Y_{\sigma(a)}^a Y_{\sigma(b)}^b \text{ with } \sigma \in S_2$$

- Continues like that: $Y_{\sigma(a)}^a Y_{\sigma(b)}^b Y_{\sigma(c)}^c$ with $\sigma \in S_3$ etc
- These correspond to products of traces of powers of Y.
- Many larger invariants are not independent.

Basis invariants: examples with Z

- One Z tensor: $Z^{ab}_{\sigma(a)\sigma(b)}$ with $\sigma \in S_2$
- Explicitly: Z^{ab}_{ab} and Z^{ab}_{ba}
- Thanks to an internal symmetry, these are identical.
- Two Z tensors: $Z^{ab}_{\sigma(a)\sigma(b)} Z^{cd}_{\sigma(c)\sigma(d)}$ with $\sigma \in S_4$
- Only two combinations are new and can be chosen to be $Z^{ab}_{bd} Z^{cd}_{ac}$ and $Z^{ab}_{cd} Z^{cd}_{ab}$

Basis invariants: General expression and CP

- A general invariant from m_Y Y and n_Z Z tensors can be written as

$$I_\sigma^{(n_Z,m_Y)} \equiv Y^{a_1}_{\sigma(a_1)} \dots Y^{a_{m_Y}}_{\sigma(a_{m_Y})} Z^{b_1 b_2}_{\sigma(b_1)\sigma(b_2)} \dots Z^{b_{2n_Z-1} b_{2n_Z}}_{\sigma(b_{2n_Z-1})\sigma(b_{2n_Z})} \text{ with } \sigma \in S_{m_Y+2n_Z}$$

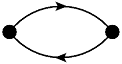
- All of the simplest examples from the previous slides were CP-even
- Recall how a CP transformation acts on Y and Z: Upper and lower indices are interchanged and the tensors are multiplied with basis transformations.
- In a basis invariant, basis transformations cancel and the CP-conjugate can be obtained by interchanging upper and lower indices:

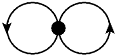
$$[I_\sigma^{(n_Z,m_Y)}]^* = Y_{a_1}^{\sigma(a_1)} \dots Y_{a_{m_Y}}^{\sigma(a_{m_Y})} Z_{b_1 b_2}^{\sigma(b_1)\sigma(b_2)} \dots Z_{b_{2n_Z-1} b_{2n_Z}}^{\sigma(b_{2n_Z-1})\sigma(b_{2n_Z})}$$

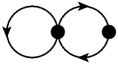
- The CP-odd part of any invariant can then be extracted:

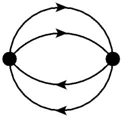
$$\mathcal{I} = I - I^*$$

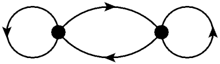
Example diagrams for small invariants

$Y^a_b Y^b_a =$ 

$Z^{ab}_{ab} =$ 

$Z^{ac}_{bc} Y^b_a =$ 

$Z^{ab}_{cd} Z^{cd}_{ab} =$ 

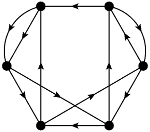
$Z^{ab}_{ac} Z^{cd}_{bd} =$ 

Note that all these diagrams do not change when inverting arrows, which means that the corresponding invariants are all CP-even.

Larger CP-odd examples

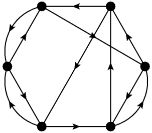
- A systematic search (with methods still to be explained) was performed and it was found there are for $m_Y = 0$ (for $m_Y \neq 0$ also a variety of invariants exist)
- no CP-odd invariants with n_Z up to 4
- only 3 different CP-odd invariants with $n_Z = 5$
- only 5 different CP-odd invariants with $n_Z = 6$.
- Two of the most useful invariants are

$$I_2^{(6)} = Z^{a_1 a_2}_{a_7 a_{10}} Z^{a_3 a_4}_{a_{11} a_6} Z^{a_5 a_6}_{a_9 a_8} Z^{a_7 a_8}_{a_3 a_{12}} Z^{a_9 a_{10}}_{a_5 a_4} Z^{a_{11} a_{12}}_{a_1 a_2} =$$



and

$$I_4^{(6)} = Z^{a_1 a_2}_{a_{11} a_{10}} Z^{a_3 a_4}_{a_5 a_8} Z^{a_5 a_6}_{a_7 a_{12}} Z^{a_7 a_8}_{a_9 a_6} Z^{a_9 a_{10}}_{a_1 a_4} Z^{a_{11} a_{12}}_{a_3 a_2} =$$



Example potentials considered

As example potentials we considered

- the most general 2HDM, where known results from the literature were reproduced by the automatic search [14]

and all potentials of

- one triplet of SM singlets
- one triplet of SM Higgs doublets
- two triplets of SM singlets
- two triplets of SM Higgs doublets

for the triplet representations from A_4, S_4, $\Delta(27)$, $\Delta(54)$, or all larger $\Delta(3n^2)$ and $\Delta(6n^2)$ groups. (And occasional U(1) symmetries to eliminate unwished for cross-terms.)

- $\Delta(3n^2)$ and $\Delta(6n^2)$ groups have a particular position among flavour symmetries[15]
- and even if they turn out to be unphysical, they are a rich group theoretical laboratory for testing ideas and methods

[14] Gunion, Haber 2005

[15] TN, King, Stuart, Luhn, Ding, Medeiros Varzielas, Altarelli, Feruglio, Hagedorn, Ziegler, Grimus, Fonseca, ...

Interesting example invariants

- one triplet of $\Delta(27)$:

$$\mathcal{I}_{4,5}^{(6)} = -\frac{3}{32}\left(d^3 - d^{*3}\right)\left(d^3 + 6dd^*s + d^{*3} - 8s^3\right)$$

- two triplets of $\Delta(3n^2)$ for $n > 3$:

$$\mathcal{I}_{2}^{(6)} = \frac{3}{512} i s_2 s_3 (-3r_2^2 + s_3^2)(-s_1^2 + s_1 s_2 + r_2(-2s_1 + s_2) + s_3^2)$$

Spontaneous CP Violation: Definitions

- VEVs transform as vectors under basis transformations:

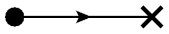
$$v_a \mapsto V_a^{a'} v_{a'} , v^{*a} \mapsto v^{*a'} V^{\dagger a}_{a'} .$$

- With this, invariants (Jnvariants) with VEVs can be written as

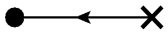
$$J_\sigma^{(n_v, m_Y, n_Z)} \equiv W^{w_1 \ldots w_{n_v}}_{\sigma(w_1)\ldots\sigma(w_{n_v})} Y^{a_1}_{\sigma(a_1)} \ldots Y^{a_{m_Y}}_{\sigma(a_{m_Y})} Z^{b_1 b_2}_{\sigma(b_1)\sigma(b_2)} \ldots Z^{b_{2n_Z-1} b_{2n_Z}}_{\sigma(b_{2n_Z-1})\sigma(b_{2n_Z})}$$

with $W^{w_1 \ldots w_{n_v}}_{w'_1 \ldots w'_{n_v}} = v_{w'_1} \ldots v_{w'_{n_v}} v^{*w_1} \ldots v^{*w_{n_v}}$.

- Additional diagrammatic rules:

$X^{a\cdot\cdot}_{\cdot\cdot} v_a =$ 

and

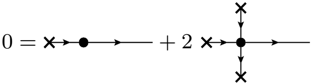
$X^{\cdot\cdot}_{a\cdot\cdot} v^{*a} =$ 

Spontaneous CP Violation: Equations of Motion

- Now, the minimisation condition can be expressed in term of diagrams:

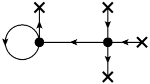
$$0 = \frac{\partial V}{\partial \phi_e} = \phi^{*a} Y_a^e + 2\phi^{*a}\phi^{*c} Z_{ac}^{ed}\phi_d$$

becomes

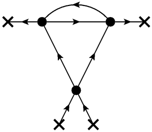


Spontaneous CP Violation: Example Diagrams

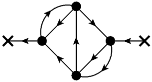
$$J_1^{(2,2)} \equiv Z^{a_1 a_2}_{a_1 a_3} Z^{a_3 a_4}_{a_5 a_6} v_{a_2} v_{a_4} v^{*a_5} v^{*a_6} =$$



$$J_1^{(3,2)} \equiv Z^{a_1 a_2}_{a_4 a_5} Z^{a_3 a_4}_{a_2 a_6} Z^{a_5 a_6}_{a_7 a_8} v_{a_1} v_{a_3} v^{*a_7} v^{*a_8} =$$



$$J_1^{(4,1)} \equiv Z^{a_1 a_2}_{a_3 a_5} Z^{a_3 a_4}_{a_7 a_8} Z^{a_5 a_6}_{a_1 a_4} Z^{a_7 a_8}_{a_2 a_9} v_{a_6} v^{*a_9} =$$



.

Spontaneous CP Violation: A $\Delta(27)$ example

$$
\begin{aligned}
\mathcal{J}_1^{(3,2)} =& \frac{1}{4}(d^{*3} - d^3)(|v_1|^4 + |v_2|^4 + |v_3|^4 - 2|v_1|^2|v_2|^2 - 2|v_1|^2|v_3|^2 - 2|v_2|^2|v_3|^2) \\
&+ \frac{1}{2}(dd^{*2} - 2d^* s^2 + d^2 s)(v_2 v_3 v_1^{*2} + v_1 v_3 v_2^{*2} + v_1 v_2 v_3^{*2}) \\
&- \frac{1}{2}(d^2 d^* - 2ds^2 + d^{*2} s)(v_2^* v_3^* v_1^2 + v_1^* v_3^* v_2^2 + v_1^* v_2^* v_3^2)
\end{aligned}
$$

Summary: CP-odd Invariants

- Additional Sources of CP violation beyond the SM are needed
- A straightforward source can be an extended scalar sector
- Besides that, most SM-extensions require an extended scalar sector
- It is non-trivial to determine if CP is violated in such potentials
- CP-odd basis invariants provide a model-independent tool for determining the CP properties of scalar potentials
- Diagrammatic methods are helpful in finding CP-odd invariants
- We further developed these methods to enable a systematic search for *all* invariants up to a given order by exploiting symmetries of diagrams
- We performed such a search up to a specific order and found new non-zero CP-odd invariants for all potentials considered
- Some of these invariants immediately provided new information about the CP situation in new and known potentials

Outlook

- CP-odd invariants
  - Next Paper: Apply this machine to spontaneous CP violation (so far only $\Delta(27)$ as minimal working example)
  - Trilinear terms?
  - Fermions?
  - Symmetries of diagrams as theoretical tool: Basis of invariants; Relation between invariance group and CPV; Relation between explicit and spontaneous CPV; New methods for minimizing potentials; etc; Eventually re-write physics in terms of invariants of internal symmetries.
- Could the CP violation found above explain the baryon asymmetry?