Discrete family symmetry models and unification

Ivo de Medeiros Varzielas

CFTP, IST, UL

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The Standard Model

Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

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Chiral spin 1/2 fermions (left and right)
Quarks: colour triplets of SU(3)_C
Left fermions are doublets of SU(2)_L
Spin 0 scalar, doublet of SU(2)_L
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The Standard Model has 3 fermion generations



The Standard Model is very successful but...

- Neutrinos have masses (*v*SM)
- Dark matter (no viable explanation)
- Matter / antimatter asymmetry (no viable explanation)
- Hierarchy problem (fine-tuning between parameters)
- Strong CP problem (fine-tuning between parameters)
- Gauge couplings (additional free parameters) GUT?
- Flavour problem (many additional free parameters) FS?

BSM solutions involve additional fields and symmetries

The Standard Model flavour problem: masses

3 fermion generations. Masses span orders of magnitude



The Standard Model flavour problem: mixing

3 generations of quarks, small mixing



3 generations of leptons, large and peculiar mixing



(mixing between weak and mass eigenstates)

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Summary of data: quark mixing

Wolfenstein parametrisation

$$V_{CKM} \simeq \left(egin{array}{cc} 1 & \lambda & \lambda^3 \ -\lambda & 1 & \lambda^2 \ \lambda^3 & -\lambda^2 & 1 \end{array}
ight)$$

 $\lambda \simeq$ 0.23 (Sine of the Cabibbo angle)

Summary of data: lepton mixing



Beyond the Standard Model with family symmetries

Without $y_f H \overline{F} f$, $\mathcal{L}_{\nu SM}$ has accidental symmetry $SU(3)^6$

FS: upgrade subgroup of $SU(3)^6$ to actual symmetry of $\mathcal L$

- Generations charged differently under FS
- Yukawa couplings no longer invariant
- FS must be broken somehow...

Non-Abelian?

3 reasons

- 3 generations explained naturally
- ν SM: FS \subset *SU*(3)⁶; *SO*(10) GUT: FS \subset *SU*(3)
- Lepton mixing strongly suggests non-Abelian FS

Why discrete symmetries?

Special angles in direct models / special alignments in indirect models (see talks by M. Tanimoto and G.-J. Ding)

Automatically arise when terms have repeated fields (see talk by R. Fonseca)

Discrete symmetries in MHDM (see talks by I. Ivanov, G. Branco)

Predictive power in CPV (see talks by T. Neder, W. Grimus, C. Hagedorn, G.-J. Ding)

Interesting testability prospects (see talks by M. Spinrath and J. Turner)

Possible string origins (see talks by P. Vaudrevange and G. Leontaris)

Brief aside: FS in colliders and in the universe

A₄ symmetry at colliders and in the universe IdMV, O. Fischer, V. Maurer https://arxiv.org/abs/1504.03955 (and refs. therein for other models, e.g. Ma's scotogenic models mentioned in talk by W. Grimus)

 A_4 triplets at EW scale; DM is A_4 triplet Observable consequences (LHC, $\mu \rightarrow e\gamma$, DM experiments)

Brief aside: dark matter family portals

Non-Abelian family symmetries as portals to dark matter IdMV, O. Fischer

https://arxiv.org/abs/1512.00869

Familon/flavon portals ($\langle \phi \rangle \neq 0$ breaks FS), basic idea:

$$\bar{f}f'\phi H + \phi\chi\chi + V(\phi, H) \tag{1}$$



Pre-pre-history

Brief pre-history in talk by M. Tanimoto

Works old enough to not have an arXiv number!

W. M. Fairbairn, T. Fulton and W. H. Klink, J. Math. Phys. 5 8, 1038 (1964)



Fermion masses and hierarchy of symmetry breaking G. C. Branco, H. P. Nilles, and V. Rittenberg Phys. Rev. D 21, 3417 (1980)

E. Ma, G. Rajasekaran 2001 https://arxiv.org/abs/hep-ph/0106291



Geometrical T Violation G. C. Branco, J.M. Gerard, W. Grimus Phys.Lett. 136B 383-386 (1984)

IdMV, S. F. King and G. G. Ross
https://arxiv.org/abs/hep-ph/0607045
E. Ma
https://arxiv.org/abs/hep-ph/0607056

More △(27) refs
IdMV
https://arxiv.org/abs/1507.00338

Direct and Indirect

Direct: residual symmetries are subgroups of family symmetry C.S. Lam https://arxiv.org/abs/hep-ph/0611017

Indirect: residual symmetry is not subgroup of the parent family symmetry (accidental due to special VEV directions)

Useful distinction for classification

Recent review by S. King https://arxiv.org/abs/1701.04413

Bottom-up approach to discrete family symmetries

J. Talbert https://arxiv.org/abs/1409.7310

IdMV, R. Rasmussen, J. Talbert https://arxiv.org/abs/1605.03581

- Start with the mixing matrix you want to get
- Discretize it using a good parametrization (e.g. $\Theta_k = \frac{a}{b}\pi$)
- Choose diagonal (residual) Z_n generators for each sector

Close group with the (diagonal) generators from one sector and the generators of the other sector rotated by the discretized mixing matrix

Top-down approach to discrete family symmetries

Top-down scans start with a group and check the possible subgroups and corresponding mixing matrices, e.g.

R. de Adelhart Toorop, F. Feruglio, C. Hagedorn https://arxiv.org/abs/1107.3486 D. Hernandez, A. Yu. Smirnov https://arxiv.org/abs/1212.2149 Martin Holthausen, Kher Sham Lim, Manfred Lindner https://arxiv.org/abs/1212.2411 S. F. King, T. Neder, A. J. Stuart https://arxiv.org/abs/1305.3200

See also complete classification of lepton mixing matrices from residual symmetries by R. Fonseca, W. Grimus https://arxiv.org/abs/1405.3678

Simple lepton models

Simple discrete models with A_4 (and near TBM)

G. Altarelli and F. Feruglio, Nucl.Phys. B741, 215 (2006), hep-ph/0512103.
G. Altarelli and D. Meloni, J.Phys. G36, 085005 (2009), 0905.0620.

VEV directions (1, 0, 0) and (1, 1, 1) aligned in the model

A_4 and TBM

 A_4 doesn't necessarily give TBM:

E. Ma, Phys.Rev. D70, 031901 (2004), hep-ph/0404199

In fact you can just add a non-trivial singlet

B. Brahmachari, S. Choubey, and M. Mitra, Phys.Rev. D77, 073008 (2008), 0801.3554.

J. Barry and W. Rodejohann, Phys.Rev. D81, 093002 (2010), 1003.2385.

Y. Shimizu, M. Tanimoto, and A. Watanabe, Prog.Theor.Phys. 126, 81 (2011), 1105.2929.

UV completions of simple lepton models

UV completions: all terms renormalisable, predictive



(a) Mass term of the τ lepton.

A₄ with trivial singlet IdMV, L. Merlo https://arxiv.org/abs/1011.6662

A₄ and non-trivial singlet IdMV, D. Pidt https://arxiv.org/abs/1211.5370

SO(10) imes SU(3)?



S. F. King and G. G. Ross, Phys. Lett. B 520 (2001) 243 [hep-ph/0108112]; S. F. King and G. G. Ross, Phys. Lett. B 574 (2003) 239 [hep-ph/0307190]; I. de Medeiros Varzielas and G. G. Ross, Nucl. Phys. B 733 (2006) 31 [hep-ph/0507176]; I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B 644 (2007) 153 [hep-ph/0512313]; S. F. King and M. Malinsky, JHEP 0611 (2006) 071 [hepph/0608021]; G. Altarelli, F. Feruglio and C. Hagedorn, JHEP 0803 (2008) 052 [arXiv:0802.0090 [hep-ph]]; P. Ciafaloni, M. Picariello, E. Torrente-Lujan and A. Urbano, Phys. Rev. D 79 (2009) 116010 [arXiv:0901.2236 [hep-ph]]; T. J. Burrows and S. F. King, Nucl. Phys. B 835 (2010) 174 [arXiv:0909.1433 [hep-ph]]; I. K. Cooper. S. F. King and C. Luhn, Phys. Lett. B 690 (2010) 396 [arXiv:1004.3243 [hepph]]; S. Antusch, S. F. King and M. Spinrath, Phys. Rev. D 83 (2011) 013005 [arXiv:1005.0708 [hep-ph]]; S. Antusch, S. F. King, C. Luhn and M. Spinrath, Nucl. Phys. B 850 (2011) 477 [arXiv:1103.5930 [hep-ph]]; D. Meloni, JHEP 1110 (2011) 010 [arXiv:1107.0221]; T. J. Burrows and S. F. King, Nucl. Phys. B 842 (2011) 107 [arXiv:1007.2310 [hep-ph]]; I. de Medeiros Varzielas, JHEP 1201 (2012) 097 [arXiv:1111.3952 [hep-ph]]; C. Hagedorn, S. F. King and C. Luhn, Phys. Lett. B 717 (2012) 207 [arXiv:1205.3114]; B. D. Callen and R. R. Volkas, Phys. Rev. D 86 (2012) 056007 [arXiv:1205.3617]; A. Meroni, S. T. Petcov and M. Spinrath, Phys. Rev. D 86 (2012) 113003 [arXiv:1205.5241 [hep-ph]]; S. F. King, C. Luhn and A. J. Stuart, Nucl. Phys. B 867 (2013) 203 [arXiv:1207.5741]; S. F. King, Phys. Lett. B 724 (2013) 92 [arXiv:1305.4846 [hep-ph]]; S. Antusch, C. Gross, V. Maurer and C. Sluka, Nucl. Phys. B 877 (2013) 772 [arXiv:1305.6612 [hep-ph]]; S. Antusch, C. Gross, V. Maurer and C. Sluka, Nucl. Phys. B 879 (2014) 19 [arXiv:1306.3984 [hep-ph]; S. F. King, JHEP 1401 (2014) 119 [arXiv:1311.3295 [hep-ph]]; S. F. King, JHEP 1408 (2014) 130 [arXiv:1406.7005 [hep-ph]].

L de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B **648** (2007) 201 [hep-ph/0607045]; S. F. King and M. Malinsky, Phys. Lett. B **645** (2007) 351 [hepph/0610250]; F. Bazzocchi and I. de Medeiros Varzielas, Phys. Rev. D **79** (2009) 093001 [arXiv:0902.3250 [hep-ph]]; B. Dutta, Y. Mimura and R. N. Mohapatra, JHEP **1005** (2010) 034 doi:10.1007/JHEP05(2010)034 [arXiv:0911.2242 [hep-ph]]; K. M. Patel, Phys. Lett. B **695** (2011) 225 [arXiv:1008.5061]; P. S. Bhupal Dev, R. N. Mohapatra and M. Severson, Phys. Rev. D **84** (2011) 03300 [arXiv:1107.2378]; P. S. Bhupal Dev, B. Dutta, R. N. Mohapatra and M. Severson, Phys. Rev. D **86** (2012) 033002 [arXiv:1202.4012]; I. de Medeiros Varzielas and G. G. Ross, JHEP **1212** (2012) 041 [arXiv:1203.6636 [hep-ph]]; A. Anandakrishnan, S. Raby and A. Wingerter, Phys. Rev. D **87** (2013) 5, 655005 [arXiv:1212.0542] [hep-ph]].

Unification with family symmetry

All fermions can have the same Dirac mass structure

P. Ramond, R.G. Roberts, G. G. Ross
https://arxiv.org/abs/hep-ph/9303320
G. G. Ross, M. Serna
https://arxiv.org/abs/0704.1248

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 + \varepsilon^3 & -a\varepsilon^2 + \varepsilon^3 \\ -\varepsilon^3 & -a\varepsilon^2 + \varepsilon^3 & 1 \end{pmatrix} \qquad \begin{array}{l} \varepsilon_d = 0.15, \ \mathbf{a}^a = -2/3 \\ \varepsilon_1 = 0.15, \ \mathbf{a}^e = -3 \\ \varepsilon_u = 0.05, \ \mathbf{a}^u = 4/3 \\ \varepsilon_v = 0.05, \ \mathbf{a}^v = 0 \end{pmatrix}$$

Seesaw distinguishes quarks and leptons

Sequential Dominance

Sequential Dominance framework very useful

S. F. King, Phys. Lett. B 439 (1998) 350 [hep-ph/9806440] S. F. King, Nucl. Phys. B 562 (1999) 57 [hep-ph/9904210]

Seesaw with very strong RH neutrino hierarchy Has implications for low energy mixing Predicts Normal hierarchy

Constrained Sequential Dominance

In particular, CSD(n) frameworks have a direct connection between aligned VEVs and mixing matrix columns CSD1 corresponds to TBM, for others see e.g. S. King https://arxiv.org/abs/1304.6264

$$m_D = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}, M_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}.$$
 (2)

 $M_1 \ll M_2$ can reverse the $a_i \ll b_i$ hierarchy required in GUTs $m_\nu = AA^T/M_1 + BB^T/M_2$

For CSD1 where A and B are orthogonal you can even read off they are also eigenvectors of m_{ν}

$SO(10) \times SU(3)$ with near TBM

A subtle example of mixing after seesaw https://arxiv.org/abs/hep-ph/0507176 Strategy to pass the mixing to low energy after seesaw:



Gives to $\mathcal{O}((\kappa^{\nu}/\kappa^{M})^{2})$ effective Majorana mass terms

$$-\frac{(\kappa^{\nu})^{2}}{\kappa_{2}^{M}}(\bar{\phi}_{\rm atm}F)(\bar{\phi}_{\rm atm}F) - \frac{(\kappa^{\nu})^{2}}{\kappa_{1}^{M}}(\bar{\phi}_{\rm sol}F)(\bar{\phi}_{\rm sol}F)$$
(4)

Another seesaw technique: screening

Screening: double seesaw and define $D = m_D (M_{RS}^{-1})^T$ $m_\nu = DM_S D^T$. $m_D \propto M_{RS}$ so $D \propto 1$, flavour structure comes from the singlets.

A. Y. Smirnov, Seesaw enhancement of lepton mixing, Phys. Rev. D 48 (1993) 3264 [hep-ph/9304205].

M. Lindner, M. A. Schmidt and A. Y. Smirnov, Screening of Dirac flavor structure in the seesaw and neutrino mixing, JHEP 0507 (2005) 048 [hep-ph/0505067].

P. Ludl, A. Y. Smirnov https://arxiv.org/abs/1507.03494

Unification and mass ratios

Different expansion parameters: GUT breaking splits the messenger masses

What about the generation mass ratios?

Alternatives to Georgi-Jarlskog factors

S. Antusch, M. Spinrath https://arxiv.org/abs/0902.4644

S. Antusch and V. Maurer, Phys. Rev. D 84 (2011) 117301 [arXiv:1107.3728 [hep-ph]]. D. Marzocca, S. T. Petcov, A. Romanino and M. Spinrath, JHEP 1111 (2011) 009 [arXiv:1108.0614 [hep-ph]]. S. Antusch, C. Gross, V. Maurer and C. Sluka, Nucl. Phys. B 866 (2013) 255 [arXiv:1205.1051 [hep-ph]]. A. Meroni, S. T. Petcov and M. Spinrath, Phys. Rev. D 86 (2012) 113003 [arXiv:1205.5241 [hep-ph]]. S. Antusch, C. Gross, V. Maurer and C. Sluka, Nucl. Phys. B 877 (2013) 772 [arXiv:1305.6612 [hep-ph]]. S. Antusch, S. F. King, M. Spinrath https://arxiv.org/abs/1311.0877

Implementations

S. Antusch, IdMV, V. Maurer, C. Sluka, M. Spinrath https://arxiv.org/abs/1405.6962

Factors $\frac{y_{\tau}}{y_b} = -\frac{3}{2}$, $\frac{y_{\mu}}{y_s} = 6$ and $\frac{y_e}{y_d} = -\frac{1}{2}$. This leads to $\left|\frac{y_{\mu}}{y_s}\frac{y_d}{y_e}\right| = 12$, in good agreement with observed value.

H. Georgi and C. Jarlskog, Phys. Lett. B 86 (1979) 297 $y_{\mu} = -3y_s$ and $y_e = \frac{1}{3}y_d$ give $\left|\frac{y_{\mu}}{y_s}\frac{y_d}{y_e}\right| = 9$, deviates from the central value by more than two sigma.

Includes a Z_4^R as in H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg and P. K. S. Vaudrevange, https://arxiv.org/abs/1102.3595

Missing partner mechanism

A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. B 115 (1982) 380.B. Grinstein, Nucl. Phys. B 206 (1982) 387

Use Missing Partner Mechanism to control Doublet-Triplet splitting in SU(5): **50** has no SU(2) doublet

$$W_{\rm MPM} = \bar{H}_5 H_{75} Z_{50} + \bar{Z}_{50} H_{75} H_5 + M_{50} Z_{50} \bar{Z}_{50} ,$$
 (5)



MPM in matrix form

SU(5) 50 has no SU(2) doublet

$$W_{\rm MPM} = \bar{H}_5 H_{75} Z_{50} + \bar{Z}_{50} H_{75} H_5 + M_{50} Z_{50} \bar{Z}_{50} , \qquad (6)$$

$$m_{Doublet} = 0$$
, $m_T = \begin{pmatrix} 0 & V \\ V & M_{50} \end{pmatrix}$, (7)

det $m_{Doublet} = 0$ and det $m_T \neq 0$

Two Abelian models

S. Antusch, IdMV, V. Maurer, C. Sluka, M. Spinrath https://arxiv.org/abs/1405.6962

Model with diagonal Y_d

$$Y_{d} = \begin{pmatrix} y_{d} & 0 & 0 \\ 0 & y_{s} & 0 \\ 0 & 0 & y_{b} \end{pmatrix}, \ Y_{u} = \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{12} & y_{22} & y_{23} \\ y_{13} & y_{23} & y_{33} \end{pmatrix} .$$
(8)

Model with θ_C from Y_d

$$Y_{d} = \begin{pmatrix} 0 & y_{d,12} & 0 \\ y_{d,21} & y_{s} & 0 \\ 0 & 0 & y_{b} \end{pmatrix}, Y_{u} = \begin{pmatrix} y_{11} & y_{12} & 0 \\ y_{12} & y_{22} & y_{23} \\ 0 & y_{23} & y_{33} \end{pmatrix}.$$
 (9)



F. Bjorkeroth, F. de Anda, IdMV, S. F. King https://arxiv.org/abs/1503.03306

 $SU(5) \times A_4$ SUSY GUT Uses the CSD3 directions aligned in A_4 (non-trivial)

$$\bar{\phi}_{\rm atm} \sim (0, 1, 1), \qquad \bar{\phi}_{\rm sol} \sim (1, 3, 1), \qquad \bar{\phi}_{\rm dec} \sim (0, 0, 1).$$
 (10)

 μ term at right scale, proton decay under control



F. Bjorkeroth, F. de Anda, IdMV, S. F. King https://arxiv.org/abs/1512.00850

 $SO(10) \times \Delta(27)$

DT splitting with Missing VEV mechanism, Dimopoulous-Wilczek (doesn't work in *SU*(5))

Intricate VEV alignment

Uses the CSD3 directions aligned in $\Delta(27)$ (non-trivial) $\Delta(27)$ F-term alignments following from IdMV https://arxiv.org/abs/1507.00338

$$\bar{\phi}_{\rm atm} \sim (0, 1, 1), \qquad \bar{\phi}_{\rm sol} \sim (1, 3, 1), \qquad \bar{\phi}_{\rm dec} \sim (0, 0, 1).$$
 (11)

CP symmetry broken spontaneously, relative phase of $2\pi/3$ in VEV (related with $\Delta(27))$

Lepton mixing completely predictive.

Obtain normal neutrino mass hierarchy, best-fit lightest neutrino mass between 0.32 – 0.38 meV, CPV phase $\delta' \approx (275 - 280)^{\circ}$ and remaining neutrino parameters all within 1σ of their best-fit experimental values.

Mixing when m_D and M_R are related by SO(10)

Using a version of the earlier seesaw strategy

$$\begin{array}{c} (\bar{\phi}_{atm}F) & (\bar{\phi}_{sol}F) & (\bar{\phi}_{atm}N^c) & (\bar{\phi}_{sol}N^c) \\ (\bar{\phi}_{atm}F) & \begin{pmatrix} 0 & 0 & \kappa^{\nu}_{atm} & 0 \\ 0 & 0 & 0 & \kappa^{\nu}_{sol} \\ (\bar{\phi}_{atm}N^c) & \kappa^{\nu}_{atm} & 0 & \kappa^{M}_{atm} & 0 \\ (\bar{\phi}_{sol}N^c) & 0 & \kappa^{\nu}_{sol} & 0 & \kappa^{M}_{sol} \\ \end{array} \right).$$
(12)

Diagonalisation gives, to $\mathcal{O}((\kappa^{\nu}/\kappa^M)^2)$, the effective Majorana mass terms

$$-\frac{(\kappa_{\rm atm}^{\nu})^2}{\kappa_{\rm atm}^M}(\bar{\phi}_{\rm atm}F)(\bar{\phi}_{\rm atm}F) - \frac{(\kappa_{\rm sol}^{\nu})^2}{\kappa_{\rm sol}^M}(\bar{\phi}_{\rm sol}F)(\bar{\phi}_{\rm sol}F)$$
(13)

Leptogenesis (BAU)

SU(5) model (and CSD Leptogenesis in general) https://arxiv.org/abs/1505.05504

SO(10) model https://arxiv.org/abs/1609.05837 (new fit added)

Getting Y_B further constrains the model parameters

Leptogenesis in SO(10) flavoured GUT

SO(10) model https://arxiv.org/abs/1609.05837 (new fit added)

Getting Y_B constrains the (unified) Dirac couplings:





Family Symmetries: best tool to address flavour problemSUSY GUTs with FS are enticing (but difficult to build)