Generalized CP invariance and co-bimaximal lepton mixing

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Program

1. Definition of CP in general gauge theories
2. Co-bimaximal lepton mixing from a generalized CP symmetry
3. Co-bimaximal lepton mixing in the scotogenic model

Definition of CP in general gauge theories

CP-type transformations

Example: QED

Electron field $e(x)$, $\widehat{x} \equiv (x^0, -\vec{x})$

$$\begin{array}{rrcl} CP: & e(x) & \to & \gamma^0 \left(C\gamma_0^T e(\widehat{x})^* \right) = -Ce^*(\widehat{x}) \\ P: & e(x) & \to & \gamma^0 e(\widehat{x}) \end{array}$$

In terms of chiral fields:

$$\begin{array}{rrcl} CP: & e_{L,R}(x) & \to & -Ce_{L,R}(\widehat{x})^* \\ P: & e_{L,R}(x) & \to & \gamma^0 e_{R,L}(\widehat{x}) \end{array}$$

Change to left-chiral fields:

$$\chi_{1L} \equiv e_L, \quad \chi_{2L} \equiv (e_R)^c = C\gamma_0^T e_L^*$$

Effect of parity:

$$\chi_{1L} = e_L \to \gamma_0 e_R = \gamma_0 (\chi_{2L})^c = \gamma_0 C\gamma_0^T \chi_{2L}^* = -C\chi_{2L}^*$$

Example: QED (continued)

$$CP: \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix} \to \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} C \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix}^*$$

$$P: \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix} \to \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} C \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix}^*$$

In this picture, the form of CP and P is only distinguished by the matrix acting on the vector of chiral fields! $\Rightarrow$

CP-type transformation:

$$\begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix} \to -UC \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix}^*$$

R. Slansky, Phys. Rep. 79 (1981) 1;
V.N. Smolyakov, Theor. and Math. Phys. 50 (1982) 225;
W. Grimus, M. Rebelo, hep-ph/0506272;
J.F. Cornwell, "Group Theory in Physics" (1984)

CP-type transformations

Gauge theories:

$\{T_a | a = 1, \ldots, n_G\}$: hermitian generators of fermion representation

f_{abc} structure constants, totally antisymmetric in a, b, c

$$[T_a, T_b] = i f_{abc} T_c \quad \text{and} \quad \mathrm{Tr}\,(T_a T_b) = k \delta_{ab}$$

$$W_\mu \equiv T_a W_\mu^a, \quad G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu] \equiv T_a G_{\mu\nu}^a$$

Pure gauge Lagrangian:

$$\mathcal{L}_G = -\frac{1}{4k}\,\mathrm{Tr}\,(G_{\mu\nu} G^{\mu\nu})$$

Fermionic Lagrangian:

$$\mathcal{L}_F = \bar{\omega}_L i \gamma^\mu \left(\partial_\mu + ig T_a W_\mu^a\right) \omega_L$$

CP-type transformations

Charge-conjugation matrix:
$C^{-1}\gamma_\mu C = -\gamma_\mu^T$, $C^T = -C$, $C^\dagger = C^{-1}$

Chiral projectors: $\gamma_L = \frac{\mathbb{1}-\gamma_5}{2}$, $\gamma_R = \frac{\mathbb{1}+\gamma_5}{2}$

Charge conjugation operation: $\psi^c \equiv C\gamma_0^T \psi^*$

$$\gamma_L \psi_L = \psi_L \quad \Rightarrow \quad \gamma_R (\psi_L)^c = (\psi_L)^c$$

Without loss of generality can confine ourselves to left-chiral fermion fields!

ω_L: vector of n_F left-chiral fermion fields

CP-type transformation:

$$\widehat{x} \equiv (x_0, -\vec{x}),\ \varepsilon(\mu) = \begin{cases} 1 & (\mu = 0) \\ -1 & (\mu = 1, 2, 3) \end{cases}$$

Require that $\int d^4x\mathcal{L}$ is invariant under

$$W^a_\mu(x) \to \varepsilon(\mu) R_{ab} W^b_\mu(\widehat{x}) \quad \text{with} \quad R \in O(n_G)$$

$$\omega_L(x) \to U\gamma^0 C \bar{\omega}_L^T(\widehat{x}) = -UC\omega_L^*(\widehat{x}) \quad \text{with} \quad U \in U(n_F)$$

Field strength tensor:

$$G^a_{\mu\nu}(x) \to \varepsilon(\mu)\varepsilon(\nu) R_{ad} \left(\partial_\mu W^d_\nu - \partial_\nu W^d_\mu - g \hat{f}_{dbc} W^b_\mu W^c_\nu \right) (\widehat{x})$$

with $\hat{f}_{dbc} = f_{a'b'c'} R_{a'd} R_{b'b} R_{c'c}$

CP-type transformations

Invariance conditions:

$$\text{(A):} \qquad f_{abc} = f_{a'b'c'} R_{a'a} R_{b'b} R_{c'c}$$

$$\text{(B):} \qquad U(-{T_b}^T R_{ab}) U^\dagger = T_a$$

Note: The T_a correspond to representations of the generators of a real Lie algebra

$$[-iT_a, -iT_b] = f_{abc}(-iT_c) \quad \text{with} \quad -iT_a \equiv D(X_a)$$

Real Lie algebra generated by $\{X_a\}$ with $[X_a, X_b] = f_{abc} X_c$

Lie algebras

Definition: Vektor space $\mathcal{L}$ over field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ with $\dim \mathcal{L} \geq 1$ and a product (Lie bracket)

$$[\ \ ,\ \ ] : \mathcal{L} \times \mathcal{L} \mapsto \mathcal{L}$$

with the properties

1. $[X, Y] = -[Y, X]$
2. $[X, c_1 Y_1 + c_2 Y_2] = c_1 [X, Y_1] + c_2 [X, Y_2]$ $(c_{1,2}, \in \mathbb{F})$
3. Jacobi identity: $[X, [Y, Z]] + [Z, [X, Y]] + [Y, [Z, X]] = 0$

Theorem of Ado (1935):
Every finite-dimensional Lie algebra has a faithful representation as square matrices, such that the Lie bracket is given by the commutator.

Consequence: We can always imagine a Lie algebra as a vector space of matrices X, Y, with $[X, Y] = XY - YX$.

Definition: An automorphism $\psi : \mathcal{L} \to \mathcal{L}$ is a linear bijective mapping such that $[\psi(X), \psi(Y)] = \psi([X, Y])$

Define automorphism ψ_R via $X_a \to R_{ba} X_b$
In order to preserve normalization $\mathrm{Tr}\,(T_a T_b) = k\delta_{ab}$, R must be orthogonal.

$$[\psi_R(X_a), \psi_R(X_b)] = f_{abc}\psi_R(X_c) \;\Rightarrow\; R_{a'a}R_{b'b}f_{a'b'c'}X_{c'} = f_{abc'}R_{c'c}X_{c'}$$
$$\Rightarrow\; R_{a'a}R_{b'b}R_{c'c}f_{a'b'c'} = f_{abc}$$

Solution of (A)

R must correspond to an automorphism ψ_R

Representation $\{T_a\}$:
In general reducible, inequivalent irreps D_r with multiplicities $m_r \Rightarrow$

$$T_a = i \bigoplus_r (\mathbf{1}_{m_r} \otimes D_r(X_a)) \quad \text{with} \quad \dim D_r = d_r$$

Theorem

Let (R, U_0) be a solution of Conditions (A) and (B) and let $(R, U_1 U_0)$ be another solution. Then

$$U_1 = \bigoplus_r (u_r \otimes \mathbf{1}_{d_r})$$

where the u_r are unitary $m_r \times m_r$ matrices.

Complex conjugate representation:

Hermiticity of the $T_a \Rightarrow$

$$(\exp(iy_a T_a))^* = \exp(-iy_a T_a^T) \;\Leftrightarrow\; (\exp(-y_a D(X_a)))^* = \exp(y_a D(X_a)^T)$$

Complex conjugate representation generated by $-D(X_a)^T$!

Reformulation of condition (B)

$$\bigoplus_r \left(\mathbf{1}_{m_r} \otimes (-D_r^T \circ \psi_{R^{-1}})\right) \sim \bigoplus_r (\mathbf{1}_{m_r} \otimes D_r)$$

Killing form

Adjoint representation:

$$\mathrm{ad}\, Y : \begin{cases} \mathcal{L} & \mapsto & \mathcal{L} \\ X & \to & [Y, X] \end{cases}$$

Jacobi identity $\Rightarrow$ Theorem: $\mathrm{ad}[Y, Z] = [\mathrm{ad}\, Y, \mathrm{ad}\, Z]$

Killing form: bilinear form on $\mathcal{L} \times \mathcal{L}$

$$\kappa(X, Y) = \mathrm{Tr}\,(\mathrm{ad} X\, \mathrm{ad}\, Y)$$

$\{X_a\}$ basis of $\mathcal{L}$ $\Rightarrow$ $\mathrm{ad}\, Y$ corresponds to matrix matrix $M(Y)$:

$$(\mathrm{ad}\, Y)\, X_a = M(Y)_{ba} X_b$$

Compute Killing form via

$$\kappa(Y, Z) = \mathrm{Tr}\,(M(Y) M(Z)) = M(Y)_{ba} M(Z)_{ab}$$

Killing form

Note: $M(X_a)_{bc} = -f_{abc}$, $\kappa(X_a, X_b) = -f_{acd} f_{bcd}$

Note:

Theorem: $\kappa([X, Y], Z) = \kappa(X, [Y, Z])$

Semisimple Lie algebras

Ideal: An ideal $\mathcal{I}$ (invariant sub-algebra) is a sub-algebra of $\mathcal{L}$ such that $[X, Y] \in \mathcal{I} \; \forall \; X \in \mathcal{I}, \; Y \in \mathcal{L}$.
Semisimple Lie algebras: Do not possess Abelian subalgebras
Simple Lie algebras: Do not possess non-trivial subalgebras
Theorem: $\mathcal{L}$ semisimple $\Leftrightarrow$ κ non-degenerate
Corollary: $\mathcal{L}$ semisimple $\Leftrightarrow$ $\kappa_{ab} \equiv \kappa(X_a, X_b)$ non-singular

Compact semisimple Lie algebra $\mathcal{L}_c$:
Killing form κ negative definite
Gauge theories: Lie algebras are of the type $\mathcal{L} = \mathcal{L}_c \oplus \mathcal{L}_A$ where $\mathcal{L}_c$ belongs to the non-Abelian part of the gauge group and $\mathcal{L}_A$ to the Abelian one.

$\mathcal{L}_A = 0$ or $\mathcal{L}_A = \underbrace{u(1) \oplus \cdots \oplus u(1)}_{r \text{ times}}$

In physics: $X_a \leftrightarrow -iT_a \Rightarrow \kappa(X_a, X_b) \propto -\delta_{ab}$

Complexification:

$\mathcal{L}$ Lie algebra over $\mathbb{R}$, its complexification denoted by $\widetilde{\mathcal{L}}$

$\mathcal{L} \to \widetilde{\mathcal{L}}$ such that $\dim_{\mathbb{R}} \mathcal{L} = \dim_{\mathbb{C}} \widetilde{\mathcal{L}}$

Cartan subalgebra (CSA):

Subalgebra $\mathcal{H}$ of $\widetilde{\mathcal{L}}$ with the following properties:

- $\mathcal{H}$ is maximally Abelian,
- ad$h$ is completely reducible for every $h \in \mathcal{H}$.

Theorem:

- Every $\widetilde{\mathcal{L}}$ possesses at least one CSA.
- All CSAs of $\widetilde{\mathcal{L}}$ are isomorphic via an automorphism of $\widetilde{\mathcal{L}}$.

Semisimple Lie algebras

Rank of $\widetilde{\mathcal{L}}$: $\ell = \dim \mathcal{H}$

Roots:

1. All ad$h$ with $h \in \mathcal{H}$ simultaneously diagonalizable
2. Therefore, $\exists\ X'_1, \ldots, X'_{n-\ell} \in \mathcal{L}_c$ such that $[h, X'_k] = \alpha_k(h) X'_k$
3. α_k linear functional on $\mathcal{H}$

Denote set Δ of such linear functionals $\alpha \Rightarrow$

$$\widetilde{\mathcal{L}} = \left(\bigoplus_{\alpha \in \Delta} \mathcal{L}_\alpha \right) \oplus \mathcal{H}$$

Some properties of this decomposition:

- $\alpha \in \Delta \Leftrightarrow -\alpha \in \Delta$
- $\dim \mathcal{L}_\alpha = 1$, basis vector e_α

Semisimple Lie algebras

Root properties:

- Theorem: The Killing form of $\widetilde{\mathcal{L}}$ provides a non-degenerate symmetric bilinear form on $\mathcal{H}$.
- Therefore, for every $\alpha \in \Delta$ it exists a $h_\alpha \in \mathcal{H}$ such that $\alpha(h) = \kappa(h_\alpha, h)$.
- For $\alpha, \beta \in \Delta$ define $\langle \alpha, \beta \rangle = \kappa(h_\alpha, h_\beta)$.
- For all $\alpha, \beta \in \Delta$ the quantity $\langle \alpha, \beta \rangle$ is real and rational.
- $\langle \alpha, \alpha \rangle > 0$

Semisimple Lie algebras

Lexicographical ordering and simple roots:
Choose ℓ linearly independent roots $\{\beta_1, \ldots, \beta_\ell\}$. Then,

$$\alpha = \sum_{j=1}^{\ell} \mu_j \beta_j \quad \text{with} \quad \mu_j \text{ real and rational}$$

for every $\alpha \in \Delta$.

Positive roots Δ_+: First non-vanishing coefficient $\mu_j > 0$.
$\alpha, \beta \in \Delta$: $\alpha > \beta$ if first non-vanishing difference $\mu_j^\alpha - \mu_j^\beta > 0$.
Simple roots: $\alpha \in \Delta_+$ simple if α cannot be expressed in the form $\alpha = \beta + \gamma$ with $\beta, \gamma \in \Delta_+$.
Theorem: There are $\ell = \text{rank}(\mathcal{H})$ simple roots. Every $\alpha \in \Delta$ can be written as

$$\alpha = \sum_{j=1}^{\ell} k_j \alpha_j$$

with non-negative integers k_j.

The Lie algebra $su(3)$ and its complexification A_2:

Basis: $X_a = i\lambda_a$ $(a = 1, \ldots, 8)$ (Gell-Mann matrices)
Killing form: $\kappa(X_a, X_b) = -12\,\delta_{ab}$
CSA: $h_1 \equiv \lambda_3$, $h_2 \equiv \lambda_8$, rank $\ell = 2$

The spaces $\widetilde{\mathcal{L}}_\alpha$:

$$\begin{array}{ll}
[h_1, (\lambda_1 + i\lambda_2)] = 2(\lambda_1 + i\lambda_2) & [h_2, (\lambda_1 + i\lambda_2)] = 0(\lambda_1 + i\lambda_2) \\
[h_1, (\lambda_6 + i\lambda_7)] = -(\lambda_6 + i\lambda_7) & [h_2, (\lambda_6 + i\lambda_7)] = \sqrt{3}(\lambda_6 + i\lambda_7) \\
[h_1, (\lambda_4 + i\lambda_5)] = (\lambda_4 + i\lambda_5) & [h_2, (\lambda_4 + i\lambda_5)] = \sqrt{3}(\lambda_4 + i\lambda_5) \\
[h_1, (\lambda_1 - i\lambda_2)] = -2(\lambda_1 - i\lambda_2) & [h_2, (\lambda_1 - i\lambda_2)] = 0(\lambda_1 - i\lambda_2) \\
[h_1, (\lambda_6 - i\lambda_7)] = (\lambda_6 - i\lambda_7) & [h_2, (\lambda_6 - i\lambda_7)] = -\sqrt{3}(\lambda_6 - i\lambda_7) \\
[h_1, (\lambda_4 - i\lambda_5)] = -(\lambda_4 - i\lambda_5) & [h_2, (\lambda_4 - i\lambda_5)] = -\sqrt{3}(\lambda_4 - i\lambda_5)
\end{array}$$

Three positive roots: α_j $(j = 1, 2, 3)$ with $\alpha_3 = \alpha_1 + \alpha_2$

$$\begin{array}{llll}
e_{\alpha_1} = \lambda_1 + i\lambda_2 & \Rightarrow & \alpha_1(h_1) = 2, & \alpha_1(h_2) = 0 \\
e_{\alpha_2} = \lambda_6 + i\lambda_7 & \Rightarrow & \alpha_2(h_1) = -1, & \alpha_2(h_2) = \sqrt{3} \\
e_{\alpha_3} = \lambda_4 + i\lambda_5 & \Rightarrow & \alpha_3(h_1) = 1, & \alpha_3(h_2) = \sqrt{3}
\end{array}$$

Semisimple Lie algebras

Cartan matrix A: $\{\alpha_1, \ldots, \alpha_\ell\}$ simple roots.

$$A_{jk} = 2\,\frac{\langle \alpha_j, \alpha_k \rangle}{\langle \alpha_k, \alpha_k \rangle}$$

Theorem: $j \neq k \Rightarrow A_{jk} \in \{0, -1, -2, -3\}$ **Representations of $\mathcal{L}_c$:**
Weight vectors: Common eigenvectors of CSA.

$$D(h)e(\lambda, q) = \lambda(h)e(\lambda, q) \quad (q = 1, \ldots, m_\lambda)$$

Weight λ, like a root α is a linear functional on $\mathcal{H}$!

Irreducible representations: Fundamental weights are defined by

$$\Lambda_j = \sum_{k=1}^{\ell} (A^{-1})_{jk} \alpha_k \quad \Rightarrow \quad 2\frac{\langle \Lambda_j, \alpha_k \rangle}{\langle \alpha_k, \alpha_k \rangle} = \delta_{jk}.$$

Theorem: For any irrep of $\mathcal{L}_c$ there is a unique highest weight Λ (with respect to the lexicographical ordering bases on the simple roots). It can be written as

$$\Lambda = n_1 \Lambda_1 + \ldots + n_\ell \Lambda_\ell$$

Root rotations and automorphisms of $\mathcal{L}_c$:

Root rotation: $\tau : \Delta \mapsto \Delta$

a) $\tau(\alpha+\beta) = \tau(\alpha) + \tau(\beta)\ \forall\ \alpha, \beta \in \Delta$ such that $\alpha + \beta \in \Delta$,

b) $\tau(-\alpha) = -\tau(\alpha)$.

Theorem: For every root rotation τ there is an automorphism ψ_τ of $\widetilde{\mathcal{L}}$ with the properties

$$\psi_\tau(h_\alpha) = h_{\tau(\alpha)} \quad \text{and} \quad \psi_\tau(e_\alpha) = \chi_\alpha e_{\tau(\alpha)},$$

where $\chi_\alpha = \pm 1\ \forall\ \alpha \in \Delta$ such that $\chi_\alpha = 1$ for all simple roots, $\chi_{-\alpha} = \chi_\alpha$, etc.

Canonical and generalized CP transformations

Reformulation of condition (B):

$$\bigoplus_r \left(\mathbf{1}_{m_r} \otimes (-D_r^T \circ \psi_{R^{-1}}) \right) \sim \bigoplus_r (\mathbb{1}_{m_r} \otimes D_r)$$

Finding a canonical CP transformation:

- Root reflexion:

$$\tau_r : \begin{cases} \Delta & \mapsto & \Delta \\ \alpha & \to & -\alpha \end{cases}$$

  The root reflexion is obviously a root rotation.
- Automorphism induced by τ_r:
  $\psi^{\triangle}$ such that $\psi^{\triangle}(h_\alpha) = h_{-\alpha} = -h_\alpha$
- Equivalent irreps: $-D_r^T \circ \psi^{\triangle} \sim D_r$
  because highest weight of $-D_r^T \circ \psi^{\triangle}$ agrees with that of D_r
- There are unitary matrices V_r such that
  $V_r(-D_r^T \circ \psi^{\triangle})V_r^\dagger = D_r \quad \forall\ r$

Canonical and generalized CP transformations

Canoncical CP transformation

$$(R^\triangle, U_0) \quad \text{with} \quad \psi_{R^\triangle} \equiv \psi^\triangle, \quad U_0 = \bigoplus_r \mathbb{1}_{m_r} \otimes V_r$$

Any gauge theory is automatically invariant under this CP transformation!

Multiplicites $m_r > 1 \Rightarrow$ freedom to perform rotations

Generalized CP transformation

$$(R^\triangle, U_1 U_0) \quad \text{with} \quad U_1 = \bigoplus_r u_r \otimes \mathbb{1}_{d_r}$$

Any gauge theory is automatically invariant under such a CP transformation!

Remarks:

- V_r determined only up to phase factor.
- CP affects only Yukawa interactions and scalar potential.

CP basis

Theorem: CP basis

For every irrep D of $\mathcal{L}_c$ there is an ON basis of $\mathbb{C}^d$ ($d = \dim D$) such that

$$D(X_a)^T = -\eta_a D(X_a), \qquad \eta_a^2 = 1 \qquad (a = 1, \ldots, n_G)$$

for the antihermitian generators of $\mathcal{L}_C$ in D. Therefore,

$$\psi^{\triangle}(X_a) = \eta_a X_a.$$

In this basis, the canonical CP transformation is represented by

$$(R^{\triangle}, \mathbb{1}) \quad \text{with} \quad R^{\triangle} = \operatorname{diag}\left(\eta_1, \ldots, \eta_{n_G}\right).$$

CP basis

The generators $\{X_a\}$ are those of the compact real form $\mathcal{L}_c$ of $\widetilde{\mathcal{L}}$. There two possiblities:

1. $D(X_a)$ imaginary and symmetric $\Rightarrow \eta_a = -1$
2. $D(X_a)$ real and antisymmetric $\Rightarrow \eta_a = 1$

In other words, the $D(X_a)$ are generalizations of $-i\sigma_a/2$ (Pauli matrices) for $SU(2)$ and $-i\lambda_a/2$ (Gell–Mann matrices) for $SU(3)$ to arbitrary irreps of semisimple compact Lie algebras.

Irreps of $\mathcal{L}_c$ in the CP basis: Note that $D(e_{-\alpha})^\dagger = -D(e_\alpha)$

- $D(-iH_j) \quad (j = 1, \ldots, \ell)$, imaginary and symmetric, $\eta_a = -1$.
- $D\left(\frac{e_\alpha - e_{-\alpha}}{i\sqrt{2}}\right) \quad (\alpha \in \Delta)$, imaginary and symmetric, $\eta_a = -1$,
- $D\left(\frac{e_\alpha + e_{-\alpha}}{\sqrt{2}}\right) \quad (\alpha \in \Delta)$, real and antisymmetric, $\eta_a = 1$.

Form of CP symmetry in CP basis

Canonical CP:

$$U_0 = \mathbb{1}_{n_F}, \quad R^{\triangle} = \mathrm{diag}\,(\eta_1, \ldots, \eta_{n_G})$$

with

$$\begin{array}{lll} T_a \quad \text{real, symmetric} & \Rightarrow & \eta_a = -1 \\ T_a \quad \text{imaginary, antisymmetric} & \Rightarrow & \eta_a = 1 \end{array}$$

Generalized CP:

sum over irreps D_r, muliplicity m_r of D_r, $u_r \in U(m_r)$

$$U_1 = \bigoplus_r u_r \otimes \mathbb{1}_{d_r}$$

Remarks on parity

Note: $\psi^\triangle$ involutive, i.e. $(\psi^\triangle)^2 = \text{id}$
Idea: Define parity via an involutive automorphism $\psi_P \neq \psi^\triangle$

- In contrast to CP, there is no canonical way to define parity.
- There are theories in which no physically meaningful definition of parity exists, e.g. the SM.
- Parity can exist within one irrep, like the **16** of $SO(10)$.
- In theories like QED, QCD we have two irreps D, $-D^T$ and $\psi_P = \text{id}$.

$$\text{QED:} \quad T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad R_P = 1 \qquad U_P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{QCD:} \quad T_a = \begin{pmatrix} \lambda_a & 0 \\ 0 & -\lambda_a^T \end{pmatrix} \quad R_P = \mathbb{1}_8 \quad U_P = \begin{pmatrix} 0 & \mathbb{1}_3 \\ \mathbb{1}_3 & 0 \end{pmatrix}$$

Co-bimaximal lepton mixing from a generalized CP symmetry

Neutrino mass matrix with μ–τ exchange symmetry

Light-neutrino Majorana mass term:

$$\mathcal{L}_{\nu\,\mathrm{mass}} = \frac{1}{2}\nu_L^T C^{-1} \mathcal{M}_\nu \nu_L + \mathrm{H.c.}$$

Defining relations for mass matrices M1, M2:

$$S \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Invariance of $\mathcal{L}_{\nu\,\mathrm{mass}}$ under

$$\begin{array}{lclcll} \nu_L & \to & S\nu_L & \Rightarrow & M1: & S\mathcal{M}_\nu S = \mathcal{M}_\nu \\ \nu_L & \to & -iSC\nu_L^* & \Rightarrow & M2: & S\mathcal{M}_\nu S = \mathcal{M}_\nu^* \end{array}$$

M2 from CP: Grimus, Lavoura, hep-ph/0305309

Phenomenology of matrix M1:

Assumption: Charged-lepton mass matrix diagonal

$$\begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = (z - w) \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \; U = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta}{\sqrt{2}} & -\frac{\cos\theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{matrix} \theta_{13} = 0^\circ \\ \theta_{23} = 45^\circ \\ \theta_{12} \equiv \theta \\ \text{arbitrary} \end{matrix}$$

M1 either ruled out or needs large corrections because $\theta_{13}|_{\mathrm{exp}} \simeq 9^\circ$.

Note: $m_3 = |z - w|$, masses are not determined by M1,

$\sin^2 2\theta_{\mathrm{atm}} = 4\,|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2\right) = 1$

Neutrino mass matrix with μ–τ exchange symmetry

Phenomenology of matrix M2:

Assumption: Charged-lepton mass matrix diagonal.

$$\text{M2:} \quad \mathcal{M}_\nu = \begin{pmatrix} a & r & r^* \\ r & s & b \\ r^* & b & s^* \end{pmatrix} \quad a, b \in \mathbb{R}, \; r, s \in \mathbb{C}$$

M2 first introduced by Babu, Ma, Valle, hep-ph/0206292 in a different context.

Lepton mixing matrix:

$$V^T \mathcal{M}_\nu V = \mathrm{diag}\,(m_1, m_2, m_3), \quad V = e^{i\hat{\alpha}} U \mathrm{diag}\left(1, e^{i\beta_1}, e^{i\beta_2}\right)$$

with

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Phenomenology of matrix M2 (continued):

- $S\mathcal{M}_\nu S = \mathcal{M}_\nu^*$ and ν mass spectrum non-degenerate

$\Rightarrow$ $SV^* = VX$ with X being a diagonal phase matrix

$\Rightarrow$ $|U_{\mu j}| = |U_{\tau j}|\ \forall j = 1, 2, 3$ (Harrison, Scott, hep-ph/0210197)

$\Rightarrow$ $s_{23}^2 = 1/2$, $s_{13} \cos\delta = 0$

$\Rightarrow$ $r^2 s^* \notin \mathbb{R}$: $s_{13} \neq 0$, $e^{i\delta} = \pm i$

Note: $\sin^2 2\theta_{\mathrm{atm}} = 4\,|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2\right) = 1 - s_{13}^4$

Notion of co-bimaximal mixing:

$\theta_{23} = 45^\circ$, $\delta = \pm 90^\circ$, Ma, arXiv:1510.02501

Seesaw extension of the SM: SM + 3 ν_R + L violation

$$\begin{aligned}\mathcal{L} &= \cdots - \textstyle\sum_j \left[\bar{\ell}_R \phi_j^\dagger \Gamma_j + \bar{\nu}_R \tilde{\phi}_j^\dagger \Delta_j \right] D_L + \text{H.c.} \\ &\quad + \left(\tfrac{1}{2}\, \nu_R^T C^{-1} M_R^* \nu_R + \text{H.c.} \right)\end{aligned}$$

$$M_R = M_R^T, \quad M_\ell = \frac{1}{\sqrt{2}} \sum_j v_j^* \Gamma_j\,, \quad M_D = \frac{1}{\sqrt{2}} \sum_j v_j \Delta_j$$

Total Majorana mass matrix for left-handed neutrino fields:

$$\mathcal{M}_{D+M} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix} \quad \text{for} \quad \begin{pmatrix} \nu_L \\ \nu_R)^c \end{pmatrix}$$

Assumption: $m_D \ll m_R$ ($m_{D,R}$ scales of $M_{D,R}$)
Seesaw formula: $\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D$

Diagonalization: $(U_R^\ell)^\dagger M_\ell U_L^\ell = \hat{m}_\ell$
Mixing matrix: $U_M = (U_L^\ell)^\dagger V$

Three sources of lepton mixing: M_ℓ, M_D, M_R

The CP model

Grimus, Lavoura, hep-ph/0305309

Features:

- M2 from a CP symmetry
- M_ℓ, M_D diagonal $\Rightarrow$ M_R sole source of lepton mixing

Multiplets:

$D_{\alpha L}$, α_R, $\nu_{\alpha R}$ $(\alpha = e, \mu, \tau)$, ϕ_j $(j = 1, 2, 3)$

Symmetries:

- Flavour lepton numbers L_α:
  broken softly by the Majorana mass terms of the ν_R
- Non-standard CP transformation:

$$
\begin{aligned}
&D_{\alpha L} \to i S_{\alpha\beta} \gamma^0 C \bar{D}^T_{\beta L} , \\
&\nu_{\alpha R} \to i S_{\alpha\beta} \gamma^0 C \bar{\nu}^T_{\beta R} , \quad \alpha_R \to i S_{\alpha\beta} \gamma^0 C \bar{\beta}^T_R , \\
&\phi_{1,2} \to \phi^*_{1,2} , \quad \phi_3 \to -\phi^*_3
\end{aligned}
$$

- $\mathbb{Z}_2^{(\mathrm{aux})}$: μ_R, τ_R, ϕ_2, ϕ_3 change sign, broken spontaneously

The CP model

Yukawa Lagrangian: y_1, y_3 real

$$\begin{aligned}\mathcal{L}_\mathrm{Y} \;=\; & -y_1 \bar{D}_e \nu_{eR} \tilde{\phi}_1 - \left(y_2 \bar{D}_\mu \nu_{\mu R} + y_2^* \bar{D}_\tau \nu_{\tau R} \right) \tilde{\phi}_1 \\ & -y_3 \bar{D}_e e_R \phi_1 - \left(y_4 \bar{D}_\mu \mu_R + y_4^* \bar{D}_\tau \tau_R \right) \phi_2 \\ & - \left(y_5 \bar{D}_\mu \mu_R - y_5^* \bar{D}_\tau \tau_R \right) \phi_3 + \text{H.c.}\end{aligned}$$

The CP model features mass matrix M2:

1. Without loss of generality $v_1 \in \mathbb{R} \Rightarrow$ $M_D = \mathrm{diag}\,(c, d, d^*)$ with $c \in \mathbb{R}$
2. $M_D^* = S M_D S$
3. $M_R^* = S M_R S$
4. Seesaw formula $\Rightarrow \mathcal{M}_\nu^* = S \mathcal{M}_\nu S$

The CP model

Interesting feature of CP model:
$m_\mu \neq m_\tau$ through CP violation

$$m_\mu = \frac{1}{\sqrt{2}} \left| y_4 v_2 + y_5 v_3 \right| , \quad m_\tau = \frac{1}{\sqrt{2}} \left| y_4^* v_2 - y_5^* v_3 \right|$$

Check the case of CP conservation:

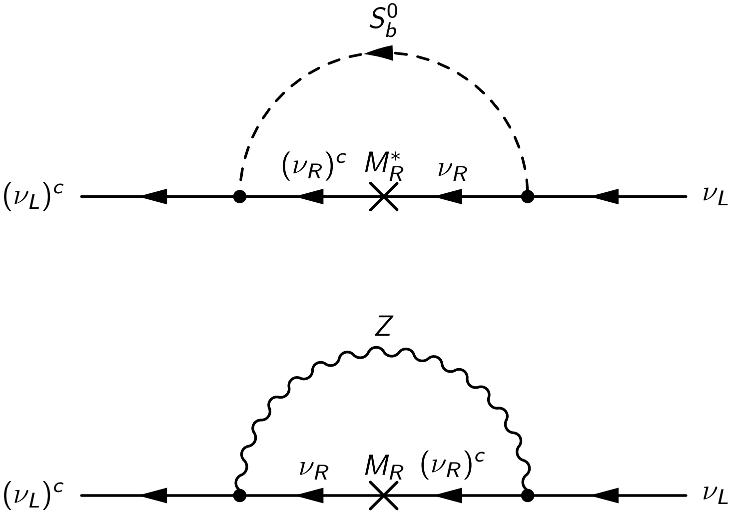
- CP conservation: $v_2 = v_2^*$, $v_3 = -v_3^*$

$\Rightarrow$ $\left| y_4^* v_2 - y_5^* v_3 \right| = \left| y_4^* v_2^* + y_5^* v_3^* \right| = \left| y_4 v_2 + y_5 v_3 \right|$

$\Rightarrow$ $m_\mu = m_\tau$

Co-bimaximal lepton mixing in the scotogenic model

Feynman diagrams for dominant seesaw corrections



Radiative corrections to the seesaw mechanism

Grimus, Lavoura, hep-ph/0207229

$$\mathcal{L}_{\nu_R \,\mathrm{Yukawa}} = -\overline{\nu_R} \left(\sum_{k=1}^{n_H} \tilde{\phi}_k^\dagger \Delta_k \right) D_L + \mathrm{H.c.}$$

Neutral-scalar mass eigenfields S_b^0: $b = 1, \ldots, 2n_H$

$$\phi_k^0 = \frac{v_k + \sum_{b=1}^{2n_H} \mathcal{V}_{kb} S_b^0}{\sqrt{2}}$$

with

$$\mathcal{V} = \mathrm{Re}\,\mathcal{V} + i \mathrm{Im}\,\mathcal{V}, \quad \begin{pmatrix} \mathrm{Re}\,\mathcal{V} \\ \mathrm{Im}\,\mathcal{V} \end{pmatrix} \; 2n_H \times 2n_H \text{ orthogonal}$$

Radiative corrections to the seesaw mechanism

$$\Delta_b \equiv \sum_{k=1}^{n_H} \mathcal{V}_{kb} \Delta_k, \quad W^\dagger M_R W^* = \widetilde{M} \equiv \mathrm{diag}\,(M_1, \ldots, M_{n_R})$$

$$\begin{aligned} \delta M_L \quad = \quad & \sum_{b \neq b_Z} \frac{m_b^2}{32\pi^2} \Delta_b^T W^* \left(\frac{\widetilde{M}}{\widetilde{M}^2 - m_b^2} \ln \frac{\widetilde{M}^2}{m_b^2} \right) W^\dagger \Delta_b \\ & + \frac{3 g^2 m_Z^2}{64\pi^2 c_w^2} M_D^T W^* \left(\frac{\widetilde{M}}{\widetilde{M}^2 - m_Z^2} \ln \frac{\widetilde{M}^2}{m_Z^2} \right) W^\dagger M_D \end{aligned}$$

$$\mathcal{M}'_{D+M} = \begin{pmatrix} \delta M_L & M_D^T \\ M_D & M_R \end{pmatrix} \quad \Rightarrow \quad \mathcal{M}'_\nu = \delta M_L - M_D^T M_R^{-1} M_D$$

The scotogenic model

Ernest Ma, hep-ph/0601225: scotogenic = caused by darkness
Two scalar doublets: $\phi_1 \equiv \phi$, $\phi_2 \equiv \eta$, three ν_R, D_L, ℓ_R
Unbroken $\mathbb{Z}_2$ symmetry: $\eta \to -\eta$, $\nu_R \to -\nu_R \Rightarrow$ dark sector
VEV of η is zero!
Scalar potential: λ_5 real without loss of generality

$$\begin{aligned} V &= \mu_1^2 \phi^\dagger \phi + \mu_2^2 \eta^\dagger \eta + \frac{1}{2}\lambda_1 \left(\phi^\dagger \phi\right)^2 + \frac{1}{2}\lambda_2 \left(\eta^\dagger \eta\right)^2 + \lambda_3 \left(\phi^\dagger \phi\right)\left(\eta^\dagger \eta\right) \\ & +\lambda_4 \left(\phi^\dagger \eta\right)\left(\eta^\dagger \phi\right) + \frac{1}{2}\lambda_5 \left[\left(\phi^\dagger \eta\right)^2 + \left(\eta^\dagger \phi\right)^2\right] \end{aligned}$$

No treelevel neutrino masses!
$\Delta_1 = 0$, $\Delta_2 \equiv \Delta$
VEV of ϕ: assume $v > 0$

The scotogenic model

Scalars S_b^0: SM Higgs S_1^0, Goldstone S_2^0, "dark" scalars $S_{3,4}^0$

$$\phi^0 = (v + S_1^0 + iS_2^0)/\sqrt{2}, \quad \eta^0 = (S_3^0 + iS_4^0)/\sqrt{2}$$

Coupling matrices to ν_R of the S_b^0: 0, 0, Δ, $i\Delta$

Masses of S_3^0 (m_R) and S_4^0 (m_I):

$$\begin{aligned} m_R^2 &= \mu_2^2 + \tfrac{v^2}{2}(\lambda_3 + \lambda_4 + \lambda_5) \\ m_I^2 &= \mu_2^2 + \tfrac{v^2}{2}(\lambda_3 + \lambda_4 - \lambda_5) \end{aligned}$$

Without loss of generality: M_R diagonal, *i.e.* $W = \mathbb{1}$

Majorana mass matrix:

$$\mathcal{M}'_\nu = \frac{1}{32\pi^2}\,\Delta^T \widetilde{M} \left(\frac{m_R^2}{\widetilde{M}^2 - m_R^2} \ln \frac{\widetilde{M}^2}{m_R^2} - \frac{m_I^2}{\widetilde{M}^2 - m_I^2} \ln \frac{\widetilde{M}^2}{m_I^2} \right) \Delta$$

The scotogenic model

Several suppression mechanisms:

- Large seesaw scale
- Small Yukawa couplings Δ
- Small λ_5
- Loop factor $(32\pi^2)^{-1}$

$\mathcal{O}(\Delta^2, \lambda_5) \sim 10^{-4} \Rightarrow$ seesaw scale $\sim 1\,\text{TeV}$ (dark matter)

Scotogenic model for co-bimaximal mixing

P.M. Ferreira, W. Grimus, D. Jurčiuconis, L. Lavoura, arXiv:1604.07777

Outline of the model:

- Type of model:
  Extension of the SM with gauge symmetry $SU(2) \times U(1)$
- Dark sector:
  Fields with eigenvalues -1 of unbroken $\mathbb{Z}_2^{(\mathrm{dark})}$
- Multiplets: $\alpha = e, \mu, \tau$

	fermions	scalar doublets
bright:	$D_{\alpha L}, \quad \alpha_R$	$\phi_j \ (j = 1, 2, 3)$
dark:	$\nu_{\alpha R}$	$\phi_4 \equiv \eta$

- Charged lepton masses:
  $\phi_1 \to m_e, \ \phi_2 \to m_\mu, \ \phi_3 \to m_\tau$

Symmetries:

- $\mathbb{Z}_2^{(\mathrm{dark})}$:
  $\eta \to -\eta,\ \nu_{eR} \to -\nu_{eR},\ \nu_{\mu R} \to -\nu_{\mu R}$, and $\nu_{\tau R} \to -\nu_{\tau R}$.
  Exact symmetry that prevents dark matter from mixing with ordinary matter.
- The flavour lepton numbers L_α:
  Broken softly by the Majorana mass terms

$$\mathcal{L}_{\mathrm{Majorana}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_{eR}}, & \overline{\nu_{\mu R}}, & \overline{\nu_{\tau R}} \end{pmatrix} M_R C \begin{pmatrix} \overline{\nu_{eR}}^T \\ \overline{\nu_{\mu R}}^T \\ \overline{\nu_{\tau R}}^T \end{pmatrix} + \mathrm{H.c.}$$

Scotogenic model for co-bimaximal mixing

Symmetries (continued):

- $$\begin{array}{lcll} \mathbb{Z}_2^{(1)} & : & \phi_1 \to -\phi_1 & e_R \to -e_R \\ \mathbb{Z}_2^{(2)} & : & \phi_2 \to -\phi_2 & \mu_R \to -\mu_R \\ \mathbb{Z}_2^{(3)} & : & \phi_3 \to -\phi_3 & \tau_R \to -\tau_R \end{array}$$

  Spontaneously broken through the VEVs $\langle 0|\phi_j^0|0\rangle = v_j/\sqrt{2}$ $(j = 1, 2, 3)$ and softly through $\mathcal{L}_{\mathrm{Majorana}}$

  - Yukawa Lagrangian:

  $$\begin{aligned} \mathcal{L}_{\ell\,\mathrm{Yukawa}} = & -y_1\, \overline{\nu_{eR}}\, \tilde{\eta}^\dagger D_{eL} - y_2\, \overline{\nu_{\mu R}}\, \tilde{\eta}^\dagger D_{\mu L} - y_3\, \overline{\nu_{\tau R}}\, \tilde{\eta}^\dagger D_{\tau L} \\ & -y_4\, \overline{e_R}\, \phi_1^\dagger D_{eL} - y_5\, \overline{\mu_R}\, \phi_2^\dagger D_{\mu L} - y_6\, \overline{\tau_R}\, \phi_3^\dagger D_{\tau L} + \mathrm{H.c.} \end{aligned}$$

  - Charged lepton masses:

  $$m_e = \left|\frac{y_4 v_1}{\sqrt{2}}\right|, \quad m_\mu = \left|\frac{y_5 v_2}{\sqrt{2}}\right|, \quad m_\tau = \left|\frac{y_6 v_3}{\sqrt{2}}\right|.$$

Symmetries (continued):

- The CP symmetry

$$CP: \begin{cases} D_L \to i\gamma_0 C\, S\, \overline{D_L}^T \\ \ell_R \to i\gamma_0 C\, S\, \overline{\ell_R}^T \\ \nu_R \to i\gamma_0 C\, S\, \overline{\nu_R}^T \\ \phi \to S\, \phi^* \\ \eta \to \eta^* \end{cases} \quad \text{with} \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$D_L = \begin{pmatrix} D_{eL} \\ D_{\mu L} \\ D_{\tau L} \end{pmatrix}, \quad \ell_R = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$\nu_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

CP spontaneously broken through the VEVs v_j

Scotogenic model for co-bimaximal mixing

Consequences of the CP symmetry:

$$S M_R S = M_R^*, \quad y_1,\ y_4 \text{ real}, \quad y_3 = y_2^*,\ y_6 = y_5^*$$

CP violation $\Rightarrow m_\mu \neq m_\tau$ because of

$$\frac{m_\mu}{m_\tau} = \left| \frac{v_2}{v_3} \right|$$

$\mathbb{Z}_2^{(\mathrm{dark})}$ and CP symmetry $\Rightarrow$

$$\Delta_1 = \Delta_2 = \Delta_3 = 0, \quad \Delta_4 = \mathrm{diag}\left(y_1, y_2, y_2^*\right) \ \Rightarrow\ S\Delta_4 S = \Delta_4^*$$

Scotogenic model for co-bimaximal mixing

Scalar potential: crucial term given by CP-invariant

$$\begin{aligned} V_\xi &= \xi_1 \left[\left(\phi_1^\dagger \eta \right)^2 + \left(\eta^\dagger \phi_1 \right)^2 \right] + \\ & \xi_2 \left[\left(\phi_2^\dagger \eta \right)^2 + \left(\eta^\dagger \phi_3 \right)^2 \right] + \xi_3 \left[\left(\phi_3^\dagger \eta \right)^2 + \left(\eta^\dagger \phi_2 \right)^2 \right] \end{aligned}$$

Hermiticity: $\xi_1 = \xi_1^*$, $\xi_3 = \xi_2^*$

$$\phi_4^0 \equiv \eta^0 = e^{i\gamma} \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$$

Generation of mass difference between φ_1 and φ_2:
Phase γ defined such that

$$\mu^2 \equiv e^{2i\gamma} \sum_{j=1}^{3} \xi_j \frac{v_j^{*2}}{2} > 0 \quad \Rightarrow \quad \mu^2 \left(\varphi_1^2 - \varphi_2^2 \right)$$

All other terms in the potential have $\left| \eta^0 \right|^2 = \left(\varphi_1^2 + \varphi_2^2 \right)/2$.

Scotogenic model for co-bimaximal mixing

Neutrino mass matrix:

$\mathcal{V}_{4\varphi_1} = e^{i\gamma}$, $\mathcal{V}_{4\varphi_2} = ie^{i\gamma}$, $W^\dagger M_R W^* = \widetilde{M}$ diagonal

$$\delta\mathcal{M}_\nu = \frac{e^{2i\gamma}}{32\pi^2}\times \left[\Delta_4 W^* \left(\frac{m^2_{\varphi_1}}{\widetilde{M}} \ln \frac{\widetilde{M}^2}{m^2_{\varphi_1}}\right) W^\dagger \Delta_4 - \Delta_4 W^* \left(\frac{m^2_{\varphi_2}}{\widetilde{M}} \ln \frac{\widetilde{M}^2}{m^2_{\varphi_2}}\right) W^\dagger \Delta_4\right]$$

Remark: $m^2_{\varphi_1} = m^2_{\varphi_2} \Rightarrow \delta\mathcal{M}_\nu = 0$

Note: $V_\xi = 0 \Rightarrow$

- $\mu = 0 \Rightarrow m^2_{\varphi_1} = m^2_{\varphi_2}$
- $U(1)$-symmetry $\quad D_L \to e^{i\psi} D_L,\ \ell_R \to e^{i\psi}\ell_R,\ \eta \to e^{-i\psi}\eta$ forbids light Majorana neutrino masses.

Scotogenic model for co-bimaximal mixing

Co-bimaximal mixing from $\delta\mathcal{M}_\nu$:

$$e^{-2i\gamma}\delta\mathcal{M}_\nu = \Delta_4 W^* \hat{A} W^\dagger \Delta_4$$

with diagonal matrix

$$\hat{A} = \frac{1}{32\pi^2}\left(\frac{m^2_{\varphi_1}}{\widetilde{M}}\ln\frac{\widetilde{M}^2}{m^2_{\varphi_1}} - \frac{m^2_{\varphi_2}}{\widetilde{M}}\ln\frac{\widetilde{M}^2}{m^2_{\varphi_2}}\right)$$

Want to prove:

$$S\left(e^{-2i\gamma}\delta\mathcal{M}_\nu\right)S = \left(e^{-2i\gamma}\delta\mathcal{M}_\nu\right)^*$$

Co-bimaximal mixing from $\delta\mathcal{M}_\nu$ (continued):

Step 1:
Assume non-degeneracy of $\widetilde{M} = \mathrm{diag}\,(M_1, M_2, M_3)$,
define $W^\dagger S W^* \equiv X$

$$\left.\begin{array}{r} W^\dagger M_R W^* = \widetilde{M} \\ S M_R S = M_R^* \end{array}\right\} \Rightarrow X^* \widetilde{M} = \widetilde{M} X \;\Rightarrow\; X \text{ diagonal sign matrix}$$

Step 2: Use $SW^* = WX$, $W^\dagger S = X W^T$

$$\begin{aligned} S\left(\Delta_4 W^* \hat{A} W^\dagger \Delta_4\right) S &= (S\Delta_4 S)\,(SW^*)\,\hat{A}\left(W^\dagger S\right)(S\Delta_4 S) \\ &= \Delta_4^* W X \hat{A} X W^T \Delta_4^* \\ &= \Delta_4^* W \hat{A} W^T \Delta_4^* \\ &= \left(\Delta_4 W^* \hat{A} W^\dagger \Delta_4\right)^* \quad \text{q.e.d.} \end{aligned}$$

Scotogenic model for co-bimaximal mixing

Concluding remarks concerning this model:

- Possible to unify the scotogenic model with co-bimaximal mixing
- CP symmetry which μ–τ flavour interchange crucial
- Proliferation of scalar gauge doublets:
  ϕ_j $(j = 1, 2, 3)$ with non-zero VEVs plus dark doublet η
- Non-trivial task to accommodate scalar with mass of 125 GeV and couplings close to that of SM Higgs because of μ–τ flavour interchange!
- Numerical result: All non-SM scalars can have masses above 600 GeV
- Consistent extension of the model to quark sector possible