Top Quark Hadroproduction at Higher Orders

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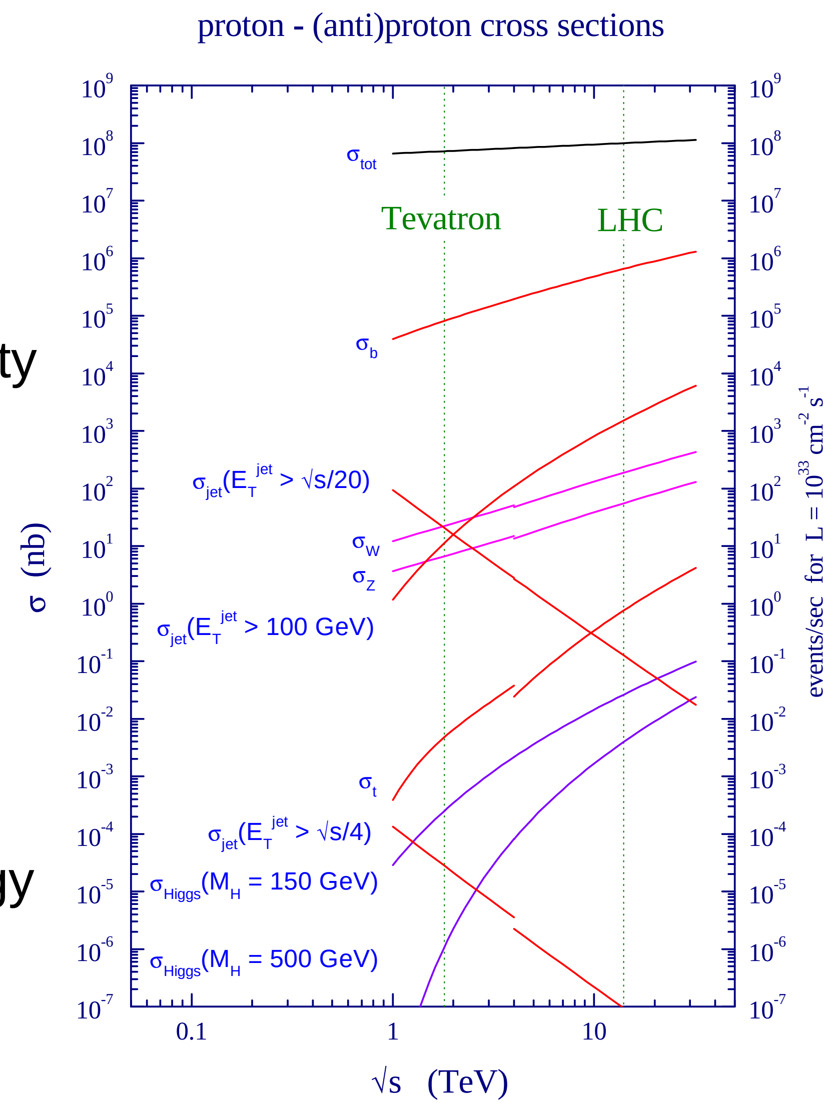
Plan

- Motivation
- Theory predictions
  - uncertainties from scale dependence, parton luminosity, etc.
  - improvements through threshold resummation
- Towards higher orders in QCD
  - soft and collinear limits of massive QCD amplitudes
  - relations between massless and massive amplitudes
- Outlook

Top-production at the LHC

Experimental expectations

- LHC will accumulate very high statistics for $t\bar{t}$-pairs
  - $8 \cdot 10^6$ events/year in low luminosity run
    (10 time more in high luminosity run)
  - mass measurement
    $\Delta m_t = \mathcal{O}(1)$GeV
    (constraints on Standard Model Higgs mass m_h)
- Use $t\bar{t}$-pairs for calibration of jet energy scale (decay $W \to 2$ jets)



Hard scattering at colliders

- QCD theory $\longrightarrow$ **factorization** of cross section
  - separate sensitivity to dynamics form different scales

$$\sigma_{\rm pp}\,(Q,m) = \sum_{\rm ij} \hat{\sigma}_{\rm ij}\,(Q/\mu, \alpha_s(\mu))\ \otimes\ PDF_{\rm i}\,(\mu,m) \otimes\ PDF_{\rm j}\,(\mu,m)$$

  - large momentum scale Q, factorization scale μ, soft scale m

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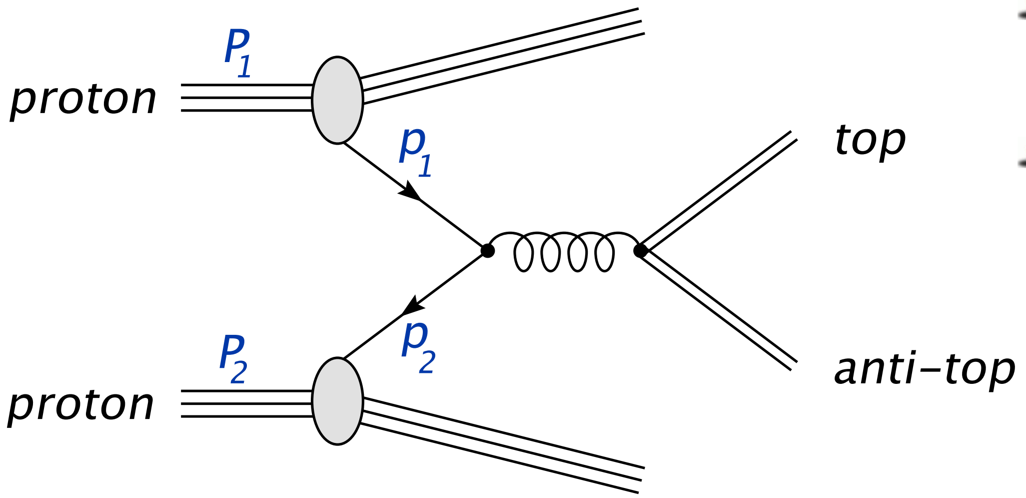


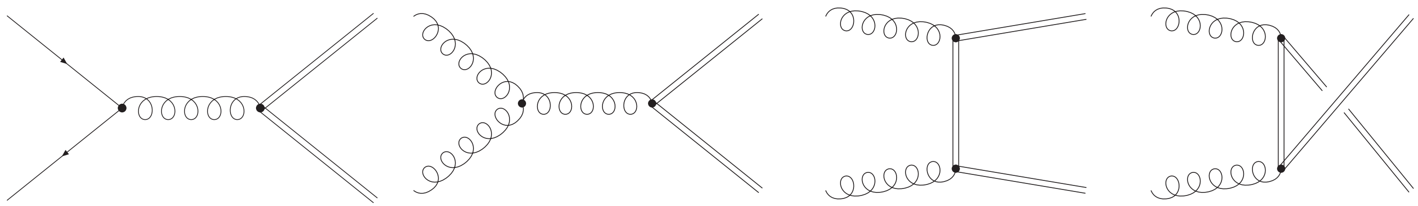
- Parton luminosity
  $PDF_{\rm i} \otimes PDF_{\rm j}$
- Theory predictions for $\hat{\sigma}_{\rm ij}$
  (uncertainties from μ-scale variation)
  - higher order QCD predictions (NLO and NNLO) needed

Theory predictions

Top-quark pair-production

- Leading order Feynman diagrams

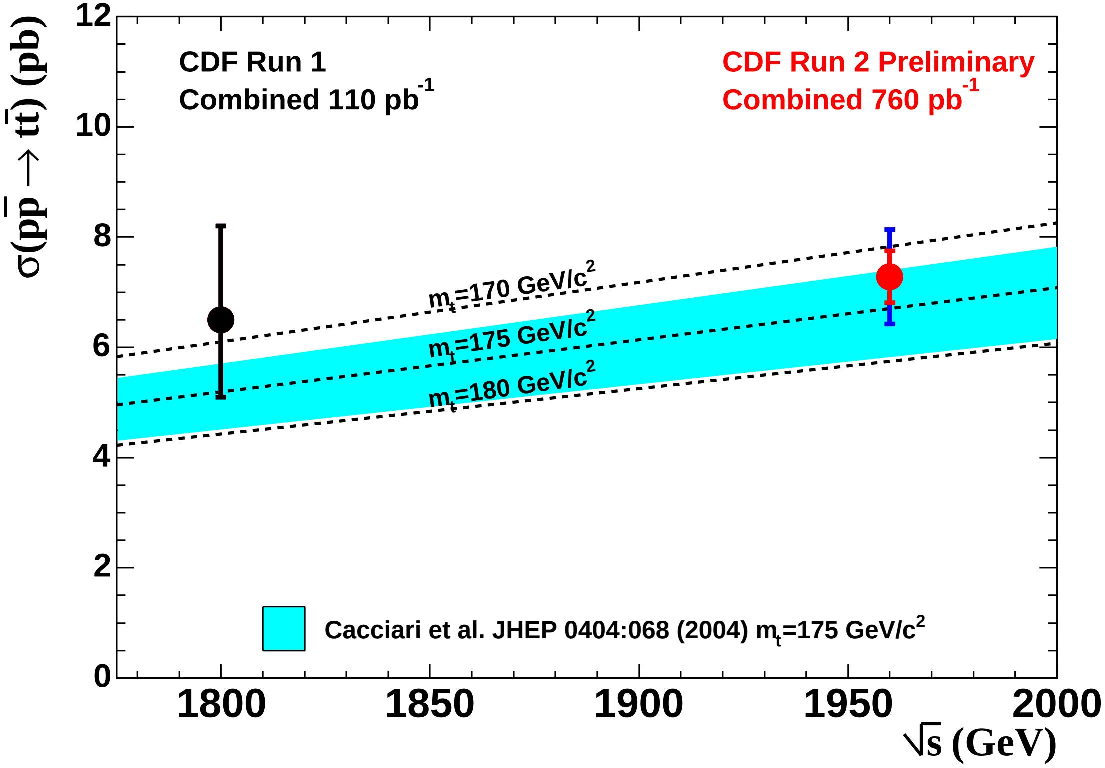
$$\mathrm{q} + \bar{\mathrm{q}} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$
$$\mathrm{g} + \mathrm{g} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$



- NLO in QCD Nason, Dawson, Ellis '88; Beenakker, Smith, van Neerven '89; Mangano,Nason, Ridolfi '92; . . .
  - $\mathrm{q\bar{q}}$ and gg dominant at NLO
  - neglect $\mathrm{qg} \longrightarrow$ at NLO only $\mathcal{O}(1\%)$
- Higher order QCD corrections essential
  - NLO in QCD accurate to $\mathcal{O}(10\% - 15\%)$ at LHC
  - threshold resummation important (however, at LHC much less than at Tevatron)

Tevatron results

- Total cross section as function of energy $\sqrt{s}$
  (theory error band from scale uncertainty)



- NNLO required for precision determinations of m_t

Renormalization / Factorization scale dependence

- If $Obs = \sum_{n=0}^{N} A_n(\mu)\alpha_s^n(\mu)$, then $\frac{\partial}{\partial \ln \mu} Obs = \mathcal{O}(\alpha_s^{N+1})$
- Scale variation produces only copies of lower order terms, i.e. dependence on $\ln \mu$ is entirely predictable

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$$\sigma_{t\bar{t}} = \alpha_s^2(\mu)A_0 + \alpha_s^3(\mu)\left\{A_1 + L f_1(A_0, \beta_0, P_0)\right\}$$

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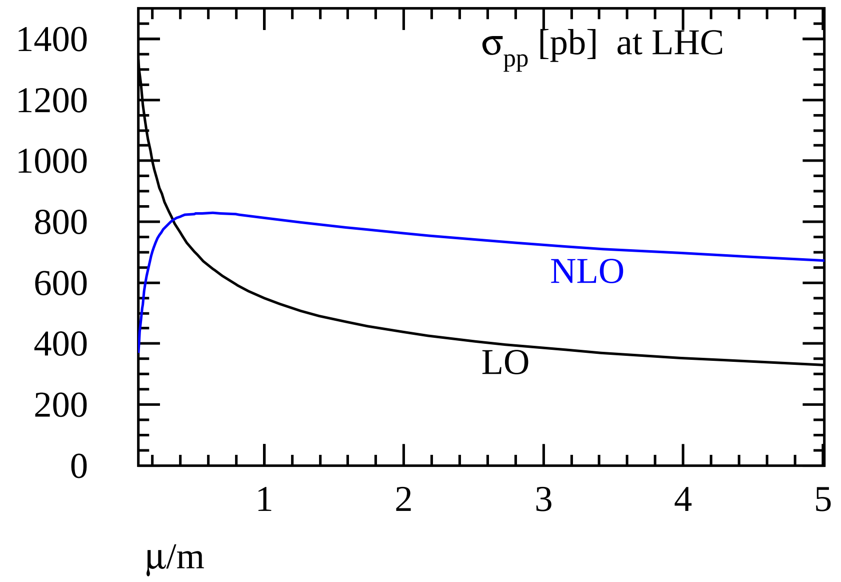
- $\sigma_{t\bar{t}}$ ($q\bar{q}$ and gg-channel) at LO, NLO



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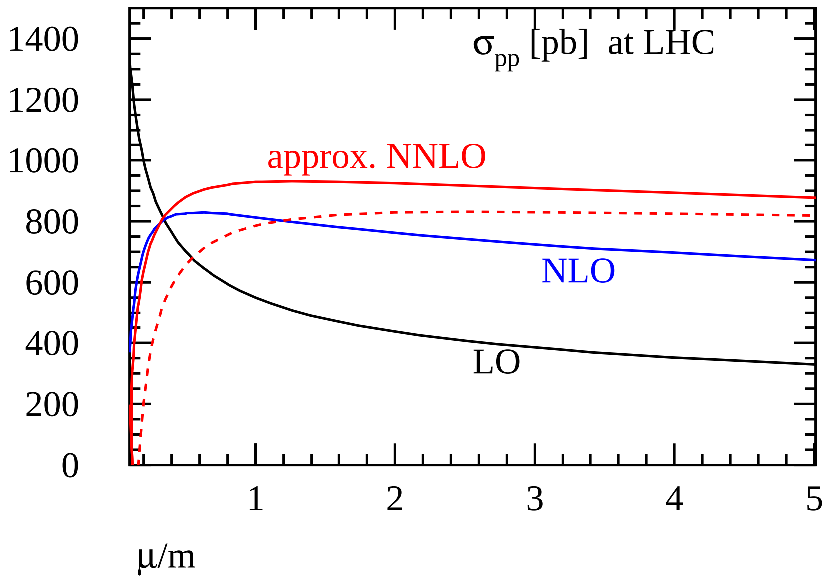
$$+\alpha_s^4(\mu)\left\{A_2 + Lf_2(A_0,A_1,\beta_0,\beta_1,P_0,P_1) + L^2 f_3(A_0,\beta_0,P_0)\right\}$$

- $\sigma_{t\bar{t}}$ ($q\bar{q}$ and gg-channel) at LO, NLO and with NNLO exact scale dependence assuming
  - $A_2 = (A_1)^2/2$ (solid) consistent with threshold exponentiation
  - $A_2 = 0$ (dotted)

  for $\sqrt{s} = 14$TeV, $m_t = 175$GeV



- Recall our master formula

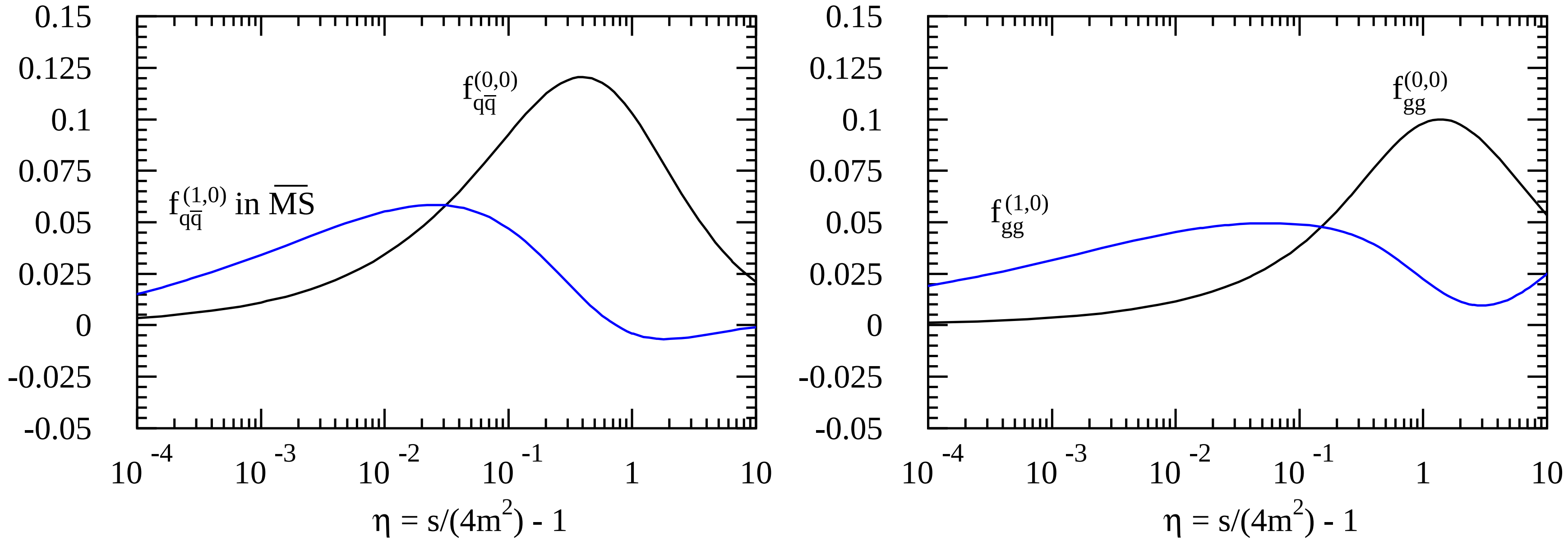
$$\sigma_{\mathrm{pp}}\left(Q,m\right) = \sum_{\mathrm{ij}} \hat{\sigma}_{\mathrm{ij}}\left(Q/\mu, \alpha_s(\mu)\right) \otimes PDF_{\mathrm{i}}\left(\mu,m\right) \otimes PDF_{\mathrm{j}}\left(\mu,m\right)$$

Parton cross section

- Expansion in terms of scaling functions $f_{\mathrm{ij}}^{(k,l)}$

$$\begin{aligned}\hat{\sigma}_{\mathrm{ij}} &= \frac{\alpha_s^2}{m^2}\Bigg[f_{\mathrm{ij}}^{(0,0)} + \\ &\quad + 4\pi\alpha_s\left(f_{\mathrm{ij}}^{(1,0)} + \ln\frac{\mu^2}{m^2} f_{\mathrm{ij}}^{(1,1)}\right) + \\ &\quad + (4\pi\alpha_s)^2\left(f_{\mathrm{ij}}^{(2,0)} + \ln\frac{\mu^2}{m^2} f_{\mathrm{ij}}^{(2,1)} + \ln^2\frac{\mu^2}{m^2} f_{\mathrm{ij}}^{(2,2)}\right)\Bigg]\end{aligned}$$

- Numerical investigation of scaling functions $f_{q\bar{q}}$ and f_{gg}
  - variable $\eta = \frac{s}{4m^2} - 1$ measures distance from $t\bar{t}$-threshold



- Resummation of threshold logarithms $\ln(\eta)$
  - reorganize perturbative expansion $\longrightarrow$ stability

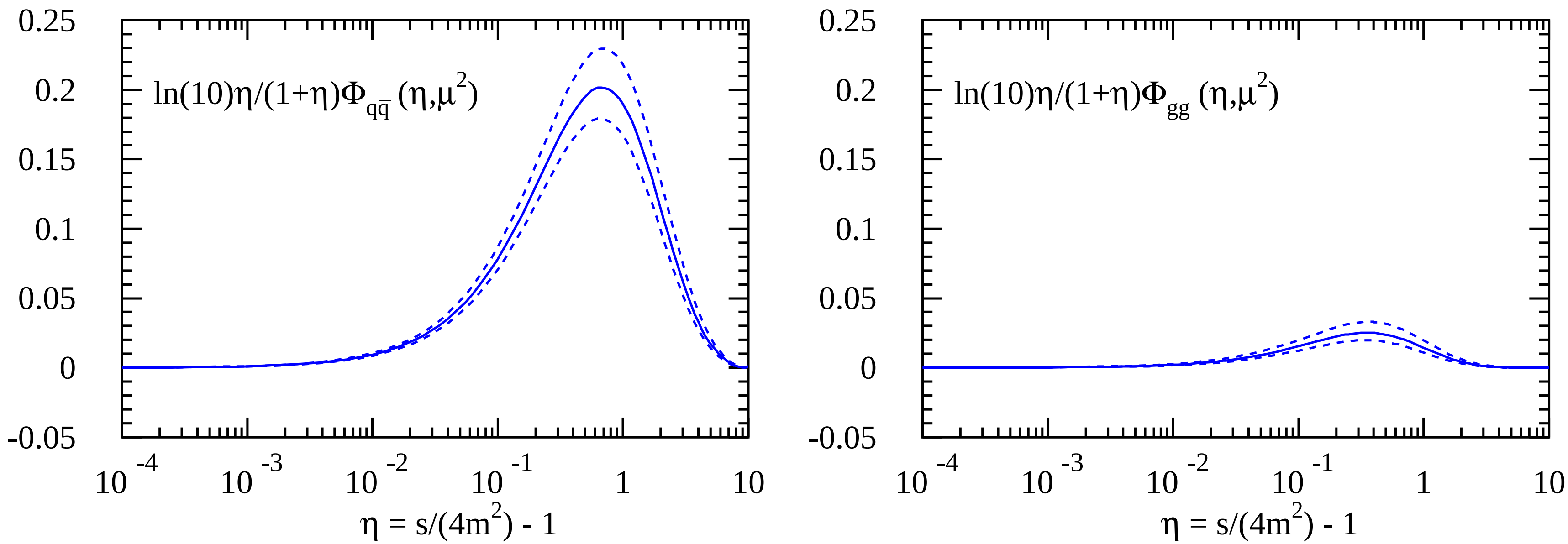
$$\mathcal{O} = 1 + \alpha_s \left(\ln^2 + \ln + 1\right) + \alpha_s^2 \left(\ln^4 + \ln^3 + \ln^2 + \ln + 1\right) + \dots$$
$$= \left(1 + \alpha_s\, 1 + \alpha_s^2\, 1 + \dots\right) \exp\left(\alpha_s \ln^2 + \alpha_s \ln + \alpha_s^2 \ln + \dots\right)$$

Parton luminosity

- Rewrite our master formula in terms of variable $\eta = \dfrac{s}{4m^2} - 1$

$$\sigma_{\rm pp}\,(Q,m) = \sum_{\rm ij} \int_{-\infty}^{\log_{10}(S/4m^2-1)} d\log_{10}\eta\, \frac{\eta}{1+\eta}\, \ln(10)\, \Phi_{ij}(\eta,\mu^2)\; \hat{\sigma}_{\rm ij}\,(Q/\mu, \alpha_s(\mu))$$

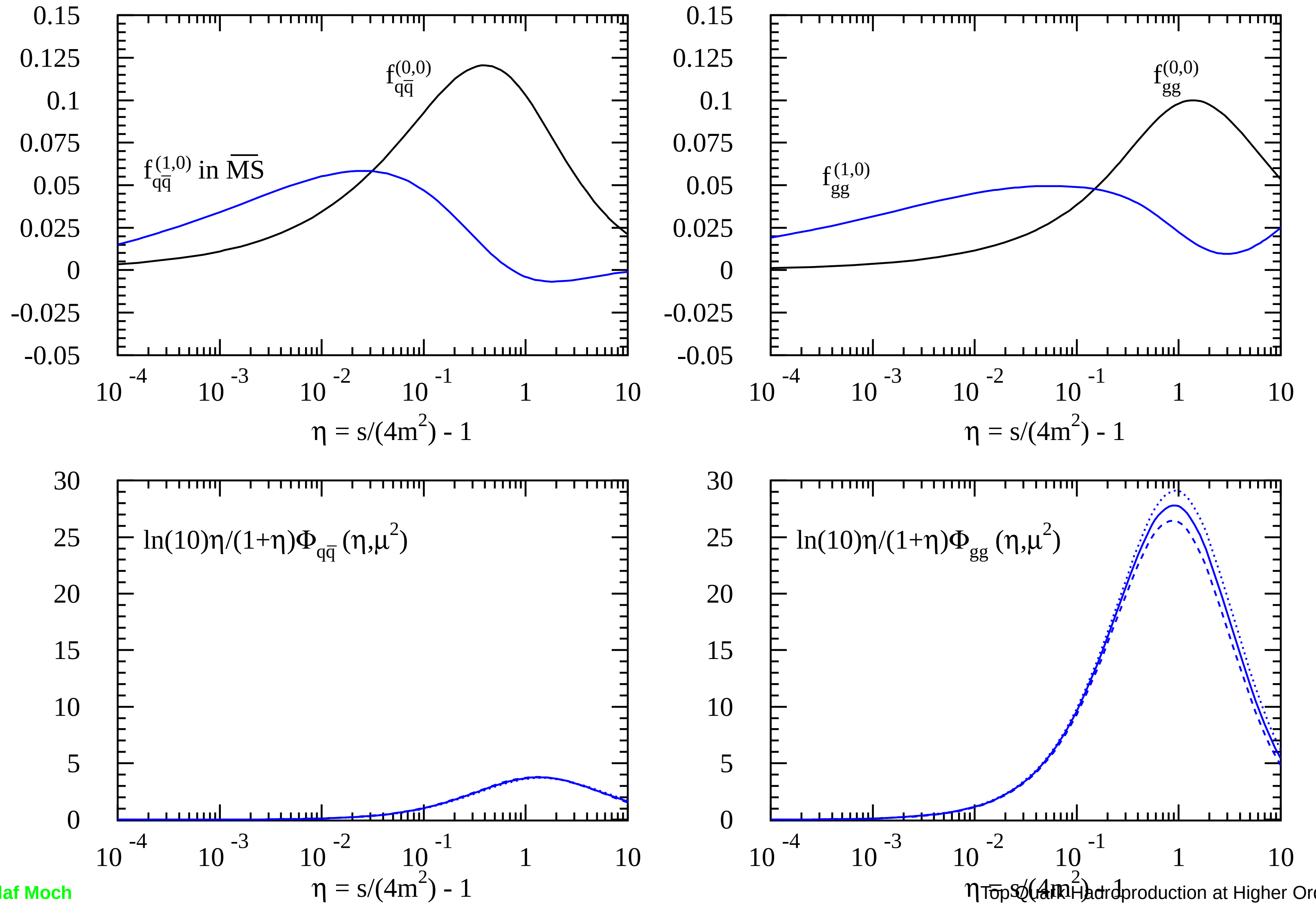
  - define parton luminosity $\Phi_{ij}(\eta,\mu^2) \;=\; PDF_{\rm i}\,(\mu,m) \otimes PDF_{\rm j}\,(\mu,m)$



- Scale variation $m/2 \leq \mu \leq 2m$
  - Tevatron kinematics with $\sqrt{S} = 1.8\mathrm{TeV}$ and $m = 175.0\mathrm{GeV}$

LHC total cross section

- LHC kinematics give less weight to threshold region
  - $\sqrt{S} = 14\text{TeV}$, $m = 175.0\text{GeV}$

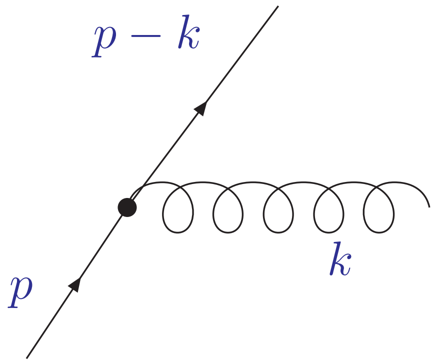


Towards higher orders in QCD

- First steps on the way to massive QCD predictions at two loops
- Study of massive QCD amplitudes
  - look at soft and collinear limits
  - relate massive to massless amplitudes in limit $m \to 0$

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons



$$\frac{1}{(p-k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

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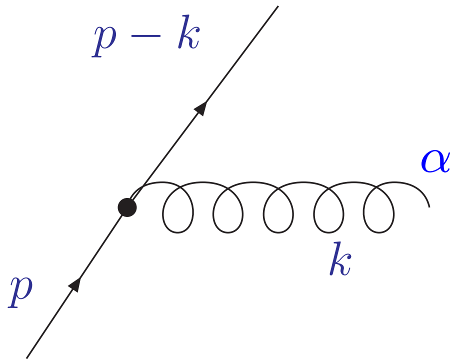

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$$\alpha_s \int d^4k \frac{1}{(p-k)^2} \longrightarrow \alpha_s \int dE_g \, d\theta_{qg} \, \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\dots)$$ in dim. reg. $D = 4 - 2\epsilon$

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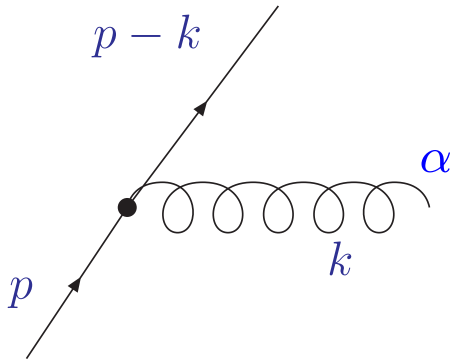

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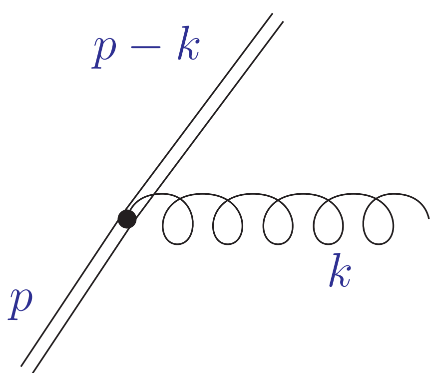
- Parton masses regulate collinear singularity



$$\frac{1}{(p-k)^2 - m_q^2} \quad = \quad \frac{1}{2p \cdot k} \quad = \quad \frac{1}{2E_q E_g (1 - \beta \cos\theta_{qg})}$$

with $\beta = \left(1 - \frac{m_q^2}{E_q^2}\right)^{1/2} < 1$

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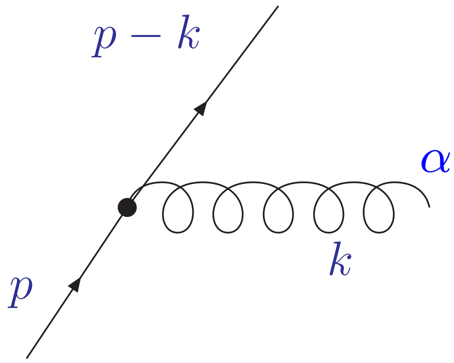

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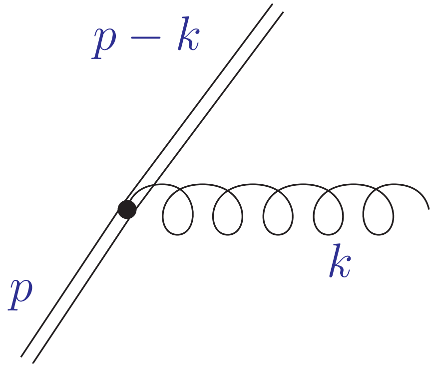

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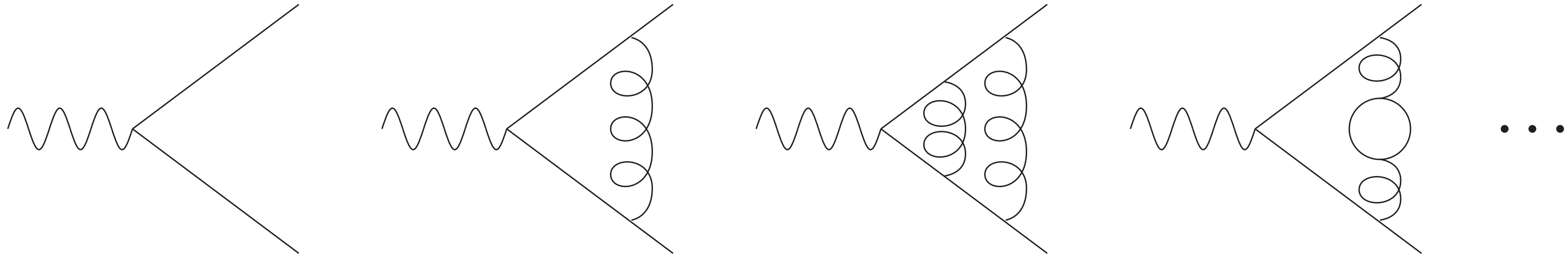
$$\alpha_s \int d^4k \frac{1}{(p-k)^2 - m_q^2} \longrightarrow \alpha_s \frac{1}{\epsilon} \ln(m_q^2) \times (\dots)$$

Form factors



- Quark form factor
  - QCD corrections to vertex $\gamma^* q\bar{q}$, i.e.

$$\Gamma_\mu(k_1, k_2) \; = \\ \mathrm{i} e_{\mathrm{q}} \, \bar{u}(k_1) \, \left(\gamma_\mu \, \mathcal{F}_1(Q^2, m^2, \alpha_s) + \frac{1}{2m} \sigma_{\mu\nu} \, q^\nu \, \mathcal{F}_2(Q^2, m^2, \alpha_s) \right) u(k_2)$$

  - gauge invariant quantity
  - infrared divergent (dimesional regularization $D = 4 - 2\epsilon$)
  - logarithms $\ln(m)$ for massive partons

Exponentiation

- Form factor $\mathcal{F}(Q^2, m^2, \alpha_s)$ exponentiates
  Collins ‘80; Sen ‘81; Korchemsky ‘88; Magnea, Sterman ‘90; Contopanagos, Laenen, Sterman ‘97; Magnea ‘00; Mitov, S.M.‘06

$$Q^2 \frac{\partial}{\partial Q^2} \ln \mathcal{F}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_{\rm s}, \epsilon\right) = \frac{1}{2} K\left(\frac{m^2}{\mu^2}, \alpha_{\rm s}, \epsilon\right) + \frac{1}{2} G\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_{\rm s}, \epsilon\right)$$

- Renormalization group equations for functions G and K
  - well-known evolution equations from factorization
  - same anomalous dimension A for G and K
    - A known to three loops from $P_{\rm qq}$ and $P_{\rm gg}$ S.M. Vermaseren, Vogt ’05

- Solution for $\ln \mathcal{F}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_{\rm s}, \epsilon\right)$ in D-dimensions
  - generating functional for Laurent-series in ϵ and $\ln(m)$ to all orders
  - boundary condition $\mathcal{F}(0, 0, \alpha_s, \epsilon) = 1$

Result

- Massless form factor
  - expansion up to two loops in terms of coefficients of A and G
    Hamberg, van Neerven, Matsuura '88; Harlander '00; Gehrmann, Huber, Maitre '05; S.M. Vermaseren, Vogt '05

$$
\begin{aligned}
\mathcal{F}_1 &= -\frac{1}{2}\frac{1}{\epsilon^2}A_1 - \frac{1}{2}\frac{1}{\epsilon}G_1 \\
\mathcal{F}_2 &= \frac{1}{8}\frac{1}{\epsilon^4}A_1^2 + \frac{1}{8}\frac{1}{\epsilon^3}A_1(2G_1 - \beta_0) + \frac{1}{8}\frac{1}{\epsilon^2}(G_1^2 - A_2 - 2\beta_0 G_1) - \frac{1}{4}\frac{1}{\epsilon}G_2
\end{aligned}
$$

- Massive form factor with logarithms $L = \ln(Q^2/m^2)$
  - expansion up to two loops in terms of coefficients of A, G, K
    Bernreuther et al. '04; Mitov, S.M. '06

$$
\begin{aligned}
\mathcal{F}_1 &= \frac{1}{\epsilon}\left\{\frac{1}{2}A_1 L + \frac{1}{2}(G_1 + K_1)\right\} - \frac{1}{4}A_1 L^2 - \frac{1}{2}G_1 L + C_1 \\
\mathcal{F}_2 &= \frac{1}{\epsilon^2}\left\{\frac{1}{8}A_1^2 L^2 + \frac{1}{4}A_1(G_1 + K_1 - \beta_0)L + \frac{1}{8}(G_1 + K_1)(G_1 + K_1 - 2\beta_0)\right\} \\
&\quad + \frac{1}{\epsilon}\left\{-\frac{1}{8}A_1^2 L^3 - \frac{1}{8}A_1(3G_1 + K_1)L^2 + \dots\right\}
\end{aligned}
$$

Massive amplitudes

- Singularity structure of massive amplitudes $|\mathcal{M}_{\mathrm{p},\{m_i\}}\rangle$
  - process p for $2 \to n$ parton scattering

$$
\begin{aligned}
|\mathcal{M}^{(0)}_{\mathrm{p},\{m_i\}}\rangle &= |\mathcal{H}^{(0)}_{\mathrm{p}}\rangle \,, \\
|\mathcal{M}^{(1)}_{\mathrm{p},\{m_i\}}\rangle &= \frac{1}{2} \sum_{i \in \, \{\text{all legs}\}} \mathcal{F}^{[i]}_1 |\mathcal{H}^{(0)}_{\mathrm{p}}\rangle + \mathcal{S}^{[\mathrm{p}]}_1 |\mathcal{H}^{(0)}_{\mathrm{p}}\rangle + |\mathcal{H}^{(1)}_{\mathrm{p}}\rangle \,, \\
|\mathcal{M}^{(2)}_{\mathrm{p},\{m_i\}}\rangle &= \frac{1}{2} \sum_{i \in \, \{\text{all legs}\}} \left(\mathcal{F}^{[i]}_2 - \frac{1}{4} \left(\mathcal{F}^{[i]}_1 \right)^2 + \frac{1}{2} \mathcal{F}^{[i]}_1 \mathcal{S}^{[\mathrm{p}]}_1 \right) |\mathcal{H}^{(0)}_{\mathrm{p}}\rangle \\
&\quad + \frac{1}{2} \sum_{i \in \, \{\text{all legs}\}} \mathcal{F}^{[i]}_1 |\mathcal{H}^{(1)}_{\mathrm{p}}\rangle + \mathcal{S}^{[\mathrm{p}]}_2 |\mathcal{H}^{(0)}_{\mathrm{p}}\rangle + \mathcal{S}^{[\mathrm{p}]}_1 |\mathcal{H}^{(1)}_{\mathrm{p}}\rangle + |\mathcal{H}^{(2)}_{\mathrm{p}}\rangle
\end{aligned}
$$

- Amplitude factorizes in terms of three known functions $\mathcal{F}$, $\mathcal{S}_{\mathrm{p}}$, $|\mathcal{H}_{\mathrm{p}}\rangle$
  - generalization of Catani's formula
    Catani '98; Sterman, Tejeda-Yeomans '02; Mitov S.M. '06

Upshot

- Simple multiplicative relation between massless and massive amplitudes to all orders

$$\mathcal{M}^{[\mathrm{p}],(m)}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) =$$

$$\prod_{i \in \{\text{all legs}\}} \left(Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right)^{\frac{1}{2}} \times \mathcal{M}^{[\mathrm{p}],(m=0)}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

- factor $Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$
  - determined by ratio of massless and massive form fator

Outlook

Higher Orders

- Theory predictions required for processes with multiple scales
  - massive particles (and jets) in final states, e.g. W, Z or t
  - differential observables, e.g. p_t or y
- Radiative corrections are important
  - NLO QCD for background and searches
  - NNLO QCD for precision measurements

Factorization

- Understand underlying factorization properties in quantum field theory
  - soft and collinear limits of massive QCD amplitudes
  - relations between massless and massive amplitudes