J.G. Körner at the workshop "Top Quark Physik am LHC",

Bad Honnef, January 16th and 17th, 2007

Polarization Effects in Top Quark Decays at NNLO

In collaboration with M. Fischer, S. Groote, W.S. Huo, A. Kadeer,
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1. Top Quark Width

Top quark decays almost 100% to $t \to X_b + W^+$
($\Gamma_{\rm Born} = 1.56$ GeV)

$\frac{1}{\Gamma_{\rm Born}} \Gamma_{t\to X_b+W^+} =$	1	Born	LO	$(\Gamma_{\rm Born} = 1.56{\rm GeV})$
	$-\ 8.5\%$	QCD	NLO	Jezabek, Kühn '89, ...
	$+\ 1.55\%$	electroweak	NLO	Denner, Sack '91, ...
	$-\ 1.56\%$	finite W^+ width		
	$-\ 2.25\%$	QCD	NNLO	Blokland, Czarnecki, Ŝlusarczyk, Tkatchov '04

2. Structure Functions

$$H^{\mu\nu}(unpolarized) = \Big(-g^{\mu\nu}\, H_1 + p_t^\mu p_t^\nu\, H_2 - i\epsilon^{\mu\nu\rho\sigma} p_{t,\rho} q_\sigma\, H_3 \Big)$$

3 unpolarized structure functions: (H_1, H_2, H_3) ; $H_i = H_i(q_0, q^2)$

$$H^{\mu\nu}(polarized) = \; - \; (q\cdot s_t)\Big(-g^{\mu\nu}\, G_1 + p_t^\mu p_t^\nu\, G_2 - i\epsilon^{\mu\nu\rho\sigma} p_{t,\rho} q_\sigma\, G_3 \Big) + \\ + \; \Big(s_t^\mu p_t^\nu + s_t^\nu p_t^\mu \Big)\, G_6 + i\epsilon^{\mu\nu\rho\sigma} p_{t\rho} s_{t\sigma}\, G_8 + i\epsilon^{\mu\nu\rho\sigma} q_\rho s_{t\sigma}\, G_9$$

only 5 polarized structure functions:
$(G_1, G_2, G_6,\; q^2 G_3 + G_8,\; q\cdot p_t G_3 - G_9)$; $G_i = G_i(q_0, q^2)$
because of Schouten identity

$$q\cdot s_t\, \epsilon^{\mu\nu\rho\sigma} p_{t,\rho} q_\sigma - q^2 \epsilon^{\mu\nu\rho\sigma} p_{t,\rho} s_{t\sigma} + q\cdot p_t\, \epsilon^{\mu\nu\rho\sigma} q_\rho s_{t,\sigma} = 0 \; .$$

3. Analysis in W^+ rest system (cascade-type analysis)

3a. Unpolarized top quark decay

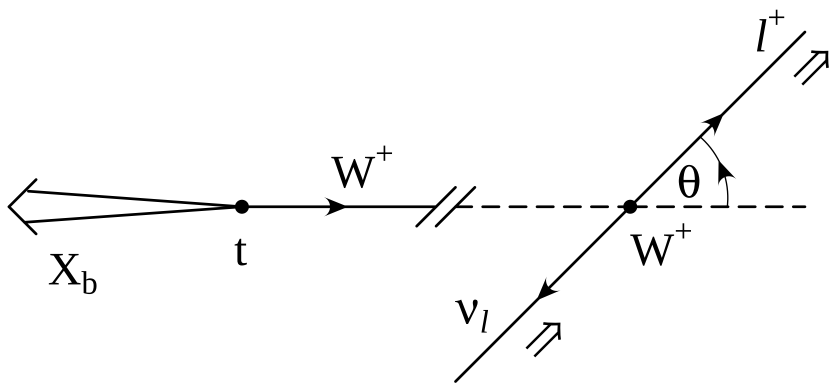


Figure 1: Definition of the polar angle θ in the rest frame decay of $W^+ \to l^+ + \nu_l$. The two lines "//" indicate a boost to the rest system of the W^+. The arrows next to the lepton lines give the helicities of the leptons.

$$
\begin{aligned}
H_+ &= H_1 + |\vec{q}| m_t \, H_3, \\
H_- &= H_1 - |\vec{q}| m_t \, H_3, \\
H_L &= m_W^2 H_1 + |\vec{q}|^2 m_t^2 \, H_2,
\end{aligned}
$$

The polar angle decay distribution is given by ($\mathcal{H}_i = H_i/\Gamma$)

$$
\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta} = \frac{3}{8}(1+\cos\theta)^2 \, \mathcal{H}_+ + \frac{3}{8}(1-\cos\theta)^2 \, \mathcal{H}_- + \frac{3}{4}\sin^2\theta \, \mathcal{H}_L
$$

Numerical results ($\hat{\Gamma}_i = \Gamma/\Gamma_{\rm Born}$):

$$
\begin{aligned}
\hat{\Gamma}_L &= 0.703\Big[1 - 0.0951(\mathsf{QCD}) + 0.0132(\mathsf{EW}) - 0.0138(\mathsf{BW}) - 0.00357(m_b \neq 0)\Big] \\
\hat{\Gamma}_- &= 0.297\Big[1 - 0.0656(\mathsf{QCD}) + 0.0206(\mathsf{EW}) - 0.0197(\mathsf{BW}) - 0.00172(m_b \neq 0)\Big] \\
\hat{\Gamma}_+ &= 0.000927(\mathsf{QCD}) + 0.0000745(\mathsf{EW}) + 0.000358(m_b \neq 0)
\end{aligned}
$$

(Work on NNLO QCD Γ_L in progress (Edmonton, Mainz))

The sum of the contributions (LO + NLO(QCD) + NLO(electroweak) + Breit-Wigner + $m_b \neq 0$) in terms of the normalized $\mathcal{H}_i = H_i/\Gamma$ $(\mathcal{H}_L + \mathcal{H}_- + \mathcal{H}_+ = 1)$

$$\begin{aligned} \mathcal{H}_L &= 0.6944 \quad (-1.2\% \quad \text{relative to Born term}) \\ \mathcal{H}_- &= 0.3041 \quad (+2.4\% \quad \text{relative to Born term}) \\ \mathcal{H}_+ &= 0.0015 \quad (+9.6\% \quad \text{relative to Born term}) \end{aligned}$$

3b. Polarized top quark decay (cascade-type analysis)

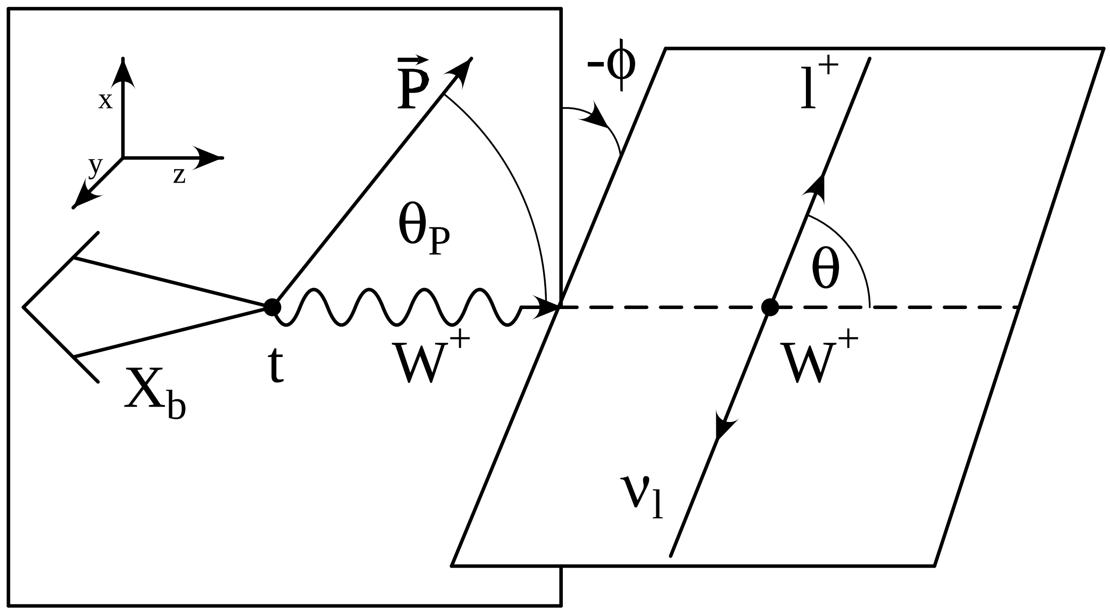


Figure 2: Definition of the polar angles θ and θ_P, and the azimuthal angle ϕ.

Angular decay distribution (cascade-type analysis)

$$
\begin{aligned}
\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta\, d\cos\theta_P\, d\phi} &= \frac{1}{4\pi}\Big\{\Big(\mathcal{H}_U + P\cos\theta_P\,\mathcal{H}_U^P\Big) \times \frac{3}{8}(1+\cos^2\theta) \\
&+ \Big(\mathcal{H}_L + P\cos\theta_P\,\mathcal{H}_L^P\Big) \times \frac{3}{4}\sin^2\theta \\
&+ \Big(\mathcal{H}_F + P\cos\theta_P\,\mathcal{H}_F^P\Big) \times \frac{3}{4}\cos\theta \\
&+ \Big(P\sin\theta_P\,\mathcal{H}_I^P\Big) \times \frac{3}{2\sqrt{2}}\sin 2\theta\cos\phi \\
&+ \Big(P\sin\theta_P\,\mathcal{H}_A^P\Big) \times \frac{3}{\sqrt{2}}\sin\theta\cos\phi\Big\},
\end{aligned}
$$

where $\mathcal{H}_U = \mathcal{H}_+ + \mathcal{H}_-$; $\mathcal{H}_F = \mathcal{H}_+ - \mathcal{H}_-$

Master formula for angular decay distribution

$$W(\theta_P, \theta, \phi) \propto \sum_{\lambda_W - \lambda'_W = \lambda_t - \lambda'_t} e^{i(\lambda_W - \lambda'_W)\phi} \, d^1_{\lambda_W 1}(\theta) \, d^1_{\lambda'_W 1}(\theta) \, H^{\lambda_t \, \lambda'_t}_{\lambda_W \lambda'_W} \, \rho_{\lambda_t \, \lambda'_t}(\theta_P) \ ,$$

Density matrix of the top quark:

$$\rho_{\lambda_t \, \lambda'_t}(\theta_P) = \frac{1}{2} \begin{pmatrix} 1 + P\cos\theta_P & P\sin\theta_P \\ P\sin\theta_P & 1 - P\cos\theta_P \end{pmatrix} ,$$

P : magnitude of the polarization of the top quark

$H^{\lambda_t \, \lambda'_t}_{\lambda_W \lambda'_W}$: helicity matrix elements of the hadronic structure function $H_{\mu\nu}$.

Relation between invariant and helicity structure functions ($\mathcal{H}_i = H_i/\Gamma$)

$$
\begin{aligned}
H_{U^P} &= 2|\vec{q}\,|\, G_1, \\
m_W^2 H_{L^P} &= |\vec{q}\,|(m_W^2\, G_1 + |\vec{q}\,|^2 m_t^2\, G_2 - 2 q_0 m_t G_6), \\
H_{F^P} &= 2\,|\vec{q}\,|^2 m_t\, G_3 - 2\, m_t\, G_8 - 2\, q_0\, G_9, \\
H_{I^P} &= \frac{1}{\sqrt{2}} \frac{m_t}{m_W} |\vec{q}\,|\, G_6, \\
H_{A^P} &= -\frac{1}{\sqrt{2}} \frac{m_t q_0}{m_W}\, G_8 - \frac{1}{\sqrt{2}} m_W G_9.
\end{aligned}
$$

Numerical NLO QCD values for polarized structure functions given in hep-ph/0101322

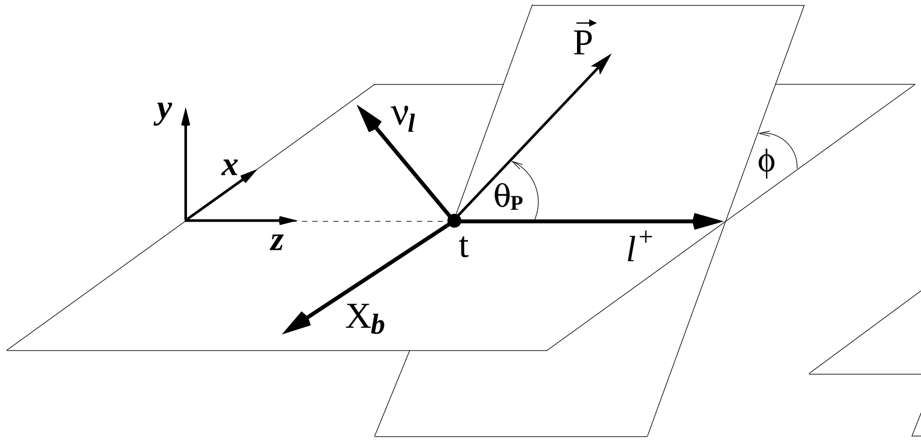
4. Analysis in top quark rest system

There are polar and azimuthal correlations between the polarization of the top quark and a given coordinate system in the event plane. We choose three different systems:

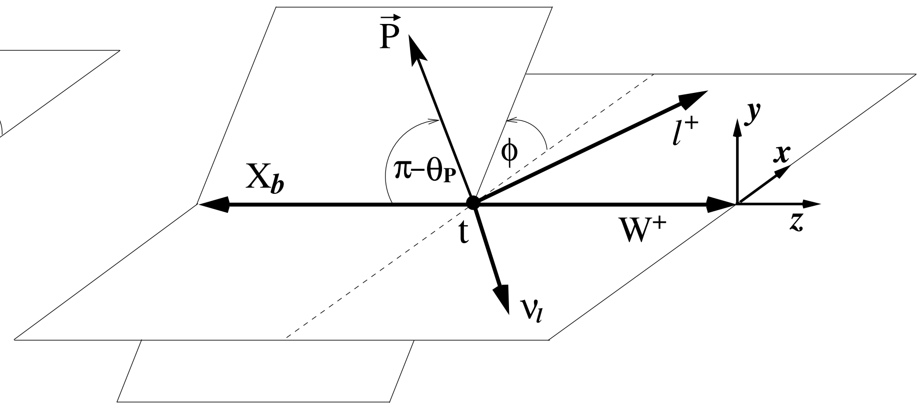
1. z–axis along lepton (system (1a))

2. z–axis along W^+–boson (system $(2'a)$)

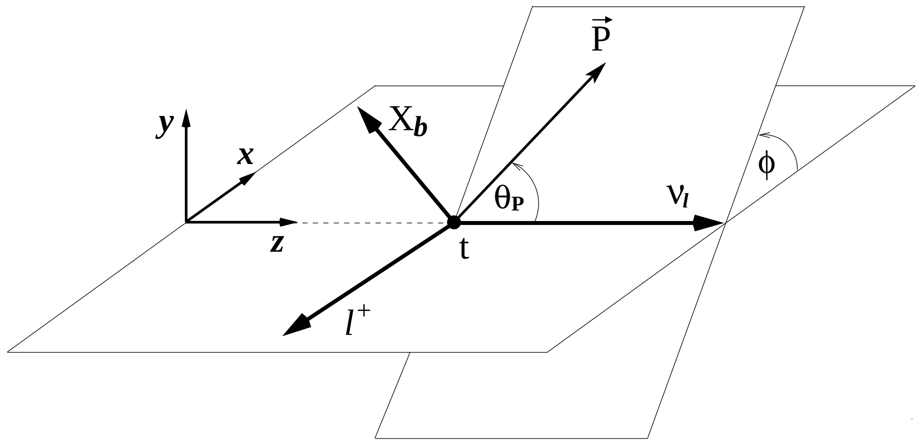
3. z–axis along neutrino (system (3a))


y
x
z
ν_l
$\vec{P}$
θ_P
ϕ
t
l^+
X_b

(1a)


$\vec{P}$
X_b
$\pi-\theta_P$
ϕ
l^+
y
x
z
t
W^+
ν_l

(2a)


y
x
z
X_b
$\vec{P}$
θ_P
ν_l
ϕ
t
l^+

(3a)

Generic form of angular decay distribution

$$\frac{d\Gamma}{dx_l d\hat{q}_0 d\cos\theta_P d\phi} = \frac{1}{4\pi}\left(\frac{d\Gamma_A}{dx_l d\hat{q}_0} + P(\frac{d\Gamma_B}{dx_l d\hat{q}_0}\cos\theta_P + \frac{d\Gamma_C}{dx_l d\hat{q}_0}\sin\theta_P\cos\phi)\right)$$

We use scaled variables: $x_l = 2E_l/m_t$, $\hat{q}_0 = q_0/m_t$

Γ_A : unpolarized rate

Γ_B : polar correlation rate

Γ_C : azimuthal correlation rate

Contribution of polarized structure function G_1 to the polar and azimuthal correlation term:

$$L^{\mu\nu}H_{\mu\nu}(G_1) = m_t q^2 G_1 \left\{ \begin{array}{l} \dfrac{x_l \hat{q}_0 - y^2}{x_l} \cos\theta_{P1} + \dfrac{y}{x_l}\sqrt{x_l(2\hat{q}_0 - x_l) - y^2}\,\sin\theta_{P1}\cos\phi \\ \sqrt{\hat{q}_0^2 - y^2}\,\cos\theta_{P2} \\ \dfrac{\hat{q}_0 - y^2}{2\hat{q}_0 - x_l}\cos\theta_{P3} + \dfrac{y}{2\hat{q}_0 - x_l}\sqrt{x_l(2\hat{q}_0 - x_l) - y^2}\,\sin\theta_{P3}\cos\phi \end{array} \right\}$$

LO and NLO QCD contribution to integrated rates:

z–axis along lepton (system (1a))

$$\frac{d\Gamma^{\rm NLO}}{d\cos\theta_P\, d\phi} = \frac{\Gamma_A^{\rm LO}}{4\pi}\Big[(1-8.54\%)+(1-8.72\%)P\cos\theta_P - 0.24\%\, P\sin\theta_P\cos\phi\Big]$$
$$= \frac{\Gamma_A^{\rm NLO}}{4\pi}\Big[1+0.998P\cos\theta_P - 0.26\%\, P\sin\theta_P\cos\phi\Big]$$

z–axis along W^+–boson (system $(2'a)$)

$$\frac{d\Gamma^{\rm NLO}}{d\cos\theta_P\, d\phi} = \frac{\Gamma_A^{\rm LO}}{4\pi}\Big[(1-8.54\%)+(0.406-11.62\%)P\cos\theta_P - (0.760-8.20\%)\, P\sin\theta_P\cos\phi\Big]$$
$$= \frac{\Gamma_A^{\rm NLO}}{4\pi}\Big[1+0.392P\cos\theta_P - 0.762\, P\sin\theta_P\cos\phi\Big]$$

Change in polar analyzing power from LO to NLO is (-3.45 %)

z–axis along neutrino (system (3a))

$$\frac{d\Gamma^{\mathrm{NLO}}}{d\cos\theta_P\,d\phi} = \frac{\Gamma_A^{\mathrm{LO}}}{4\pi}\Big[(1-8.54\%)-(0.318-1.02\%)P\cos\theta_P-(0.919-8.61\%)\,P\sin\theta_P\cos\phi\Big]$$

$$= \frac{\Gamma_A^{\mathrm{NLO}}}{4\pi}\Big[1+0.344\,P\cos\theta_P-0.918\,P\sin\theta_P\cos\phi\Big]$$

Change in polar analyzing power from LO to NLO is (-8.18 %)

More details can be found in:

S. Groote, W.S. Huo, A. Kadeer, J.G. Körner, D. Kubistin
"Polar and azimuthal correlations in the rest frame decay of a polarized top quark at $O(\alpha_s)$" (to be published)

and in

hep-ph/0602026
hep-ph/0209185
hep-ph/0101322
hep-ph/0011075
hep-ph/9811482