NLO QCD corrections to $pp/p\bar{p} \to t\,\bar{t}$ + jet + X

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Introduction / Motivation — Top as signal

Why is top quark physics important ?

Top-quark properties and dynamics still not precisely known

- Important signal process at the Tevatron and the LHC
- Δm_t = 1 GeV challenging task, only possible with detailed theoretical predictions
- Important tool to search for new physics

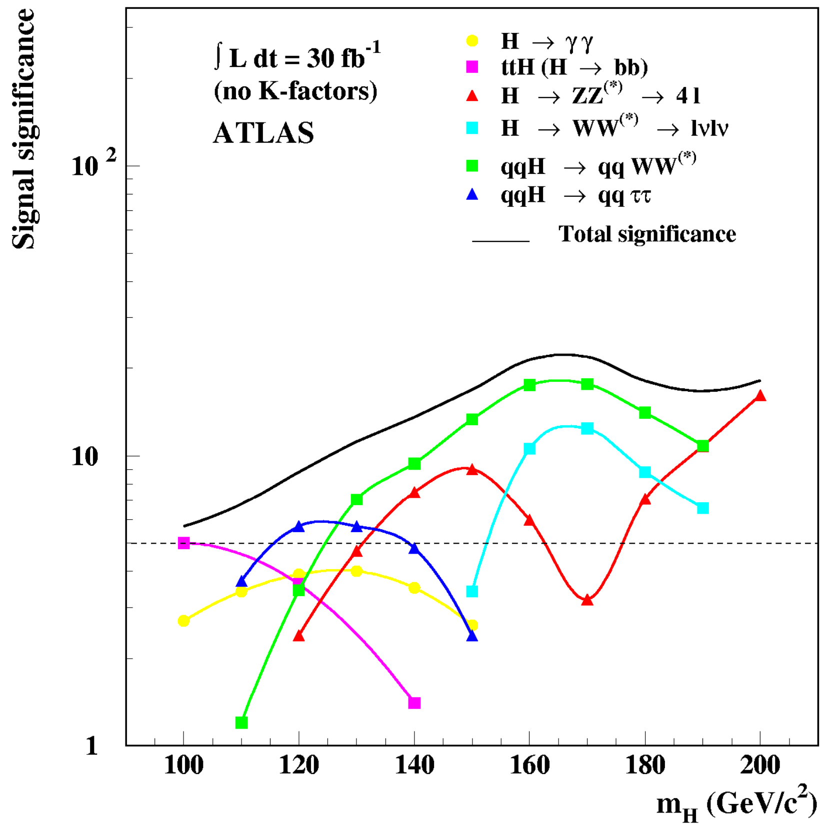
To explore the top quark additional observables useful

tt + 1-Jet can be used for

- Search of anomalous top gluon couplings
- Forward-Backward charge asymmetry (Tevatron) [Halzen, Hoyer Kim '87 Kühn, Rodrigo '98]
- Might also be useful for mass determination

Introduction / Motivation — Top as background

Higgs searches at LHC



[Atlas `03]

The WBF process

$$qq \rightarrow WWqq \rightarrow qqH$$

→ WW

is important over a wide Higgs mass range

Important backgrounds:

channel	$e^{\pm}\mu^{\mp}$	$e^{\pm}\mu^{\mp}$ w/minijet veto	$e^{\pm}e^{\mp}, \mu^{\pm}\mu^{\mp}$	$e^{\pm}e^{\mp}, \mu^{\pm}\mu^{\mp}$ w/minijet veto
$70 < m_h < 300$ GeV	1.90	1.69	1.56	1.39
SM, $m_h = 155$ GeV	5.60	4.98	4.45	3.96
$t\bar{t}$	0.086	0.025	0.086	0.025
$t\bar{t}j$	7.59	2.20	6.45	1.87
$t\bar{t}jj$	0.83	0.24	0.72	0.21
single-top (tbj)	0.020	0.015	0.016	0.012
$b\bar{b}jj$	0.010	0.003	0.003	0.001
QCD $WWjj$	0.448	0.130	0.390	0.113
EW $WWjj$	0.269	0.202	0.239	0.179
QCD $\tau\tau jj$	0.128	0.037	0.114	0.033
EW $\tau\tau jj$	0.017	0.013	0.016	0.012
QCD $\ell\ell jj$	–	–	0.114	0.033
EW $\ell\ell jj$	–	–	0.011	0.008
total bkg	9.40	2.87	8.04	2.49
S/B	1/5.0	1/1.7	1/5.1	1/1.8
$\mathcal{L}_{5\sigma}^{\rm obs}$[fb^{-1}]	65	25	82	32

[Alves, Eboli, Plehn, Rainwater '04]

→ Precise predictions for pp→ tt + jet are necessary

Introduction / Motivation — Top as test ground

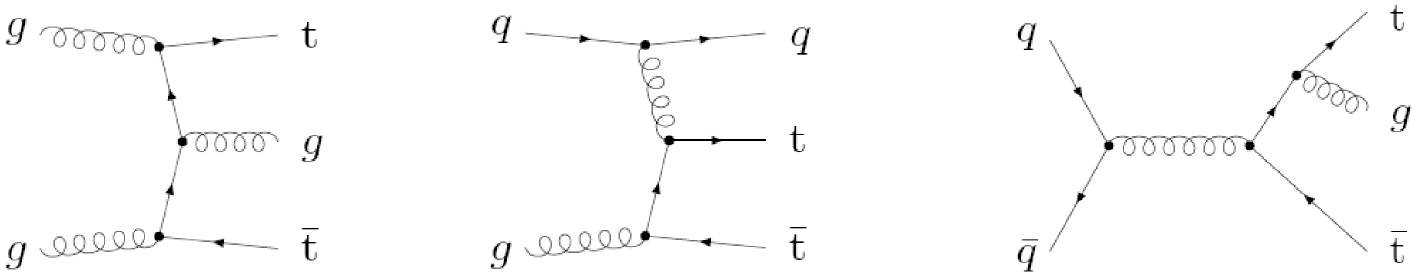
Ideal test ground for developing and testing of new methods for loop calculations

- Top quark physics not just a toy application
- Significant complexity due to
  - additional mass scale
  - infrared structure still complicated
  - large expressions, many diagrams
- ttj may serve as benchmark process for new methods

Top quark physics also useful in the commissioning phase of the experiments

Leading-order results — some features

Sample diagrams:



Partonic processes:

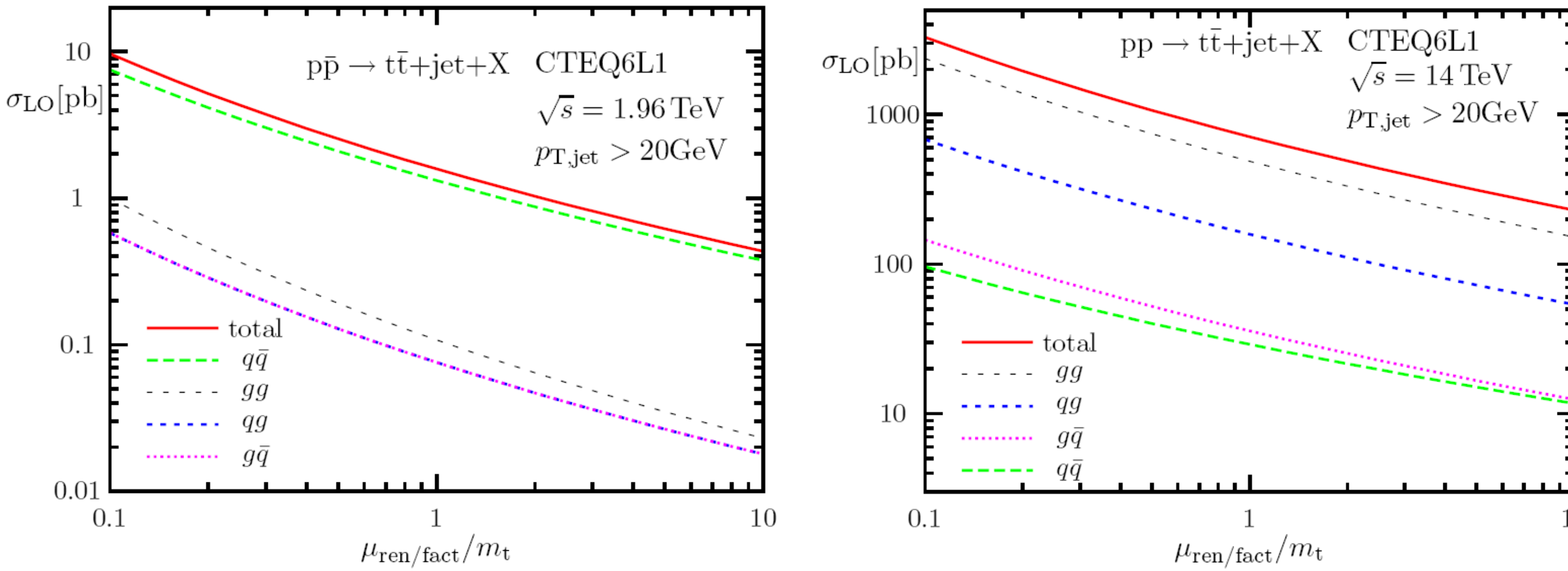
$$gg \to t\bar{t}g,\; \underbrace{q\bar{q} \to t\bar{t}g,\; qg \to t\bar{t}q,\; g\bar{q} \to t\bar{t}\bar{q}}_{\text{related by crossing}}$$

Many different methods for LO exist, we used:

1. Berends-Giele recurrence relation + spinor helicity formalism
2. Feynman-Diagram based approach + spinor helicity formalism
3. Feynman-Diagram based approach + "Standard Matrix Elements"

We also compared with Madgraph…

Leading-order results — some features



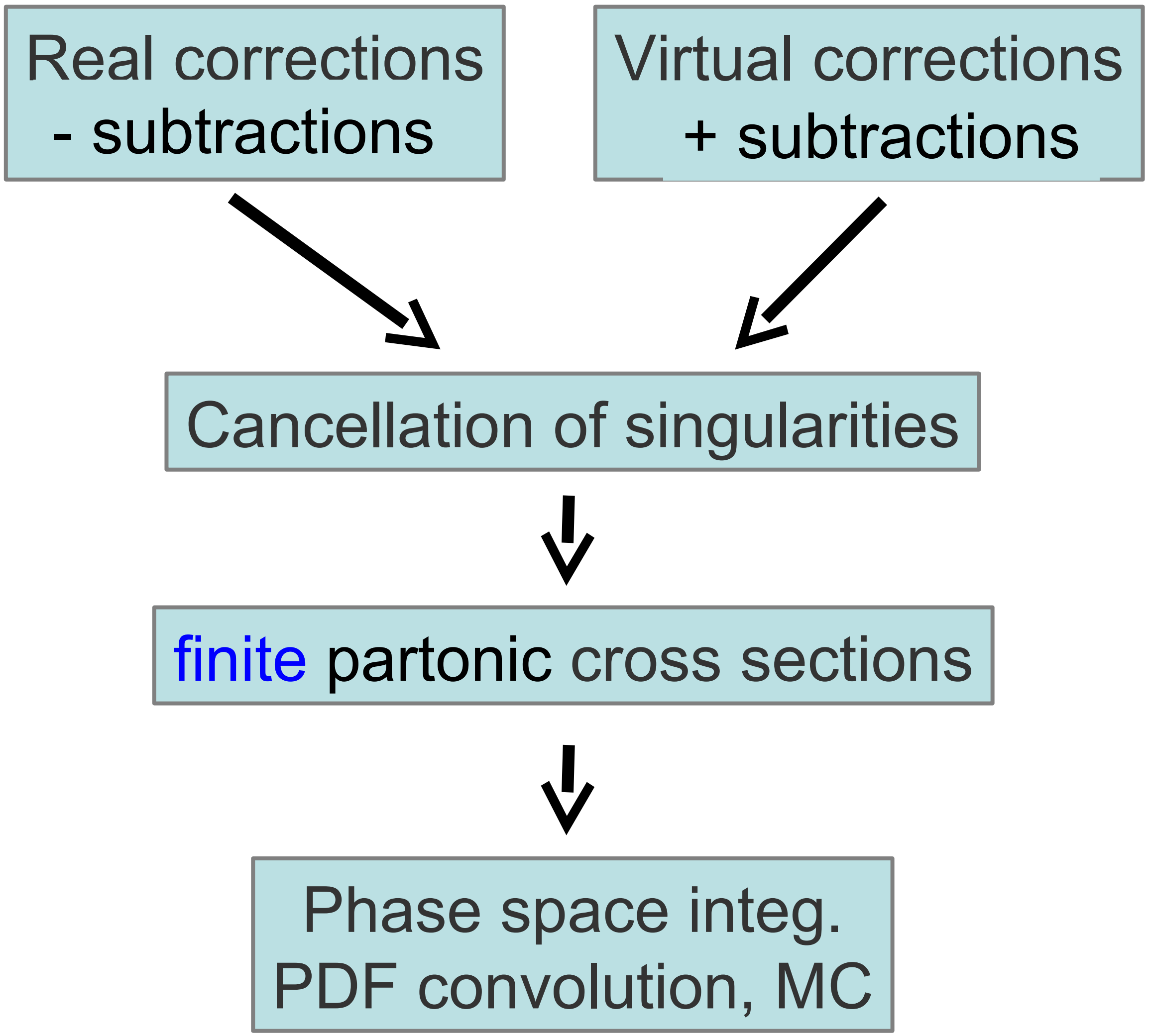
Observable:

- Cross section for ttbar + 1 additional Jet + X
- assume t and tbar as tagged
- to resolve additional jet demand min k_t of 20 GeV
  → k_t-cut renders observable IR finite

Note:

- in LO no recombination → no dependence on jet-alg.
- strong scale dependence of LO result
- Cross section is **NOT** small

Outline of the NLO calculation



Good: no conceptual issues, bad: no general library available

Conceptually solved ≠ practical solution

Issues:

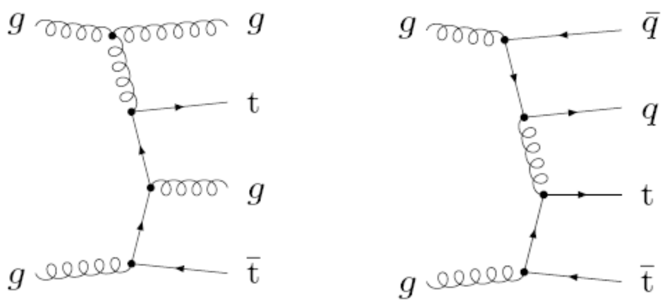
- complexity → automatization, efficient algorithms
- numerical efficiency / speed
- numerical stability

If there is a problem somewhere in the phase space
Sooner or later you will hit it…

That these issues are indeed non-trivial is reflected in the small number of NLO calculations for 2→3 and 2→4 proesses

Real corrections

Sample diagrams:



Partonic subprocesses:

$$gg \to t\bar{t}gg, gg \to t\bar{t}q\bar{q}, gq \to t\bar{t}gq$$

$$g\bar{q} \to t\bar{t}\bar{q}g, q\bar{q} \to t\bar{t}q'\bar{q}', \ldots$$

Related to 3 basic processes

$$0 \to t\bar{t}gggg, t\bar{t}q\bar{q}gg, t\bar{t}q\bar{q}q'\bar{q}'$$

Many subprocesses → runtime is an issue…

We used again (as in the LO case):

- Berends/Giele recurrence + FDH
- Feynman diagramatic approach + FDH

} checked against Madgraph

IR/coll. singularities treated using dipole subtraction formalism

[Frixione, Kunszt, Signer ´95; Catani, Seymour '96; Dittmaier '99
Phaf, Weinzierl '01, Catani, Dittmaier, Seymour, Trocsanyi '02]

General idea of subtraction method:

→ Add and subtract a counterterm which is enough to be integrated analytically:

$$
\begin{aligned}
& \int_0^\alpha dx \frac{1}{x} f(x) x^\epsilon \\
= \quad & \int_0^\alpha dx \frac{1}{x} (f(x) - f(0)) x^\epsilon + \frac{1}{x} f(0) x^\epsilon \\
= \quad & + \frac{1}{\epsilon} \alpha^\epsilon + \int_0^\alpha \frac{1}{x} (f(x) - f(0)) + O(\epsilon)
\end{aligned}
$$

Construction of subtraction for real corrections more involved, Fortunately a general solution exists:

→**”Catani-Seymour” subtraction formalism**

Real corrections: Dipole subtraction method

How it works in practise:

$$\sigma_{\rm NLO} = \int_{m+1} \sigma_{\rm real} + \int_m \sigma_{\rm virt.} + \int dx \int_m \sigma_{\rm fact.}(x)$$

$$\sigma_{\rm NLO} = \underbrace{\int_{m+1} [\sigma_{\rm real} - \sigma_{\rm sub}]}_{\rm finite} + \underbrace{\int_m [\sigma_{\rm virt.} + \bar{\sigma}^1_{\rm sub}]}_{\rm finite} + \underbrace{\int dx \int_m [\sigma_{\rm fact.}(x) + \bar{\sigma}_{\rm sub}(x)]}_{\rm finite}$$

Requirements:

$$0 = -\int_{m+1} \sigma_{\rm sub} + \int_m \bar{\sigma}^1_{\rm sub} + \int dx \int_m \bar{\sigma}_{\rm sub}(x)$$

$\sigma_{\rm sub} \longrightarrow \sigma_{\rm real}$ in all single-unresolved regions

Due to universality of soft and collinear factorization, general algorithms to construct subtractions exist

Real corrections: Some issues

- Subtraction term = sum over dipoles

  Dipoles have non-trivial structure in color and spin space,
  And there are many of them, i.e. 36 for gg->ttgg

  → Need general library to calculate subtraction terms

- Accuracy…

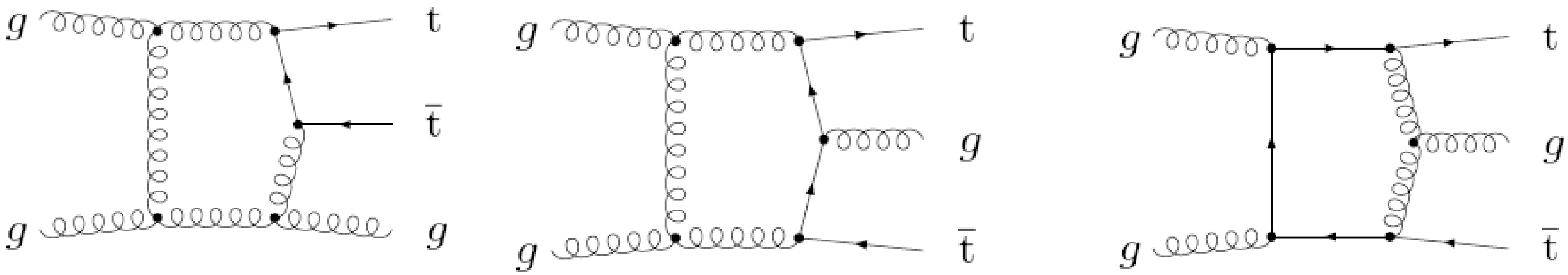
  Dipole subtraction relies on numerical cancellation
  of large numbers !

  → how precise are the matrix elements in sing. regions ?

Virtual corrections

Number of 1-loop diagrams ~ 350 (100) for $gg\ (q\bar{q})$

Most complicated 1-loop diagrams – pentagons of the type:



Algebraic decomposition of amplitudes:

color, i.e. $C_1 = (T_{a_1} T_{a_2} T_{a_3})_{\bar{t}t}$

$$\mathcal{A}(gg \to t\bar{t}g) = \sum_{k,l} f_{kl}(\{(p_i \cdot, p_j\}) \times S_k \times C_l$$

standard matrix

elements, i.e. $S_1 = \langle k_{\bar{t}} | \varepsilon_1 | k_t \rangle (\varepsilon_2 \cdot \varepsilon_3),$

→ Calculation similar to pp → ttH @ NLO

[Beenakker, Dittmaier, Krämer, Plümper, Spira, Zerwas ´03; Dawson, Jackson, Orr, Reina, Wackeroth 0

Virtual corrections

1. Feynmandiagram Generation — Feynarts, QGRAF
2. Standard Matrix Elements — Mathematica, Form
3. Reduction of Tensorintegrals
4. Numerical Integration

Steps 1,2,4 more or less standard, no particular difficulties...

Step 3 is the tricky part:

How to reduce tensor integrals fast and numerically stable, how to get a finite answer*) in a finite amount of time ?

Many methods developed in the last years

↕

Only a few non-trivial examples exist…

which essentially rely on very few methods…

*) and correct

Virtual corrections: Reduction of tensor integrals

Four and lower-point tensor integrals:

Reduction à la Passarino-Veltman,
with special reduction formulae in singular regions,
→ two complete independent implementations !

Five-point tensor integrals:

- Apply 4-dimensional reduction scheme, 5-point tensor integrals are reduced to 4-point tensor integrals

  → No dangerous Gram determinants! [Denner, Dittmaier 02]

  Based on the fact that in 4 dimension 5-point integrals can be reduced to 4 point integrals [Melrose ´65, v. Neerven, Vermaseren 84]

- Reduction à la Giele and Glover [Duplancic, Nizic 03, Giele, Glover 04]

  Use integration-by-parts identities to reduce loop-integrals

  nice feature: algorithm provides diagnostics and rescue system

Results: Checks

- leading-order amplitudes checked with Madgraph
- Subtractions checked in singular regions
- structure of UV singularities checked
- structure of IR singularities checked

In addition:

- two complete independent programs using a complete different tool chain and different algorithms

For example:

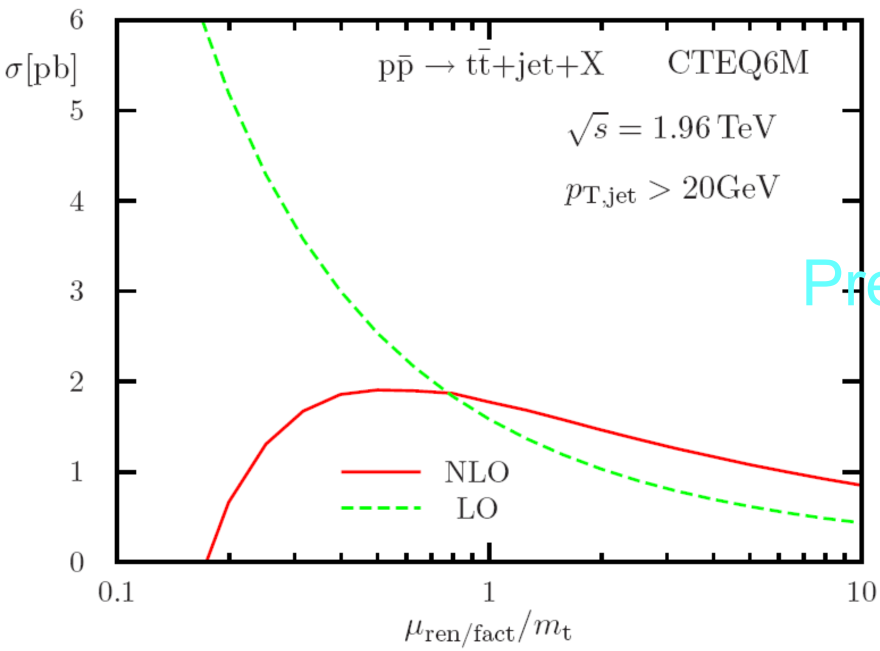
virtual corrections:

→ Feynarts 1.0 — Mathematica — Fortran77

→ QGraf — Form3 — C,C++

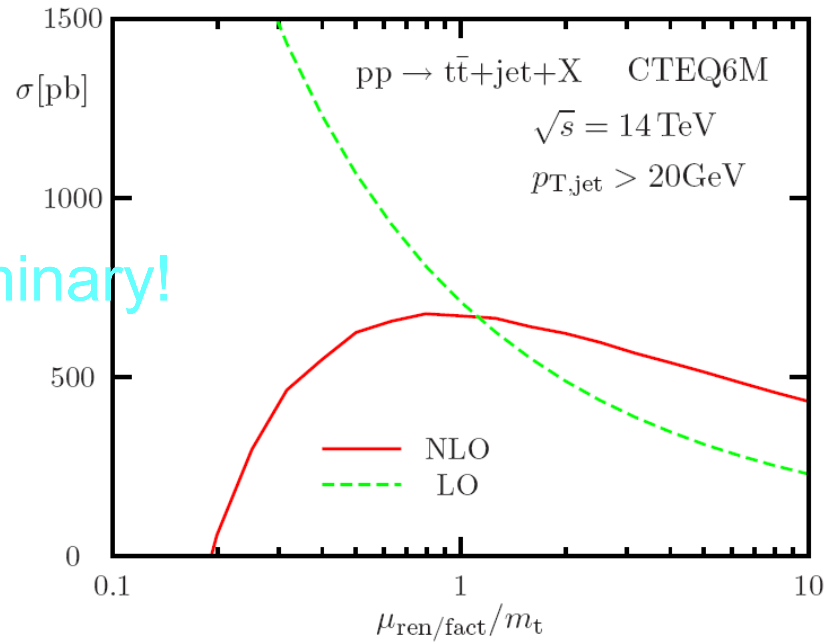
Results:

Tevatron



Preliminary!

LHC



- jet algorithm of Ellis and Soper used in the real corrections
- scale dependence is stabilized
- other observables are possible

Summary

- Top quark physics play an important role at the Tevatron an the LHC
- Precise theoretical predictions required for signal and background studies →NLO is needed
- NLO calculations for multi (> 2) particle final states still difficult
- NLO corrections to tt + 1 Jet serves as important benchmark process for new methods

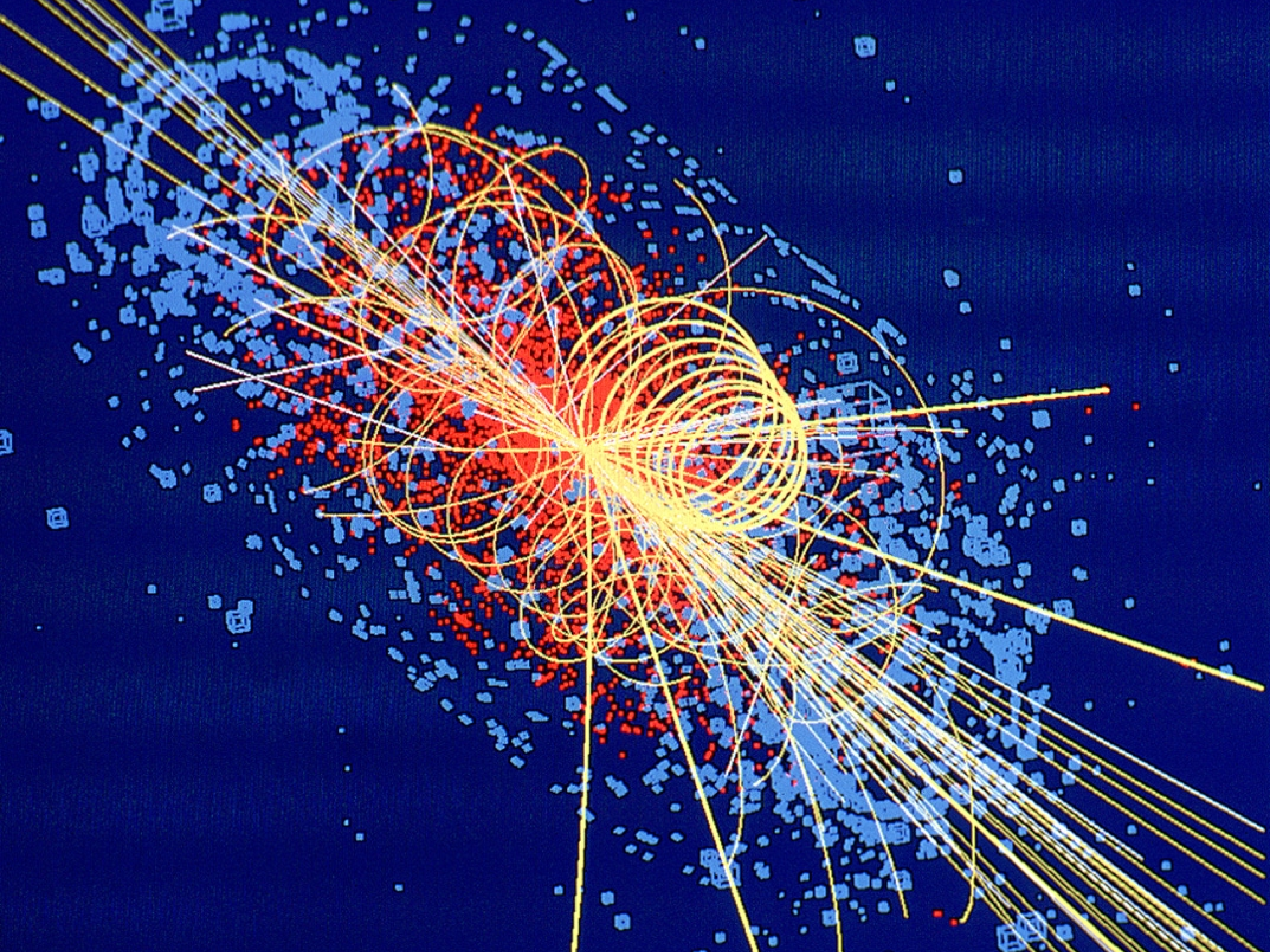
Summary

Status of $pp/p\bar{p} \rightarrow t\bar{t} + jet + X$ at NLO QCD:

- Calculation completed, but not yet fully cross-checked
- Preliminary numerical results shown
- Last checks are running
- Methods can be used to address more complicated processes
- Further improvements possible and underway

Outlook

- Charge asymmetry for the Tevatron (calculated but not yet cross checked)
- Differential distributions for the LHC and Tevatron (Program/algorithm completely flexible)
- If you are interested in specific distribution we should discuss it now…



Virtual corrections: Evaluation of 5-point Tensorintegrals

In 4 dimensions the loop-momentum can be expressed in p_1–p_4:

$$0 = \det \begin{pmatrix} 2\ell^2 & 2\ell\cdot p_1 & \ldots & 2\ell\cdot p_4 \\ 2p_1\cdot\ell & 2p_1\cdot p_1 & \ldots & 2p_1\cdot p_4 \\ \vdots & \vdots & \ddots & \vdots \\ 2p_4\cdot\ell & 2p_4\cdot p_1 & \ldots & 2p_4\cdot p_4 \end{pmatrix}$$

To regularize spurious UV singularities in individual terms

[Denner, Dittmaier ´02]

$$0 = \frac{1}{i\pi^2}\int d^4\ell \frac{\ell_{\mu_1}\cdots\ell_{\mu_P}}{N_0 N_1 \ldots N_4}\,\frac{-\Lambda^2}{\ell^2-\Lambda^2} \det \begin{pmatrix} 2\ell^2 & 2\ell\cdot p_1 & \ldots & 2\ell\cdot_4 \\ 2p_1\cdot\ell & 2p_1\cdot p_1 & \ldots & 2p_1\cdot p_4 \\ \vdots & \vdots & \ddots & \vdots \\ 2p_4\cdot\ell & 2p_4\cdot p_1 & \ldots & 2p_4\cdot p_4 \end{pmatrix}$$

with $N_i = (\ell+p_i)^2 - m_i^2 + i\varepsilon,\ p_0 = 0$

Can be expressed in terms of N_i

Reduction of $E_{\mu_1\mu_2\ldots}$ in terms of $D_{\mu_1\mu_2\ldots}$

Reduction of singular 5-point integrals

Dress all the mass less propagators with small mass λ:

$$E^{d} \to E^{\lambda,d}$$

[Dittmaier ´03]

Consider:

$$E^{\lambda,d} = E^{\lambda,d}_{\text{sing.}} + \underbrace{\left(E^{\lambda,d} - E^{\lambda,d}_{\text{sing.}} \right)}_{\text{regularization scheme indep.}}$$

For λ→0, $E^{\lambda,d}_{\text{sing.}}$ reproduces the singular behaviour of E^{d}

$E^{\lambda,d}_{\text{sing.}}$ obtained from soft and collinear limits of $E^{\lambda,d}$

→ simple combination of 3-point integrals

$$E^{d} = E^{\lambda=0,d}_{\text{sing.}} + \left(E^{\lambda,d=4} - E^{\lambda,d=4}_{\text{sing.}} \right)$$

can now be reduced to lower-point integrals

Alternative reduction procedure

From Schwinger or Feynman parametrization of tensor integrals:

$$\int d\ell \frac{\ell_{\mu_1} \dots \ell_{\mu_r}}{((\ell+q_1)^2 - m_1^2)((\ell+q_2)^2 - m_2^2)\dots((\ell+q_n)^2 - m_n^2)}$$

$$= \sum_{\lambda, z_1, \dots, z_n} \delta(2\lambda + \sum_i z_i - r) \left(-\frac{1}{2}\right) z_1! \dots z_n! \{g^\lambda q_1^{z_1} \dots q_n^{z_n}\}^{\mu_1 \dots \mu_r}$$

$$\times \; I(d + 2(m - \lambda), \{1 + z_i\})$$

[Davydychev]

→ Reduction to scalar integrals with raised powers of the propagators and shifted dimension!

Integration by parts (IBP)

[Chetyrkin, Kataev, Tkachov]

$$0 \quad = \quad \int \frac{d^d\ell}{(2\pi)^d}\frac{\partial}{\ell^\mu}\left(\frac{\sum_i^n y_i(\ell+q_i)}{((\ell+q_1)^2-m_1^2)^{\nu_1}\ldots((\ell+q_n)^2-m_n^2)^{\nu_n}}\right)$$

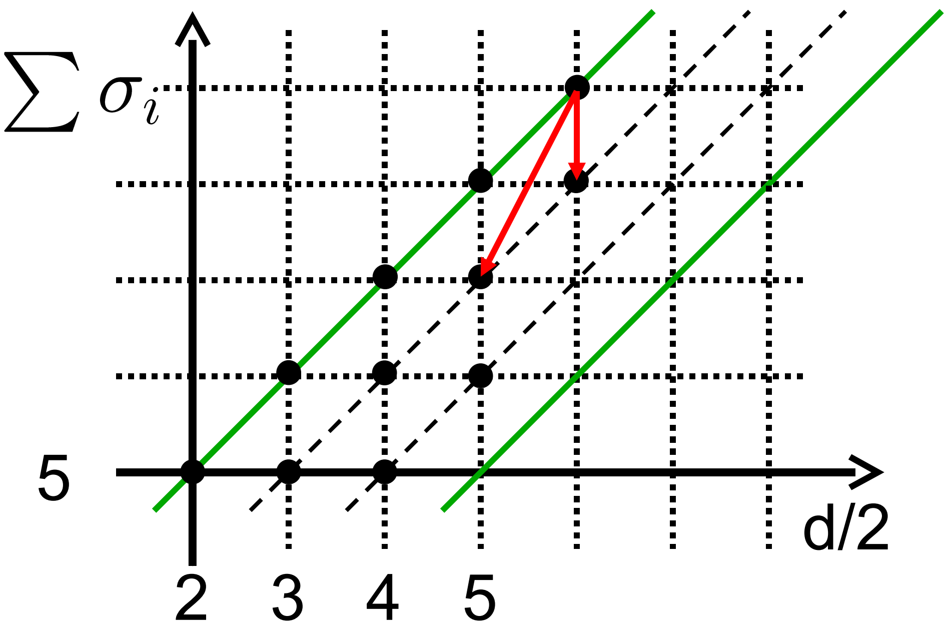
[Duplancic, Nizic 03, Giele, Glover 04]

$$(\nu_k-1)I(d;\nu_l) =$$

$$\sum_{i=1}^n S_{ki}^{-1} I(d-2;\{\nu_l-\delta_{li}-\delta_{lk}\}) - b_k(d-\sigma)I(d,\{\nu_l-\delta_{lk}\})$$

$$\sigma = \sum \sigma_i$$

$$S_{ki} = (q_i - q_j)^2$$



5-point