

# Cosmological Hints for Supersymmetric Grand Unification

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# UV Completion of the Standard Model

- Structure of Standard Model points towards “grand unification” of strong and electroweak interactions (quark and lepton content, gauge group, “unification” of gauge couplings, small neutrino masses ...). Also strong theoretical arguments for supersymmetry (gravity, extra dimensions, string theory)
- Energy scale of grand unification:  $\Lambda_{\text{GUT}} \simeq 10^{15} \dots 10^{16}$  GeV, energy scale of supersymmetry breaking:  $\Lambda_{\text{SB}} \simeq ??$
- Can cosmology help to provide evidence for grand unification and supersymmetry? Interplay of inflation and reheating (GUT symmetry breaking), baryogenesis (decays of GUT scale RH neutrinos) and dark matter (supersymmetry breaking)

# Seesaw & Leptogenesis

Lepton number can be violated by masses and couplings of heavy Majorana neutrinos to light neutrinos and charged leptons; after electroweak symmetry breaking, 3 light and 3 heavy neutrinos (seesaw mechanism):

$$\begin{aligned} N &\simeq \nu_R + \nu_R^c , & \nu &\simeq V_\nu^T \nu_L + \nu_L^c V_\nu^* \\ m_N &\simeq M , & m_\nu &\simeq -V_\nu^T m_D^T \frac{1}{M} m_D V_\nu \end{aligned}$$

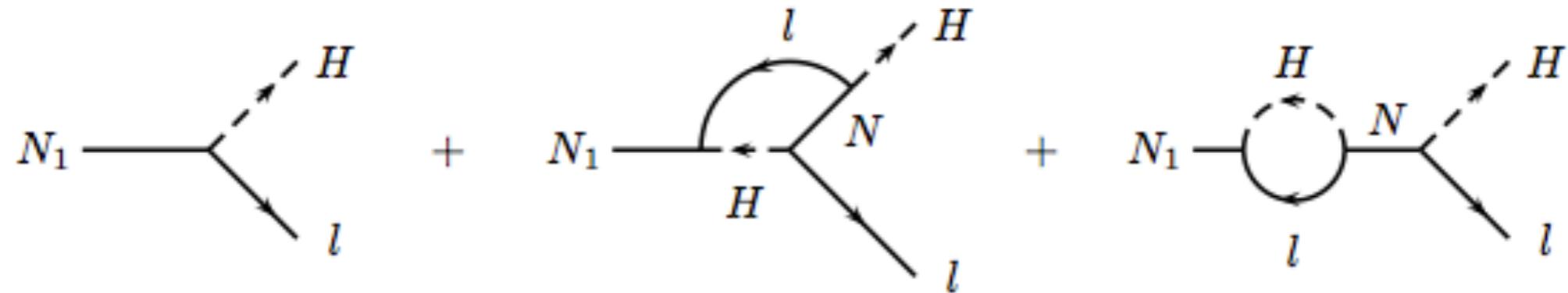
For hierarchical right-handed neutrinos, after electroweak symmetry breaking, light neutrino masses naturally related to the GUT scale:

$$M_3 \sim \Lambda_{\text{GUT}} \sim 10^{15} \text{ GeV} , \quad m_3 \sim \frac{v^2}{M_3} \sim 0.01 \text{ eV}$$

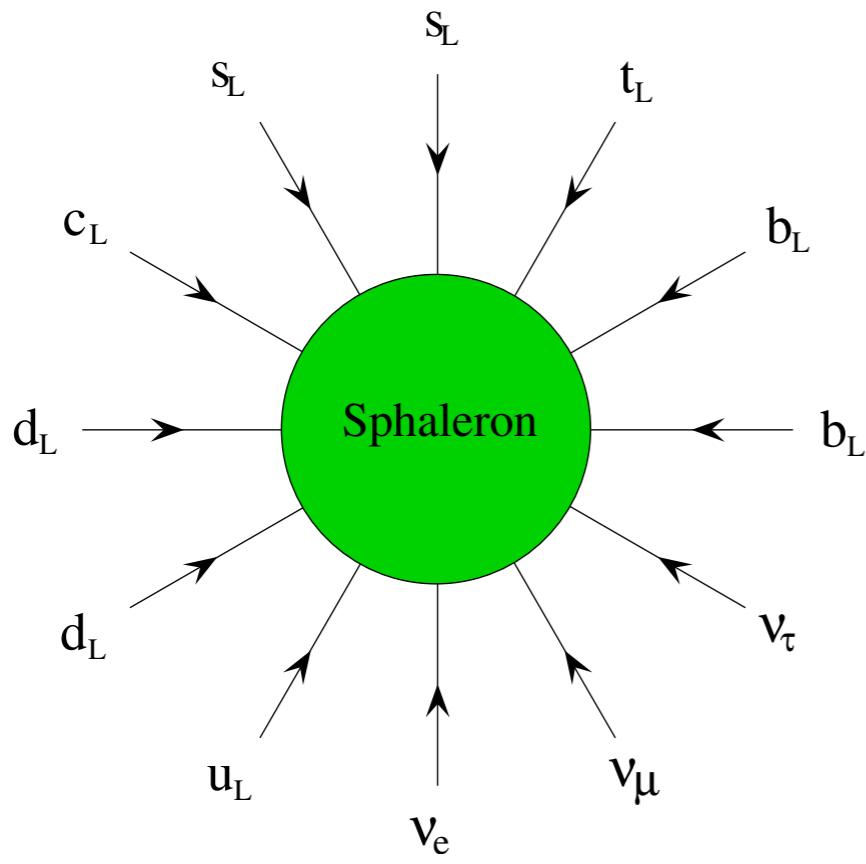
suggests that neutrino physics probes the mass scale of grand unification (but there are also other examples ...)

## CP violating heavy neutrino decays:

$$\begin{aligned}\varepsilon_1 &= \frac{\Gamma(N_1 \rightarrow H + l_L) - \Gamma(N_1 \rightarrow H^\dagger + l_L^\dagger)}{\Gamma(N_1 \rightarrow H + l_L) + \Gamma(N_1 \rightarrow H^\dagger + l_L^\dagger)} \\ &\simeq -\frac{3}{16\pi} \frac{M_1}{(hh^\dagger)_{11} v_F^2} \text{Im} (h^* m_\nu h^\dagger)_{11}\end{aligned}$$



Leptogenesis relates successfully neutrino masses and matter-antimatter asymmetry; CP asymmetry is small: suppressed by small Yukawa couplings, and loop effect (quantum interference); natural explanation of very small observed baryon asymmetry [Fukugita, Yanagida '86]



Rough estimate for CP asymmetry  
(hierarchical heavy neutrinos) and  
baryon asymmetry:

$$\begin{aligned}\varepsilon_1 &\sim 0.1 \frac{M_1 m_3}{v_F^2} \\ &\sim 0.1 \frac{M_1}{M_3} \sim 10^{-5} \dots 10^{-6}\end{aligned}$$

$$\eta_B \simeq -\frac{c_s}{f} N_{B-L} \simeq 10^{-2} \varepsilon_1 \kappa_f$$

Neutrino masses suggest that leptogenesis is process close to thermal equilibrium, i.e.  $\Gamma_1 \sim H|_{T=M_1}$  ; in terms of neutrino masses:

$$\tilde{m} = \frac{(m_D m_D^\dagger)_{11}}{M_1} \sim m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v_F^2}{M_P} \simeq 10^{-3} \text{ eV}$$

confirmed by solution of Boltzmann equations; significant progress in QFT description; leptogenesis temperature:  $T_L \sim M_1 \sim 10^{10} \text{ GeV}$

# Leptogenesis & Supersymmetry

WB, Domcke, Kamada, Schmitz '13, '14

*Leptogenesis & gravitinos:* for (thermal) leptogenesis and ‘typical’ superparticle masses, thermal production yields observed amount of dark matter:

$$\Omega_{3/2} h^2 = C \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( \frac{100 \text{ GeV}}{m_{3/2}} \right) \left( \frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2 , \quad C \sim 0.5$$

$\Omega_{3/2} h^2 \sim 0.1$  is natural value for  $T_R \sim T_L \sim 10^{10}$  GeV; but why should reheating temperature be close to leptogenesis temperature ?

*Simple observation:* heavy neutrino decay width (for typical LG parameters)

$$\Gamma_{N_1} = \frac{\tilde{m}_1}{8\pi} \left( \frac{M_1}{v_F} \right)^2 \sim 10^3 \text{ GeV} , \quad \tilde{m}_1 \sim 0.01 \text{ eV} , \quad M_1 \sim 10^{10} \text{ GeV}$$

yields reheating temperature (for gas of decaying heavy neutrinos)

$$T_R \sim 0.2 \cdot \sqrt{\Gamma_{N_1}^0 M_P} \sim 10^{10} \text{ GeV}$$

wanted for gravitino DM. *Intriguing hint or misleading coincidence?*

# Spontaneous B-L breaking & false vacuum decay

Supersymmetric SM with right-handed neutrinos:

$$W_M = h_{ij}^u \mathbf{10}_i \mathbf{10}_j H_u + h_{ij}^d \mathbf{5}_i^* \mathbf{10}_j H_d + h_{ij}^\nu \mathbf{5}_i^* n_j^c H_u + h_i^n n_i^c n_i^c S_1$$

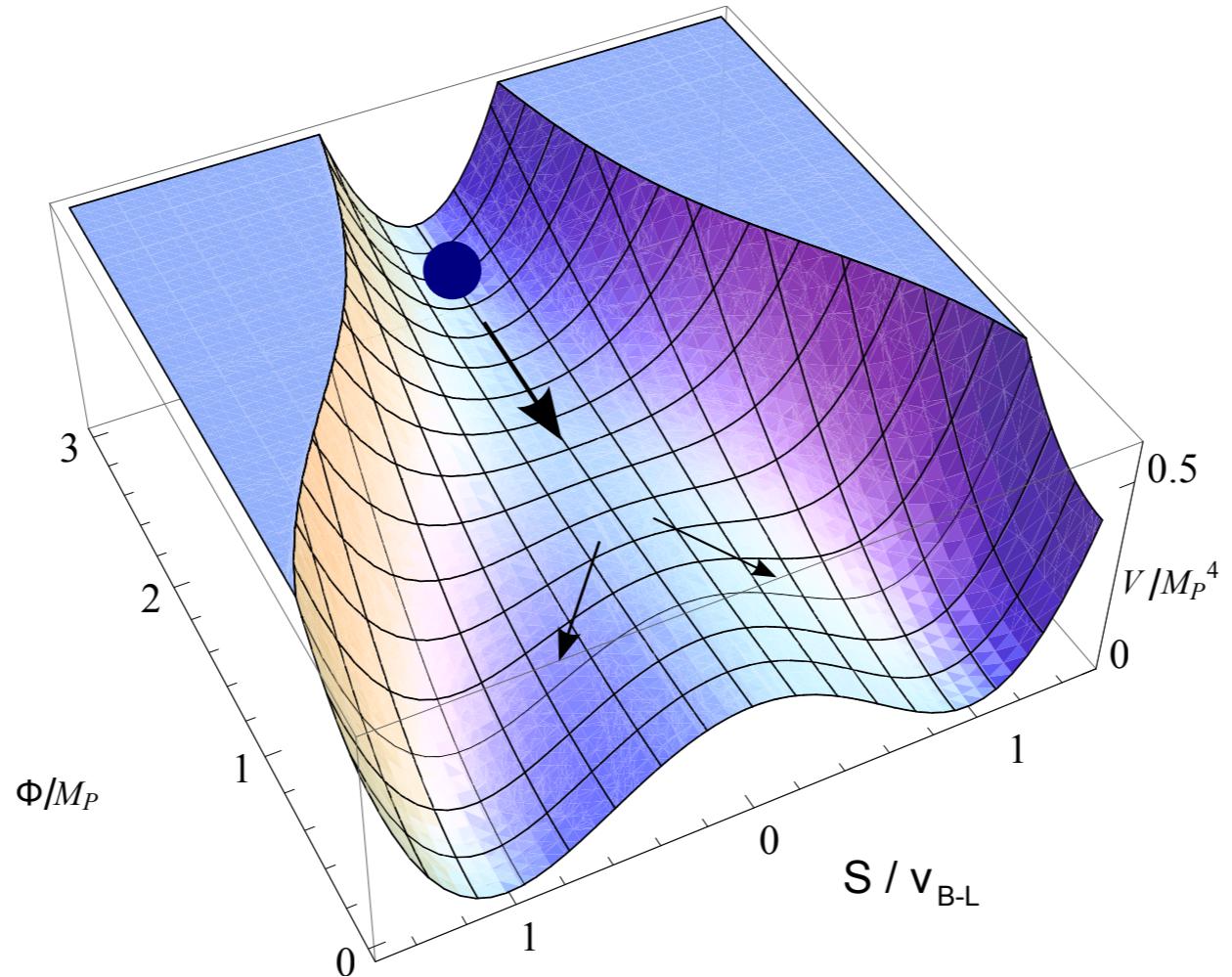
in  $SU(5)$  notation:  $\mathbf{10} \supset (q, u^c, e^c)$ ,  $\mathbf{5}^* \supset (d^c, l)$ ,  $n^c \supset (\nu^c)$ ; B-L breaking:

$$W_{B-L} = \lambda \Phi \left( \frac{1}{2} v_{B-L}^2 - S_1 S_2 \right)$$

$\langle S_{1,2} \rangle = v_{B-L}/\sqrt{2}$  yields heavy neutrino masses.

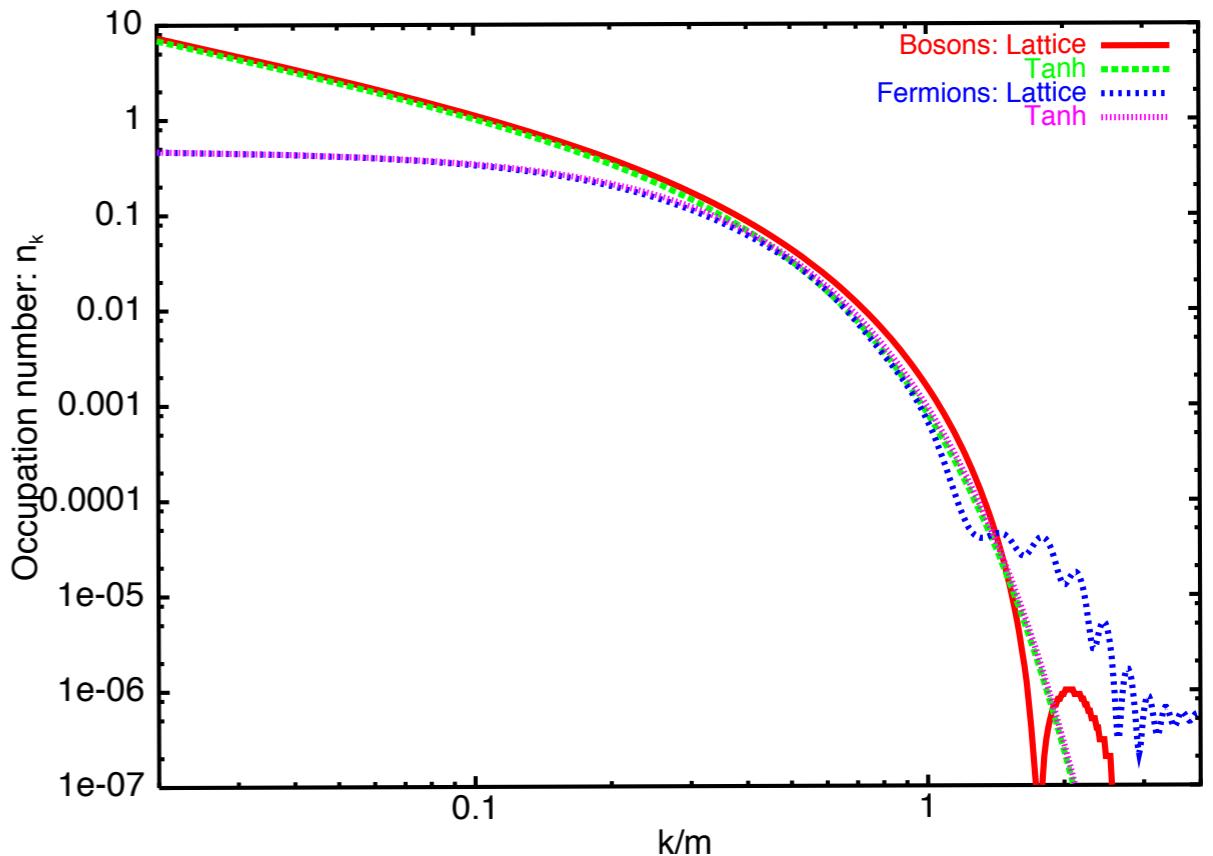
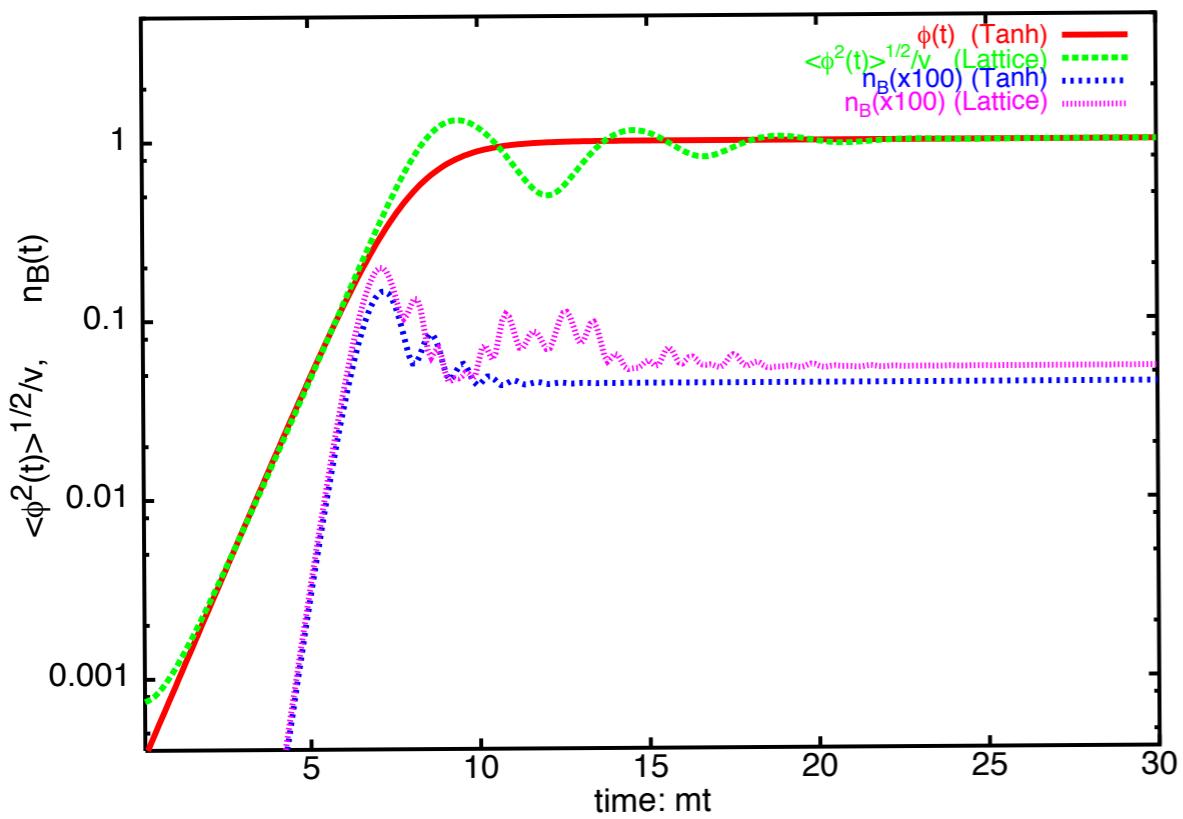
Lagrangian is determined by low energy physics: quark, lepton, neutrino masses etc, but it *contains all ingredients wanted in cosmology*: inflation, leptogenesis, dark matter, ..., all related! [F-term hybrid inflation: Copeland et al. '94, Dvali et al. '94].

Technically: Abelian Higgs model in unitary gauge; inflation ends with phase transition (“tachyonic preheating”, “spinodal decomposition”)

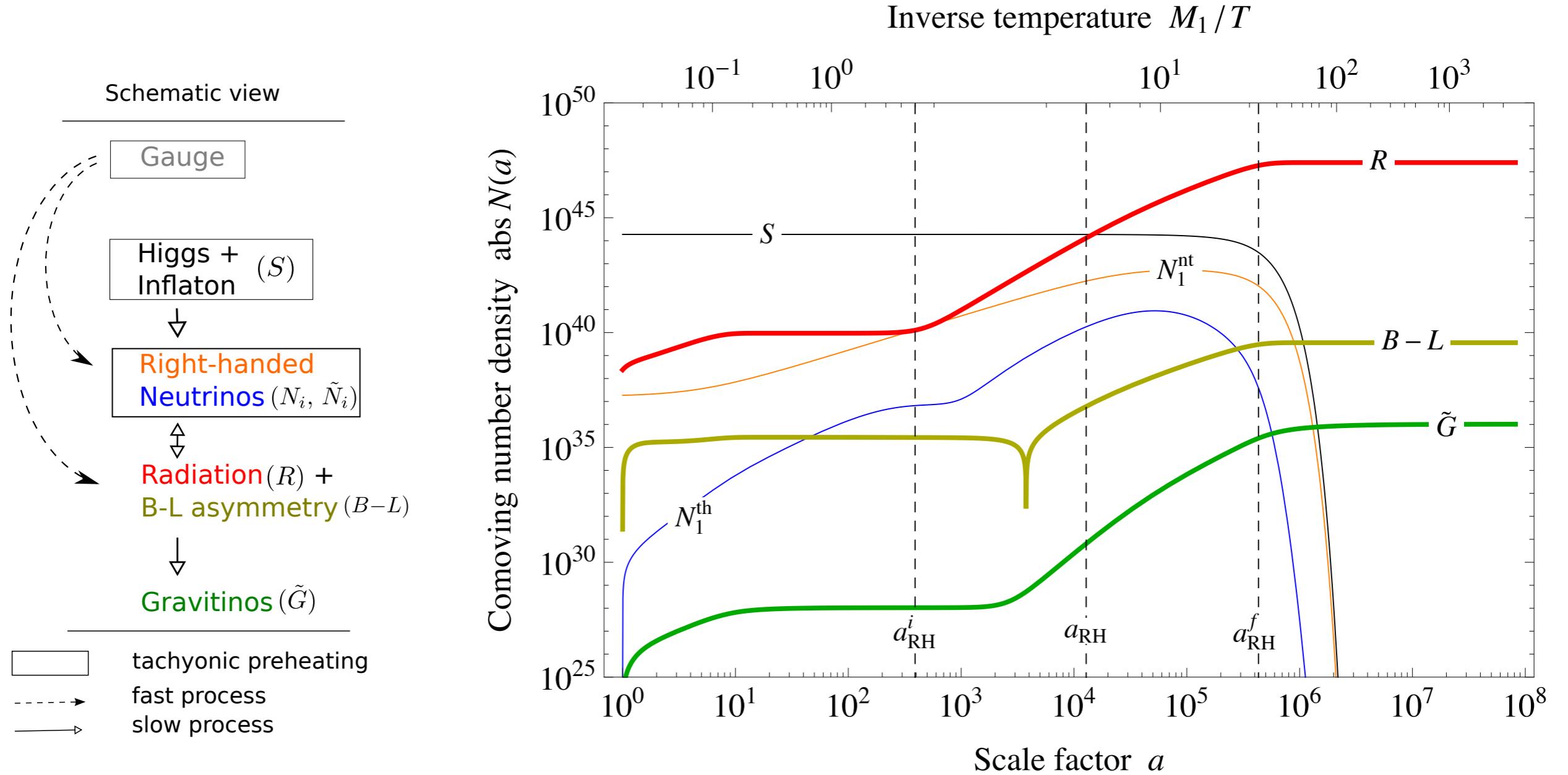


time-dependent masses of B-L Higgs, inflaton, heavy neutrinos ... (bosons and fermions):

$$m_\sigma^2 = \frac{1}{2}\lambda(3v^2(t) - v_{B-L}^2) , \quad m_\phi^2 = \lambda v^2(t) , \quad M_i^2 = (h_i^n)^2 v^2(t) \dots$$



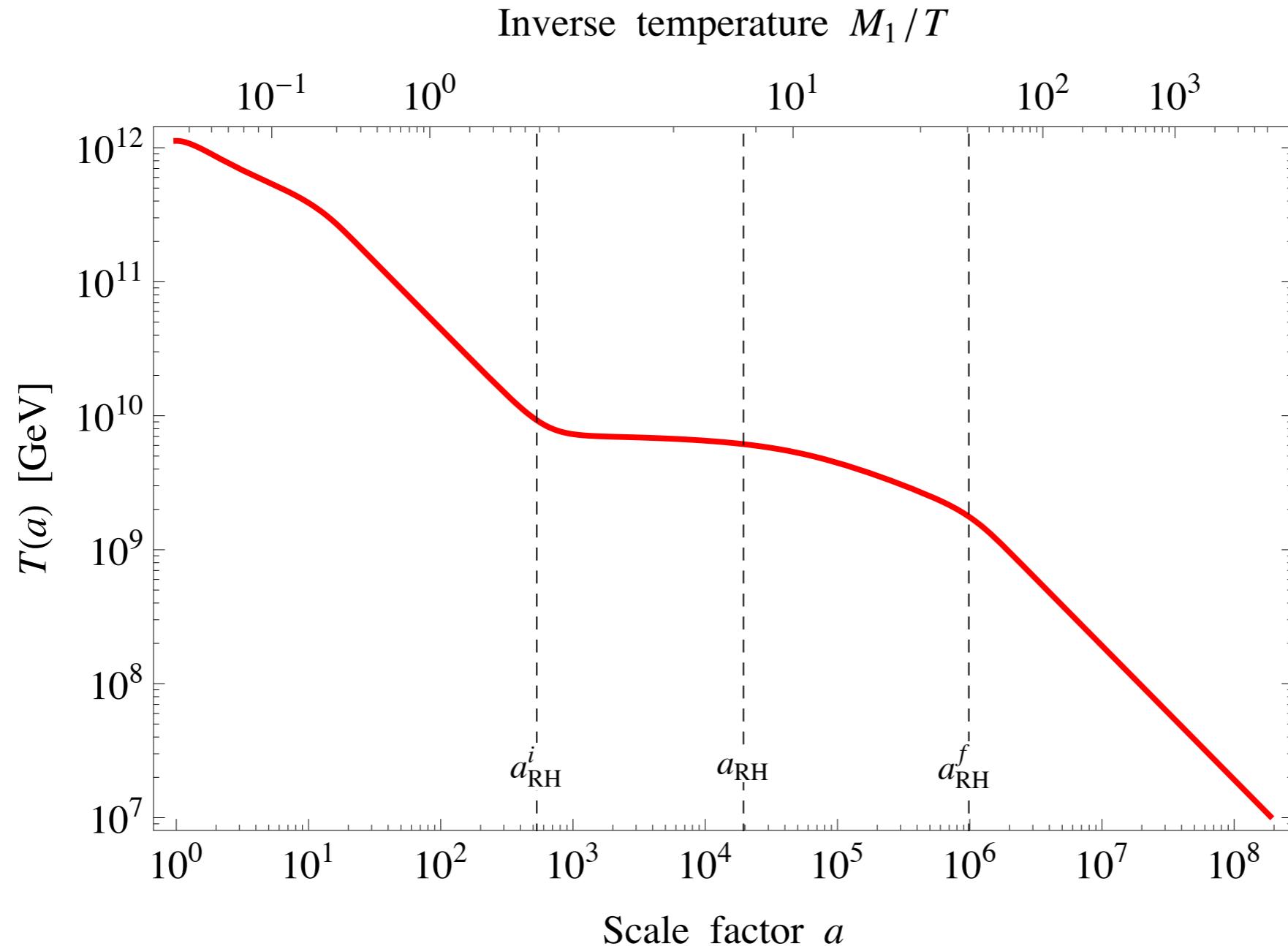
Rapid transition from false to true vacuum by fluctuations of ‘waterfall’ B-L Higgs field; production of low momentum Higgs bosons (contain most energy), also other bosons and fermions coupled to B-L Higgs field [Garcia-Bellido, Morales ’02], production of cosmic strings: initial conditions for reheating



Transition from end of inflation to hot early universe (typical parameters), calculated by means of Boltzmann equations:

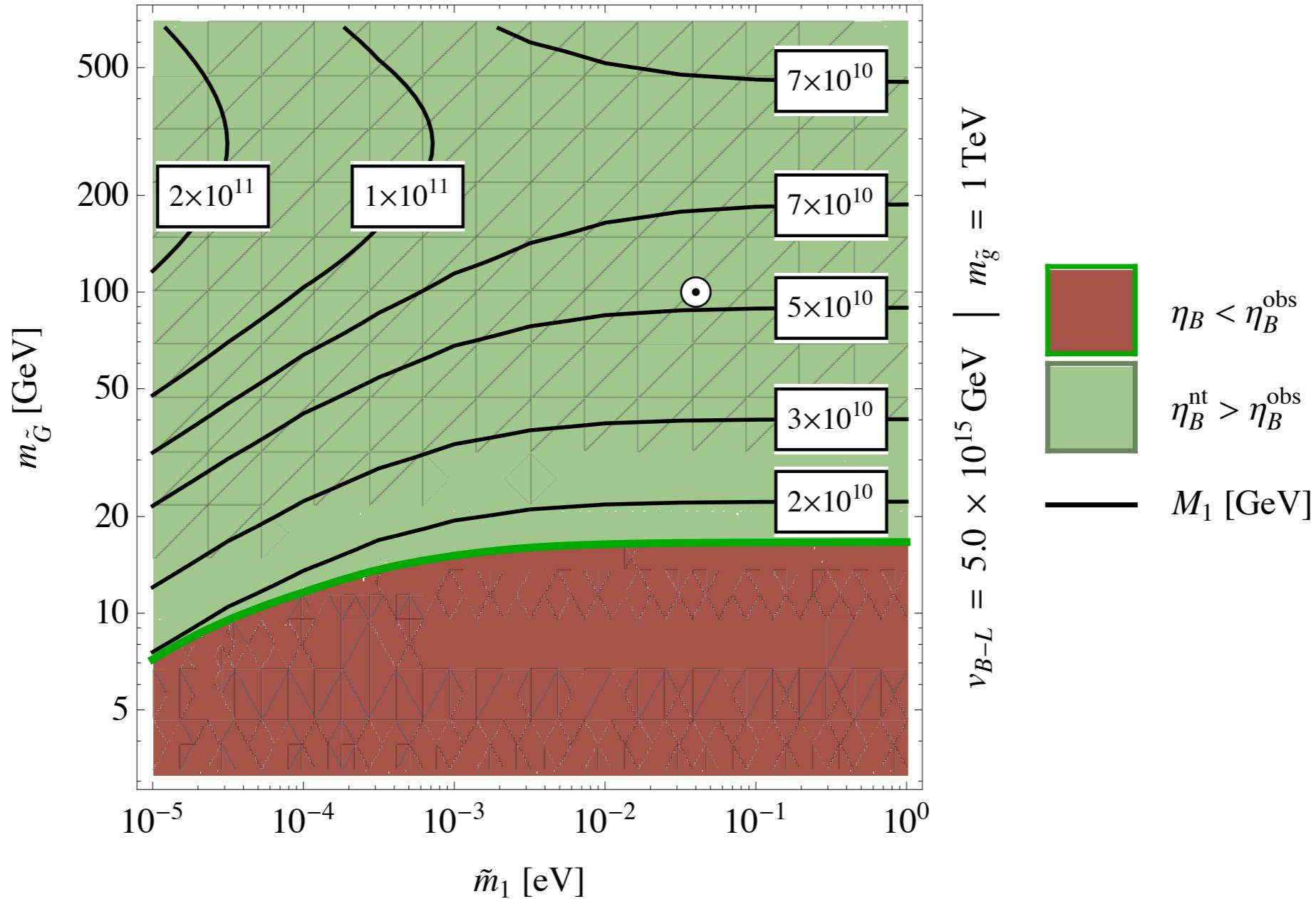
$$E \left( \frac{\partial}{\partial t} - H p \frac{\partial}{\partial p} \right) f_X(t, p) = \sum_{i'j'..} \sum_{ij..} C_X (Xi'j'.. \leftrightarrow ij..)$$

yields correct baryon asymmetry and dark matter abundance

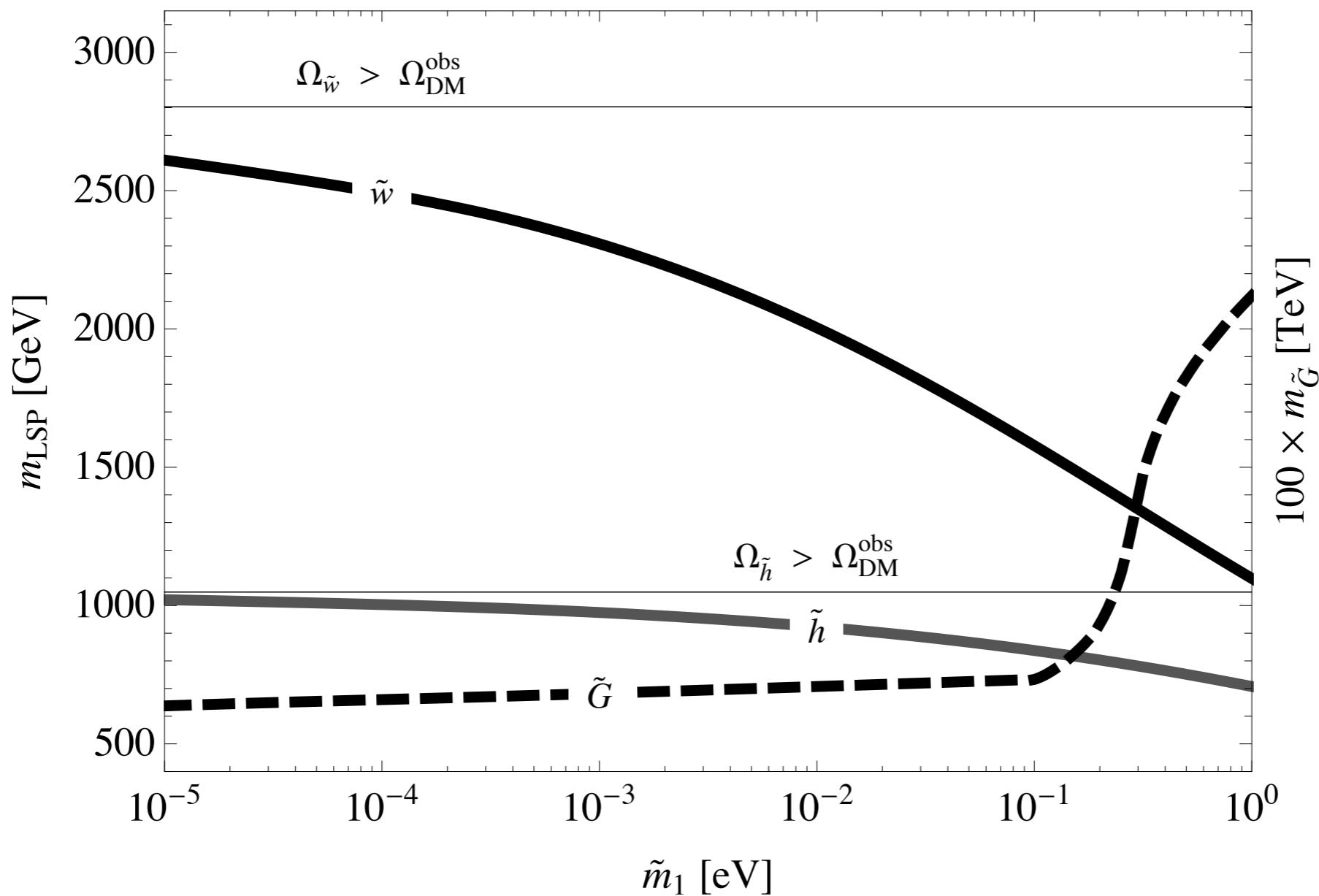


Time evolution of temperature: intermediate plateau (“maximal temperature”), determined by neutrino properties! Yields gravitino abundance when combined with ‘standard formula’ for thermal gravitino production

$M_1$  [GeV] such that  $\Omega_{\tilde{G}} h^2 = 0.11$



parameter scan: successful leptogenesis (thermal & non-thermal) and **gravitino** dark matter; allowed gravitino masses:  $m_{\tilde{G}} \sim 10 \dots 500$  GeV; search at LHC: neutralino decay outside detector, i.e. missing energy events; no direct DM detection!



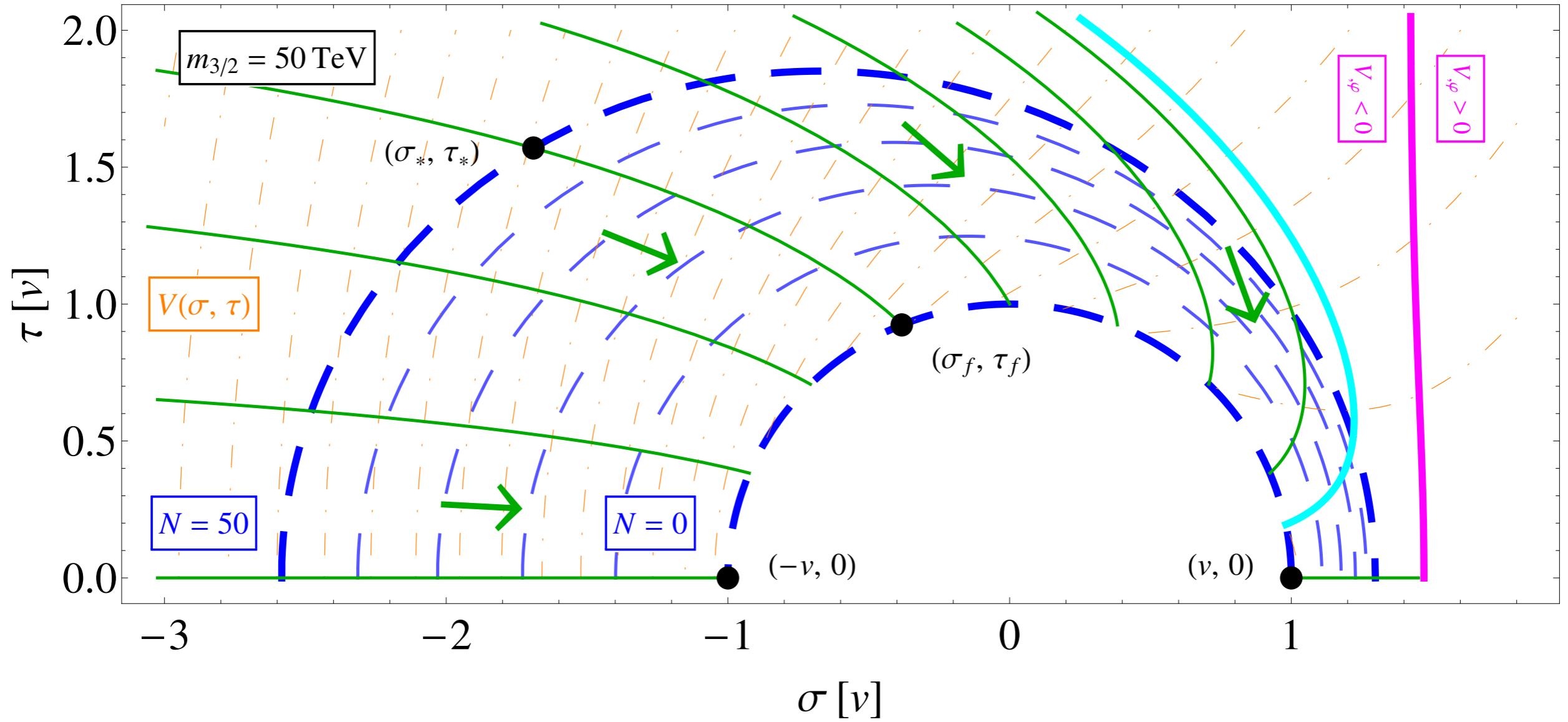
parameter scan: successful leptogenesis and **higgsino/wino** dark matter [non-thermally produced in decays gravitinos]; upper bounds on higgsino/wino masses, lower bound on gravitino mass; search at LHC: missing energy events; direct DM detection possible.

# Hybrid Inflation in the Complex Plane

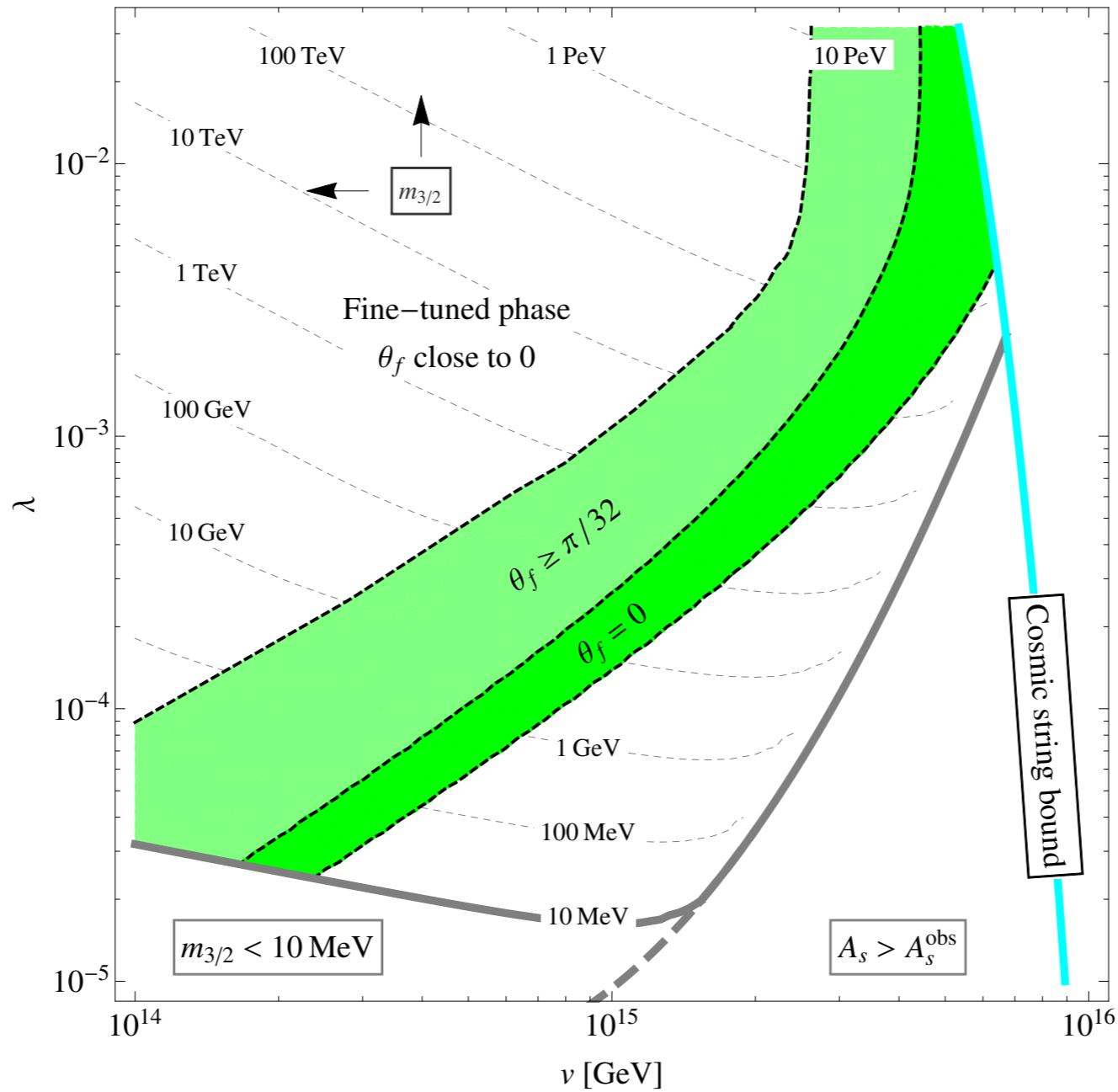
Inflationary potential can be strongly affected by supersymmetry breaking (supergravity correction [WB, Covi, Delepine '00]):

$$\begin{aligned} V(\phi) &= V_0 + V_{\text{CW}}(\phi) + V_{\text{SUGRA}}(\phi) + V_{3/2}(\phi) , \\ V_0 &= \frac{\lambda^2 v^4}{4} , \\ V_{\text{CW}}(\phi) &= \frac{\lambda^4 v^4}{32\pi^2} \ln \left( \frac{|\phi|}{v/\sqrt{2}} \right) + \dots , \\ V_{\text{SUGRA}}(\phi) &= \frac{\lambda^2 v^4}{8M_{\text{P}}^4} |\phi|^4 + \dots , \\ V_{3/2}(\phi) &= -\lambda v^2 m_{3/2}(\phi + \phi^*) + \dots \end{aligned}$$

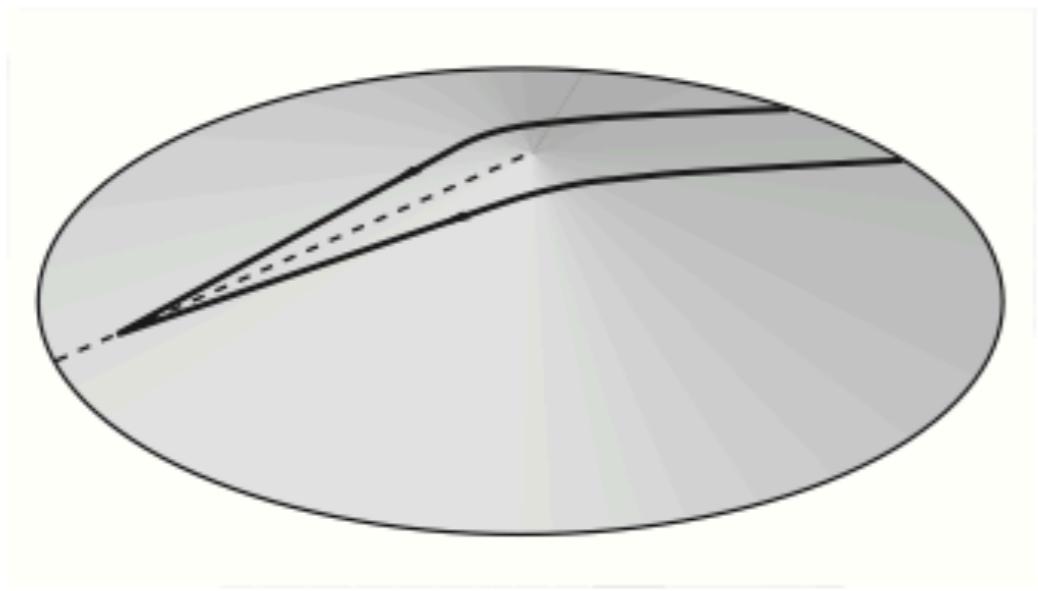
linear term turns hybrid inflation into two-field model in complex plane;  
strong effect on inflatonary observables, now dependent on trajectory, i.e.  
initial conditions! Inflation consistent with Planck data now possible  
(otherwise very difficult)



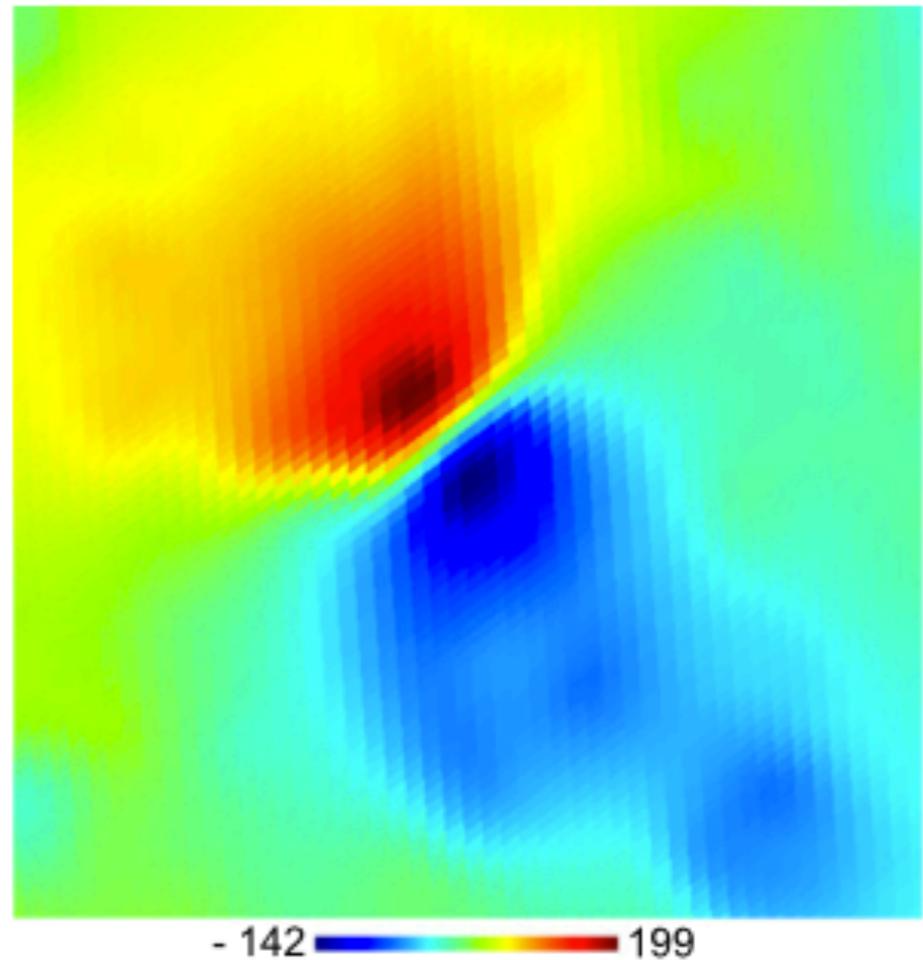
F-term hybrid inflation ( $\lambda \ll 1$ ) with SUSY breaking: resulting linear term in inflaton potential leads to two-field dynamics of complex inflaton in field space, observables depend on inflationary trajectory, range for spectral index consistent with  $n_s \simeq 0.96$



Parameter scan: relations between scale of B-L breaking, gravitino mass and scalar spectral index (correct in green band) cosmic string bound automatically fulfilled! Prediction for tensor-scalar ratio:  $r \lesssim 2 \times 10^{-6}$ ; gravitino masses consistent with gravitino DM or higgsino DM; further prediction: characteristic spectrum of gravitational waves



**Fig. 1.** The spacetime around a cosmic string is conical, as if a narrow wedge were removed from a flat sheet and the edges identified. For this reason cosmic strings can create double images of distant objects. Strings moving across the line of sight will cause line-like discontinuities in the CMB radiation.



**Fig. 2.** Characteristic CMB temperature discontinuity created by a cosmic string. Here, the simulated Nambu-Goto string has produced a cusp, a small region on the string that approaches the speed of light, which has generated a localised CMB signal.

from two sets of abelian-Higgs simulations are given (Moore et al. 2002; Bevis et al. 2007b). The evolution can be fitted with a VOS model Eq. (6) with  $\tilde{c} = 0.57$ , which is 150% higher than for Nambu-Goto strings. Field theory simulations have further important applications, particularly for describing delocalised

Abelian Higgs string:  $G\mu/c^2 < 3.2 \times 10^{-7}$ , consistent with GUT scale strings

# Gravitational Waves

Relic gravitational waves are window to very early universe; contributions from inflation, preheating & cosmic strings (see Maggiore '07, Dufaux et al '10, Hindmarsh '11); cosmological B-L breaking: prediction of GW spectrum with all contributions! Perturbations in flat FRW background:

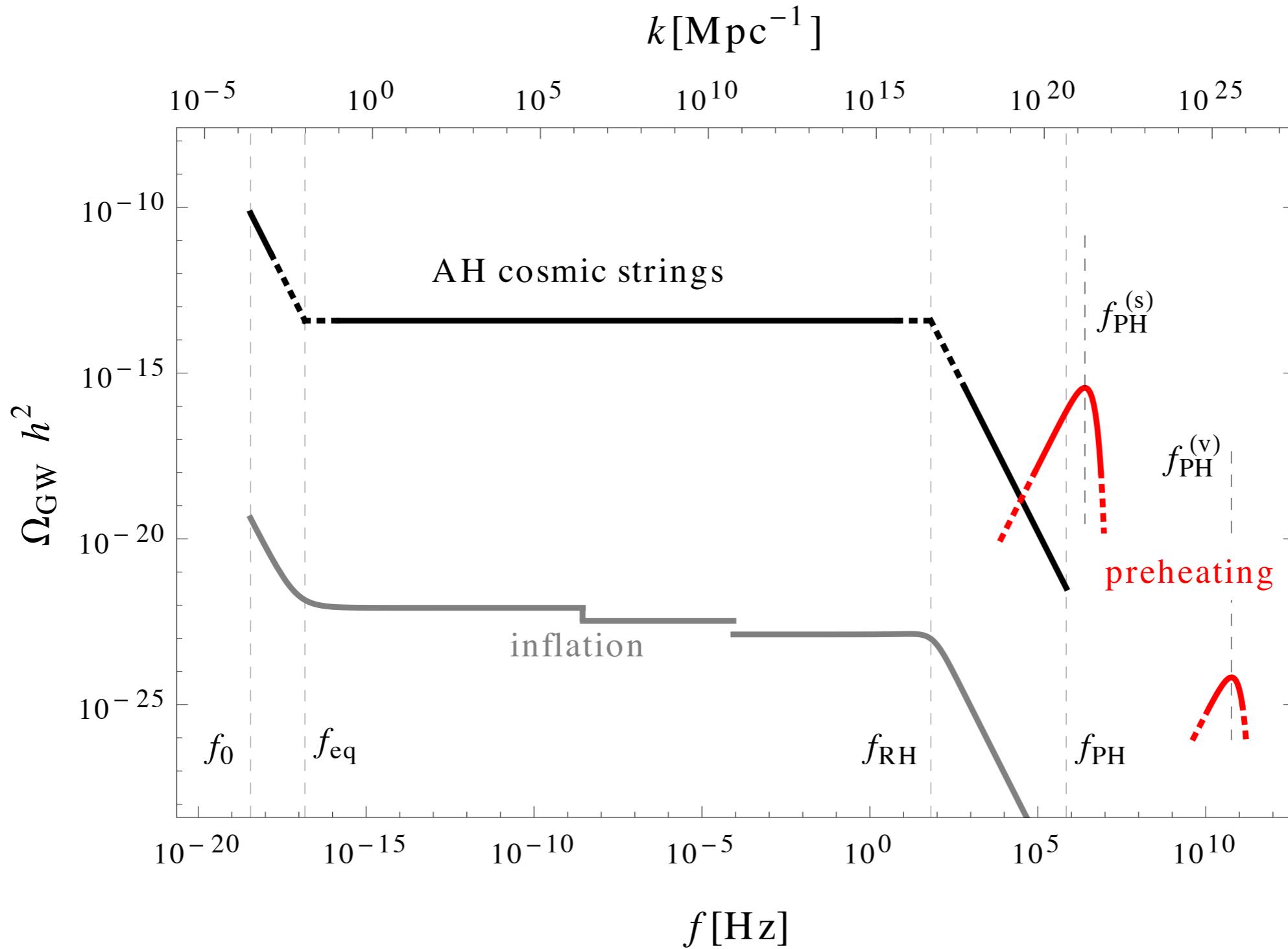
$$ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu , \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h_\rho^\rho$$

determined by linearized Einstein equations,

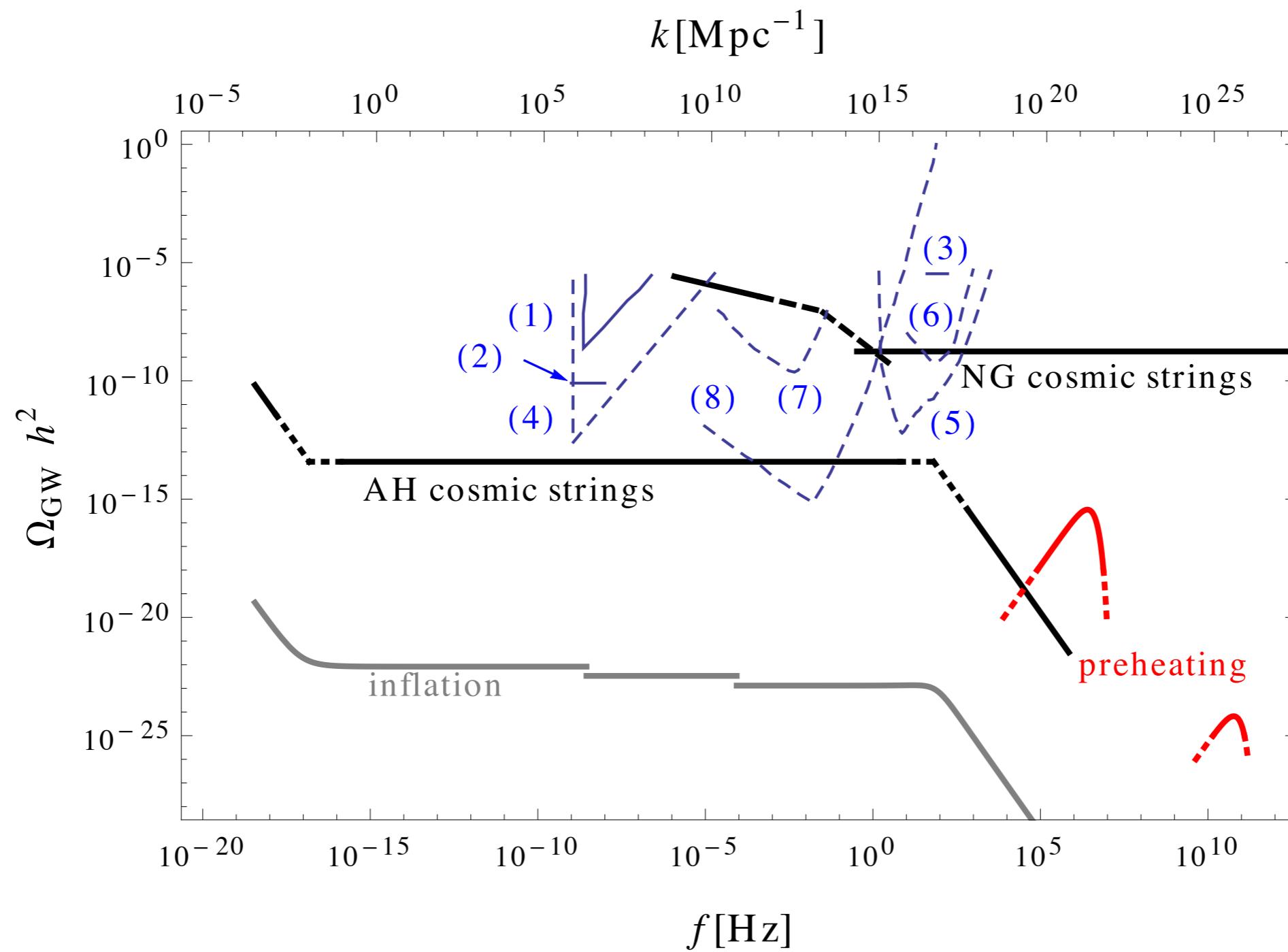
$$\bar{h}_{\mu\nu}''(\mathbf{x}, \tau) + 2\frac{a'}{a}\bar{h}_{\mu\nu}'(\mathbf{x}, \tau) - \nabla_{\mathbf{x}}^2\bar{h}_{\mu\nu}(\mathbf{x}, \tau) = 16\pi G T_{\mu\nu}(\mathbf{x}, \tau)$$

yields spectrum of GW background:

$$\Omega_{GW}(k, \tau) = \frac{1}{\rho_c} \frac{\partial \rho_{GW}(k, \tau)}{\partial \ln k} ,$$
$$\int_{-\infty}^{\infty} d \ln k \frac{\partial \rho_{GW}(k, \tau)}{\partial \ln k} = \frac{1}{32\pi G} \left\langle \dot{h}_{ij}(\mathbf{x}, \tau) \dot{h}^{ij}(\mathbf{x}, \tau) \right\rangle$$



Result: GWs from inflation, preheating and cosmic strings (Abelian Higgs), for typical parameters consistent with leptogenesis and dark matter; similar spectrum from inflation and cosmic strings [Figueroa, Hindmarsh, Urrestilla '13], .... but different normalization!!



Observational prospects (for typical parameters); ‘soon’: Advanced Ligo (6) [100 Hz], eLISA [0.01 Hz]; ‘later’: Einstein Telescope [100 Hz], BBO/DECIGO [0.01 Hz]; eventually determination of reheating temperature (leptogenesis)?!

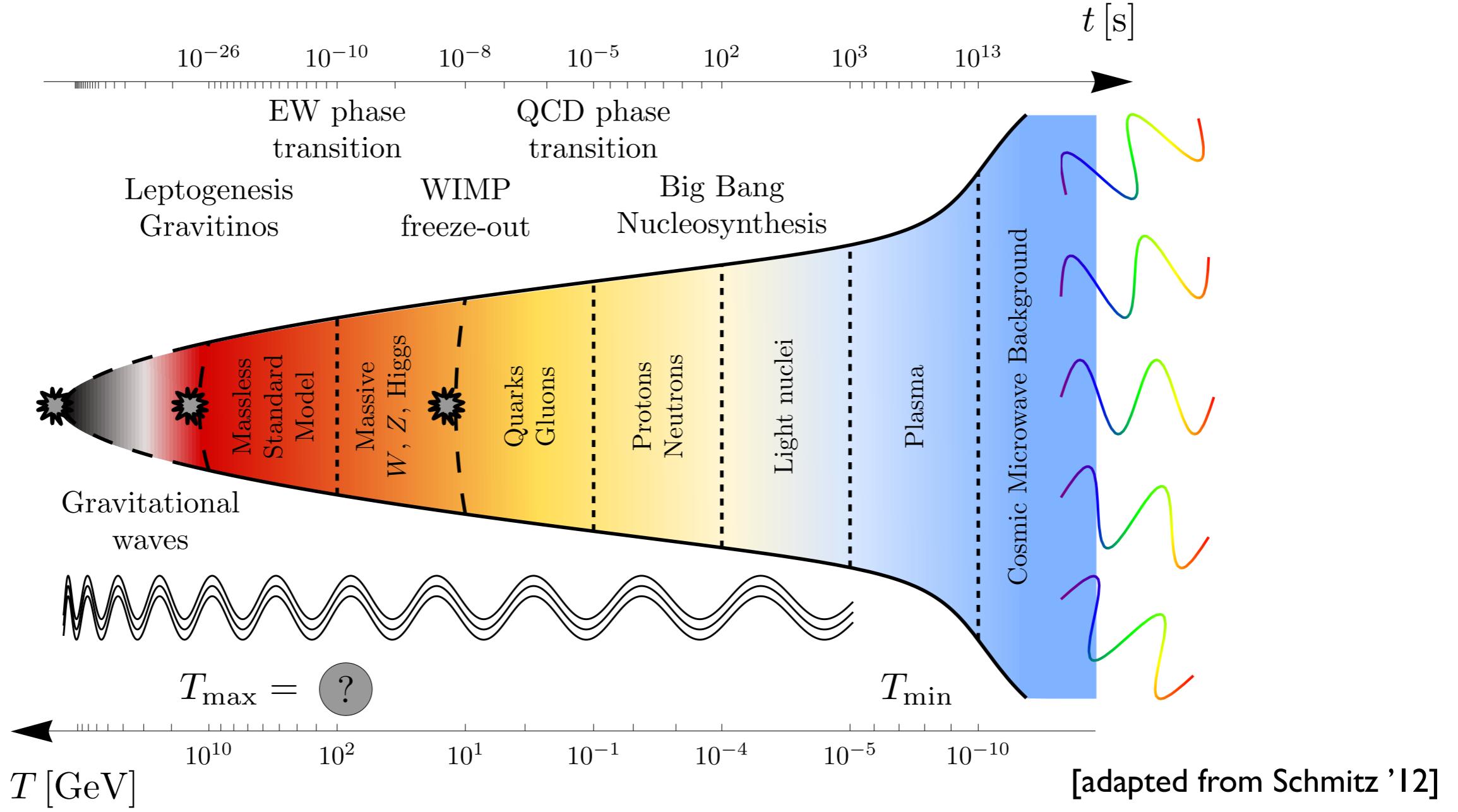
# Conclusions

- Strong theoretical motivation for grand unification; together with supersymmetry all ingredients needed for early universe cosmology

Observational hints:

- Successful predictions of GUT-scale leptogenesis: neutrino masses below 0.1 eV; standard mSUGRA with TeV gravitino and thermal neutralino WIMP inconsistent (gravitino problem)
- LHC: gravitino DM or higgsino DM via monojet searches
- Hope: cosmic strings in CMB or large scale structure
- Ultimate hope: gravitational waves - signal from cosmic strings, measurement of leptogenesis temperature, evidence for preheating

# **BACKUP SLIDES**



**Connection to particle physics in very early universe, before formation of CMB: nucleosynthesis, formation of hadrons, generation of mass (electroweak phase transition), dark matter, leptogenesis ... preceded by inflation (?) which generates initial conditions of hot early universe; important: global picture of very early universe!**

# Slow-Roll Inflation

Inflation is driven by slowly varying potential energy  $V(\phi)$  of inflaton field;  
small slow-roll parameters determine spectral indices:

$$\epsilon = \frac{M_{\text{pl}}^2}{2} \left( \frac{\partial_\phi V}{V} \right)^2, \quad \eta = M_{\text{pl}}^2 \frac{\partial_\phi^2 V}{V}$$

$$n_s - 1 = 2\eta_* - 6\epsilon_*, \quad n_t = -2\epsilon_*$$

Tensor-to-scalar ratio yields energy density during inflation:

$$r = \frac{\Delta_s^2}{\Delta_t^2} = \frac{2}{3\pi^2 \Delta_s} \frac{V(\phi_*)}{M_{\text{pl}}^4}, \quad V^{1/4} \approx \left( \frac{r}{0.01} \right)^{1/4} 10^{16} \text{ GeV}$$

Tensor-to-scalar ratios  $r \gtrsim 0.005$  require super-Planckian field values  
[Lyth '97; Antusch, Nolde '14]; theoretical challenge, ongoing theory activities...