# Neutrino CP violation in the minimal seesaw model

## - Towards the minimal seesaw model via CP violation of neutrinos -

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arXiv:1709.02136

Launch17, MPIK, Heidelberg, Germany, September 14, 2017

Particle Physics group at Niigata and Kanazawa Universities deeply appreciate for collaboration with Manfred and MPIK colleague.

2010 – 2012 supported by JSPS

with Jisuke Kubo (Kanazawa U)

Ryo Takahashi (Tohoku U)
Yusuke Shimizu (Hiroshima U)
Atsushi Watanabe (YITP, Kyoto)
Sintaro Eijima (EPFL, Lausanne)
Takashi Shimomura (Miyazaki U)
Takehiko Asaka (Niigata U)
Mayumi Aoki (Kanazawa U)
Takashi Toma (TU, Munich)

**Thank you, Manfred !**

Lieber Manfred,

Herzlichen Glückwunsch zum 60. Geburtstag.

Vielen  Dank für die langjährige Freundschaft.
Wir wünschen Dir weiterhin Erfolg.

60 ist nicht alt, aber auch nicht jung,
um Deine Gesundheit vernachlässigen zu  dürfen.
60 ist auch alt genug, um Dich auf  Deine Enkelkinder freuen zu dürfen.

Schade, dass wir  nicht dabei sein konnten.

Ich freue mich auf weitere Zusammenarbeit.

Annette und Kubo von der Ishigaki Insel.

# Plan of my talk

1 Introduction

2 Towards minimal seesaw model

3 Flavor Symmetry

4 Prediction of CP violation

5 Summary and Discussions

# 1 Introduction

The origin of flavor is still unknown !

What determines the flavor structure of quarks and leptons ?

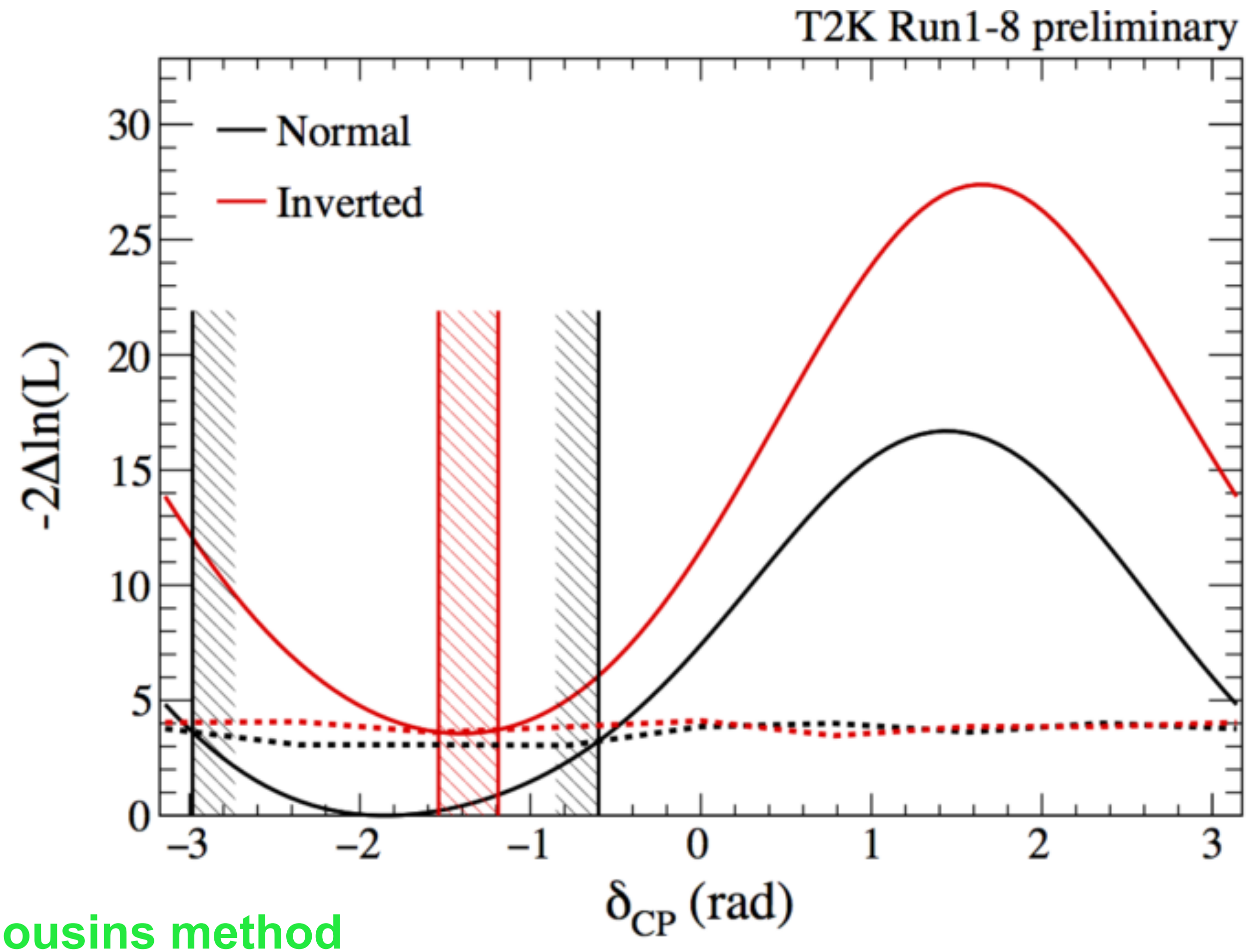
Symmetry ?  Zeros ?  Anarchy ?

In order to answer this question, we search for a simple scheme to examine the flavor structure of quark/lepton mass matrices, because the number of available data is much less than unknown parameters:

For neutrinos, 2 mass square differences,
3 mixing angles in experimental data
however, 9 parameters in neutrino mass matrix

How can we reduce the number of unkown parameters ?

# T2K reported the constraint on $\delta_{CP}$ August 4, 2017




Feldman-Cousins method

*The 2σ CL confidence interval:* Normal hierarchy: [-2.98, -0.60] radians
Inverted hierarchy: [-1.54, -1.19] radians

**CP conserving values (0,π) fall outside of 2σ intervals**

# 2 Towards minimal seesaw model

We try to remove a certain of parameters
in neutrino mass matrix.

This work is non-trivial.

At present, we know a simple, but non-trivial way to remove them:

- texture zeros in the neutrino mass matrix
- 2 Right-handed Majorana Neutrinos

  $m_1$ or $m_3$ vanishes
- Flavor Symmetry $A_4$, $S_4$, $A_5$ ……

# Example : Occam's Razor Approach

Harigaya, Ibe, Yanagida, PRD86, 2012, 013002

- **Texture Zeros:**
  One put maximal zeros in entries of mass matrices.

- **Two right-handed Majorana neutrinos** ( Det $M_\nu$=0 )
  Minimal number of unobserved particles

2x2 right-handed Majorana neutrino mass matrix
$(3x2)_{LR}$ Dirac neutrino mass matrix

Case A　　　　Case B

$$m_{\nu D} = \begin{pmatrix} 0 & * & * \\ * & 0 & * \end{pmatrix}_{RL}, \quad m_{\nu D} = \begin{pmatrix} * & * & 0 \\ 0 & * & * \end{pmatrix}_{RL}$$

$$M_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}_{RR}$$ Real diagonal

3 phases are absorbed by 3 left-handed doublets.

only one phase remains !

After seesaw, neutrino mass matrix is given by 5 parameters.

Experimental data are 3 mixing angles and 2 masses.

$\delta_{CP}=\pm 90°$ One phase controlls both $\delta_{cp}$ and Baryon Asymmetry !

However, inverted neutrino mass hierarchy !
Normal hierarchy is never reproduced !

## In our approach of this talk, we take

- **Trimaximal mixing basis:** instead of Texture Zeros
  This basis is reproduced by $A_4$, $S_4$ flavor symmetry.

- **Two right-handed Majorana neutrinos** ( Det $M_\nu$=0 )
  Minimal number of unobserved particles

Yusuke Simizu, Kenta Takagi, M.T, arXiv:1709.02136

2x2 right-handed Majorana neutrino mass matrix
$(3\text{x}2)_{LR}$ Dirac neutrino mass matrix

# Trimaximal mixing is still available !

$$V_{\mathrm{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

TM$_1$ TM$_2$

**Tri-bimaximal mixing**

Harrison, Perkins, Scott
2002

Named by Albright and Rodejohann, arXiv:0812.0436

**Trimaximal mixing**

**TM$_1$**

$$U_{\mathrm{PMNS}} = V_{\mathrm{TBM}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & e^{-i\sigma}\sin\phi \\ 0 & -e^{i\sigma}\sin\phi & \cos\phi \end{pmatrix}$$

$$\sin^2\theta_{12} = 1 - \frac{2}{3}\frac{1}{\cos^2\theta_{13}} \le \frac{1}{3}\,, \qquad \cos\delta_{CP}\tan 2\theta_{23} \simeq -\frac{1}{2\sqrt{2}\sin\theta_{13}}\left(1 - \frac{7}{2}\sin^2\theta_{13}\right)$$

**TM$_2$**

$$U_{\mathrm{PMNS}} = V_{\mathrm{TBM}} \begin{pmatrix} \cos\phi & 0 & e^{-i\sigma}\sin\phi \\ 0 & 1 & 0 \\ -e^{i\sigma}\sin\phi & 0 & \cos\phi \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{1}{3}\frac{1}{\cos^2\theta_{13}} \ge \frac{1}{3}\,, \qquad \cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1 - \frac{5}{4}\sin^2\theta_{13}\right)$$

Taking both the charged lepton mass matrix and
the right-handed Majorana neutrino one to be real diagonal:

$$M_R = M_0 \begin{pmatrix} p^{-1} & 0 \\ 0 & 1 \end{pmatrix}, \qquad M_D = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}_{LR}$$

$p = M_{R2} / M_{R1}$

After seesaw, we can take p=1 by rescaling prameters.

## At first, let us consider the case of $TM_1$

One can take a,b,c to be complex and d,e,f to be real
by using the freedom of phase-redefinitions of left-handed leptons.

# $TM_1$ with NH $m_1=0$

After seesaw, we obtain

$$M_\nu = M_D M_R^{-1} M_D^T = \frac{1}{M_0}\begin{pmatrix} a^2p+d^2 & abp+de & acp+df \\ abp+de & b^2p+e^2 & bcp+ef \\ acp+df & bcp+ef & c^2p+f^2 \end{pmatrix}$$

And then, move to TBM basis:

$$\hat{M}_\nu \equiv V_{\rm TBM}^T M_\nu V_{\rm TBM} \qquad V_{\rm TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Finally, we find

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix}$$

gives $TM_1$ with NH.
$m_1=0$

$$\hat{M}_\nu = \frac{1}{M_0}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}\left((b+c)^2p+(e+f)^2\right) & \frac{1}{2}\sqrt{\frac{3}{2}}\left((c^2-b^2)p-e^2+f^2\right) \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}\left((c^2-b^2)p-e^2+f^2\right) & \frac{1}{2}\left((b-c)^2p+(e-f)^2\right) \end{pmatrix}$$

$$m_2^2+m_3^2 = \frac{f^4}{16}\left[B^4(5j^2+2j+5)^2+2B^2(5jk+j+k+5)^2\cos 2\phi_B+(5k^2+2k+5)^2\right]$$

$$m_2^2 m_3^2 = \frac{9}{4}(j-k)^4 B^4 f^8 \qquad \frac{e}{f}=k\ , \qquad \frac{b}{c}=j\ , \qquad \frac{c}{f}=Be^{i\phi_B}$$

# $TM_1$ with IH

**$m_3=0$**

After taking
$$M_D = \begin{pmatrix} -2b & \frac{e+f}{2} \\ b & e \\ b & f \end{pmatrix}$$
, we get

$$\hat{M}_\nu = \frac{1}{M_0}\begin{pmatrix} 6b^2 & 0 & 0 \\ 0 & \frac{3}{4}(e+f)^2 & -\frac{1}{2}\sqrt{\frac{3}{2}}(e-f)(e+f) \\ 0 & -\frac{1}{2}\sqrt{\frac{3}{2}}(e-f)(e+f) & \frac{1}{2}(e-f)^2 \end{pmatrix}$$

$$\frac{e}{f} = ke^{i\phi_k}$$

$$= \frac{6b^2}{M_0}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{f^2}{M_0}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}(ke^{i\phi_k}+1)^2 & -\frac{1}{2}\sqrt{\frac{3}{2}}(k^2e^{2i\phi_k}-1) \\ 0 & -\frac{1}{2}\sqrt{\frac{3}{2}}(k^2e^{2i\phi_k}-1) & \frac{1}{2}(ke^{i\phi_k}-1)^2 \end{pmatrix}$$

**Mixing angles and CP phase are given only by k and $\Phi_k$**

# $TM_2$ with NH or IH

$m_1=0$ or $m_3=0$

After taking

$$M_D = \begin{pmatrix} b & -e-f \\ b & e \\ b & f \end{pmatrix}$$

, we get

$$\frac{e}{f} = ke^{i\phi_k}$$

$$\hat{M}_\nu = \frac{1}{M_0} \begin{pmatrix} \frac{3}{2}(e+f)^2 & 0 & \frac{\sqrt{3}}{2}(e^2-f^2) \\ 0 & 3b^2 & 0 \\ \frac{\sqrt{3}}{2}(e^2-f^2) & 0 & \frac{1}{2}(e-f)^2 \end{pmatrix}$$

$$= \frac{3b^2}{M_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{f^2}{M_0} \begin{pmatrix} \frac{3}{2}(ke^{i\phi_k}+1)^2 & 0 & \frac{\sqrt{3}}{2}(k^2e^{2i\phi_k}-1) \\ 0 & 0 & 0 \\ \frac{\sqrt{3}}{2}(k^2e^{2i\phi_k}-1) & 0 & \frac{1}{2}(ke^{i\phi_k}-1)^2 \end{pmatrix}$$

Mixing angles and CP phase are given only by k and $\Phi_k$

# 3 Flavor Symmetry

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix}$$

**$TM_1$ with NH**

$$M_D = \begin{pmatrix} -2b & \frac{e+f}{2} \\ b & e \\ b & f \end{pmatrix}$$

**$TM_1$ with IH**

$$M_D = \begin{pmatrix} b & -e-f \\ b & e \\ b & f \end{pmatrix}$$

**$TM_2$ with NH or IH**

# Realization of Dirac neutrino mass matrix for TM$_1$ with NH in $S_4$ flavor symmetry

$S_4$ : irreducible representations 1, 1', 2, 3, 3'

Introduce two flavons (gauge singlet scalars) 3' in $S_4$: $\Phi_{atm}$ , $\Phi_{sol}$

Lepton doublets L : 3'   Right-handed neutrinos $\nu_R$ : 1

Consider
specific vacuum alignments for 3'

$$\langle\phi_{\rm atm}\rangle \sim \begin{pmatrix} \frac{b+c}{2} \\ c \\ b \end{pmatrix}, \qquad \langle\phi_{\rm sol}\rangle \sim \begin{pmatrix} \frac{e+f}{2} \\ f \\ e \end{pmatrix}$$

$S_4$ generators : S, T, U

preserves $Z_2$ {1, SU} symmetry for 3'.

$$SU\ (US) = \mp\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{pmatrix}$$ for 3 and 3'. $$SU \begin{pmatrix} \frac{b+c}{2} \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{b+c}{2} \\ b \\ c \end{pmatrix}$$ for 3'

3'×3'×1 3'×3'×1

$$\frac{y_{\rm atm}}{\Lambda}\phi_{\rm atm} L H_u \nu^c_{R1} + \frac{y_{\rm sol}}{\Lambda}\phi_{\rm sol} L H_u \nu^c_{R2}$$ give

$$L(\mathbf{3'})\phi(\mathbf{3'}) = L_1\phi_1 + L_2\phi_3 + L_3\phi_2$$

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix}$$

## ★ TM$_1$ with IH in $S_4$ flavor symmetry

3×3×1     3'×3×1'

$$\frac{y_{\rm atm}}{\Lambda}\phi_{\rm atm}LH_u\nu^c_{R1}+\frac{y_{\rm sol}}{\Lambda}\phi_{\rm sol}LH_u\nu^c_{R2}$$

$$M_D=\begin{pmatrix}-2b & \frac{e+f}{2}\\ b & e\\ b & f\end{pmatrix}$$

$$\langle\phi_{\rm atm}\rangle\sim\begin{pmatrix}-2\\1\\1\end{pmatrix},\qquad \langle\phi_{\rm sol}\rangle\sim\begin{pmatrix}\frac{e+f}{2}\\ f\\ e\end{pmatrix}$$

3     3'

$\begin{pmatrix}-2\\1\\1\end{pmatrix}$ preserves $Z_2$ (SU) symmetry for 3.

## ★ TM$_2$ with NH or IH in $A_4$ or $S_4$ flavor symmetry

$\begin{pmatrix}1\\1\\1\end{pmatrix}$ preserves S symmetry for 3.

$$S=\frac{1}{3}\begin{pmatrix}-1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1\end{pmatrix}$$ for 3 and 3'.

**S is a generator of $A_4$ and $S_4$ generator**

$\begin{pmatrix}-e-f\\ e\\ f\end{pmatrix}$ **breaks S, T , U, SU unless e=f .**

**We need auxiliary $Z_2$ symmetry to obtain**

$$M_D=\begin{pmatrix}b & -e-f\\ b & e\\ b & f\end{pmatrix}$$

# 4 Prediction of CP violation

## Input Data (Global Analyses)

| observable | $3\sigma$ range for NH | $3\sigma$ range for IH |
|---|---|---|
| $\lvert\Delta m^2_{13}\rvert$ | $(2.407 \sim 2.643) \times 10^{-3}\mathrm{eV}^2$ | $(2.399 \sim 2.635) \times 10^{-3}\mathrm{eV}^2$ |
| $\lvert\Delta m^2_{12}\rvert$ | $(7.03 \sim 8.09) \times 10^{-5}\mathrm{eV}^2$ | $(7.03 \sim 8.09) \times 10^{-5}\mathrm{eV}^2$ |
| $\sin^2\theta_{23}$ | $0.385 \sim 0.635$ | $0.393 \sim 0.640$ |
| $\sin^2\theta_{12}$ | $0.271 \sim 0.345$ | $0.271 \sim 0.345$ |
| $\sin^2\theta_{13}$ | $0.01934 \sim 0.02392$ | $0.01953 \sim 0.02408$ |

Esteban, Gonzalez-Garcia, Maltoni, Martinez-Soler, Schwetz JHEP1701 (2017) 087

## Output: CP violating phase $\delta_{CP}$

Since we take trimaximal mixing, $\sin^2\theta_{12}$ is strongly constrained independent of parameters of Dirac mass matrices:

$$\sin^2\theta_{12} = 1 - \frac{2}{3}\frac{1}{\cos^2\theta_{13}} \le \frac{1}{3} \quad \text{for } TM_1 \qquad \sin^2\theta_{12} = \frac{1}{3}\frac{1}{\cos^2\theta_{13}} \ge \frac{1}{3} \quad \text{for } TM_2$$

**$\sin^2\theta_{12}$=0.317-0.320 for TM1 and 0.34 for TM2**

# $TM_1$ with NH $m_1=0$

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix}$$

4 real parameters
2 phases

**Specific three cases to remove 2 parameters**

**Case I** $b+c=0$

$$M_D = \begin{pmatrix} 0 & \frac{e+f}{2} \\ b & e \\ -b & f \end{pmatrix}$$

**Case 2** $c=0$

$$M_D = \begin{pmatrix} \frac{b}{2} & \frac{e+f}{2} \\ b & e \\ 0 & f \end{pmatrix}$$

**Case 3** $b=0$

$$M_D = \begin{pmatrix} \frac{c}{2} & \frac{e+f}{2} \\ 0 & e \\ c & f \end{pmatrix}$$

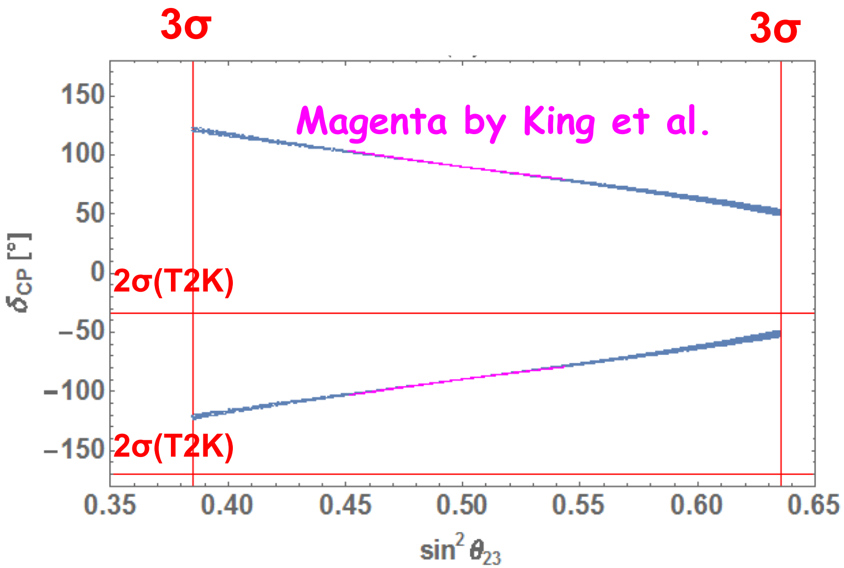
**3 parameters + 1 phase** e, f are real : b is complex

$$M_D = \begin{pmatrix} 0 & f \\ b & 3f \\ -b & -f \end{pmatrix}$$

$e/f=-3$, 2 real parameters + 1 phase

Littlest seesaw model by King et al.

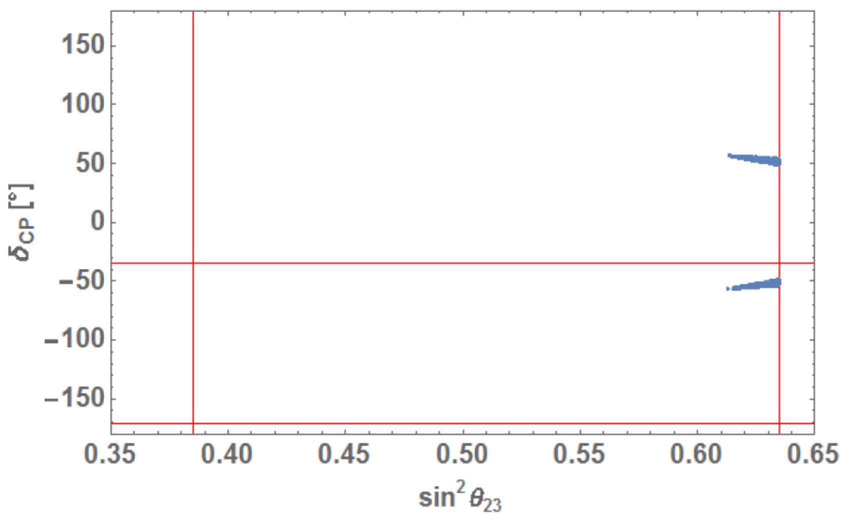
## Case 1



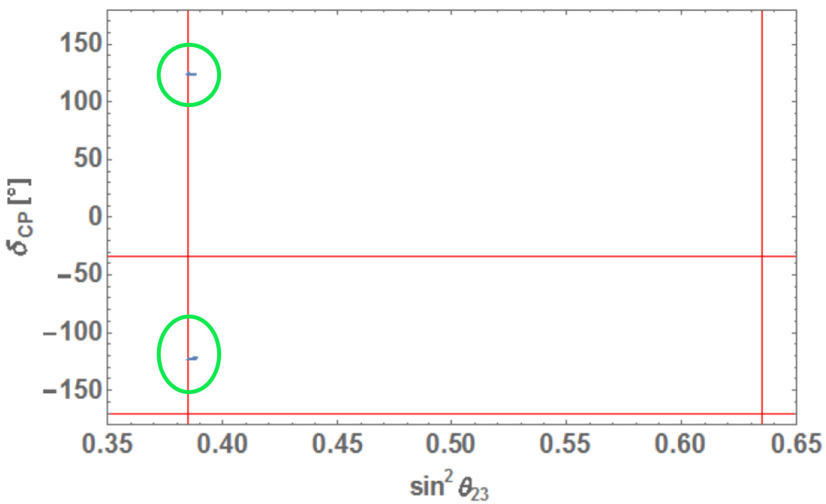

## $TM_1$ sum rule

$$\cos\delta_{CP}\tan 2\theta_{23} \simeq -\frac{1}{2\sqrt{2}\sin\theta_{13}}\left(1-\frac{7}{2}\sin^2\theta_{13}\right)$$

## Case 2

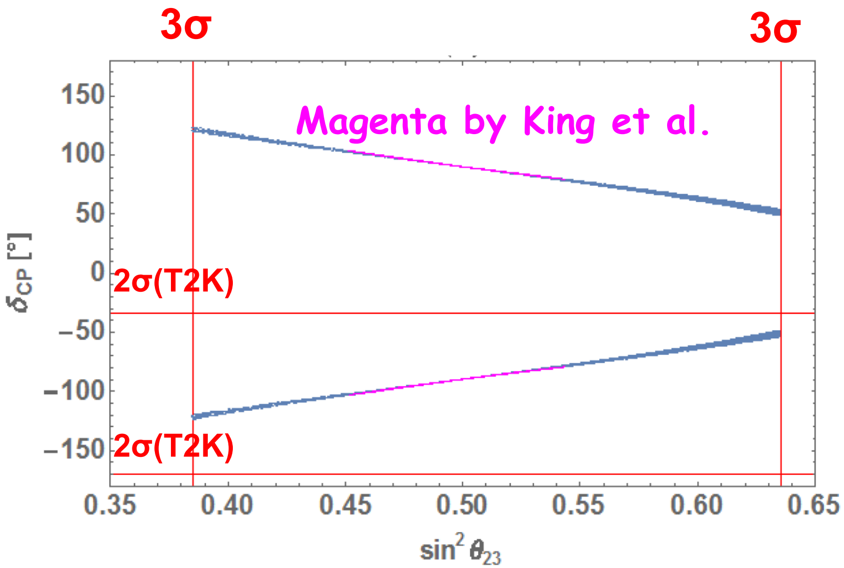


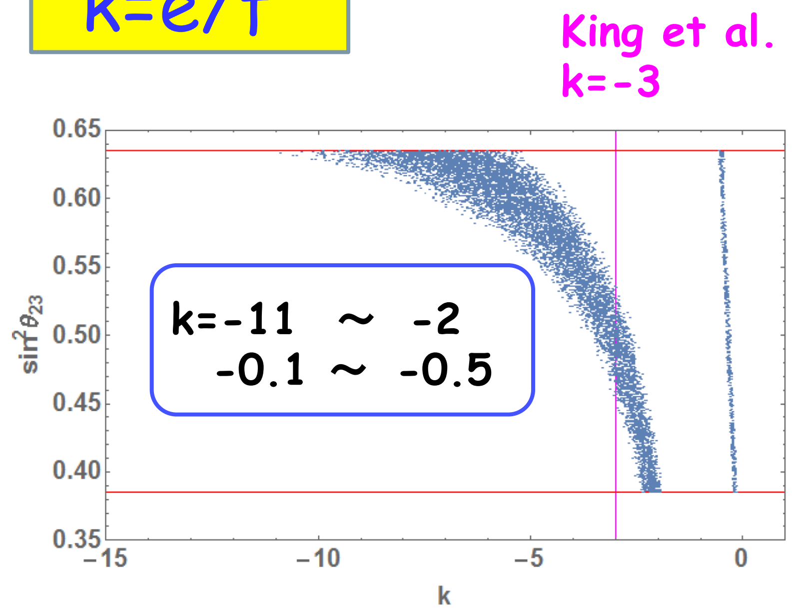

## Case 3

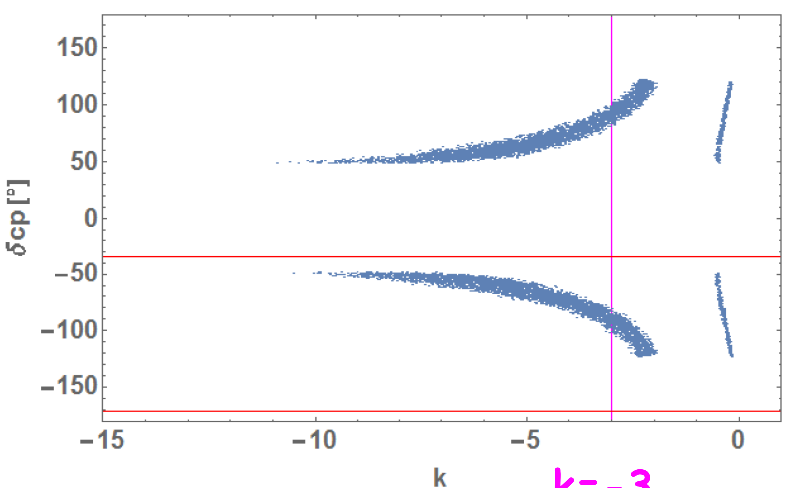


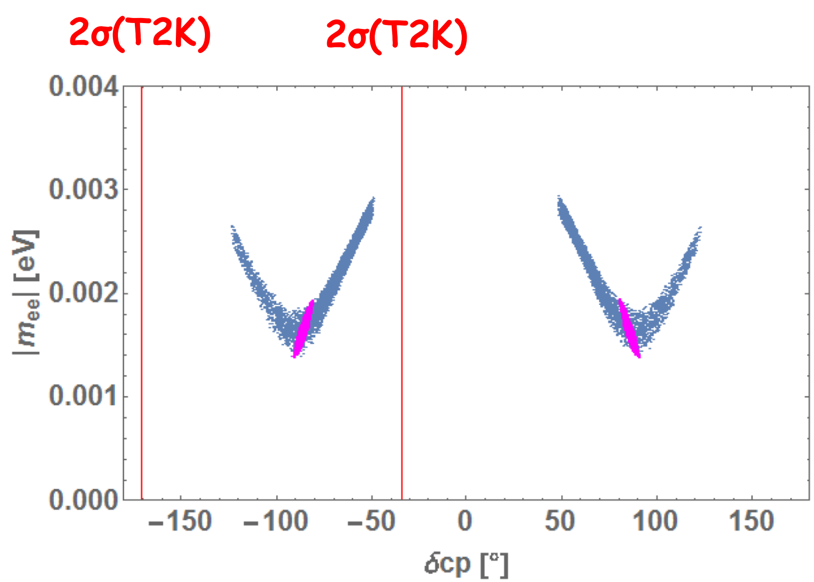
# Case 1

k=e/f


3σ
3σ
Magenta by King et al.
2σ(T2K)
2σ(T2K)
$\delta_{CP}$ [°]
$\sin^2\theta_{23}$


King et al.
k=-3
k=-11 ~ -2
-0.1 ~ -0.5
$\sin^2\theta_{23}$
k


δcp [°]
k
k=-3
King et.al.


2σ(T2K)
2σ(T2K)
$|m_{ee}|$ [eV]
δcp [°]

# New simple Dirac neutrino mass matrices with different k=e/f

$$M_D = \begin{pmatrix} 0 & 2f \\ b & 5f \\ -b & -f \end{pmatrix}, \quad \begin{pmatrix} 0 & 2f \\ b & -f \\ -b & 5f \end{pmatrix}, \quad \begin{pmatrix} 0 & f \\ b & 4f \\ -b & -2f \end{pmatrix}, \quad \begin{pmatrix} 0 & f \\ b & -2f \\ -b & 4f \end{pmatrix}$$

| k= -5 | k= -1/5 | k= -2 | k= -1/2 |
|---|---|---|---|
| $\delta_{CP}=\pm(50\text{-}70)°$ | $\delta_{CP}=\pm 120°$ | $\delta_{CP}\sim\pm 120°$ | $\delta_{CP}=\pm(50\text{-}70)°$ |
| $\sin^2\theta_{23}\geqq 0.55$ | $\sin^2\theta_{23}\sim 0.4$ | $\sin^2\theta_{23}\sim 0.4$ | $\sin^2\theta_{23}\geqq 0.55$ |

## Littlest seesaw model by King et al.

$$M_D = \begin{pmatrix} 0 & f \\ b & 3f \\ -b & -f \end{pmatrix}$$

k=-3

$\delta_{CP}=\pm(80\text{-}105)°$

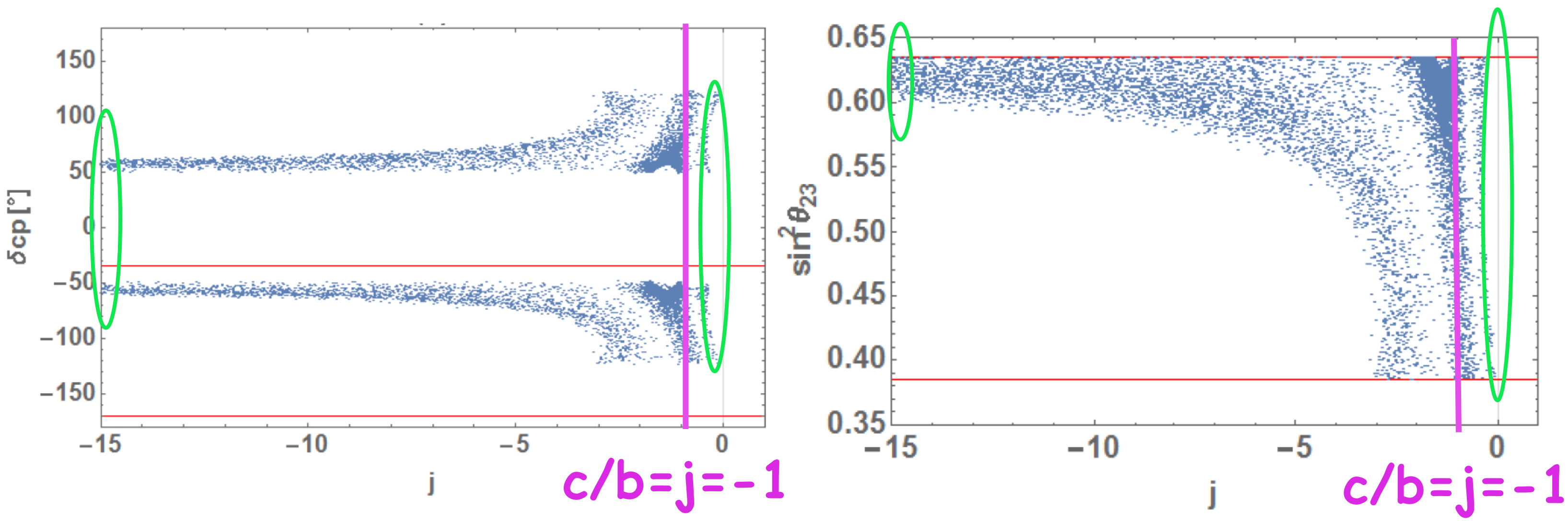
$\sin^2\theta_{23}=0.45\sim 0.55$

# Predictions at arbitrary c/b=j

$$\frac{e}{f} = k \qquad \frac{b}{c} = j \,, \qquad \frac{c}{f} = Be^{i\phi_B}$$

5 parameters by supposing j to be real

$$\hat{M}_\nu = \frac{f^2}{M_0}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}\left[B^2e^{2i\phi_B}(1+j)^2+(k+1)^2\right] & \frac{1}{2}\sqrt{\frac{3}{2}}\left[B^2e^{2i\phi_B}(1-j^2)-k^2+1\right] \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}\left[B^2e^{2i\phi_B}(1-j^2)-k^2+1\right] & \frac{1}{2}\left[B^2e^{2i\phi_B}(1-j)^2+(k-1)^2\right] \end{pmatrix}$$

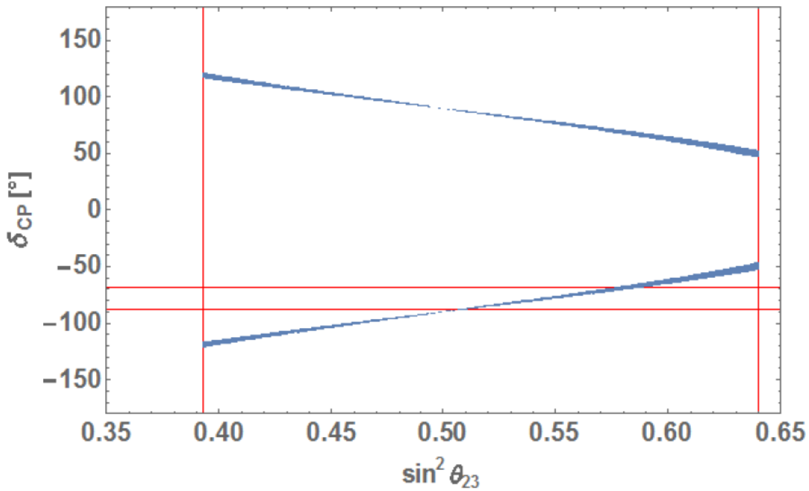


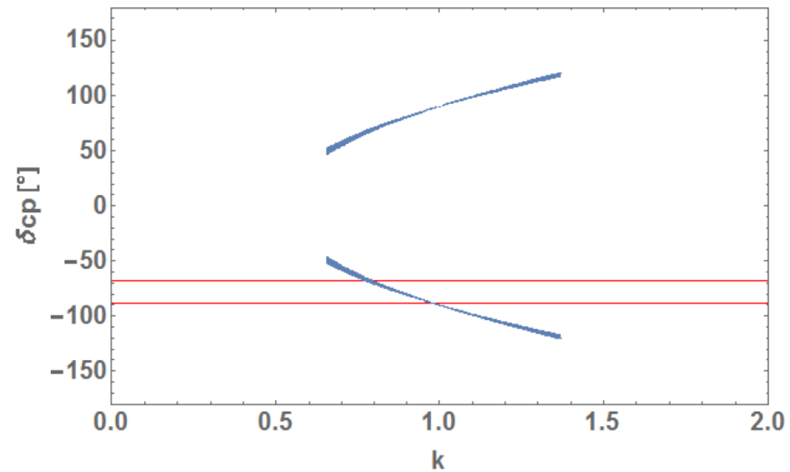

**case 1: c/b=-1, case 2: c/b=-∞, case 3: c/b=0**

# TM$_1$ with IH $m_3=0$

$$M_D = \begin{pmatrix} -2b & \frac{e+f}{2} \\ b & e \\ b & f \end{pmatrix}$$

$$\cos\delta_{CP}\tan 2\theta_{23} \simeq -\frac{1}{2\sqrt{2}\sin\theta_{13}}\left(1-\frac{7}{2}\sin^2\theta_{13}\right)$$




$$\frac{e}{f} = k e^{i\phi_k}$$

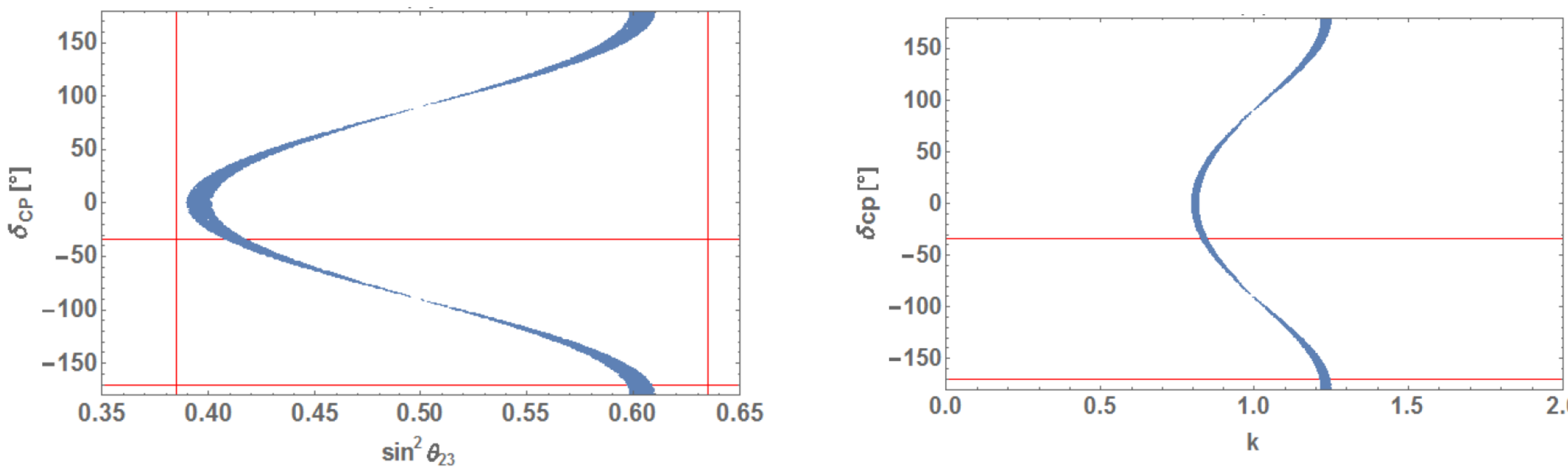
**$k=|e/f|=0.65\sim1.37$ $\Phi_k=\pm(25°\sim38°)$**

**$|m_{ee}|\sim 50$ meV**

# $TM_2$ with NH  $m_1=0$

$$M_D = \begin{pmatrix} b & -e-f \\ b & e \\ b & f \end{pmatrix}$$

$$\cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1-\frac{5}{4}\sin^2\theta_{13}\right)$$




**Predicted $\delta_{CP}$ is very sensitive to k.**

$$\frac{e}{f} = ke^{i\phi_k}$$

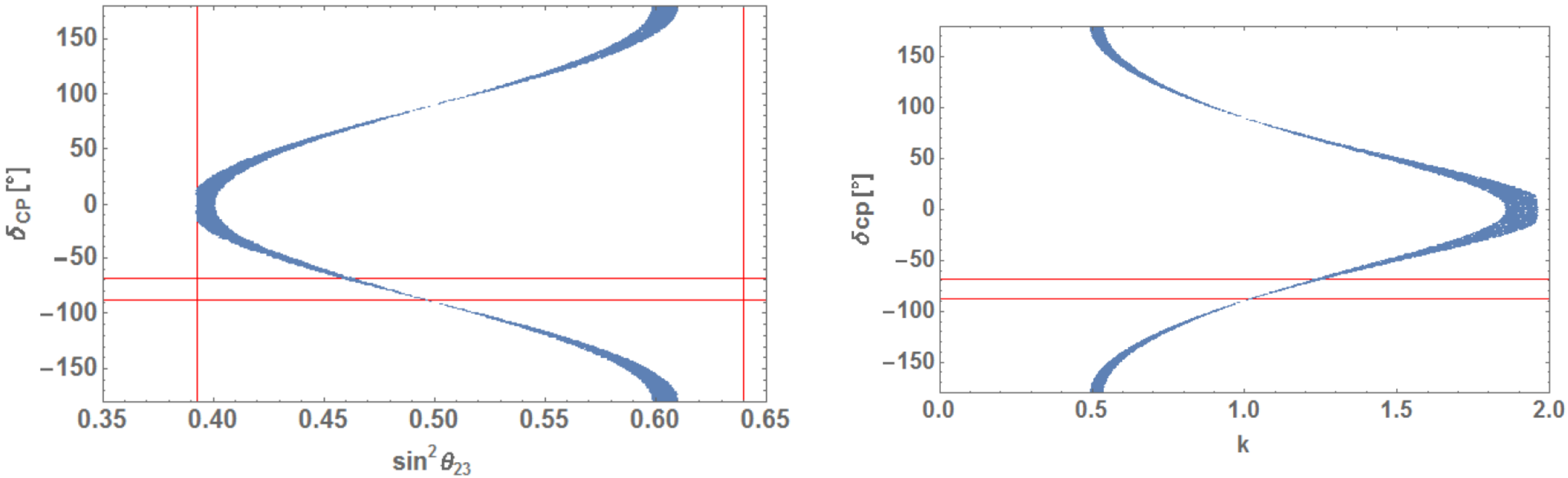
**k=|e/f|=0.78～1.24**    **$\Phi_k$=±(165°～180°)**

**$|m_{ee}|$=(2～4) meV**

# $TM_2$ with IH $m_3=0$

$$M_D = \begin{pmatrix} b & -e-f \\ b & e \\ b & f \end{pmatrix}$$

$$\cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1-\frac{5}{4}\sin^2\theta_{13}\right)$$

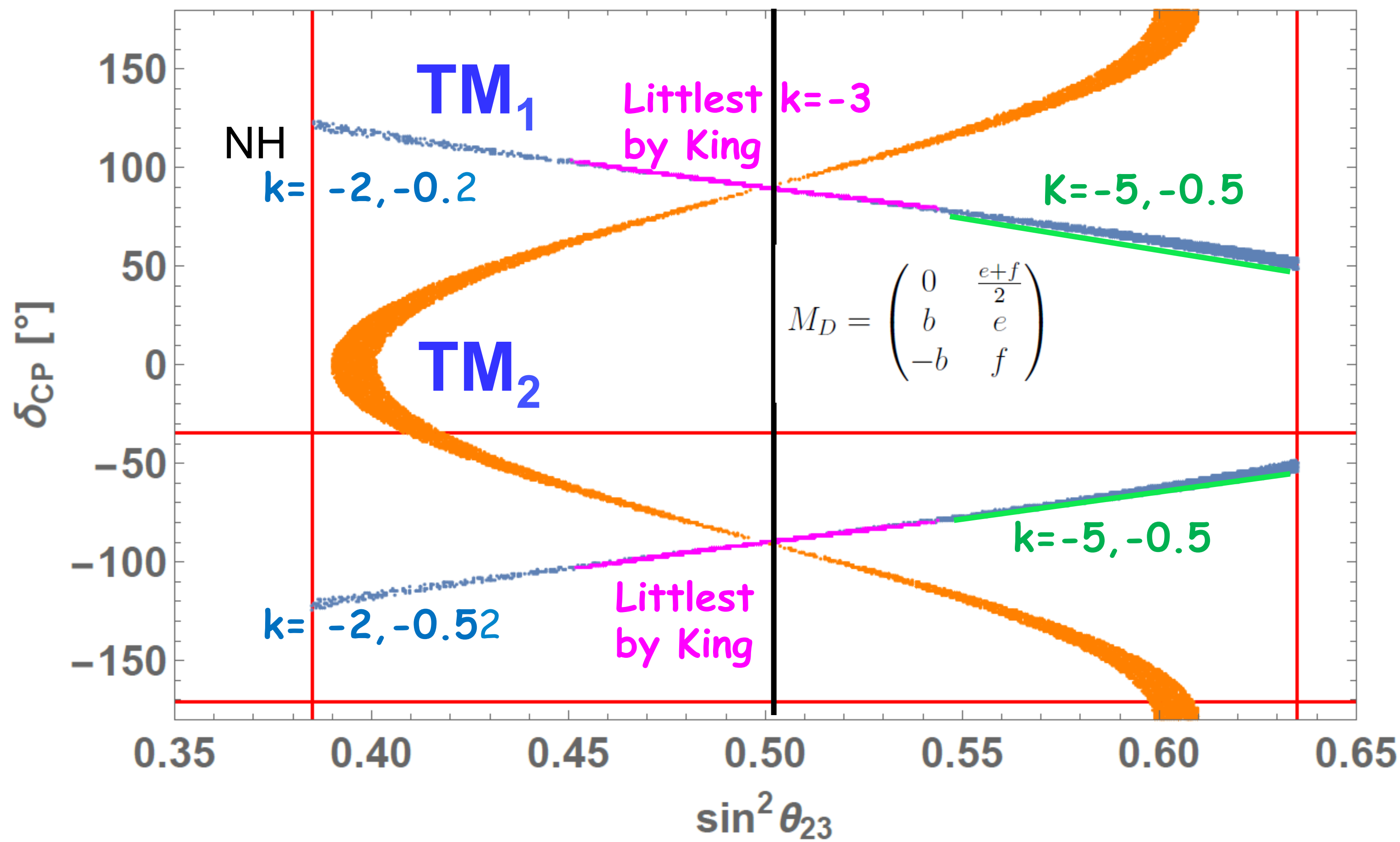



$$\frac{e}{f} = k e^{i\phi_k}$$

**k=|e/f|=0.49∼1.95** **$\Phi_k$= -40°∼40°**

**$|m_{ee}|$∼50 meV**

# Combined result




**e/f will be fixed by the observation of $\delta_{cp}$ .**

# 5 Summary and Discussions

- **Trimaxial mixing basis**

  This basis is reproduced by $A_4$, $S_4$ flavor symmetry.

- Two right-handed Majorana neutrinos

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} \qquad M_D = \begin{pmatrix} -2b & \frac{e+f}{2} \\ b & e \\ b & f \end{pmatrix} \qquad M_D = \begin{pmatrix} b & -e-f \\ b & e \\ b & f \end{pmatrix}$$

**$TM_1$ with NH** | **$TM_1$ with IH** | **$TM_2$ with NH or IH**

**Minimal seesaw mass matrices will be tested by $\delta_{CP}$ and $\sin^2\theta_{23}$ .**

$$M_D = \begin{pmatrix} 0 & 2f \\ b & 5f \\ -b & -f \end{pmatrix}, \quad \begin{pmatrix} 0 & 2f \\ b & -f \\ -b & 5f \end{pmatrix}, \quad \begin{pmatrix} 0 & f \\ b & 4f \\ -b & -2f \end{pmatrix}, \quad \begin{pmatrix} 0 & f \\ b & -2f \\ -b & 4f \end{pmatrix} \qquad M_D = \begin{pmatrix} 0 & f \\ b & 3f \\ -b & -f \end{pmatrix}$$

The sign of by $\delta_{CP}$ is not determined in our talk. Baryon asymmetry of the universe can investigate its sign by using the leptogenesis.

# Back up slides

# Neutrino mixing vs. quark mixing

## Neutrino mixing

(3σ C.L. range)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\begin{pmatrix} 0.800\text{-}0.844 & 0.515\text{-}0.581 & 0.139\text{-}0.155 \\ 0.229\text{-}0.516 & 0.438\text{-}0.699 & 0.614\text{-}0.790 \\ 0.249\text{-}0.528 & 0.462\text{-}0.715 & 0.595\text{-}0.776 \end{pmatrix}$$

I. Esteban et al., JHEP 01 (2017) 087

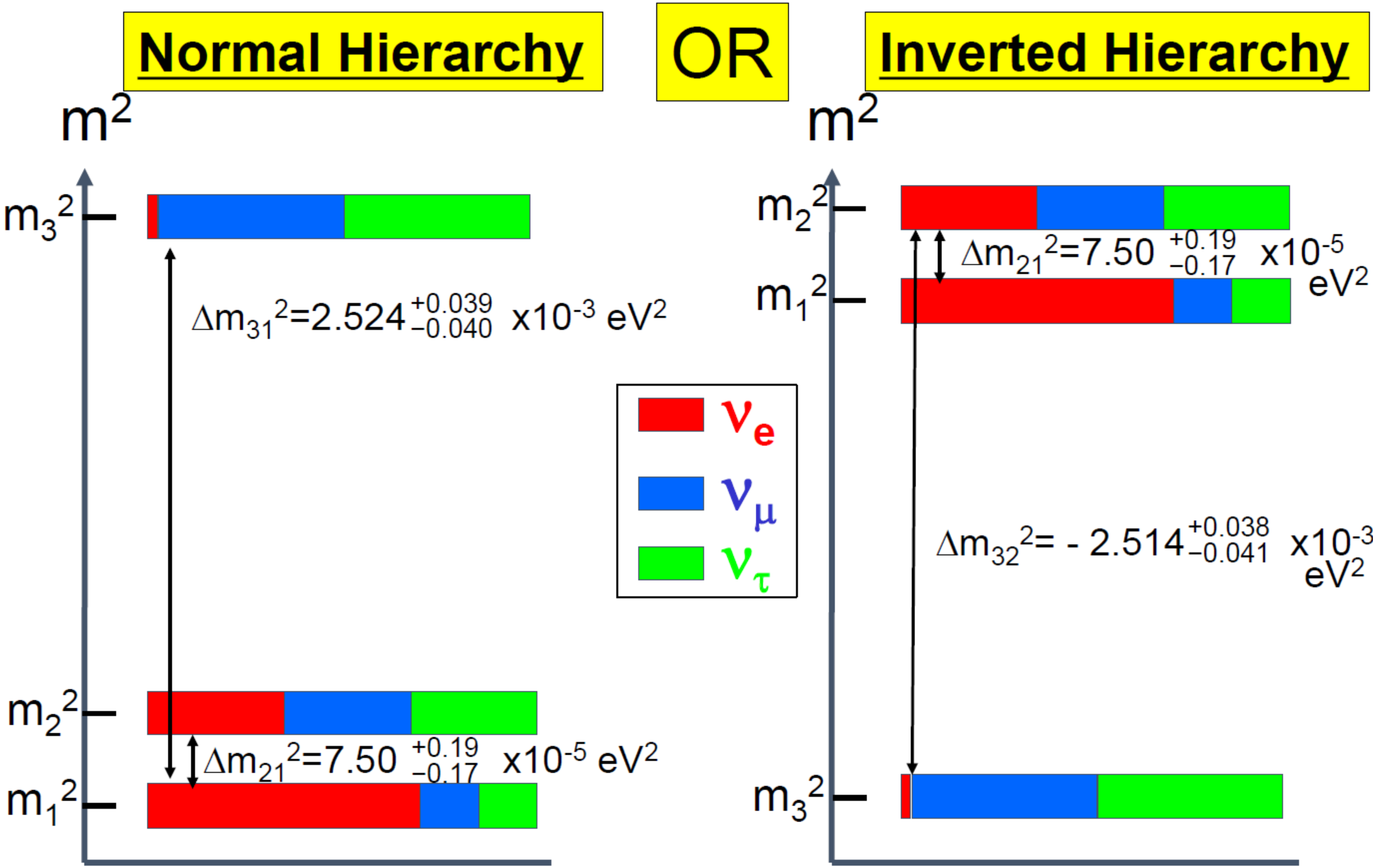
## Quark mixing (CKM matrix)

$$\begin{pmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0414 \\ 0.00875 & 0.0403 & 0.99915 \end{pmatrix}$$

Particle Data Group (2016)

They are so much different!

# Neutrino mass and mixing (what we know now)




Numbers from I. Esteban et al., JHEP 01 (2017) 087

After seesaw

$$(m_\nu)_{\alpha\beta} = \sum_{i=1}^{n_N} \lambda^T_{\alpha i} M_i^{-1} \lambda_{i\beta}\, v^2$$

**Neutrino mass matrix has one zero.**

$$\begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$$

$$0 = c_{12}c_{13}(-s_{12}c_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}})e^{2i\alpha}m_1 + s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}})e^{2i\beta}m_2 + s_{13}s_{23}c_{13}e^{-i\delta_{CP}}m_3$$

$m_1$(NH) or $m_3$(IH) = 0 because of 2 right-handed Majorana neutrinos

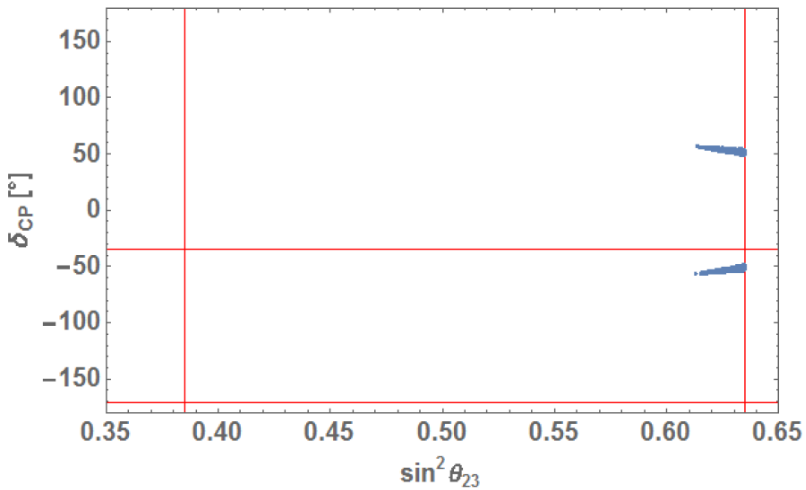
**In the case of $m_3$=0 (IH), this relation is easily satisfied.**
**In the case of $m_1$=0 (NH), this relation is significantly broken.**

$$U_{\text{PMNS}} \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\frac{\alpha}{2}} & 0 & 0 \\ 0 & e^{i\frac{\beta}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$J_{CP} = \text{Im}\left[U_{e1}U_{\mu 2}U^*_{e2}U^*_{\mu 1}\right] = s_{23}c_{23}s_{12}c_{12}s_{13}c^2_{13}\sin\delta_{CP}$$

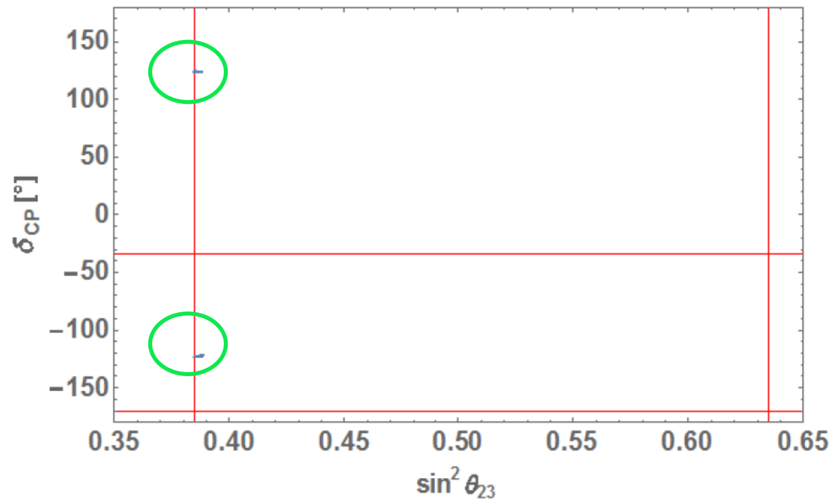
$$V_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

## Case 2

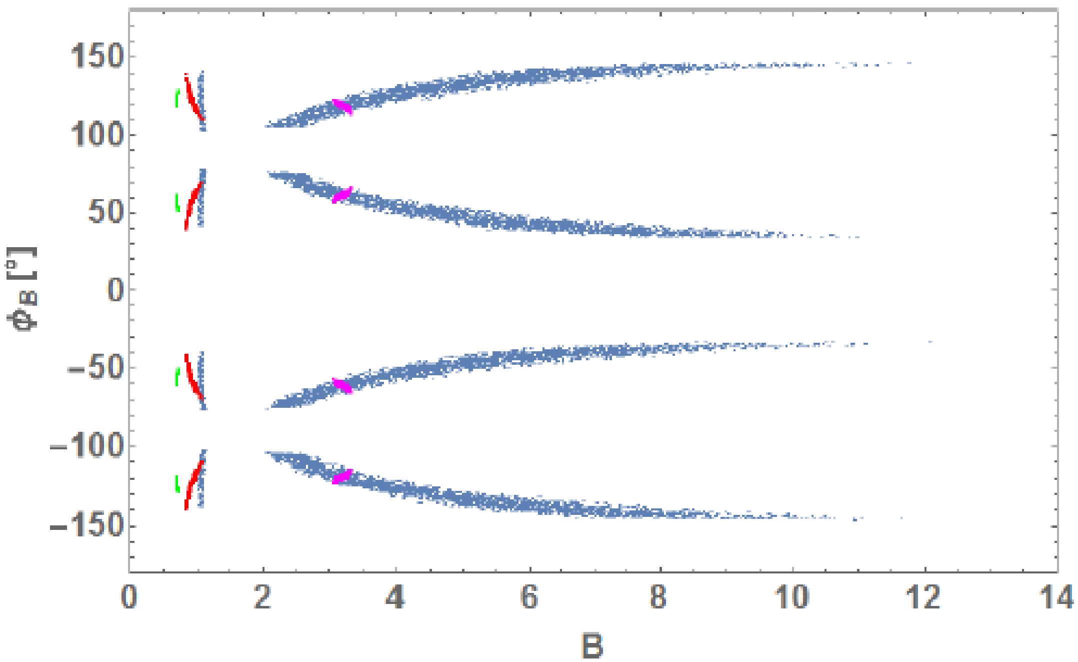



k=e/f=-1.7~ -1.1

## Case 3




k=e/f=-0.84~ -0.73



150
100
50
0
−50
−100
−150
$\phi_B$ [°]
0
2
4
6
8
10
12
14
B

# Case 1