Flux Compactifications & the Hierarchy Problem

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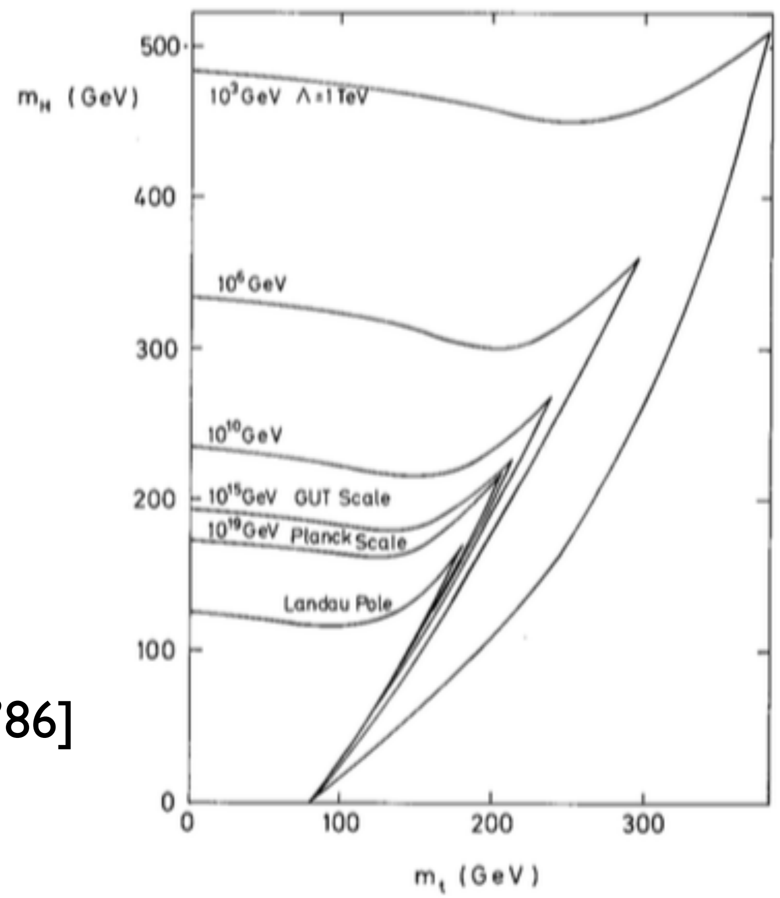
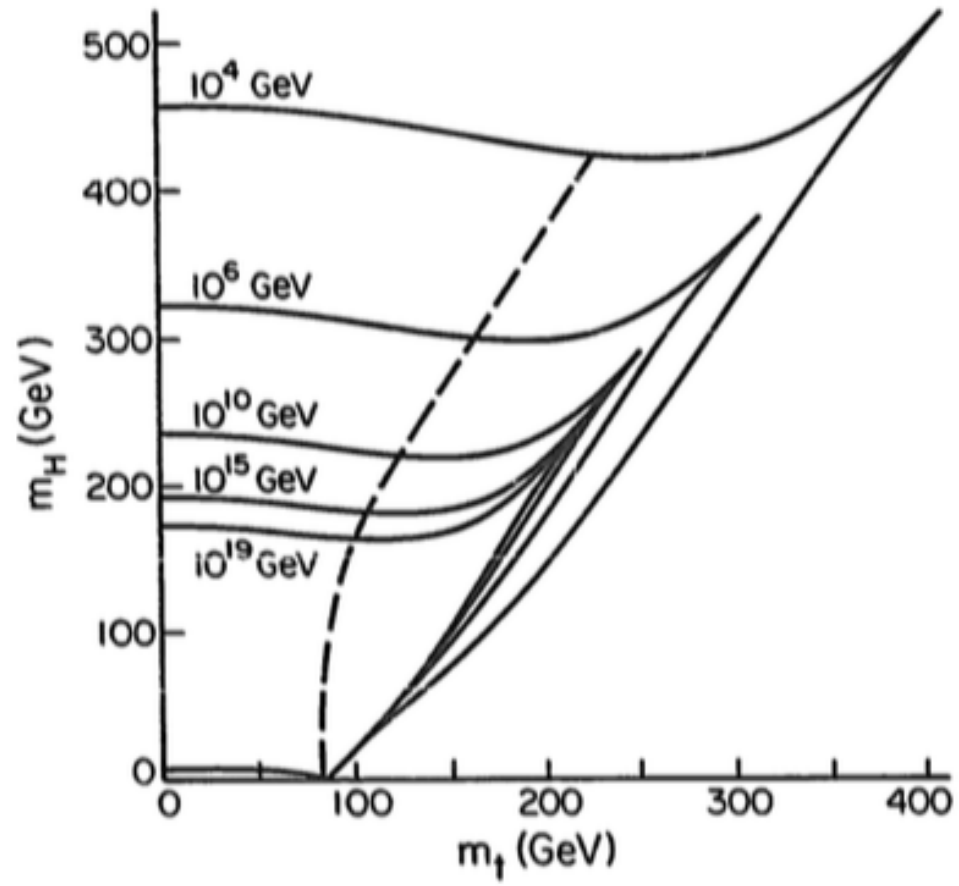
MPIK Heidelberg, LindnerFest, September 2017

Manfred & conformal symmetry

My first encounter with Manfred: Santa Barbara, 1990, UV completion of SM



[Lindner '86]



[WB, Busch '90]

Triviality and vacuum stability limit allowed range of Higgs and top masses; adding dilaton yields `hidden scale invariance' with stringent vacuum stability bound $m_t < 100\ \mathrm{GeV}$ 😢 (still interesting topic ...)

UV Completion of the Standard Model

- Structure of Standard Model points towards "grand unification" of strong and electroweak interactions (quark and lepton content, gauge group, "unification" of gauge couplings, small neutrino masses ...)

- Strong theoretical arguments for supersymmetry at "high" energy scales (gravity, extra dimensions, string theory)

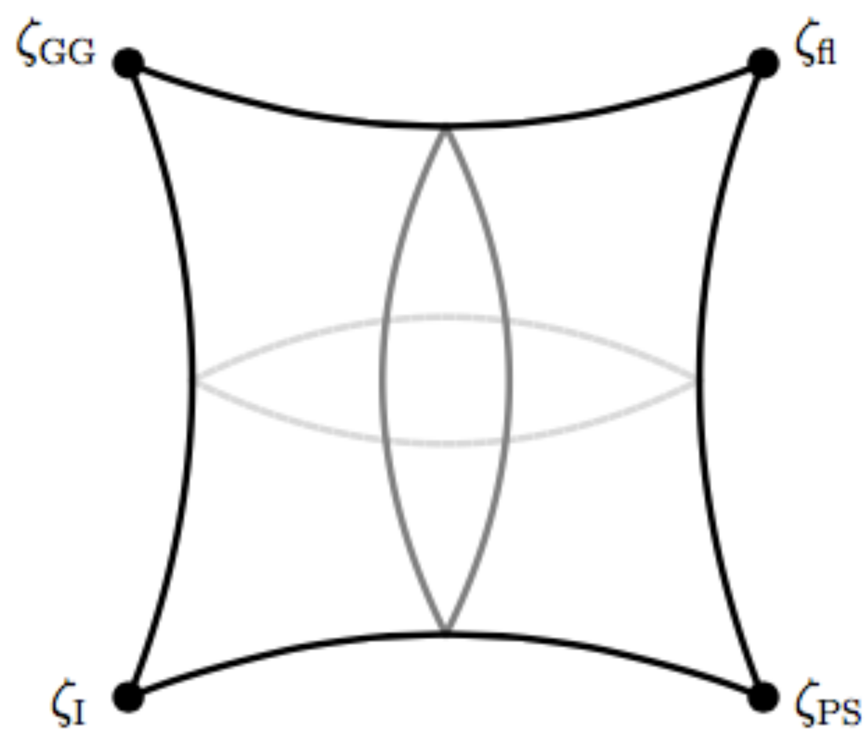
- Energy scale of grand unification: $\Lambda_{\mathrm{GUT}} \simeq 10^{15} \ldots 10^{16}\ \mathrm{GeV}$
  energy scale of supersymmetry breaking: $\Lambda_{\mathrm{SB}} \simeq$??

- This talk: 6d supergravity GUTs, $\Lambda_{\mathrm{SB}} \sim R_{\mathrm{c}}^{-1} \sim \Lambda_{\mathrm{GUT}}$

Split symmetries

WB, Dierigl, Ruehle, Schweizer '15, '16

Consider SO(10) GUT group in 6d, broken at orbifold fixed points to standard SU(5)xU(1), Pati-Salam SU(4)xSU(2)xSU(2) and flipped SU(5)xU(1), with SM group as intersection; bulk fields 45, 16, 16*, 10's [Asaka, WB, Covi '02; Hall, Nomura et al '02; ...]; full 6d gauge symmetry:



$$SO(10) \times U(1)_A$$

N 16's from charged bulk 16-plet and N flux quanta:

$$\mathbf{16}\ [SO(10)] \sim \mathbf{5}^* + \mathbf{10} + \mathbf{1}\ [SU(5)] \sim \mathbf{q}, \mathbf{l}, \mathbf{u^c}, \mathbf{e^c}, \mathbf{d^c}, \nu^{\mathbf{c}}\ [G_{\mathrm{SM}}]$$

Higgs fields from uncharged bulk 10-plets, form split multiplets:

$$H_1 \supset H_u\,, \quad H_2 \supset H_d\,, \quad \Psi \supset D^c, N^c\,, \quad \Psi^c \supset D, N$$

Flux **breaks supersymmetry** [Bachas '95], soft SUSY breaking only for quark-lepton families:

$$M^2 = m_{\tilde{q}}^2 = m_{\tilde{l}}^2 = \frac{4\pi N}{V_2} \sim (10^{15}\ \mathrm{GeV})^2$$

$$m_{3/2} \sim 10^{14}\ \mathrm{GeV}\,, \quad m_{\tilde{q}}^2 = m_{\tilde{l}}^2 > m_{3/2} \sim m_{1/2} \gg m_{\tilde{h}}$$

Emerging picture of **Split Symmetries** (reminiscent of "split/spread SUSY" [Arkani-Hamed, Dimopoulos; Giudice, Romanino '04; Hall, Nomura '11]):

- supersymmetry breaking is large for scalar quarks and leptons *because* they form *complete* GUT multiplets

- supersymmetry breaking can be small for gauge and Higgs fields *because* they form *incomplete* GUT multiplets (THDM)

Can GUT-scale SUSY breaking be viable?

- Is a matching of THDMs to SUSY at GUT scale consistent with RG running and vacuum stability? What can we hope for at LHC?

- Can all moduli be stabilized (D-term breaking, F-term breaking ...) with de Sitter (Minkowski) vacua?

- How do **quantum corrections** change the tree-level picture? (`screening' of divergencies by magnetic flux?)

- Can the 6d SO(10)xU(1) SUGRA models be embedded into string theory?

Quantum Corrections: toy model

WB, Dierigl, Dudas, Schweizer '16; Lee, Ghilencea `17

Simple example: **6d SUSY QED**, compactified on torus:

$$S_6 = \int d^6x \Big\{ \tfrac{1}{4} \int d^2\theta \, W^\alpha W_\alpha + \text{h.c.} + \int d^4\theta \left(\partial V \bar{\partial} V + \phi\bar{\phi} + \sqrt{2} V \left(\bar{\partial}\phi + \partial\bar{\phi} \right) \right)$$

$$+ \int d^2\theta \, \tilde{Q}(\partial + \sqrt{2} g q \phi) Q + \text{h.c.} + \int d^4\theta \left(\overline{Q} e^{2gqV} Q + \overline{\tilde{Q}} e^{-2gqV} \tilde{Q} \right) \Big\} ,$$

$$\partial = \partial_5 - i\partial_6 \, , \quad \phi|_{\theta=\bar{\theta}=0} = \tfrac{1}{\sqrt{2}} (A_6 + iA_5)$$

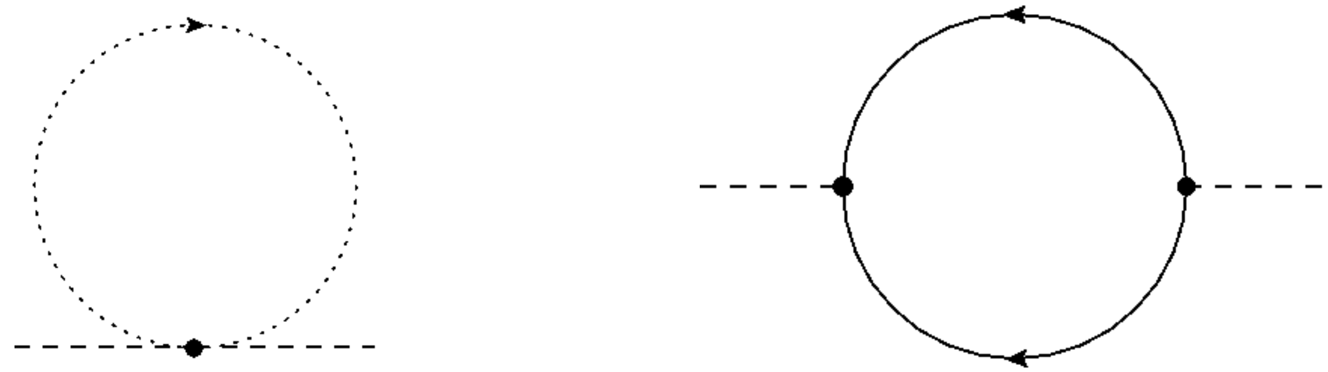
Wilson line and flux background, mode expansion of superfields:

$$\phi_0|_{\theta=\overline{\theta}=0} = \frac{f}{2\sqrt{2}} (x_5 - ix_6) + \varphi \, , \quad \varphi = \tfrac{1}{\sqrt{2}} (a_6 + ia_5) \, ,$$

$$Q(x_M, \theta, \bar{\theta}) = \sum_{n,j} Q_{n,j}(x_\mu, \theta, \bar{\theta}) \, \psi_{n,j}(x_m) \, , \quad \tilde{Q}(x_M, \theta, \bar{\theta}) = \ldots$$

Bachas '95: Landau levels

→ effective 4d action, compute Wilson line potential



simplest case: no gauge background, bosonic and fermionic zero modes with small mass terms:

$$\mathcal{L}_m = -m_b^2(|\tilde{Q}|^2 + |Q|^2) - (m_f \tilde{\chi}\chi + c.c.)$$

one-loop quantum corrections with cutoff Λ:

$$\delta m_\varphi^2 = \frac{q^2}{4\pi^2}\left(\Lambda^2 - 2m_b^2 \ln \Lambda + \ldots\right)$$

$$\delta m_\varphi^2 = -\frac{q^2}{4\pi^2}\left(\Lambda^2 - 4m_f^2 \ln \Lambda + \ldots\right)$$

remaining logarithmic divergence for softly broken SUSY:

$$\delta m_\varphi^2 = -\frac{q^2}{2\pi^2}\left(m_b^2 - 2m_f^2\right) \ln \Lambda + \ldots$$

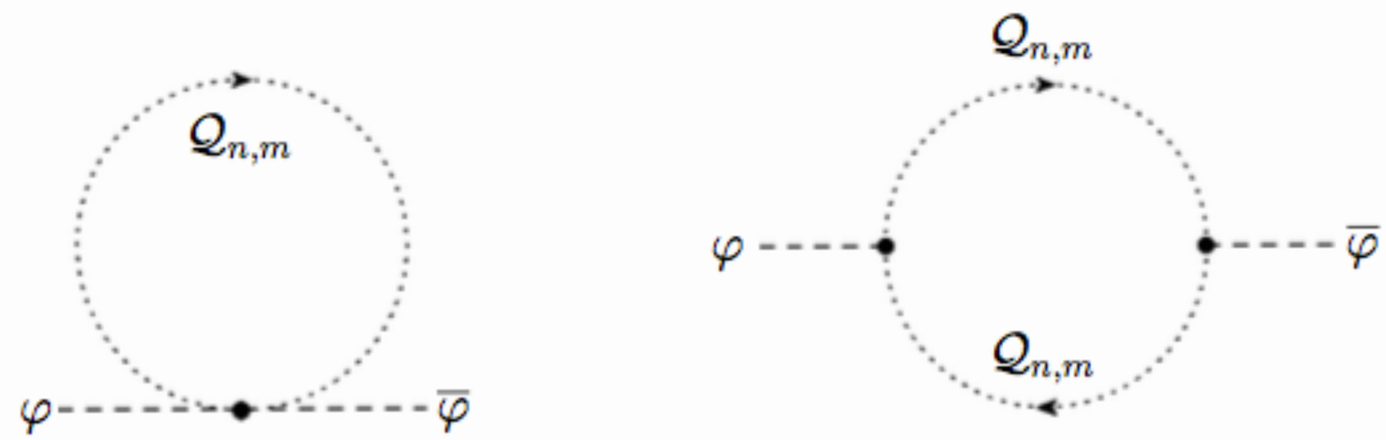


Figure 1: Bosonic contributions to the Wilson line mass without flux.

$$\begin{aligned}
\delta m_b^2 &= 2g^2q^2 \sum_{n,m} \int_0^\infty dt\, t\, e^{-|M_{n,m}|^2 t} \int \frac{d^4k}{(2\pi)^4} k^2 e^{-k^2 t} \\
&= \frac{g^2q^2L^2}{16\pi^3} \int_0^\infty \frac{dt}{t^3}\ \Theta_3\left(0; \frac{iL^2}{4\pi t}\right)^2 \\
&= \frac{g^2q^2L^2}{16\pi^3} \int_0^\infty du\, u\, \Theta_3\left(0; \frac{iL^2 u}{4\pi}\right)^2 \\
&= \frac{g^2q^2}{\pi^3L^2} \sum_{r,s} \frac{1}{(r^2+s^2)^2} \ \sim\ \frac{g^2q^2}{\pi^3L^2}\left(\frac{1}{0^4}+1\right)
\end{aligned}$$

Antoniadis, Benakli, Quiros '01, ...

mass correction finite, after subtraction of divergent constant (remnant of 6d gauge symmetry, invariance w.r.t. large gauge transformations)

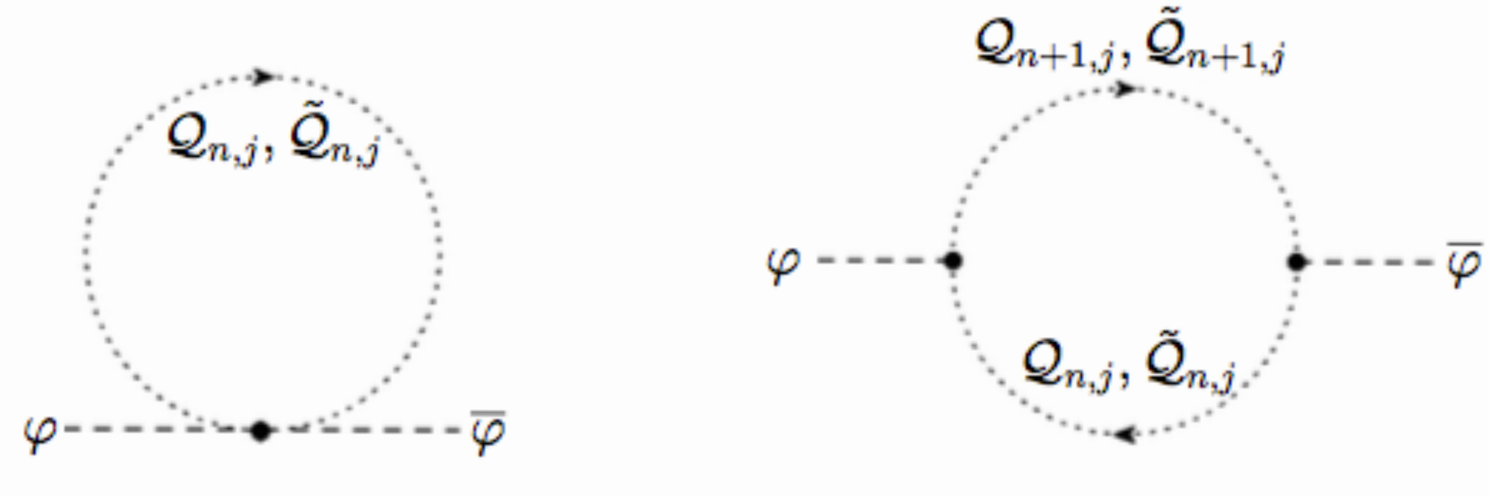


Figure 3: Bosonic contributions to the Wilson line mass with flux.

$$
\begin{aligned}
\delta m_b^2 &= -4q^2 g^2 |N| \sum_n \int \frac{d^4k}{(2\pi)^4} \left(\frac{n}{k^2 + \alpha(n+\frac{1}{2})} - \frac{n+1}{k^2 + \alpha(n+\frac{3}{2})} \right) \\
&= -\frac{q^2 g^2}{4\pi^2} |N| \sum_n \int_0^\infty dt\, \frac{1}{t^2} \left(n e^{-\alpha(n+\frac{1}{2})t} - (n+1) e^{-\alpha(n+\frac{3}{2})t} \right) \\
&= -\frac{q^2 g^2}{4\pi^2} |N| \int_0^\infty dt\, \frac{1}{t^2} \left(\frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t}-1)^2} - \frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t}-1)^2} \right) \;=\; 0
\end{aligned}
$$

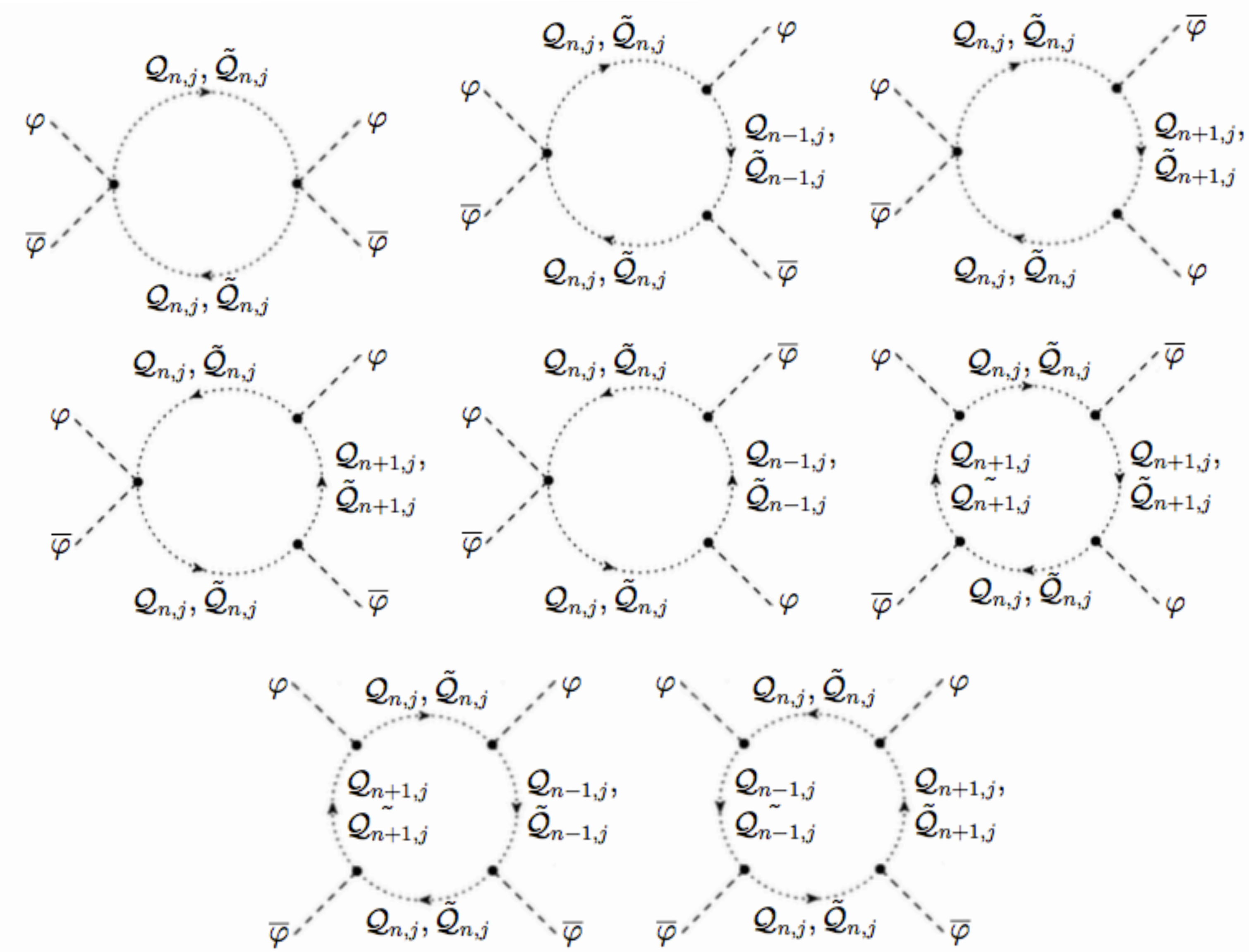


Figure 6: Bosonic contributions to the Wilson line quartic coupling with flux.

$$\delta\lambda_f = -\frac{q^4 g^4}{2\pi^2}|N| \int_0^\infty dt \sum_n \Big[\frac{1}{t}\Big(-n^2 e^{-\alpha n t} + (n+1)^2 e^{-\alpha(n+1)t}\Big) + \frac{1}{\alpha t^2}\Big(n(n+1)e^{-\alpha n t} - (n+1)(n+2)e^{-\alpha(n+1)t} - n(n+1)e^{-\alpha(n+1)t} + (n+1)(n+2)e^{-\alpha(n+2)t}\Big)\Big] = 0$$

Wilson lines as Goldstone bosons

Action of charged matter field invariant w.r.t. translations,

$$S_6 = \int d^6x \left(-D_M\bar{Q}D^M Q\right) , \quad D_M Q = (\partial_M + iqgA_M)Q$$

$$\delta Q = \epsilon^m \partial_m Q \, , \quad \delta A_n = \epsilon^m \partial_m A_n$$

Symmetries for constant Wilson line background $\varphi = \frac{1}{\sqrt{2}}\left(a_6 + ia_5\right)$,

$$\delta Q = \epsilon^m \partial_m Q \, , \quad \delta a_n = 0$$

Flux background breaks translational symmetries spontaneously,

$$D_m Q = \left(\partial_m + iqg\left(a_m + \frac{f}{2}\epsilon_{mn}x_n\right)\right) Q \, , \quad \langle A_m \rangle = \frac{f}{2}\epsilon_{mn}x_n$$

Translational symmetries now nonlinearly realized with Wilson lines as Goldstone bosons,

$$\delta Q = \epsilon^m \partial_m Q \, , \quad \delta a_n = \epsilon^m \frac{f}{2}\epsilon_{nm}$$

[Veneziano: IUVC ?] More realistic case under investigation ...

6d SO(10) F-theory vacua

WB, Dierigl, Oehlmann, Ruehle 1709.xxxxx

Start from $K3$ manifold, torus (in Weierstrass form) fibered over base $\mathbb{P}^1$,

$$F = -y^2 + x^3 + fx + g = 0$$

with dependence on base coordinates $z_0,\ d_i(z_0, z_1)$,

$$\begin{aligned} f =& z_0^2 \left(-\frac{1}{3} d_5^2 d_7^2 + z_0 R_1 + \mathcal{O}(z_0^2) \right) , \\ g =& z_0^3 \left(-\frac{2}{27} d_5^3 d_7^3 + z_0 R_2 + \mathcal{O}(z_0^2) \right) \end{aligned}$$

torus is singular at point (x, y) if discriminant vanishes,

$$F = \frac{\partial F}{\partial x} = \frac{\partial F}{\partial y} = 0 ,$$

$$\Delta = 4f^3 + 27g^2 = 0$$

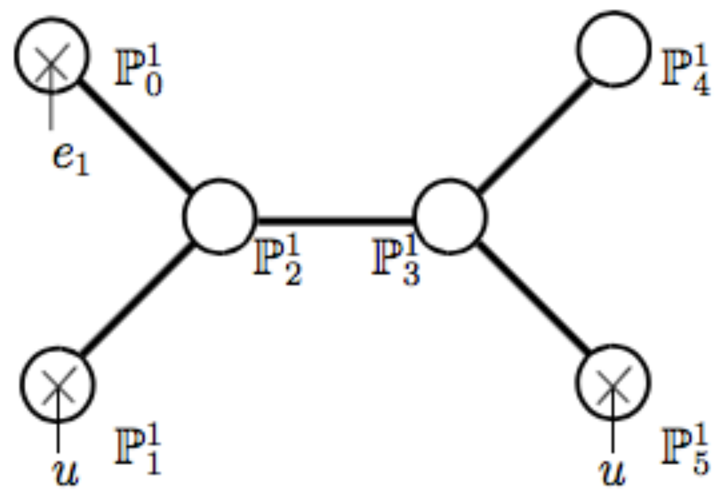
Kodaira classification: order of singularity determines non-Abelian gauge group, $\mathrm{Ord}(f, g, \Delta) = (2,3,7)$ yields $SO(10)$:

$$\Delta = z_0^7 (P + z_0 R + \mathcal{O}(z_0^2)) \,,$$
$$P = -d_5^3 d_7^3 (d_3 d_5 - d_1 d_7)^2 d_9^2$$

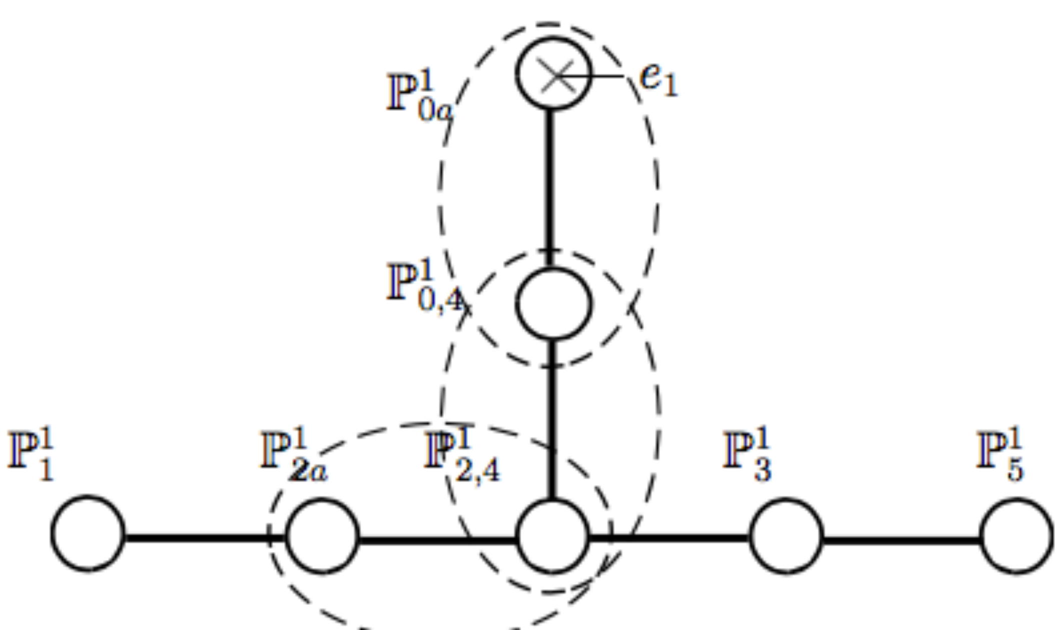
Vanishing of P at some points of basis leads to stronger singularities, and therefore larger symmetries, at these points:

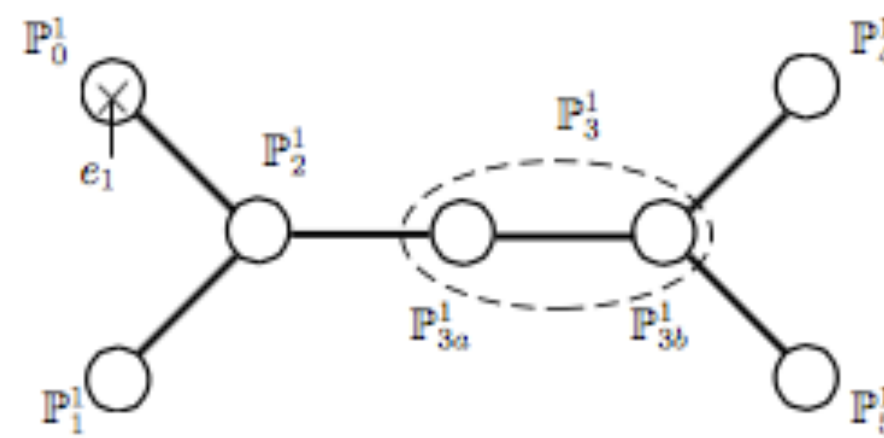
locus	$\mathrm{Ord}(f, g, \Delta)$	fiber singularity
$z_0 = 0$	$(2,3,7)$	$SO(10)$
$z_0 = d_9 = 0$	$(2,3,8)$	$SO(12)$
$z_0 = d_5 = 0$	$(3,4,8)$	E_6
$z_0 = d_7 = 0$	$(3,4,8)$	E_6
$z_0 = d_3 d_5 - d_1 d_7 = 0$	$(2,3,8)$	$SO(12)$

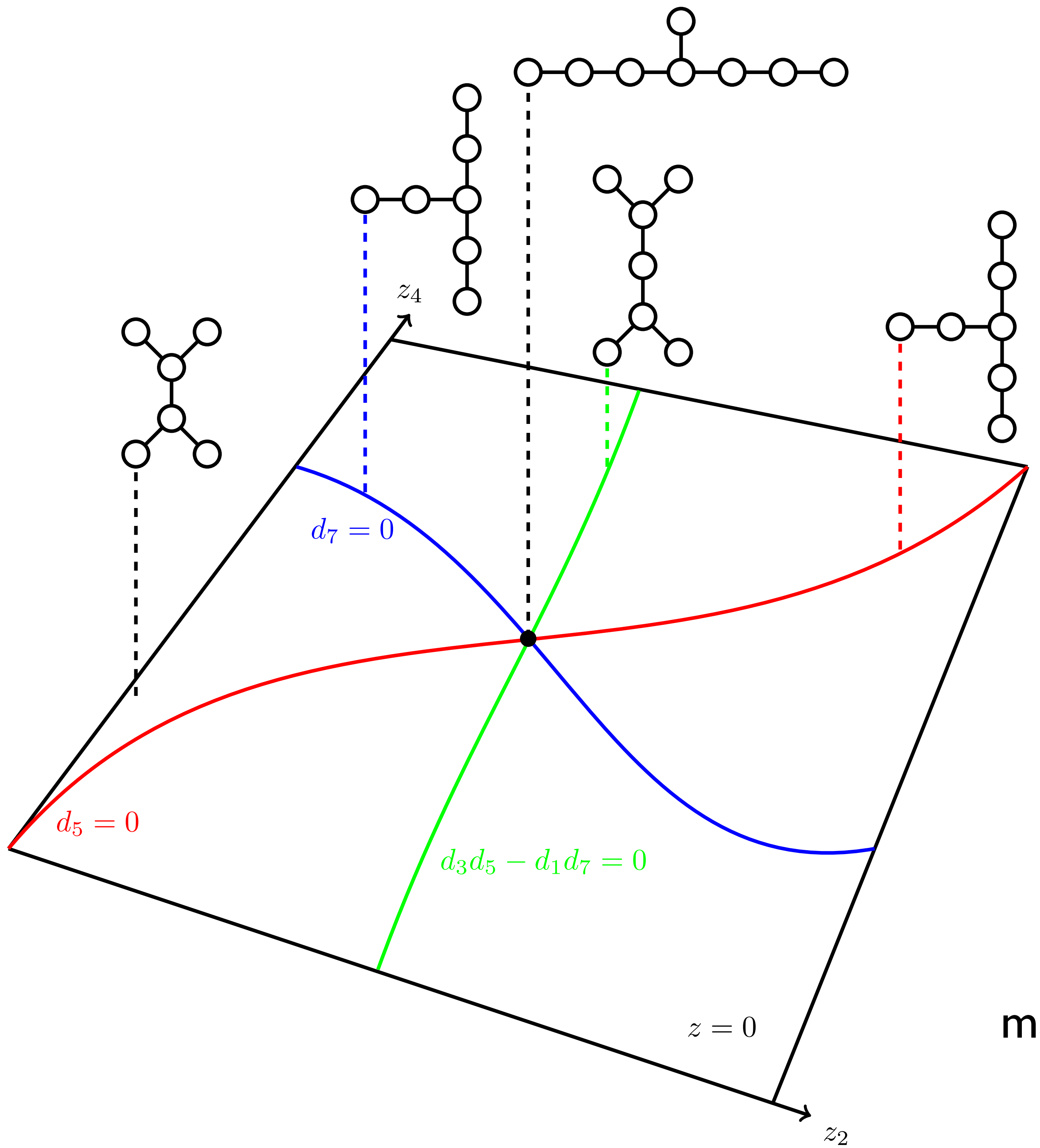
Intersection pattern at resolved $SO(10)$ singularity:



Global GUT model building, starting from toric geometry [Morrison, Taylor, Schafer-Nameki, Weigand, Grimm, Palti, Cvetic, Klevers, Ruehle, Oehlmann, ... '12 ...]; at enhanced symmetry points `coset matter' is generated, i.e. 16's and 10's:







matter curves and Yukawa coupling

Result: classification of all 6d SO(10) supergravity models in terms of elliptically fibered (toric) CY threefolds (full geometric determination of all model properties: gauge symmetry, matter representations, anomalies, ...)

Conclusions

- Supersymmetric extensions of Standard Model strongly motivated, but what is the scale of SUSY breaking?
- Higher-dimensional GUT models with flux lead to GUT scale for SUSY breaking; emerging low energy spectrum reminiscent of `spread' SUSY (THDM + higgsino + ...)
- Flux and F-term breaking allows for moduli stabilization
- Effect of flux on quantum corrections? Fine-tuning of electroweak scale?
- Embedding of 6d SUGRA with SO(10)xU(1) symmetry into F-theory possible

Backup Slides

representation	locus	multiplicity
$\mathbf{10}_{3/2}$	$z = d_9 = 0$	2
$\mathbf{16}_{3/4}$	$z = d_5 = 0$	0
$\mathbf{16}_{-1/4}$	$z = d_7 = 0$	4
$\mathbf{10}_{-1/2}$	$z = d_3 d_5 - d_1 d_7 = 0$	4
$\mathbf{45}$	$z = 0$	0
$\mathbf{1}_3$	$d_8 = d_9 = 0$	2
$\mathbf{1}_2$	$V(2)$	36
$\mathbf{1}_1$	$V(3)$	76
$\mathbf{1}_0$	moduli	$51 + 1$
T	tensor	1