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Spontaneous versus explicit breaking of scale invariance

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Outline

- Why scale invariance ?
- Conformal anomaly
- Minimal models
- Exact quantum scale invariance
- Naturalness and quantum scale invariance
- The minimal model - scale invariant νMSM
- Conclusions

Why scale invariance?

If the mass of the Higgs boson is put to zero in the SM, the Lagrangian has a wider symmetry: it is scale and conformally invariant:

Dilatations - global scale transformations ($\sigma = const$)

$$\Psi(x) \rightarrow \sigma^n \Psi(\sigma x) \; ,$$

$n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

It is tempting to use this symmetry for solution of the hierarchy problem Bardeen '95: why the Higgs mass is so small in comparison with the Planck scale?

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Bardeen '95: why the Higgs mass is so small in comparison with the Planck scale?

But what about quantum corrections?

Conformal anomaly

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Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a} \ ,$$

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No go theorem?

Conformal anomaly

Softer version, Helmboldt et al, logarithmic: “ The conceptual difficulty in the conformal model building is the nature of the symmetry, which is sometimes misleadingly called classical scale invariance. This symmetry is anomalous, since generically the renormalization-group running of the parameters leads to a non-vanishing trace of the energy-momentum tensor, which enters the divergence of the scale current. ...

...The anomalous Ward identity thus allows only logarithmic dependence of physical quantities on the renormalization scale. Any quadratically divergent contributions to the Higgs mass must therefore be purely technical and are typically introduced by explicitly breaking the conformal invariance by regulators. ”

Radiative symmetry breaking

Lagrangian is invariant at the classical level, and scale symmetry is broken by quantum corrections (conformal anomaly) a'la Coleman-Weinberg:

Linde '76; Weinberg '76; Buchmuller, Dragon '88; Hempfling '96; Meissner, Nicolai '06; Foot et al '07, '11; Iso, et al '09; Boyle et al '11; Salvio, Strumia '14; Manfred and collaborators, '14, '15, '17

Does not work for the SM - top quark is too heavy. Enlarging SM is necessary. Extra gauge bosons? Extra fermions? Extra scalars?

Minimal extension

Helmbold, Humbert, Lindner, Smirnov

The same gauge group, minimal number of extra fields. Extension of the Higgs sector : SM + 2 scalar singlets, one with non-zero vev

Interesting features:

- Self-consistent weakly coupled theory up to the Planck scale
- Relatively light PGB (few GeV), testable LHC phenomenology
- Scalar dark matter candidate, $\mathbf{300 - 370}$ GeV
- Neutrino masses can be accommodated with RH neutrinos

Exact anomaly free quantum scale invariance

Toy model

Classically scale-invariant Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi, \chi)$$

Potential (χ - “dilaton”, φ - “Higgs”):

$$V(\varphi, \chi) = \frac{\lambda}{4}\left(h^2 - \frac{\alpha}{\lambda}\chi^2\right)^2 + \beta\chi^4,$$

$\beta < 0$: vacuum is unstable

$\beta = 0$: flat direction, $h^2 = \frac{\alpha}{\lambda}\chi^2$. Choice of parameters:

$\alpha \sim \left(\frac{M_W}{M_P}\right)^2 \sim 10^{-32}$, to get the Higgs-Planck hierarchy correctly.

Standard reasoning

Dimensional regularisation $d = 4 - 2\epsilon$, $\overline{MS}$ subtraction scheme:

mass dimension of the scalar fields: $1 - \epsilon$,

mass dimension of the coupling constant: 2ϵ

Counter-terms:

$$\lambda = \mu^{2\epsilon} \left[\lambda_R + \sum_{k=1}^{\infty} \frac{a_n}{\epsilon^n} \right] ,$$

μ is a dimensionful parameter!!

One-loop effective potential along the flat direction:

$$V_1(\chi) = \frac{m_H^4(\chi)}{64\pi^2} \left[\log \frac{m_H^2(\chi)}{\mu^2} - \frac{3}{2} \right] ,$$

Result: explicit breaking of the dilatation symmetry. Dilaton acquires a nonzero mass due to radiative corrections.

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Idea: Replace $\mu^{2\epsilon}$ by combinations of fields χ and h, which have the correct mass dimension:

$$\mu^{2\epsilon} \to \chi^{\frac{2\epsilon}{1-\epsilon}} F_\epsilon(x) \ ,$$

where $x = h/\chi$. $F_\epsilon(x)$ is a function depending on the parameter ϵ with the property $F_0(x) = 1$.

Zenhäusern, M.S '08

Englert, Truffin, Gastmans, '76

Field-dependent cutoff Wetterich '88

Almost trivial statement - by construction: Quantum effective action is scale invariant in all orders of perturbation theory.

Less trivial statement Gretsch, Monin: Quantum effective action is conformally invariant in all orders of perturbation theory.

The main problem with this construction: theory is not renormalisable, one needs to add infinite number of counter-terms.

However:

- For $\alpha \ll 1$ all counter-terms are suppressed by the dimensionful parameter $\langle\chi\rangle$
- We get an effective field theory valid up to the energy scale fixed by $\langle\chi\rangle$
- Gravity is non-renormalisable anyway, and making $\langle\chi\rangle \sim M_P$ does not make a theory worse

Origin of Λ_{QCD}

Consider the high energy ($\sqrt{s} \gg v$ but $\sqrt{s} \ll \chi_0$) behaviour of scattering amplitudes on the example of Higgs-Higgs scattering (assuming, that $\zeta_R \ll 1$). In one-loop approximation

$$\Gamma_4 = \lambda_R + \frac{9\lambda_R^2}{64\pi^2}\left[\log\left(\frac{s}{\xi_\chi \chi_0^2}\right) + \mathrm{const}\right] + \mathcal{O}\left(\zeta_R^2\right) \ .$$

This implies that at $v \ll \sqrt{s} \ll \chi_0$ the effective Higgs self-coupling runs in a way prescribed by the ordinary renormalization group!

For QCD:

$$\Lambda_{QCD} = \chi_0 e^{-\frac{1}{2b_0\alpha_s}}, \quad \beta(\alpha_s) = b_0\alpha_s^2$$

Hierarchy problem

$$V(\varphi,\chi) = \frac{\lambda}{4}\left(h^2 - \frac{\alpha}{\lambda}\chi^2\right)^2 + \beta\chi^4,$$

For $\alpha = \beta = 0$ the classical Lagrangian has an extra symmetry : $\chi \to \chi + const$. Therefore, there are no large perturbative corrections to the Higgs mass: those proportional to χ contain necessarily α or β, those proportional to λ contain only logs of χ. This construction leads to “natural” hierarchy $\chi \gg h$. However, no explanation of why $\alpha \ll 1$.

Important ingredient for naturalness: almost exact shift symmetry.

Requirement of the shift symmetry $\equiv$ requirement of absence of heavy particles with sufficiently strong interaction with the Higgs field and the dilaton, e.g.

$$\lambda_h h^2 \phi^2 + \lambda_\chi \chi^2 \phi^2$$

$\lambda_h \sim \lambda_\chi \sim 1$ spoils the argument!

Conjecture: natural theory should not have heavy particles between the Fermi and Planck scales

Inclusion of gravity

Planck scale: through non-minimal coupling of the dilaton to the Ricci scalar,

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + \xi_h h^2\right) \frac{R}{2} \, ,$$

This term, for $\xi_\chi \sim 1$, does break the shift symmetry. However, this is a coefficient in front of graviton kinetic term. Since the graviton stays massless in any constant scalar background, the perturbative computations of gravitational corrections to the Higgs mass in scale-invariant regularisation are suppressed by M_P. There are no corrections proportional to M_P!

Consequences

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- Theory is “natural” in perturbative sense: Higgs mass is stable against radiative corrections
- The dilaton is massless in all orders of perturbation theory
- Since it is a Goldstone boson of spontaneously broken symmetry it has only derivative couplings to matter (inclusion of gravity is essential!)
- Fifth force or Brans-Dicke constraints are not applicable to it

Problems

- What happens beyond perturbation theory?
- What leads to selection of parameter $\beta = 0$ $\equiv$ existence of flat direction $\equiv$ absence of the cosmological constant ?
- Unitarity and high-energy behaviour: What is the high-energy behaviour $(E > M_{Pl})$ of the scattering amplitudes? Is the theory unitary? Can it have a scale-invariant UV completion?

The minimal model - scale invariant νMSM

Particle content

Particles of the SM

+

graviton

+

dilaton

+

3 Majorana leptons

Scale-invariant Lagrangian

$$\mathcal{L}_{\nu\mathrm{MSM}} = \mathcal{L}_{\mathrm{SM}[\mathrm{M}\to 0]} + \mathcal{L}_G + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi,\chi)$$
$$+ \left(\bar{N}_I i\gamma^\mu \partial_\mu N_I - h_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\varphi} - f_I \bar{N}_I{}^c N_I \chi + \mathrm{h.c.}\right) \ ,$$

Potential (χ - dilaton, φ - Higgs, $\varphi^\dagger\varphi = 2h^2$):

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4,$$

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger\varphi\right)\frac{R}{2}\ ,$$

Roles of different particles

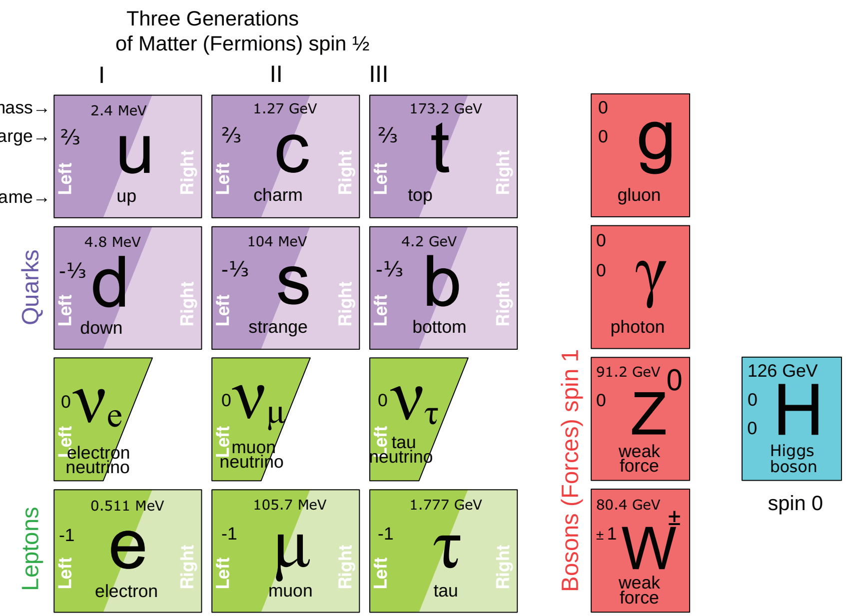
The roles of dilaton:

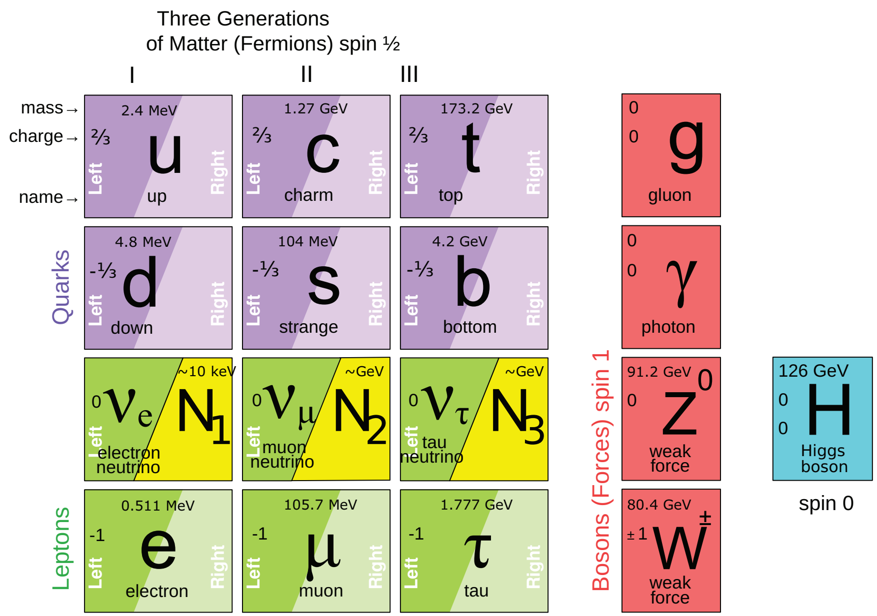
- determine the Planck mass
- give mass to the Higgs
- give masses to 3 Majorana leptons
- may lead to dynamical dark energy

Roles of the Higgs boson:

- give masses to fermions and vector bosons of the SM
- provide inflation

New physics below the Fermi scale: the νMSM





Role of N_1 with mass in keV region: dark matter. Search - with the use of X-ray telescopes. Already found? Bulbul et al., Boyarsky et al

Role of $N_2, \ N_3$ with mass in 100 MeV – GeV region: “give” masses to neutrinos and produce baryon asymmetry of the Universe. Search - intensity and precision frontier, SHiP at CERN.

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  - Unique source for all mass scales.
  - Higgs mass is stable against radiative corrections (scale symmetry + approximate shift symmetry $\chi \to \chi + const$)
  - The massless sector of the theory contains dilaton, which has only derivative couplings to matter
  - All observational drawbacks of the SM can be solved by new physics below the Fermi scale

Problems to solve

- Though the stability of the electroweak scale against quantum corrections may be achieved, it is unclear *why* the electroweak scale is so much smaller than the Planck scale (or why $\alpha \lll 1$).

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- Though the stability of the electroweak scale against quantum corrections may be achieved, it is unclear *why* the electroweak scale is so much smaller than the Planck scale (or why $\alpha \lll 1$).
- Why eventual cosmological constant is zero (or why $\beta = 0$)?
- High energy limit