Pion-pion scattering resonances and the timelike pion form factor from $N_f=2+1$ lattice QCD

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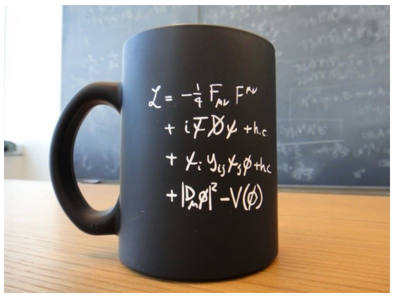
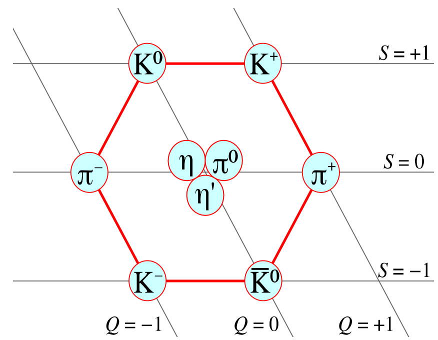
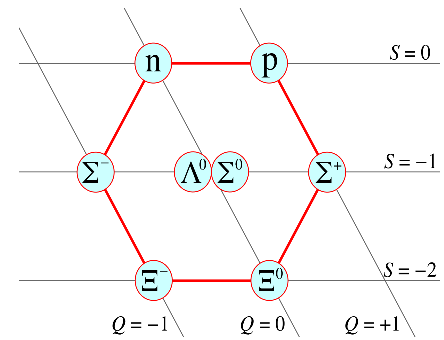
CP3-Origins

Lattice QCD at the physical pion mass
DESY Zeuthen
Apr. 11th, 2017

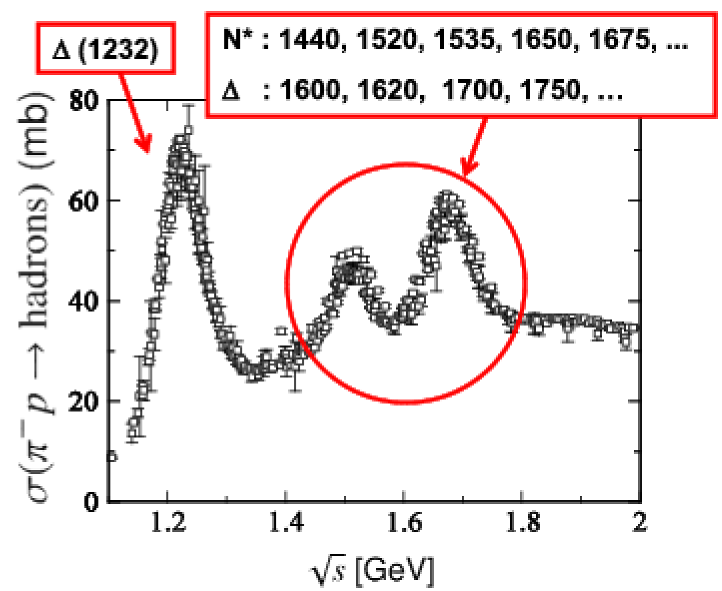
QCD Phenomenology

-  → hundreds of (known) hadrons!
- QCD-stable hadrons:





- Unstable hadrons from scattering amplitudes:



Excited Baryon Analysis Center (EBAC), JLab

Scattering amplitudes in lattice QCD

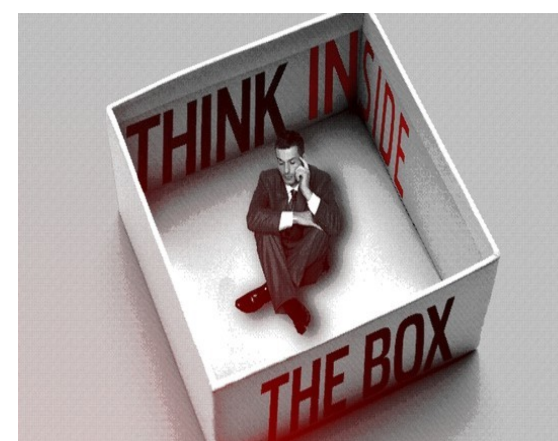
- In imaginary time, $\langle 0|T[\hat{\mathcal{O}}'(x')\hat{\mathcal{O}}^\dagger(x)]|0\rangle$ generally contains no info about on-shell amplitudes. L. Maiani, M. Testa, *Phys. Lett.* **B245** (1990) 585

- However, below $n \geq 3$ hadron thresholds (identical particles),

$$E_{\rm cm} = \sqrt{E_{\rm lat}^2 - \boldsymbol{P}^2}, \qquad \boldsymbol{q}_{\rm cm}^2 = \frac{1}{4}E_{\rm cm}^2 - m^2$$

$$\det[1 + F(q_{\rm cm}L)\{S(q_{\rm cm}) - 1\}] + {\rm O}({\rm e}^{-ML}) = 0$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531



- Determinant over partial wave and channel indices
  - S diagonal in partial waves, may couple channels
  - F couples partial waves, but not channels

Energies on the lattice

- Observables: temporal correlation functions

$$C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle = \langle 0|\hat{\mathcal{O}}_i\, \mathrm{e}^{-\hat{H}t}\, \hat{\mathcal{O}}_j^\dagger|0\rangle$$

$$= \sum_n \langle 0|\hat{\mathcal{O}}_i|n\rangle\langle n|\hat{\mathcal{O}}_j^\dagger|0\rangle\, \mathrm{e}^{-E_n t}$$

- Single- and multi-hadron operators composed of:
  - Mesons: $$\mathcal{O}_M(\vec{p},t) = \sum_{\vec{x}} \mathrm{e}^{i\vec{p}\cdot\vec{x}}\, \bar{\psi}(\vec{x},t)\,\Gamma\,\psi(\vec{x},t)$$
  - Baryons:

$$\mathcal{O}_B(\vec{p},t) = \sum_{\vec{x}} \mathrm{e}^{i\vec{p}\cdot\vec{x}}\, \Gamma_{\alpha\beta\gamma}\,\epsilon_{abc}\,\psi_{\alpha a}(\vec{x},t)\psi_{\beta b}(\vec{x},t)\psi_{\gamma c}(\vec{x},t)$$

Theoretical advances

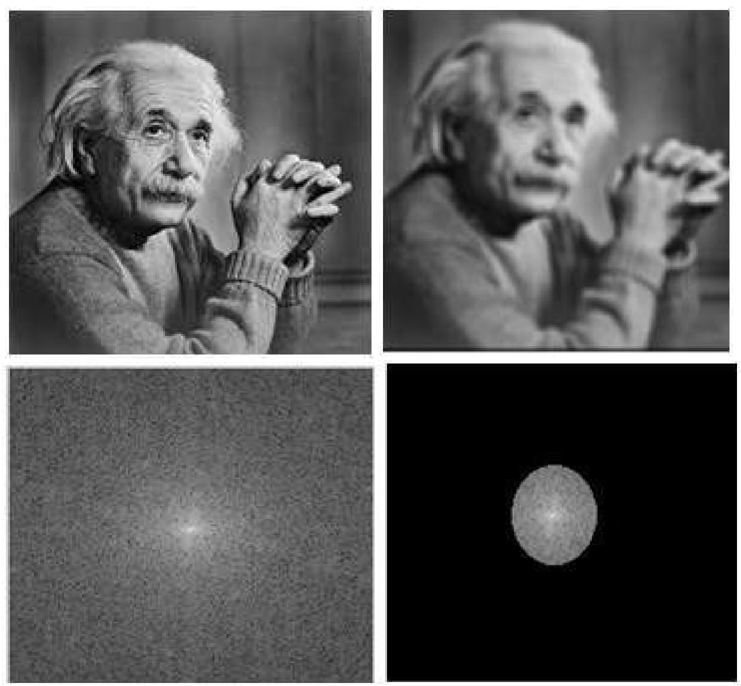
View M_{xy}^{-1} as an 'image', apply low-pass filter: distillation

M. Peardon, JB, J. Foley, C. Morningstar, J. Dudek, R. Edwards, B. Joo, H.-W. Lin, D. Richards, K. Juge, *Phys. Rev.* **D80** (2009) 054506

original | filtered

config. space

Fourier space



Filter/quark smearing: low-modes of gauge-covariant 3-D Laplacian

$$\tilde{\Delta}[U]v_n = \lambda_n v_n$$

Stochastic LapH: stochastic estimation gives improved scaling with volume.

C. Morningstar, JB, J. Foley, K. Juge, D. Lenkner, M. Peardon, C. H. Wong, *Phys. Rev.* **D83** (2011) 114505

First Test/Results

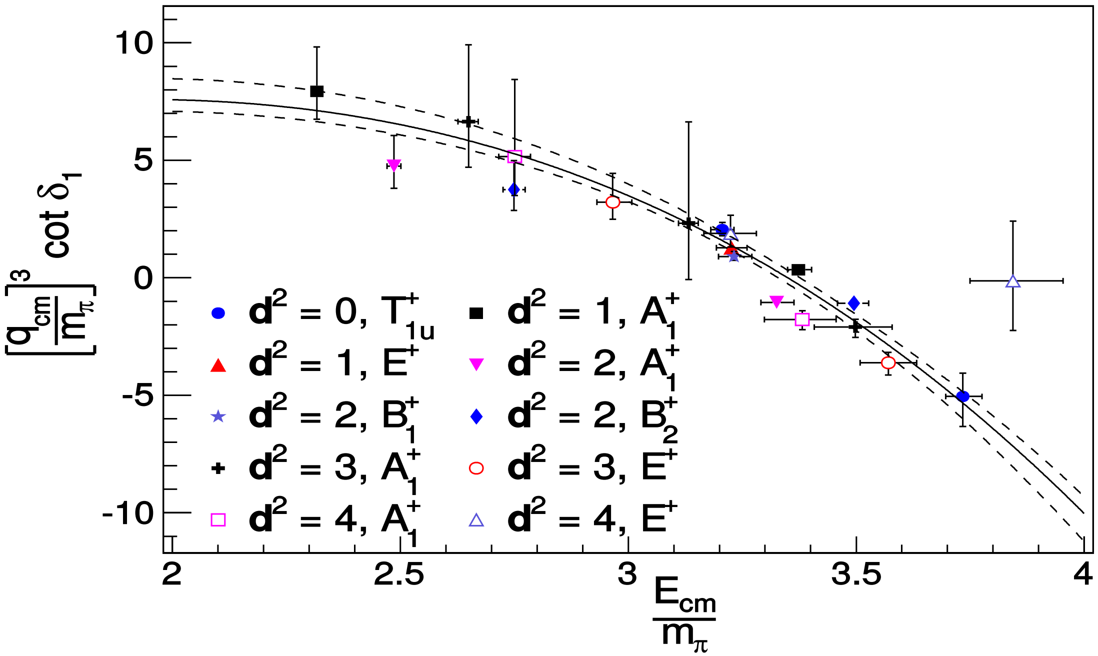
- Anisotropic Wilson-clover lattice:
  - Dynamical light and strange quarks: $N_\mathrm{f} = 2 + 1$
  - Large volume, fine temporal resolution: $a_s/a_t \approx 3.5$

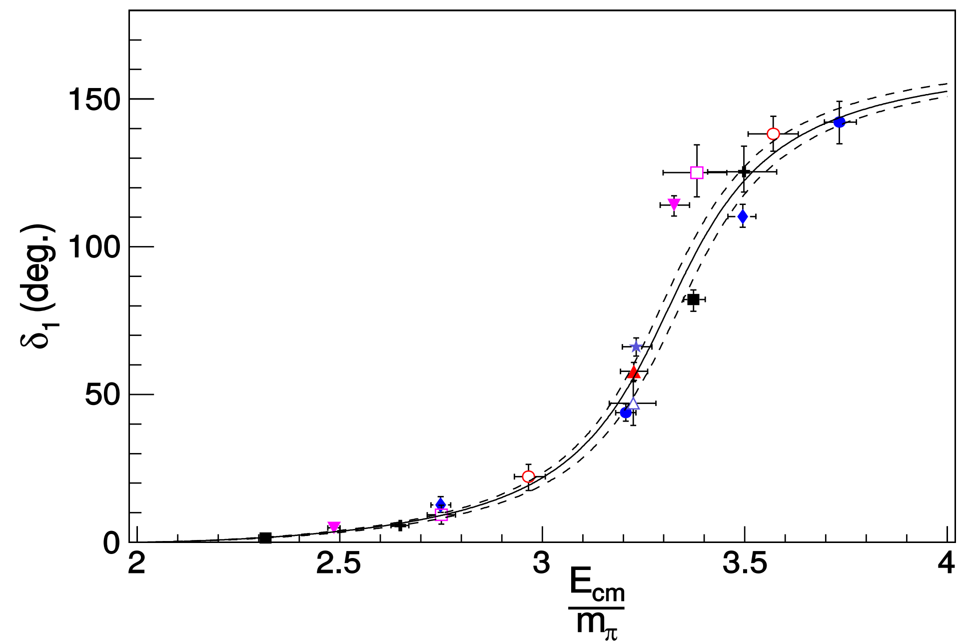
    $$32^3 \times 256,\ m_\pi \approx 240\mathrm{MeV},\ a_s \approx 0.12\mathrm{fm},\ L \approx 4\mathrm{fm}$$
  - Safe from 'thermal effects': $m_\pi T \approx 10$

H.-W. Lin, S. Cohen, J. Dudek, R. Edwards, B. Joo, D. Richards, JB, J. Foley, C. Morningstar, E. Engelson, S. Wallace, K. J. Juge, N. Mathur, M. Peardon, S. Ryan, *Phys. Rev.* **D79** (2009) 034502

- Elastic pion-pion scattering:
  - Total Isopin: $I = 0, 1, 2$
  - Resonances: $\rho(770), f_0(500), \ldots$

Isovector p-wave results





JB, B. Fahy, B. Hörz, K. J. Juge, C. Morningstar, C. H. Wong, *Nucl. Phys.* **B910** (2016) 114513

Breit-Wigner fit: $$q_{\mathrm{cm}}^3 \cot\ \delta_1 = (m_\rho^2 - s)\frac{6\pi\sqrt{s}}{g_{\rho\pi\pi}^2}$$

$$\frac{m_\rho}{m_\pi} = 3.350(24), \qquad g_{\rho\pi\pi} = 5.99(26), \qquad \frac{\chi^2}{d.o.f} = 1.04$$

Chiral limit, discretization errors

>cls

M. Bruno, D. Djukanovic, G. Engel, A. Francis, G. Herdoiza, H. Horch, P. Korcyl, T. Korzec, M. Papinutto, S. Schaefer, E. Scholz, J. Simeth, H. Simma, W. Söldner, JHEP **1502** (2015) 043
G. Bali, E. Scholz, J. Simeth, W. Söldner, *Phys. Rev.* **D94** (2016), 074501

- Coordinated Lattice Simulations (CLS): a broad EU community effort

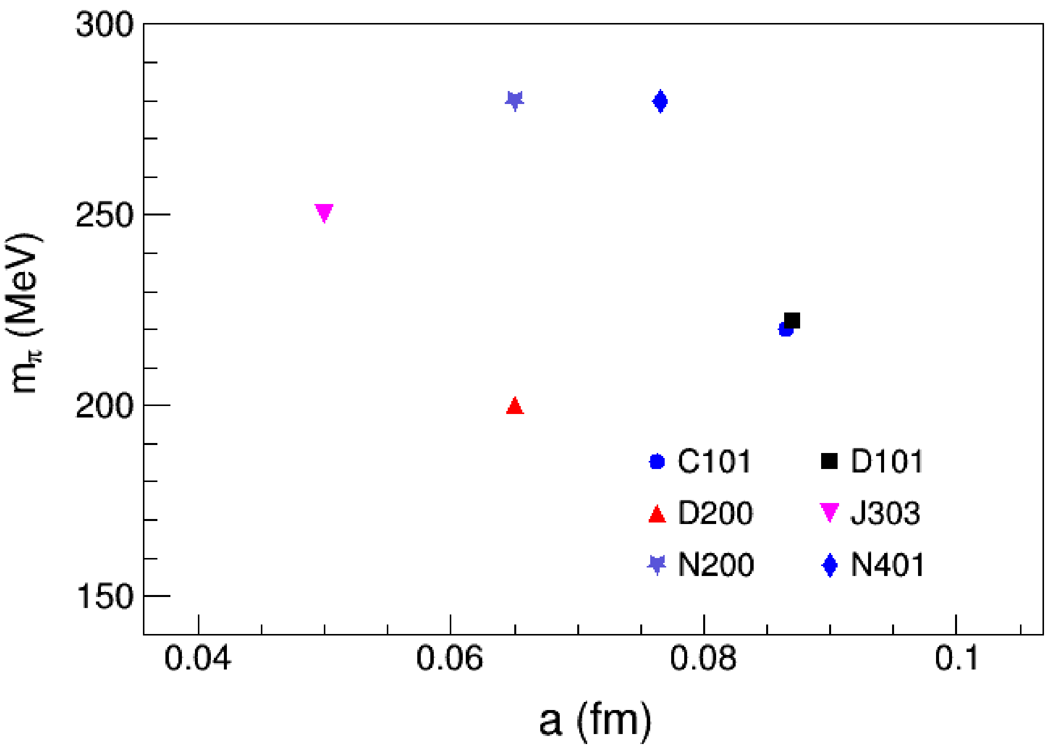
- ~20-30 researchers at ~5-10 institutions across the EU

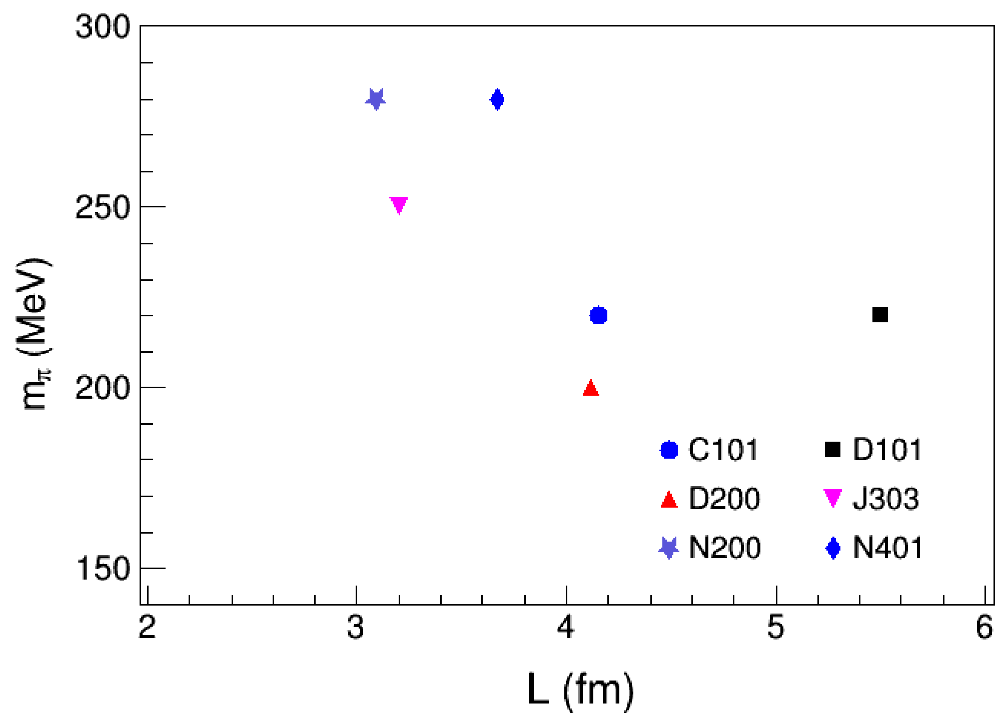
- >200M core-hr on PRACE/EU supercomputers

- Nearly completed, will be made available to the community.

CLS ensembles

- Isotropic: simpler operator renormalization
- 5 lattice spacings $a \geq 0.03\mathrm{fm}$, pion masses $m_\pi \gtrsim 190\mathrm{MeV}$
- Two $N_\mathrm{f} = 2+1$ chiral limits: $m_s = const.$ $\mathrm{Tr}M = const.$





First CLS results

- Two ensembles:

$$\mathrm{Tr}M = const., \quad a = 0.064\mathrm{fm}, \quad t \in [T/4, 3T/4]$$

  - N200: $m_\pi = 280\mathrm{MeV}, \quad 48^3 \times 128$
  - D200: $m_\pi = 200\mathrm{MeV}, \quad 64^3 \times 128$

- Correlation function measurement cost = roughly half of gauge generation. Inversions ~70-80%.

Correlator Measurements

- $N_{\text{cfg}} = 852\,(\text{N200}),\ 559\,(\text{D200})$
- Dilution Scheme:

N_{ev}	line type	N_r	scheme	N_{t_0}	N_{inv}
192 (N200),	fixed	5	(TF, SF, LI8)	2	320
448 (D200)	relative	2	(TI16, SF, LI8)	-	1024

- 3-4 two-pion operators and 1 (local) rho-like operator in each irrep.

C. Morningstar, JB, B. Fahy, J. Foley, Y. C. Jhang, K. J. Juge, D. Lenkner, C. H. Wong, Phys. Rev. D88 (2013) 014511

- All possible irreps with total momenta:

$$\boldsymbol{d}^2 = 0, 1, 2, 3, 4$$

Energy determination

- Form a matrix of correlation functions:

$$C_{mn}(t) = \langle \mathcal{O}_m(t)\mathcal{O}_n^\dagger(0)\rangle$$

- Solve the GEVP once for a single $\{t_0, t_d\}$ and rotate

$$C(t_d)v_n = \lambda_n C(t_0)v_n \qquad \hat{C}_n(t) = v_n^\dagger C(t)v_n$$

C. Micheal, I. Teasdale, *Nucl. Phys.* **B215** (1983) 433

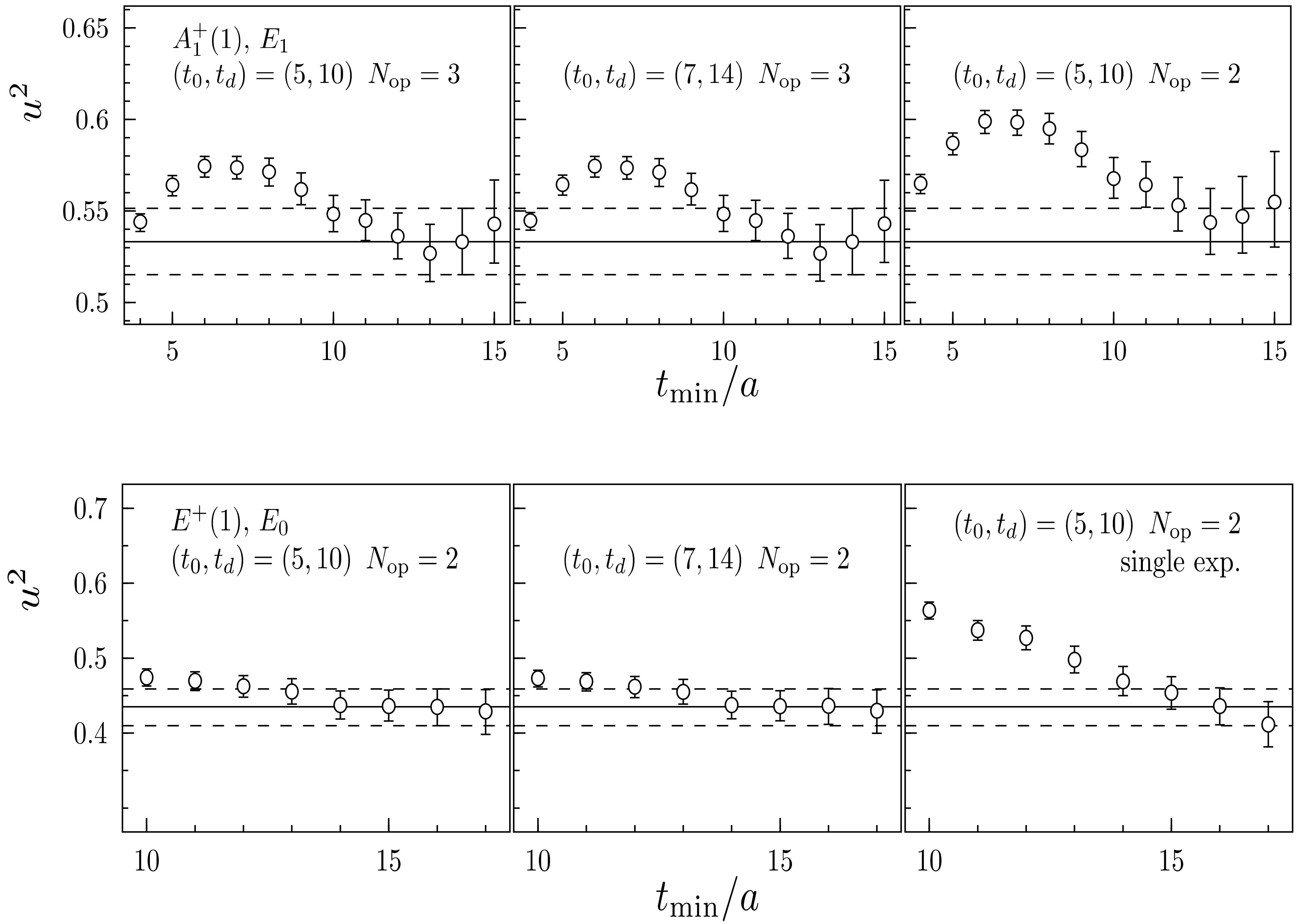
- Perform single-exponential fits to ratios and obtain energy shifts.

$$R(t) = \frac{\hat{C}_n(t)}{C_{\pi,1}(t)C_{\pi,2}(t)} \xrightarrow[t\to\infty]{} A\mathrm{e}^{-\Delta E t}$$

Energy determination

B. Hörz, Ph.D. Thesis (2016)

- Vary GEVP basis, (t_0, t_d)
- 'Bumps' in ratio fits near resonance
- Excited state effects smaller in ratio fits



Using the quantization condition

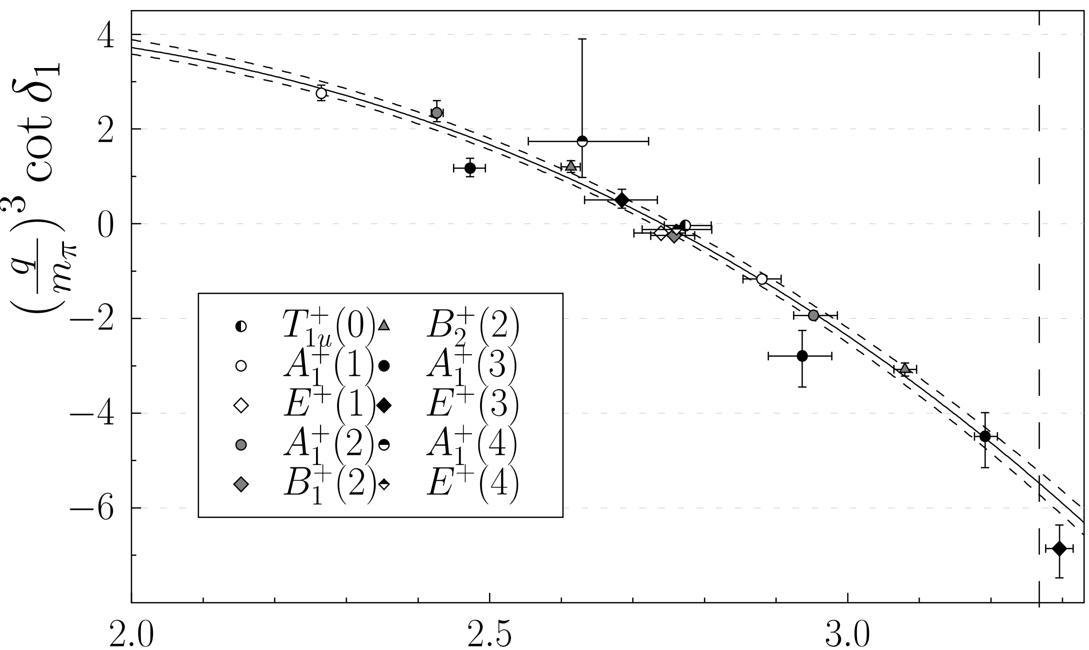
$$\mathbf{P}_{\rm tot} = \frac{2\pi}{L}\mathbf{d}, \qquad \mathbf{q}_{\rm cm} = \frac{2\pi}{L}\mathbf{u}, \qquad w_{\ell m} = \frac{8Z_{\ell m}(\mathbf{d},\gamma,\mathbf{u}^2)}{\gamma\pi^{-3/2}L^3u^{\ell-2}}$$

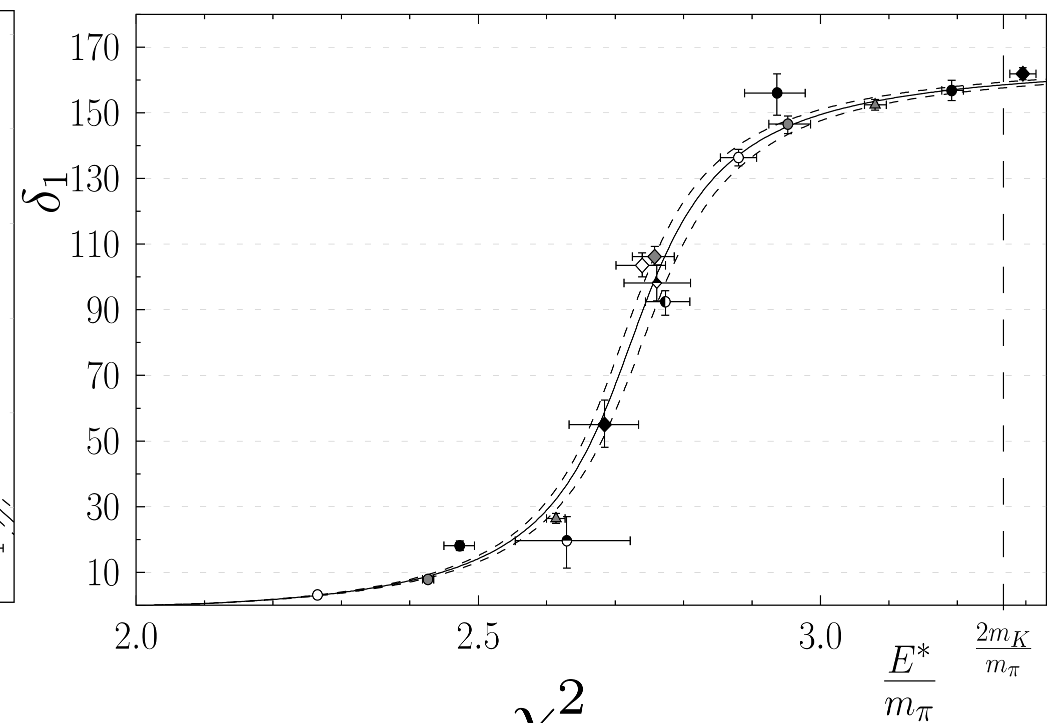
$\mathbf{d}$	Λ	$\mathbf{q}_{\rm cm}^3 \cot\,\delta_1$
$(0,0,0)$	T_{1u}^+	w_{00}
$(0,0,n)$	A_1^+	$w_{00} + \frac{2}{\sqrt{5}}w_{20}$
	E^+	$w_{00} - \frac{1}{\sqrt{5}}w_{20}$
$(0,n,\pm n)$	A_1^+	$w_{00} + \frac{1}{2\sqrt{5}}w_{20} - \sqrt{\frac{6}{5}}iw_{21} - \sqrt{\frac{3}{10}}w_{22}$
	B_1^+	$w_{00} - \frac{1}{\sqrt{5}}w_{20} + \sqrt{\frac{6}{5}}w_{22}$
	B_2^+	$w_{00} + \frac{1}{2\sqrt{5}}w_{20} + \sqrt{\frac{6}{5}}iw_{21} - \sqrt{\frac{3}{10}}w_{22}$
$(n,\pm n,\pm n)$	A_1^+	$w_{00} + 2\sqrt{\frac{6}{5}}iw_{22}$
	E^+	$w_{00} - \sqrt{\frac{6}{5}}iw_{22}$

Isovector *p*-wave results: N200

B. Hörz, Ph.D. Thesis (2016)

$(m_\pi = 280\mathrm{MeV})$





$$\frac{m_\rho}{m_\pi} = 2.741(16), \qquad g_{\rho\pi\pi} = 5.97(10), \qquad \frac{\chi^2}{d.o.f} = 0.75$$

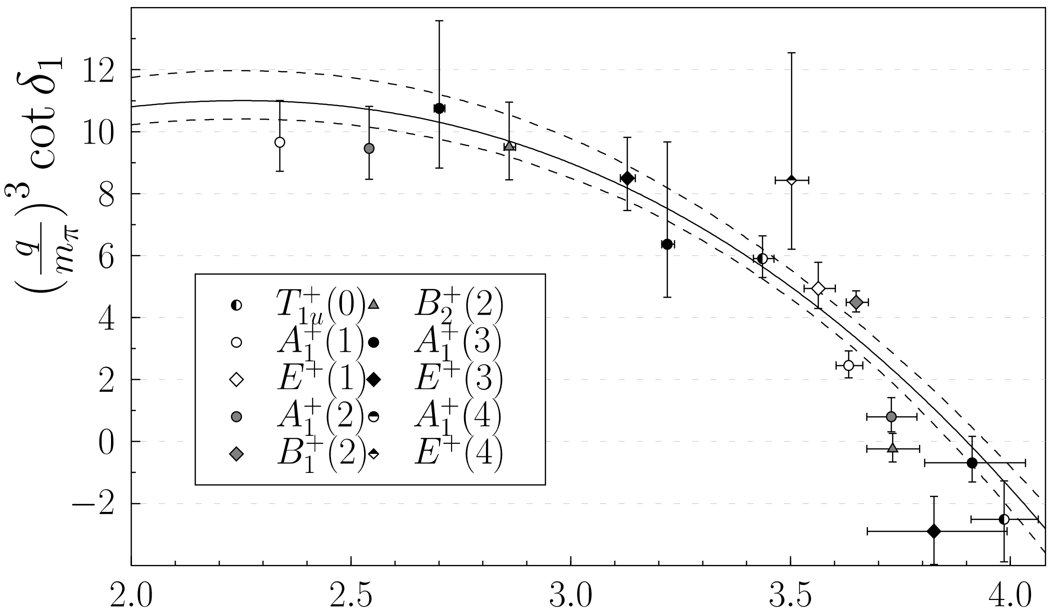
$$m_\rho = 775(4)(8)\mathrm{MeV}$$

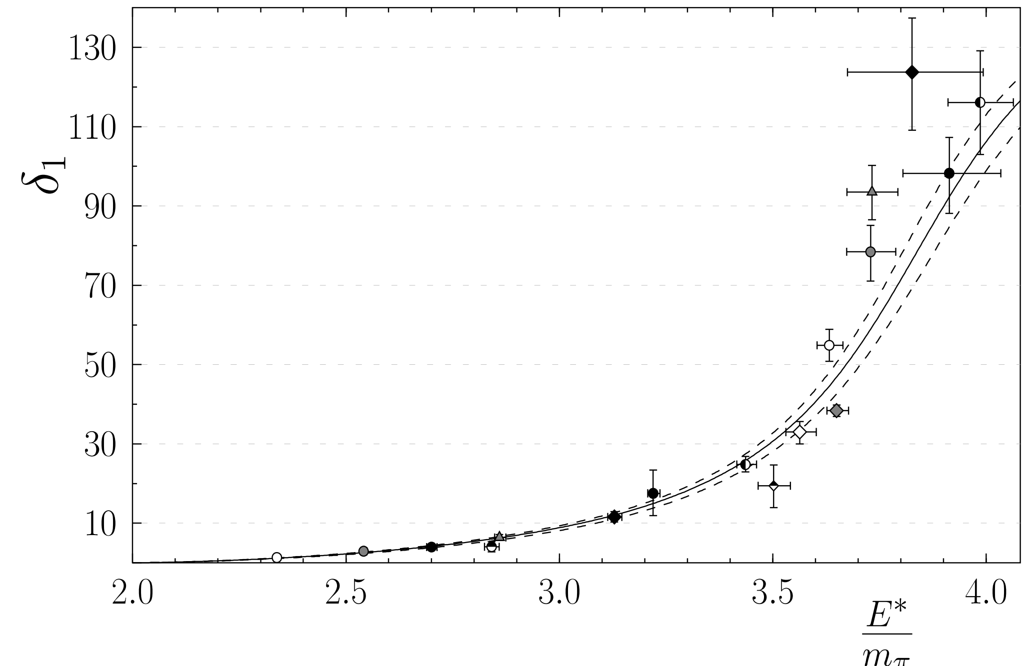
Scale determination/uncertainties from M. Bruno, T. Korzec, S. Schaefer, arXiv:1608.08900 [hep-lat]

Isovector p-wave results: D200

$(m_\pi = 200\text{MeV})$

B. Hörz, Ph.D. Thesis (2016)



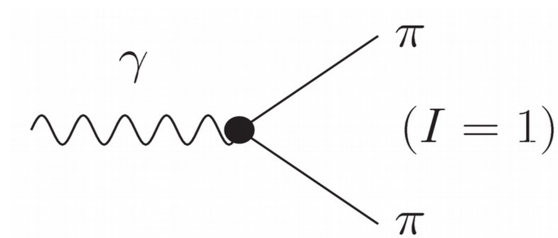


$$\frac{m_\rho}{m_\pi} = 3.901(41), \qquad g_{\rho\pi\pi} = 6.25(25), \qquad \frac{\chi^2}{d.o.f} = 1.34$$

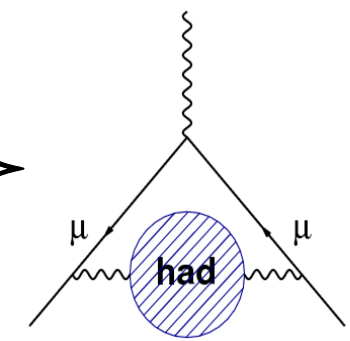
$$m_\rho = 780(8)(8)\text{MeV}$$

Timelike pion form factor

- Low-energy hadron vacuum polarization $\Pi(q^2)$: important theoretical uncertainty in $(g-2)_\mu$



$\Longrightarrow$



- Optical Theorem:

$$\mathrm{Im}\,\Pi(s) = \frac{\alpha(s)}{3} R(s)$$

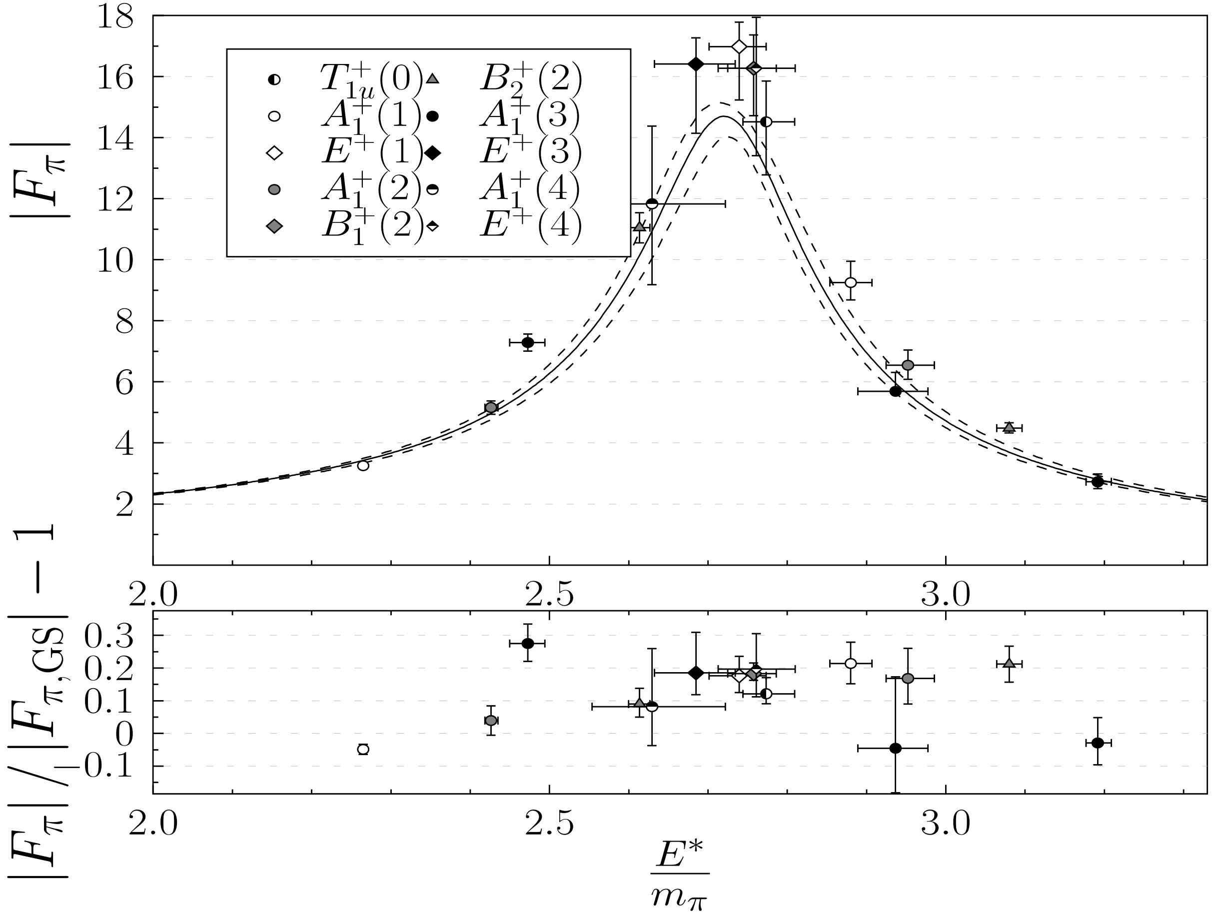
$$R(s) = \sigma_{\mathrm{tot}}(\mathrm{e}^+\mathrm{e}^- \to \text{ hadrons}) \left(\frac{4\pi\alpha(s)^2}{3s}\right)^{-1}$$

- At low energies, given by the time-like pion form-factor

$$R(s) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi(s)|^2,\ 4m_\pi^2 < s < 9m_\pi^2$$

Jegerlehner and Nyffeler '09; Meyer '11;
Feng, et al. `15

Form factor results: N200



$$(m_\pi = 280\mathrm{MeV})$$

B. Hörz, Ph.D. Thesis (2016)

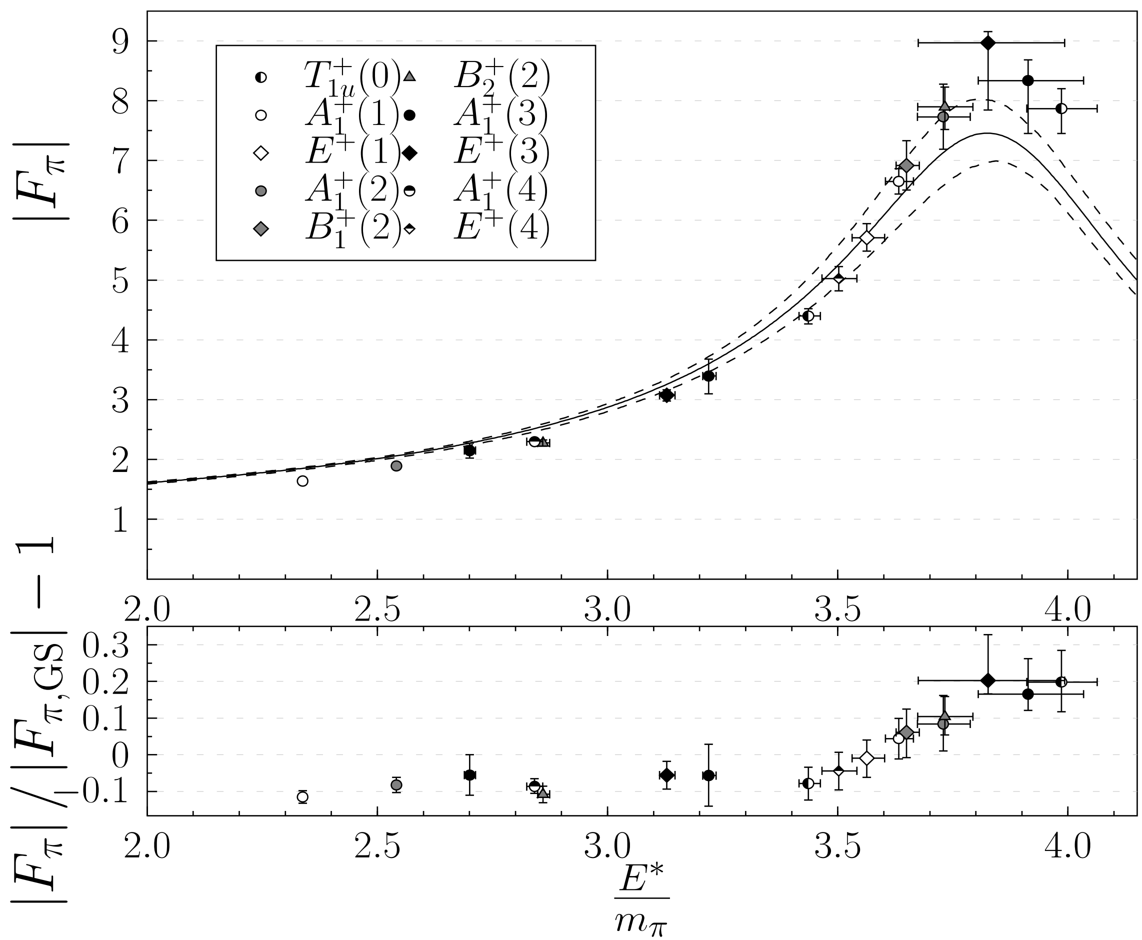
'Fit' is the Gounaris-Sakurai parametrization

$$|F_\pi(E_{\mathrm{cm}})|^2 = \frac{2\pi E_{\mathrm{cm}}}{2L^3p^5} g(\gamma) \left(q\phi'(q) + p\frac{\partial\delta_1}{\partial p} \right) |\langle 0|\hat{j}_{\mathrm{em}}|\pi(\vec{p}_1)\pi(\vec{p}_2)\rangle|^2$$

Meyer, `11 See also Feng, et al `15

Form factor results: D200

$(m_\pi = 200\mathrm{MeV})$



B. Hörz, Ph.D. Thesis (2016)

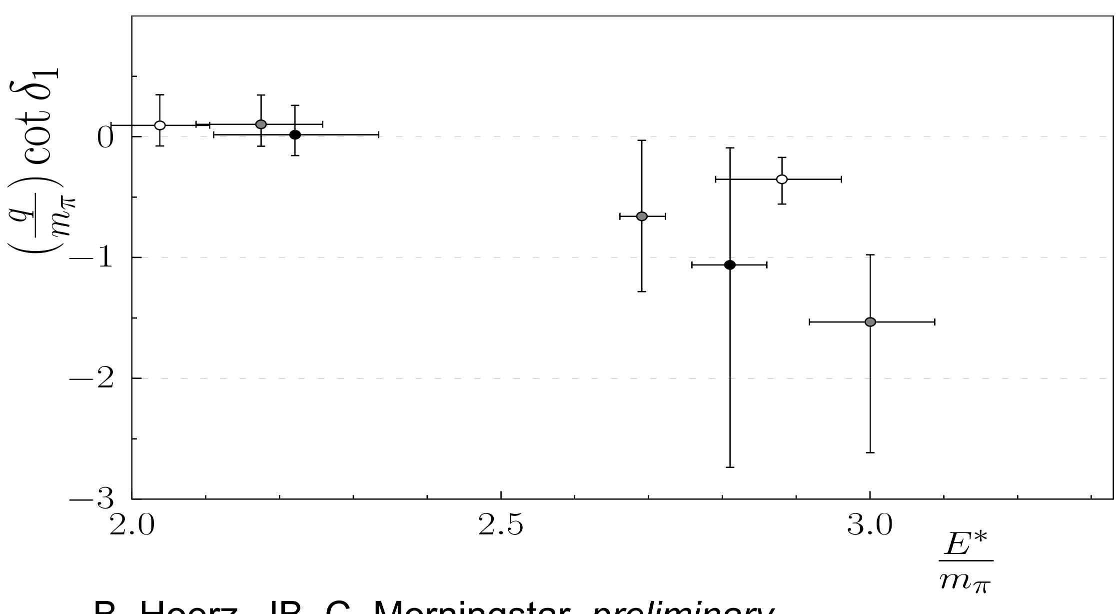
TODO:

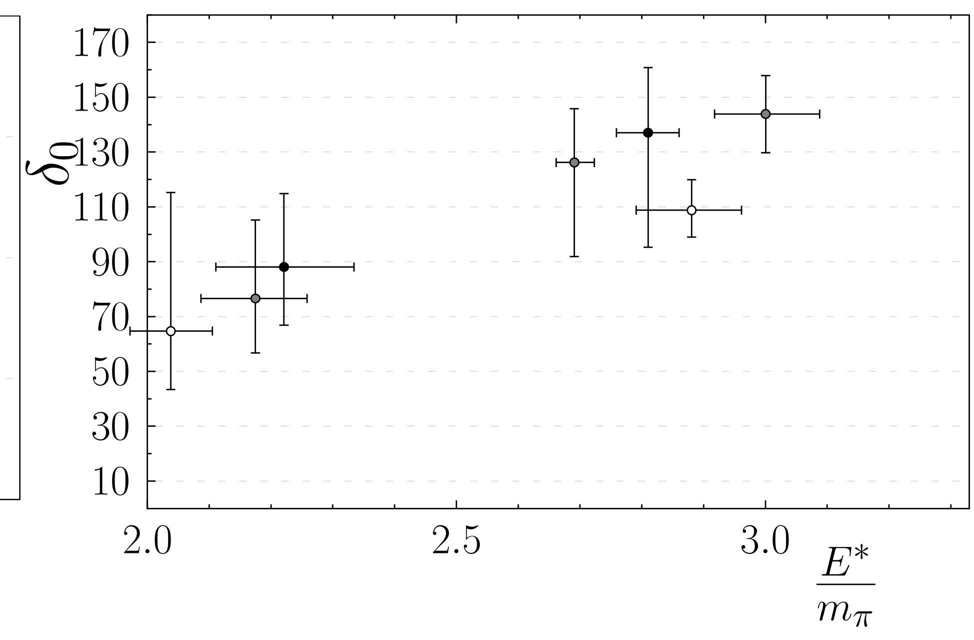
- Investigate deviation from GS form
- Combined determination of a_μ^{HVP}

Preliminary isoscalar results

- Difficult: v.e.v. subtraction, mixing with pure glue (not done yet, only non-zero total momenta)
- N200: not yet all noise combinations
- Consistent with recent ansio. calculation

R. Briceno, J. Dudek, R. G. Edwards, D. Wilson, 2016





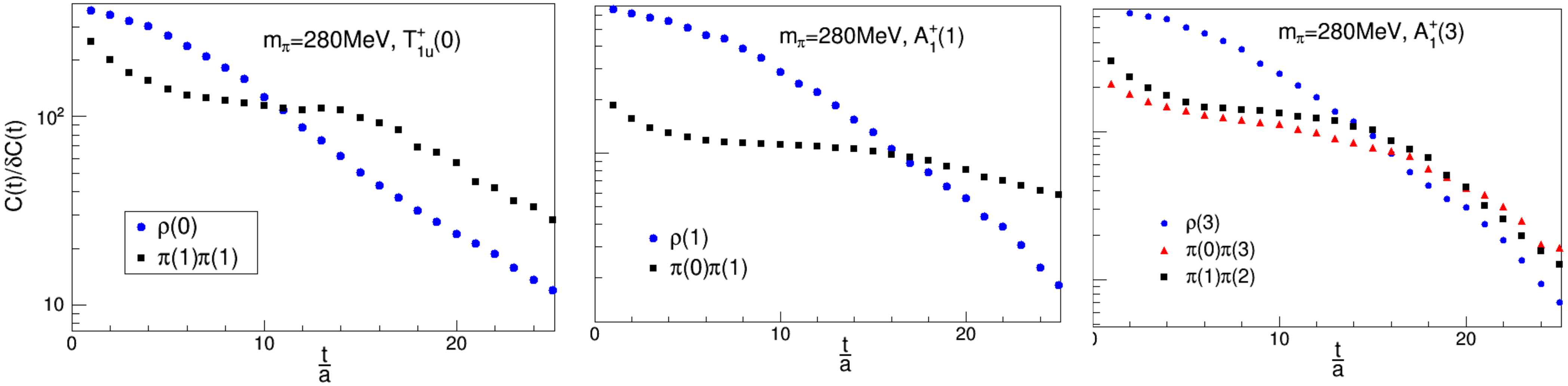
B. Hoerz, JB, C. Morningstar, *preliminary*

Approaching the physical point

- Inelastic thresholds!

M. Hansen, S. Sharpe, *Phys. Rev.* **D90** (2014) 116003
K. Polejaeva, A. Rusetsky, *Eur. Phys. J.* **A48** (2012) 67

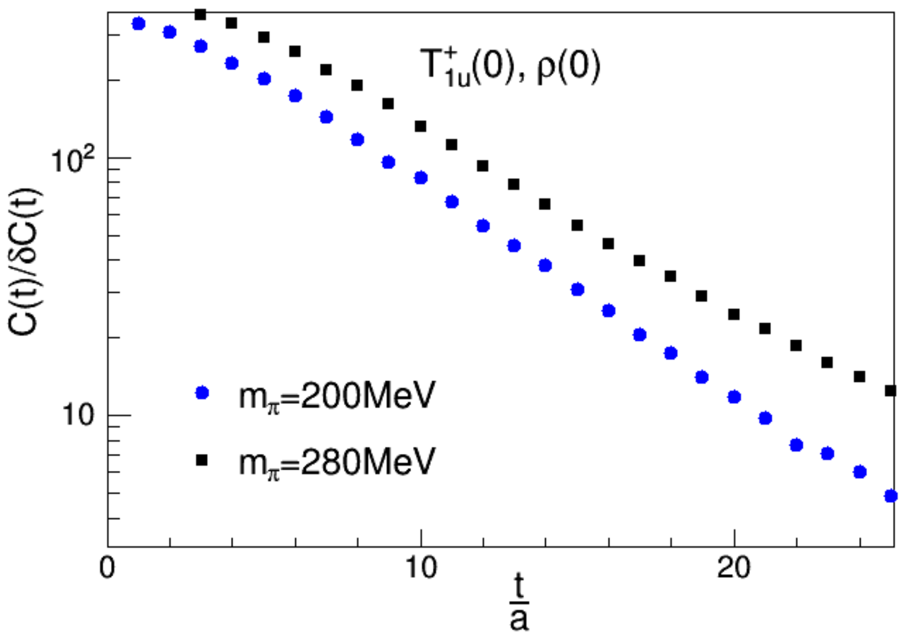
- Signal-to-noise ratio:

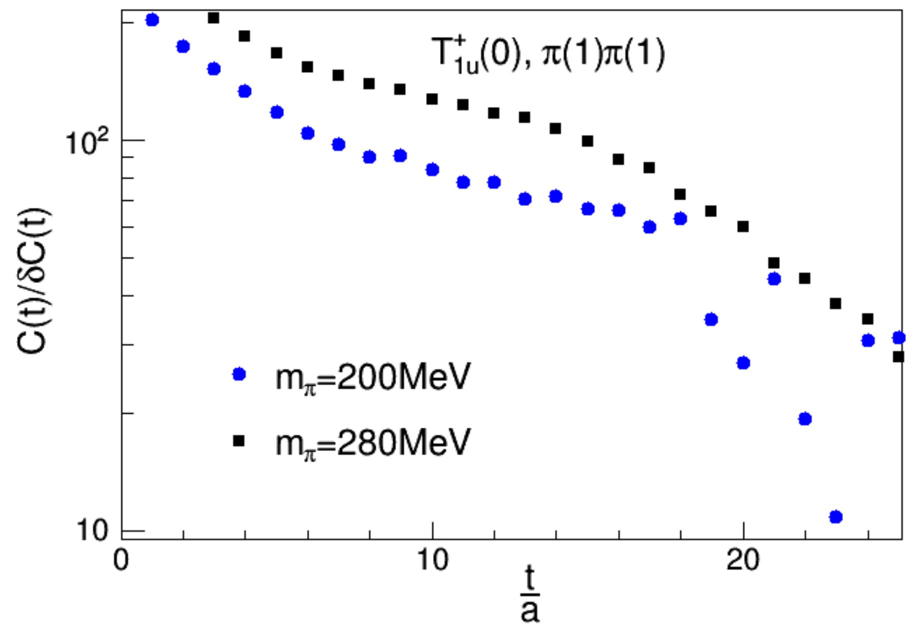


- Two-pion correlators show 'plateaux', even if state is high-lying

Approaching the physical point

- Plateaux persist at lighter pion mass





- Mitigating factors to s/n breakdown: (decrease by ~30-40%)
  - S/n plateaux
  - Volume average
  - Increased density of states

Higher partial waves

- Additional systematic error in scattering amplitude calculations.

- Finite-volume energies determined by

$$\det[1 - KB] = 0$$

where

$$S = (1 + iK)(1 - iK)^{-1}$$

and B is the 'box matrix', linear combinations of

$$R_{\ell m} = \mathrm{Re}\, w_{\ell m}, \qquad I_{\ell m} = \mathrm{Im}\, w_{\ell m}$$

- Block-diagonal when expressed in finite-volume irreps. Each block is infinite-dimensional.

- So far: truncate to leading partial wave in each block.

Higher partial waves

- Automated determination of box matrix elements

R. Brett, JB, J. Fallica, A. Hanlon, B. Hoerz, C. Morningstar, B. Singh, *preliminary*

- For all partial waves $\ell \leq 6$, all total spin $s \leq 7/2$, all irreps, non-identical particles.

- Plan to publish C++ code for evaluation. Example box matrix element:

$$B^{A_1,\mathrm{oa}}(\ell_1 = \ell_2 = 6, n_1 = n_2 = 1) = R_{00} - \frac{2\sqrt{5}}{55} R_{20} - \frac{96}{187} R_{40} - \frac{80\sqrt{13}}{3553} R_{60} + \frac{445\sqrt{17}}{3553} R_{80} + \frac{15\sqrt{24310}}{3553} R_{88} - \frac{498\sqrt{21}}{7429} R_{10,0} + \frac{6\sqrt{510510}}{7429} R_{10,8} + \frac{2178}{37145} R_{12,0} + \frac{66\sqrt{277134}}{37145} R_{12,8}$$

First application

- Influence of $\ell = 3$ partial wave on p-wave pion-pion scattering.

- K-matrix ansatz: $$\left(\frac{q_{\mathrm{cm}}}{m_\pi}\right)^7 (K^{-1})_{33} = \frac{1}{m_\pi^7 a_3}$$

- Three types of determinant conditions (e.g.):
  - $B_1^+(1):\ n_1 = 0, n_3 = 1$
  - $T_{1u}^+(0):\ n_1 = 1,\ n_3 = 1$
  - $E^+(1):\ n_1 = 1,\ n_3 = 2$

- Multiply out each determinant condition: $$\sum_i f_i(K) b_i(B) = 1$$

First application

- Fit results w/o f-wave contribution: (aniso. data)

$$\frac{m_\rho}{m_\pi} = 3.354(24), \qquad g_{\rho\pi\pi} = 6.01(26), \qquad \frac{\chi^2}{d.o.f} = 1.02$$

- Fit results with f-wave contribution:

$$\frac{m_\rho}{m_\pi} = 3.353(24), \qquad g_{\rho\pi\pi} = 6.00(26),$$

$$m_\pi^7 a_3 = -0.0003(24), \qquad \frac{\chi^2}{d.o.f} = 1.02$$

- Experimentally: $m_\pi^7 a_3 = 6.3(4) \times 10^{-5}$

Pelaez, Yndurian ‘05

Conclusions

- Algorithmic advances enable precise finite volume energies, methods are (more-or-less) set
- CLS ensembles enable exploration of continuum, chiral, and infinite volume limits
- Simple resonance photoproduction amplitude: timelike pion form factor
- Higher partial wave contributions
  - Systematic evaluation of box matrices for all irreps, partial waves up to 6.
  - Will be useful also for coupled scattering channels.

- Approaching the physical point:
  - Plateaux in s/n of pion-pion correlators mitigate signal-to-noise problem
  - Volume average/denser spectrum also help