Observables probing nucleon structure and BSM system

Aurora Scapellato

Mid-term report, DESY - Zeuthen

April 18, 2017

BERGISCHE UNIVERSITÄT WUPPERTAL

University of Cyprus

HPC–LEAP
EUROPEAN JOINT DOCTORATES

Supervisors and Collaborators

Supervisors:

- Prof. Constantia Alexandrou (University of Cyprus)
- Prof. Francesco Knechtli (University of Wuppertal)

Collaborators:

- Dr. Karl Jansen, Dr. Fernanda Steffens (NIC- DESY Zeuthen)
- Dr. Giannis Koutsou, Kyriakos Hadjiyiannakou (The Cyprus Institute)
- Prof. Martha Constantinou (Temple University)
- Prof. Haralambos Panagopoulos (University of Cyprus)

Collaboration with HPC-LEAP fellows:

- ESR 5 (Simone Bacchio)
- ESR 12 (Salvatore Calì)

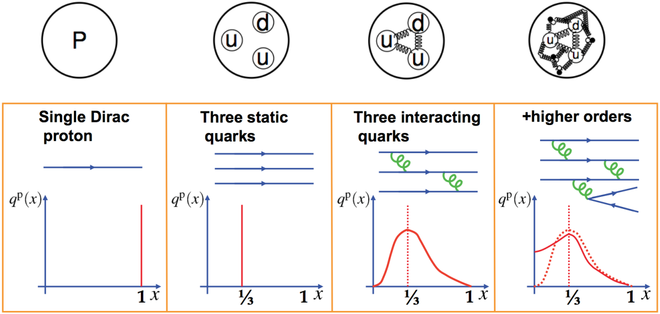
What do we study?

Parton Distribution Functions (PDFs)

Hadrons are complex systems, composed by *quarks* and *gluons*.

- PDFs $\rightarrow$ probability density for a parton (quark/gluon) to carry the fraction x of the proton momentum

Open question: expected form of the parton distribution?



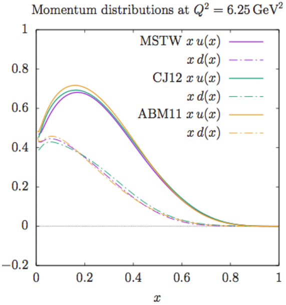
Motivation

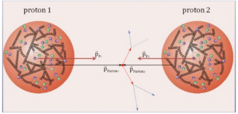
- PDFs cannot be directly measured directly in the experiments
- we have only some indirect estimate from collision experiments (Jlab, DESY,...)
  $\rightarrow$ results depend on the fitting scheme and selected data

> *Ab initio* results would be desirable to:
> 1. make predictions on scattering experiments
> 2. understand inner hadron structure

Our investigation tool: Lattice QCD
(New method by Ji in 2013)

See previous works by 2 groups:
- USQCD collaboration
- European Twisted Mass Collaboration

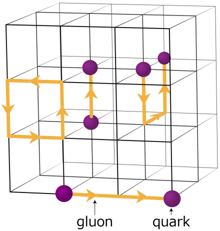




PDFs from Lattice QCD...crucial test of QCD

Why Lattice QCD?

- PDFs are non-perturbative objects
- observables can be computed on the computer in terms of quarks and gluons
- the only input parameters are: coupling constant α_s, quark masses

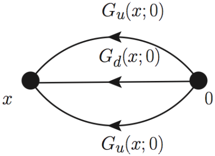


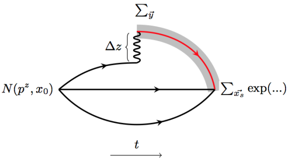
Degrees of freedom:

- quark: $\psi_f(x)^a_\alpha$, with f flavor index
- gluon: $U_\mu(x)_{ab}, \quad a, b = 0, 1, 2$

Strategy and Results

- We compute the PDFs numerically in terms of quarks and gluons

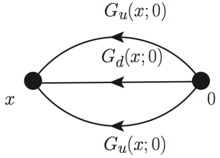


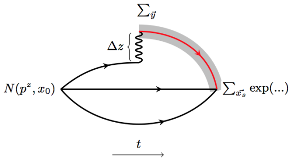


PROBLEM: we need to simulate proton moving with very large momentum!
$\rightarrow$ find a strategy to reduce the noise!

Strategy and Results

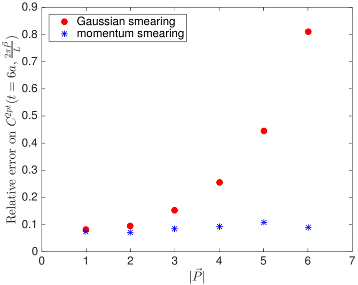
- We compute the PDFs numerically in terms of quarks and gluons



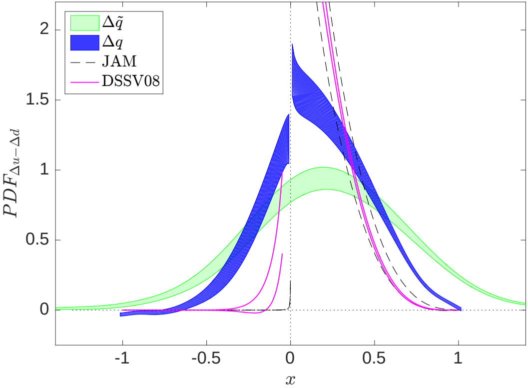


PROBLEM: we need to simulate proton moving with very large momentum!
→ find a strategy to reduce the noise!

Momentum smearing method
[G.Bali et al.] at the physical point, essential to reduce the noise on the lattice. That is crucial because to get the physical parton distribution we need to use very high proton momenta. When using the momentum smearing, the error is approximately constant with increasing P!



First estimate of PDF at the physical point (p=0.7GeV)



1. the phenomenological data have an error
2. our estimate has to be improved
   - higher proton momenta ($p > 2$ GeV)
   - renormalization of our diagram (not simple task! and work still going on)

Conclusions and Outlook

- conclude the computation of PDFs from the lattice (very new field of research)
  - explore other proton momenta (work going on)
  - renormalize our lattice data
- investigate hadron structure through computation of other observables (charge radii proton and other proton form factors)

Poster

For other details and results about this work, a poster has been presented at the thematic workshop in LQCD (DESY- Zeuthen)

Computer facilities

- SuperMUC (CPU machine with more than 241.000 cores)
- Titan (AMD Opteron CPUs in conjunction with Nvidia Tesla GPUs).

Outreach

Researcher's night, September 30th, 2016, Nicosia (Cyprus).

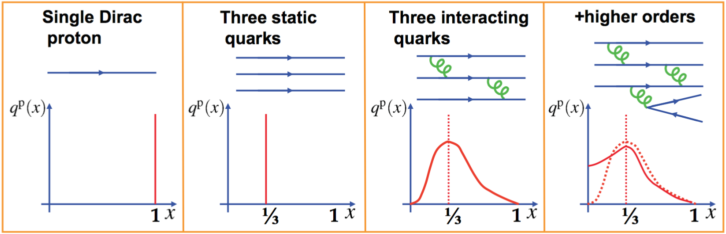
Thank you for your attention.

Definition and properties of PDFs

$f_q(x) \equiv$ probability density of finding a parton q with momentum fraction x of the parent hadron's momentum

- universal functions
- depend on the type of parton: $u(x)$, $\bar{u}(x)$, $d(x)$, $\bar{d}(x)$, ...
- $\int_0^1 dx\, x\,[u(x) + \bar{u}(x) + \ldots + g(x)] = 1$

Expected form?



Phenomenological PDFs

- *Ansatz* for $f_q(x)$ distribution

$\rightarrow xu(x) = A_u x^{n_1}(1-x)^{n2} P_u(x)$

$\rightarrow xd(x) = A_d x^{n_3}(1-x)^{n4} P_d(x)$

Note 1:

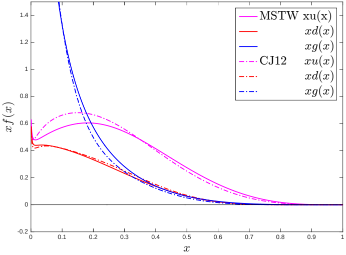
- $u(x) \approx 2d(x)$
- for small x gluon predominate

Note 2:

Results depend on: data sets, form of the fit!

$\rightarrow$ "Ab initio" results would be desirable to have a prediction of quark distributions

Momentum Distributions at $Q^2 = 10\ GeV^2$



[Durham High Energy Physics Database]

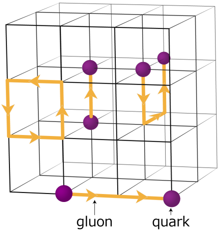
PDFs from Lattice QCD...crucial test of QCD

Why Lattice QCD?

- PDFs are non-perturbative objects
- observables can be computed on the computer in terms of quarks and gluons
- the only input parameters are: coupling constant α_s, quark masses

Degrees of freedom:

- quark: $\psi_f(x)^a_\alpha$, with f flavor index
- gluon: $U_\mu(x)_{ab}, \quad a, b = 0, 1, 2$

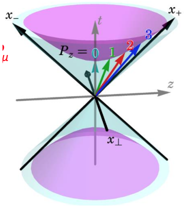


Extraction of PDFs on the lattice

- Definition as light-cone distribution

$$q(x) = \int_{-\infty}^{\infty} \frac{d\xi}{4\pi} e^{-ix\xi P} \langle P|\bar{\psi}(0)\lambda \cdot \gamma W(0,\xi\lambda)\psi(\xi\lambda)|P\rangle$$

  - $0 \leq x \leq 1$: parton momentum
  - $\lambda = (1,0,0,-1)/\sqrt{2}$
  - $W(0,\xi\lambda) = e^{-ig\int_0^{\xi\lambda} d\eta\, A(\eta)}$
  - $P = (P_0, 0, 0, P_z)$



$\rightarrow$ issue in Euclidean space: $(\xi\lambda)^2 = t^2 + \vec{x}^2 = 0$

New Method [Ji, 2013]

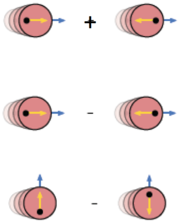
$P_z \rightarrow 0,\ \lambda^2 = 0 \quad \Leftrightarrow \quad P_z \rightarrow \infty,\ \lambda = (0,0,0,-1)$

Goals of our work

- *PDFs* at the physical quark mass

use $N_f = 2$ twisted mass clover ensemble [ETM collaboration: arXiv:1507.05068]

	β	$L/a, T/a$	k	$a\mu$	a [fm]	m_π [MeV]
$cA2.09.48$	2.1	48, 96	0.13729	0.0009	0.093	130



Operators

Unpolarized γ_i $q(x)_\downarrow + q(x)_\uparrow$

Helicity $\gamma_i\gamma_5$ $q(x)_\downarrow - q(x)_\uparrow$

Transversity σ_{ij} $q(x)_\perp - q(x)_\top$

Ideally we should boost the nucleon to infinite momentum!

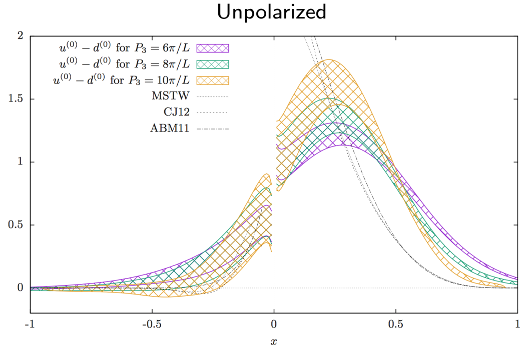
What do we do in practise?

- choose a sufficiently large proton momentum P_3
  $P_3 = \frac{2\pi}{L} n, \qquad n \in [-\frac{N}{2} + 1, \frac{N}{2}]$
- compute the *quasi distribution*
  $\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi}\, e^{-izxP_3} \langle P|\bar{\psi}(0)\gamma W(0,z)\psi(z)|P\rangle$
- make sure to observe convergence increasing P_3
- relate the *quasi* to the *real* distribution $\tilde{q}(x, P_3) \to q(x)$
  (but previous renormalization!)

Note:
in $\tilde{q}(x, P_3)$: $x < 0$ and $x > 1$ is possible

Previous results

- ensemble: $32^3 \times 64$, $m_\pi \approx 370$ MeV [ETM collaboration],[arxiv:1610.03689v1]

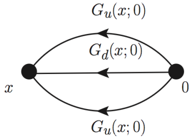


- asymmetry quark-antiquark PDF
- expected behaviour for $x \to 1$
- distributions move to the parametrizations as P increases

Proton two point function

- $C^{2pt}(\vec{x'}, t'; \vec{x_0}, t) \equiv \langle \Omega | N(\vec{x'}, t') \bar{N}(\vec{x_0}, t) | \Omega \rangle$
  probability amplitude of annihilating a proton in $(\vec{x'}, t')$, once created in $(\vec{x_0}, t)$

- $N_\alpha(x) = \epsilon^{abc} u^a_\mu(x) (C\gamma_5)_{\mu\nu} d^b_\nu(x) u^c_\alpha(x)$



- $C^{2pt}(\vec{P}, t'; t) = \sum_{\vec{x'}} e^{-i\vec{P}\cdot(\vec{x'} - \vec{x_0})} \Gamma_{\alpha\beta} C^{2pt}(\vec{x'}, t'; \vec{x_0}, t)$
  $= \sum_n \frac{|\Omega | N(\vec{x_0}, t) | n\vec{P}|^2}{2E_n(\vec{P})} e^{-E_n(\vec{P})(t'-t)}$

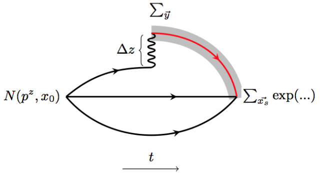
$\rightarrow$ for large t' the sum is dominated by the proton ground state
$\rightarrow$ the signal to noise ratio decrease increasing P

Evaluation of three point functions

- we also need to compute the relevant three point function

$\rightarrow$ $C^{3pt}(x';\tau;x) = \langle\Omega|N(\vec{x'},t')\underbrace{\bar{\psi}(0)\gamma W(0,z)\psi(z)}_{O(z,\tau)}\bar{N}(\vec{x},t)|\Omega\rangle$

- $O(z,\tau,Q=0) = \sum_{\vec{y}}\bar{\psi}(y)\gamma W(y,y+z)\psi(y)$

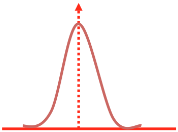


$\leftarrow$ all-to-all propagator needed
(due to momentum projection)

- $C^{3pt}(\vec{P};\tau;t') = f_{00}e^{-E_0t'}\left(1+\frac{f_{01}}{f_{00}}e^{-E_{01}\tau}+\frac{f_{10}}{f_{00}}e^{-E_{01}(t'-\tau)}+...\right)$

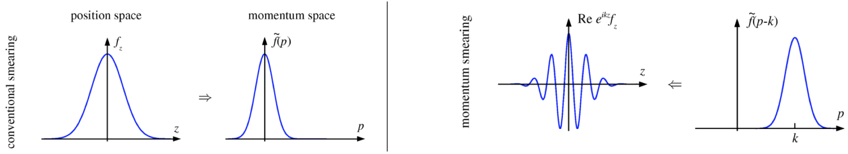
$\Rightarrow$ choose τ, $t'-\tau$ sufficiently large

Methods for reducing the noise



- step1. increase the overlap with the proton ground state
  - Gaussian smearing on quarks fields
    $\psi^{smear}(\vec{x}, t) = \sum_{\vec{y}} F(\vec{x}, \vec{y}, U(t))\psi(\vec{y}, t)$
- step2. improve the signal for high momenta
  - Momentum smearing [Bali *et. al*, 2016]

    $U_j(x) \to e^{i\xi P} U_j(x)$



Test momentum smearing on proton correlator

Lattice Setup

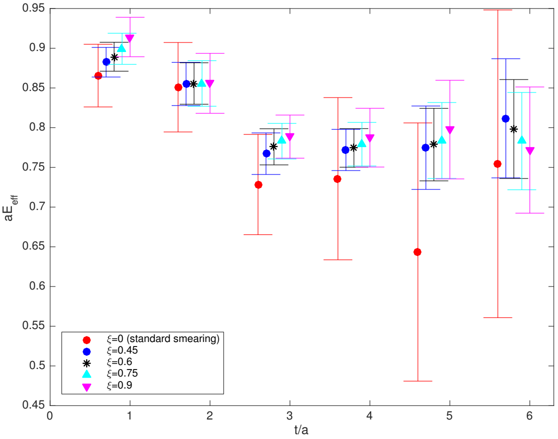
Ensemble: $48^3 x 96$ at the physical point

- 50 configurations
- Proton momentum: $P = \frac{2 \cdot \pi}{L}, ..., \frac{12 \cdot \pi}{L}$ (0.3 GeV, ..., 1.7 GeV)
- Momentum smearing: $\xi \in [0, 1]$

Methods

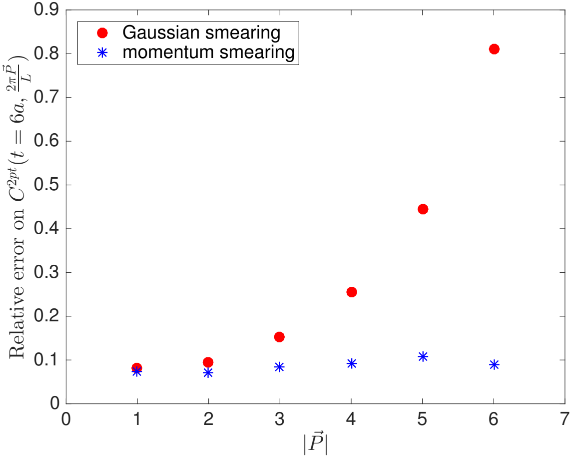
- error on the effective energy: $E_p(t) = \log \frac{C^{2pt}(\vec{P}, t)}{C^{2pt}(\vec{P}, t+1)}$
- relative error on the correlator

- Effective energy for $\vec{P} = (4, 0, 0)$, corresponding to $|\vec{P}| = 1.1$ GeV



- we choose $\xi = 0.6$ as optimal parameter

Relative error of the correlator



Exponential growth v.s. roughly constant error!

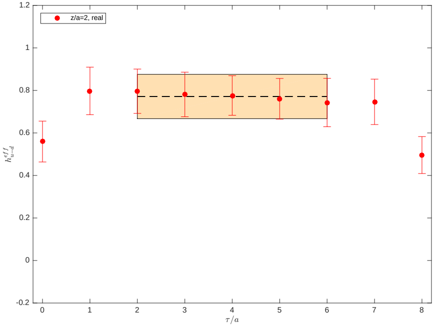
Exploratory study

Lattice Setup

- Lattice: $48^3 \times 96$
- 40 configurations
- $t_s = 8a \approx 0.74$ fm
- $P = \frac{6\pi}{L} \approx 0.8 GeV$

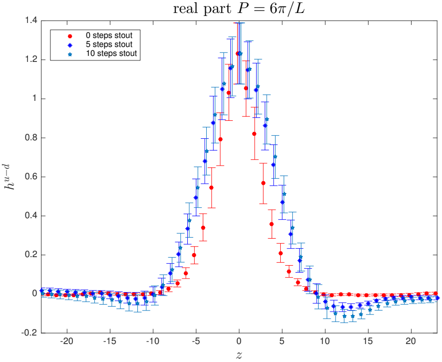
insertion operator:
$O(z;\tau) = \bar{\psi}(0)\gamma_i W_i(0,z)\psi(z)$

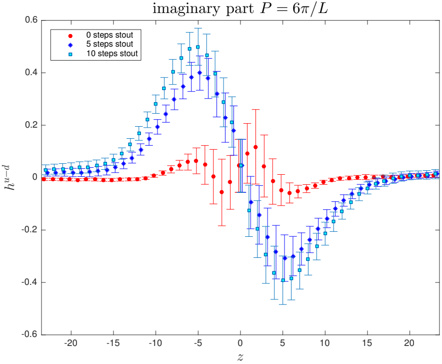
$$\frac{C^{3pt}(t';\tau;t)}{C^{2pt}(t';t)} \underset{0 \gg \tau \gg t'}{=} \frac{-iP}{E} h(P,z)$$



Unpolarized PDF

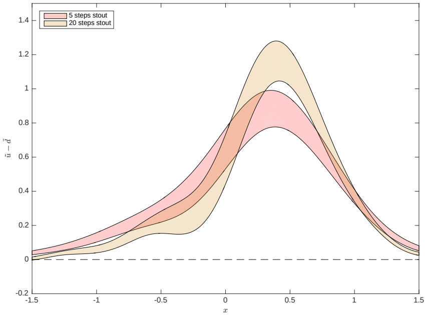
- we try to estimate the influence of the renormalization by applying stout smearing to the links in the operator





Bare unpolarized quasi distribution

- $\tilde{q}(x,P) = 2P\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dz}{4\pi} e^{-izxP} h(z,P)$



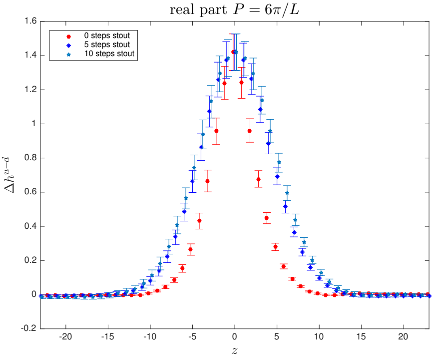
- smearing amplifies the distribution for $x > 0$
- asymmetry between the quark-antiquark PDF

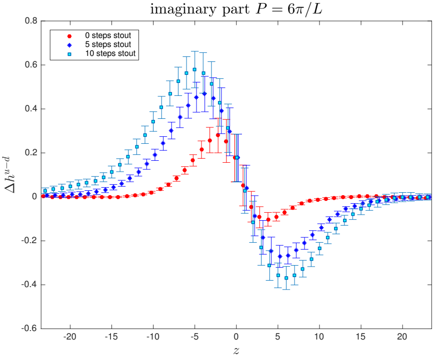
PDF is different from zero at $x = 1$

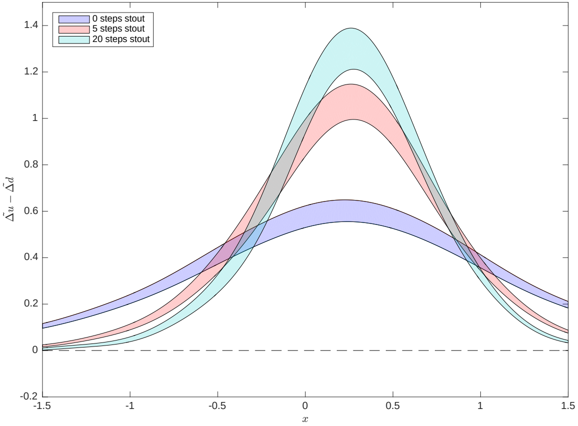
$\rightarrow$ quite low P and matching coefficients are missing!

Helicity distribution

- matrix element: $h(P,z) = \langle P|\bar{\psi}(0)\gamma_5\gamma_i W_i(0,z)\psi(z)|P\rangle$







- crossing relation $\Delta q(-x) = \Delta \bar{q}(x)$
- quark-antiquark asymmetry

Conclusions and outlook

- we showed how to compute momentum and spin distribution in the nucleon
- in this framework the *momentum smearing* helps to access to large momenta
- we showed to effect of stout smearing on quasi-distributions

Outlook

- use a large source-sink separation (at least 1 fm)
- go to higher momenta (at least 2.3 GeV)
- include the *renormalization* of the bare matrix elements
- why not simulations at others physical ensembles?($N_f = 2 + 1$?)

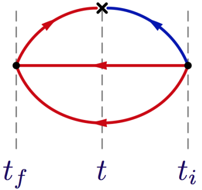
I thank...

Supervisor: Constantia Alexandrou

Collaborators:
Giannis Koutsou, Christian Wiese, Kyriakos Hadjiyiannakou, Haralambos Panagopoulos, Martha Constantinou, Karl Jansen, Fernanda M. Steffens.

Thank you for your attention.

Sequential method (through the sink)



$$= \sum_{\vec{x}'} e^{i\vec{P}\cdot\vec{x}'} \sum_y \langle \chi(x') O(y,\tau) \chi(x) \rangle$$

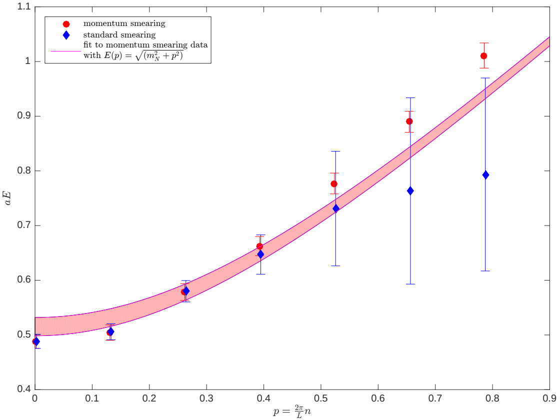
step1. compute point-to all propagator from the source

step2. define a source vector
$K^{ab}_{\rho\sigma}(\vec{x}', t'; \vec{P}; \Gamma) = \Gamma_{\rho\sigma'} G^{ab}_{\sigma'\sigma}(\vec{x}', t'; \vec{0}, 0)\delta(t' - t_{sink})e^{i\vec{x}'\vec{P}}$

step3. invert the source vector
$S(\vec{y}, t_y; t_{sink}; \vec{P}; \Gamma) = \sum_{\vec{x}'} G(\vec{y}, t_y; \vec{x}, t)\Gamma G(\vec{x}, t; 0, 0)e^{-i\vec{x}'\vec{P}}$

$\Longrightarrow$ the sum over $\vec{x}'$ is carried out through an inversion!

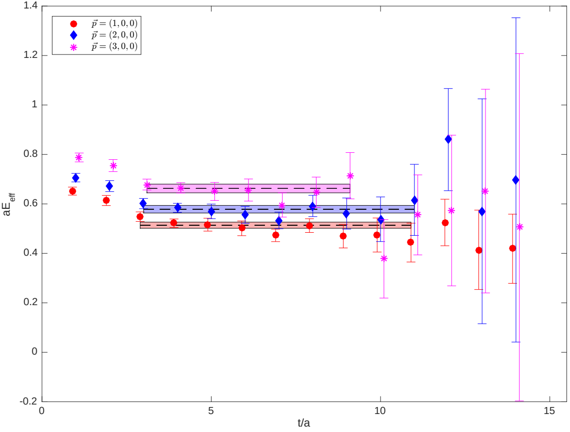
Dispersion relation



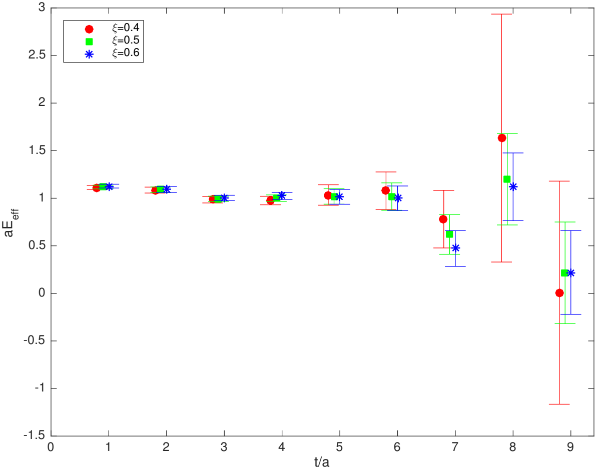
- fit result $m_N \approx 0.51 \to 1$ GeV

Fit in the plateau region

- for large t, $E_{eff}(t)$ provides us the ground state energy



- Effective energy for $\vec{P} = (6, 0, 0)$, corresponding to $|\vec{P}| = 1.7$ GeV



- we choose $\xi = 0.6$ as optimal parameter