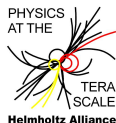


Monte Carlo Simulation of Collider Physics

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Hamburg SFB lectures
April/May 2017

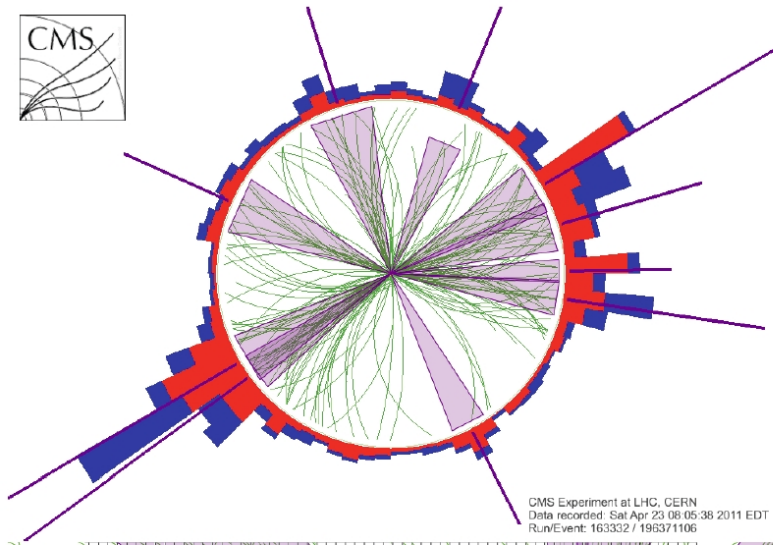


Motivation: jets



[Google Images]

Motivation: jets (at LHC of course)



Why Monte Carlos?

We want to understand

$$\mathcal{L}_{\text{int}} \longleftrightarrow \text{Final states} .$$

Why Monte Carlos?

LHC experiments require
sound understanding of signals and *backgrounds*.



Full detector simulation.



Fully exclusive hadronic final state.

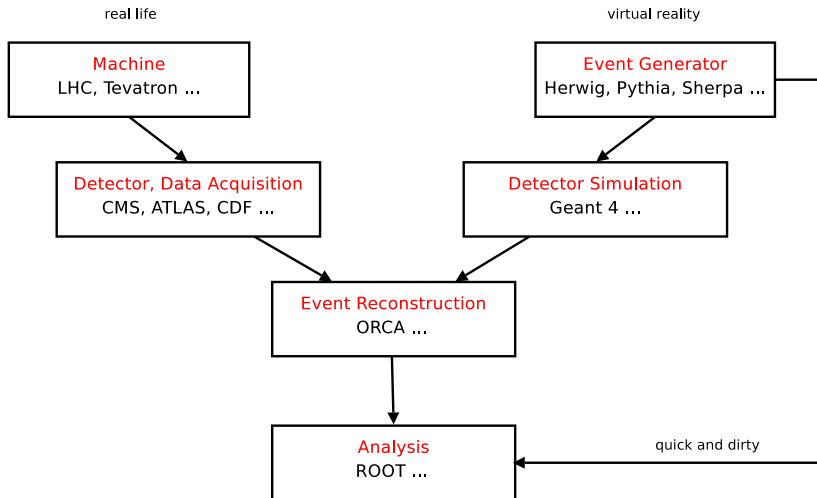


Monte Carlo event generator with
parton shower, hadronization model, decays of unstable
particles.



Parton level computations.

Experiment and Simulation

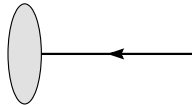
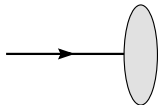


Monte Carlo Event Generators

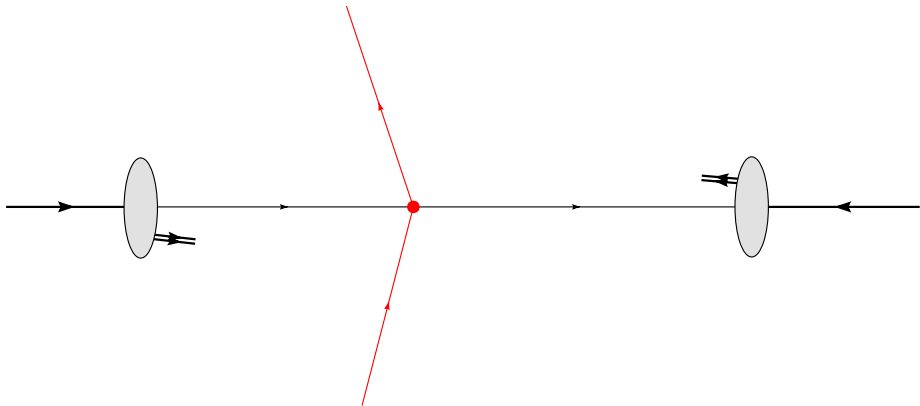
- Complex final states in full detail (jets).
- Arbitrary observables and cuts from final states.
- Studies of new physics models.
- Rates and topologies of final states.
- Background studies.
- Detector Design.
- Detector Performance Studies (Acceptance).
- *Obvious* for calculation of observables on the quantum level

$$|A|^2 \longrightarrow \text{Probability.}$$

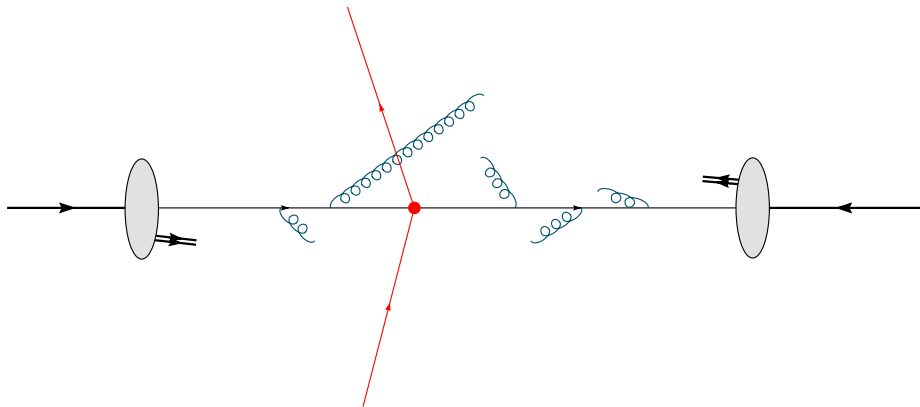
pp Event Generator



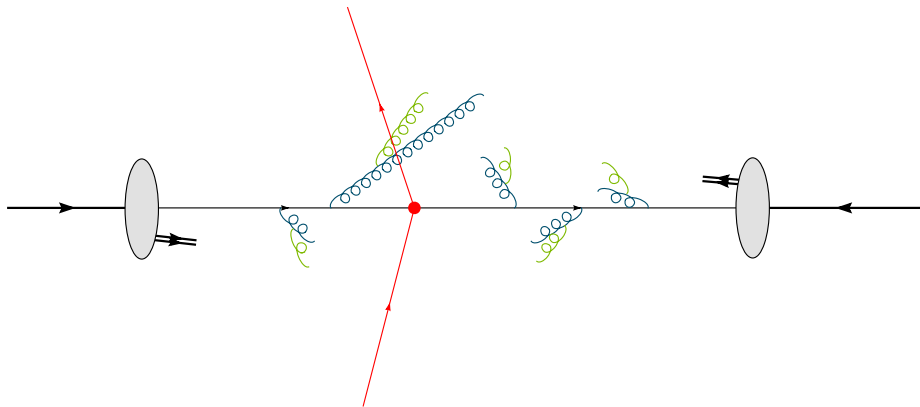
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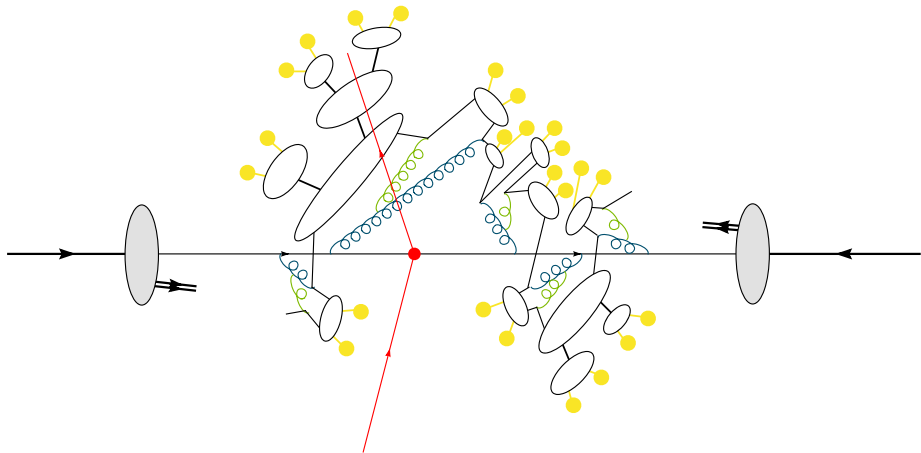
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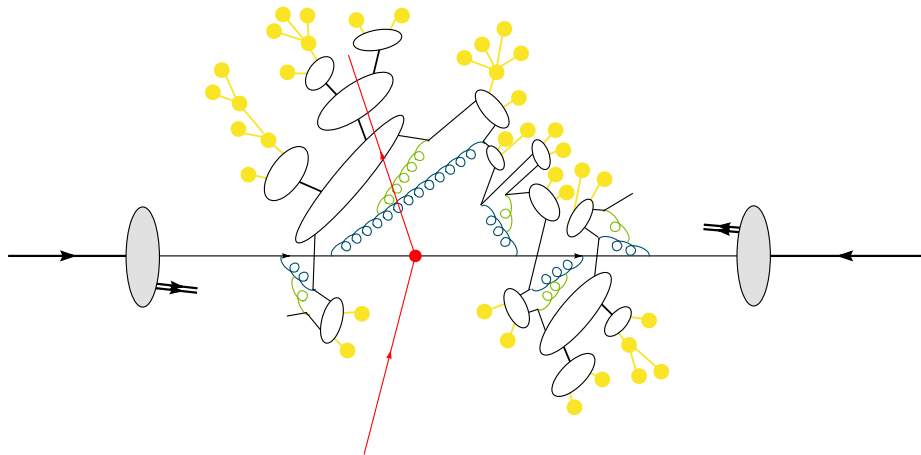
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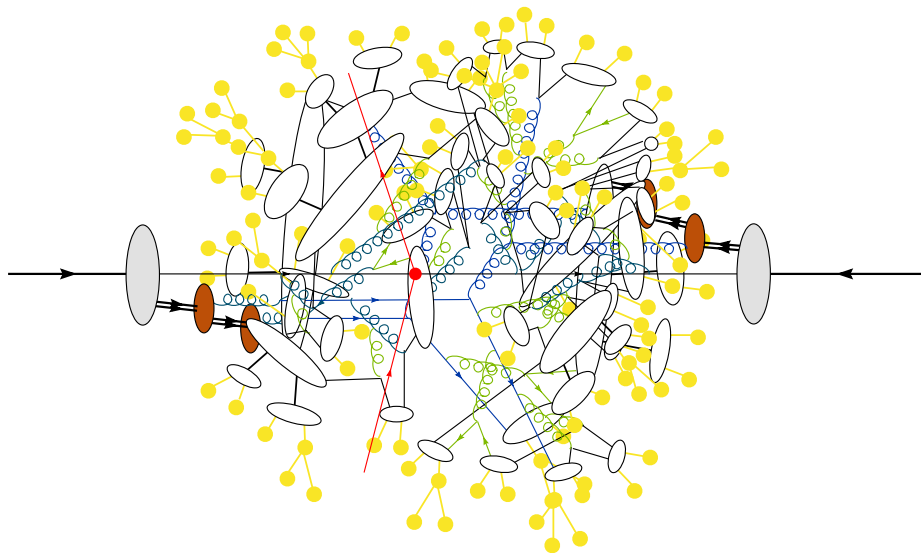
pp Event Generator



pp Event Generator



pp Event Generator



Divide and conquer

Partonic cross section from Feynman diagrams

$$d\sigma = d\sigma_{\text{hard}} dP(\text{partons} \rightarrow \text{hadrons})$$

$$\begin{aligned} dP(\text{partons} \rightarrow \text{hadrons}) = & dP(\text{resonance decays}) && [\Gamma > Q_0] \\ & \times dP(\text{parton shower}) && [\text{TeV} \rightarrow Q_0] \\ & \times dP(\text{hadronisation}) && [\sim Q_0] \\ & \times dP(\text{hadronic decays}) && [O(\text{MeV})] \end{aligned}$$

Underlying event from multiple partonic interactions

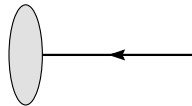
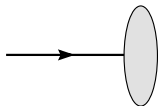
$$d\sigma \longleftarrow d\sigma(\text{QCD } 2 \rightarrow 2)$$

Plan for these lectures

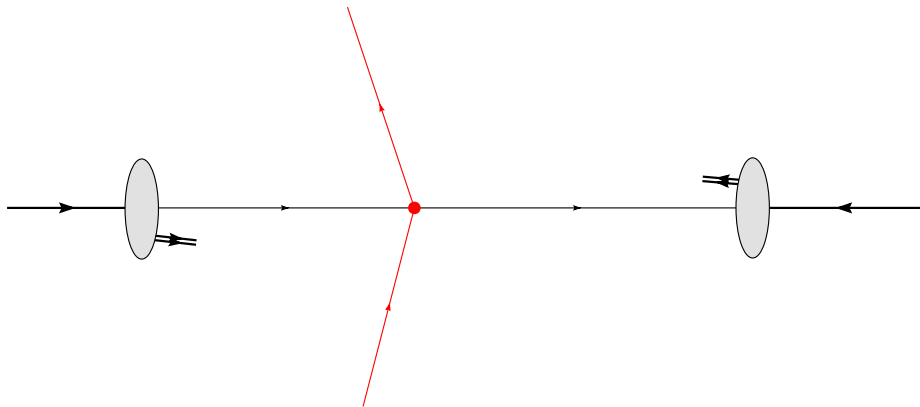
- Monte Carlo Methods
- Hard Scattering
- Parton Showers
- Hadronization and Hadronic Decays
- NLO Matching
- Merging with Higher Orders
- Underlying Event
- Multiple Parton Interactions (MPI) Modelling

Hard Scattering

Hard scattering

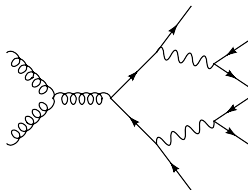


Hard scattering



Matrix elements

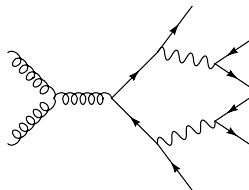
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- OK for very inclusive observables.

Matrix elements

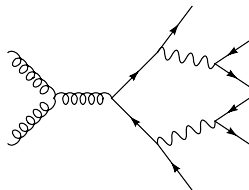
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- Starting point for further simulation.
- Want exclusive final state at the LHC ($O(100)$).

Matrix elements

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- OK for very inclusive observables.
- Starting point for further simulation.
- Want exclusive final state at the LHC ($O(100)$).
- Want arbitrary cuts.
- $\rightarrow$ use Monte Carlo methods.

Matrix elements

Where do we get (LO) $|M|^2$ from?

- Most/important simple processes (SM) are ‘built in’.
- Calculate yourself (≤ 3 particles in final state).
- Matrix element generators:
 - MadGraph/MadEvent.
 - Comix/AMEGIC (part of Sherpa).
 - HELAC/PHEGAS.
 - Whizard.
 - CalcHEP/CompHEP.

generate code or event files that can be further processed.

- $\rightarrow$ FeynRules interface to ME generators.

Also NLO mostly automatically available.

See “Matching and Merging”.

Cross section formula

From Matrix element, we calculate

$$\sigma = \int f_i(x_1, \mu^2) f_j(x_2, \mu^2) \frac{1}{F} \sum |M|^2 \, dx_1 dx_2 d\Phi_n ,$$

Cross section formula

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now,

$$\frac{1}{F} dx_1 dx_2 d\Phi_n = J(\vec{x}) \prod_{i=1}^{3n-2} dx_i \quad \left(d\Phi_n = (2\pi)^4 \delta^{(4)}(\dots) \prod_{i=1}^n \frac{d^3 \vec{p}}{(2\pi)^3 2E_i} \right)$$

such that

$$\begin{aligned} \sigma &= \int g(\vec{x}) d^{3n-2} \vec{x} , \quad \left(g(\vec{x}) = J(\vec{x}) f_i f_j \sum |M|^2 \Theta(\text{cuts}) \right) \\ &= \frac{1}{N} \sum_{i=1}^N \frac{g(\vec{x}_i)}{p(\vec{x}_i)} = \frac{1}{N} \sum_{i=1}^N w_i . \end{aligned}$$

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We generate **events** $\vec{x}_i$ with **weights** w_i .

Mini event generator

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Generate events with same frequency as in nature!

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$$P_i = \frac{w_i}{w_{\max}} ,$$

where $w_{\max}$ has to be chosen sensibly.

→ reweighting, when $\max(w_i) = \bar{w}_{\max} > w_{\max}$, as

$$P_i = \frac{w_i}{\bar{w}_{\max}} = \frac{w_i}{w_{\max}} \cdot \frac{w_{\max}}{\bar{w}_{\max}} ,$$

i.e. reject events with probability $(w_{\max}/\bar{w}_{\max})$ afterwards.

Mini event generator

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Matrix elements

Some comments:

- Use common Monte Carlo techniques to generate events efficiently. Goal: small variance in w_i distribution!

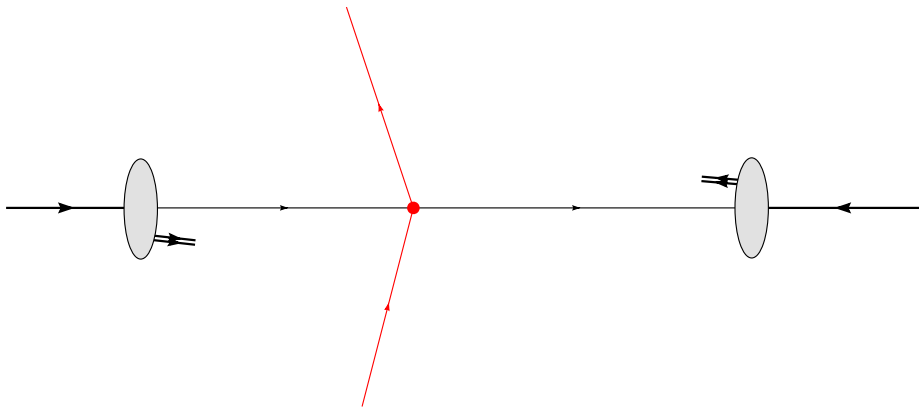
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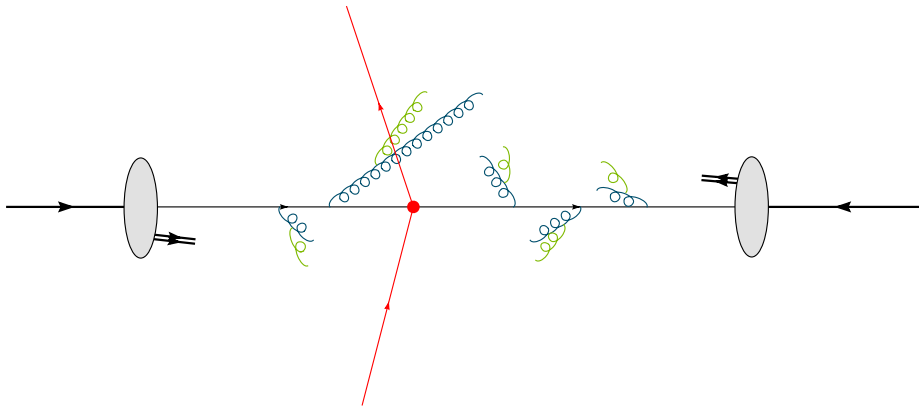
- Use common Monte Carlo techniques to generate events efficiently. Goal: small variance in w_i distribution!
- Efficient generation closely tied to knowledge of $f(\vec{x}_i)$, *i.e.* the matrix element's propagator structure.
→ build phase space generator already while generating ME's automatically.

Parton Showers

Hard matrix element



Hard matrix element $\rightarrow$ parton showers



Parton showers

Quarks and gluons in final state, pointlike.

Parton showers

Quarks and gluons in final state, pointlike.

- Know short distance (short time) fluctuations from matrix element/Feynman diagrams: $Q \sim \text{few GeV to } O(\text{TeV})$.
- Measure hadronic final states, long distance effects, $Q_0 \sim 1 \text{ GeV}$.

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Dominated by large logs, terms

$$\alpha_s^n \log^{2n} \frac{Q}{Q_0} \sim 1 .$$

Generated from emissions *ordered* in Q .

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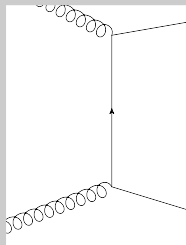
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Generated from emissions *ordered* in Q .

Soft and/or collinear emissions.

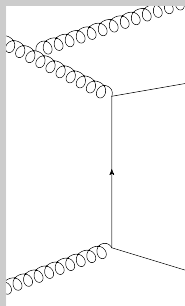
ME approximated by parton cascade

Evolution in scale, typically $Q \sim 1 \text{ TeV}$ down to $Q \sim 1 \text{ GeV}$.



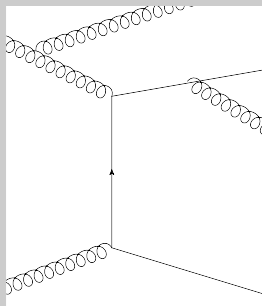
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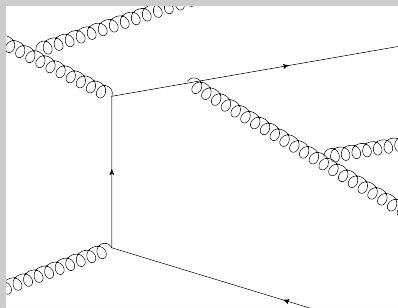
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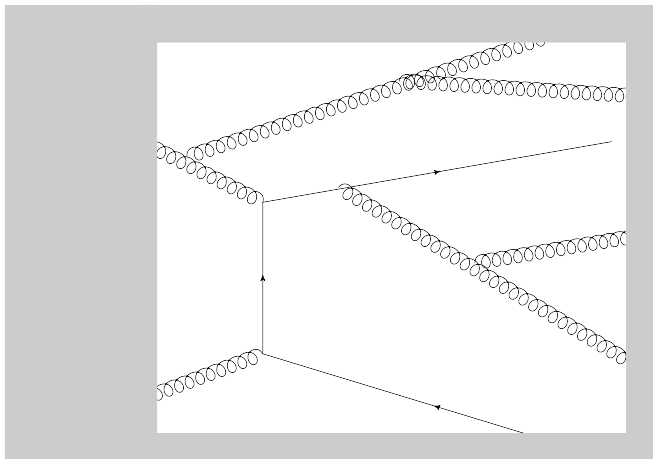
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e^+e^- annihilation

Good starting point: $e^+e^- \rightarrow q\bar{q}g$:

Final state momenta in one plane (orientation usually averaged).

Write momenta in terms of

$$x_i = \frac{2p_i \cdot q}{Q^2} \quad (i = 1, 2, 3),$$

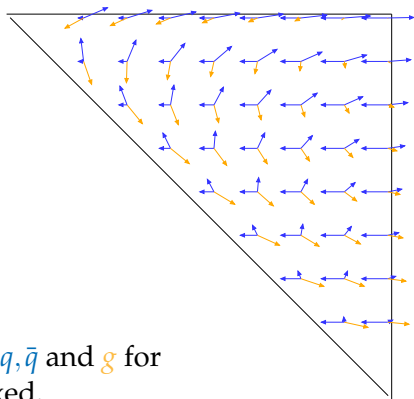
$$0 \leq x_i \leq 1, x_1 + x_2 + x_3 = 2,$$

$$q = (Q, 0, 0, 0),$$

$$Q \equiv E_{cm}.$$

Fig: momentum configuration of $q, \bar{q}$ and g for given point (x_1, x_2) , $\bar{q}$ direction fixed.

$(x_1, x_2) = (x_q, x_{\bar{q}})$ -plane:

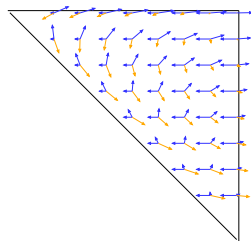
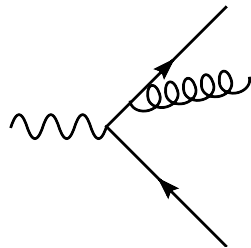


e^+e^- annihilation

Differential cross section:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \frac{x_1 + x_2}{(1-x_1)(1-x_2)}$$

Collinear singularities: $x_1 \rightarrow 1$ or $x_2 \rightarrow 1$. Soft singularity: $x_1, x_2 \rightarrow 1$.



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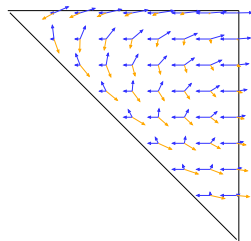
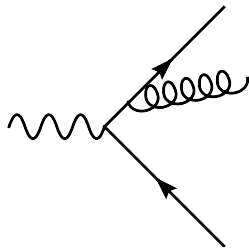
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Collinear singularities: $x_1 \rightarrow 1$ or $x_2 \rightarrow 1$. Soft singularity: $x_1, x_2 \rightarrow 1$.

Rewrite in terms of x_3 and $\theta = \angle(q, g)$:

$$\frac{d\sigma}{d\cos\theta dx_3} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \left[\frac{2}{\sin^2\theta} \frac{1 + (1-x_3)^2}{x_3} - x_3 \right]$$

Singular as $\theta \rightarrow 0$ and $x_3 \rightarrow 0$.



e^+e^- annihilation

Can separate into two jets as

$$\begin{aligned}\frac{2d\cos\theta}{\sin^2\theta} &= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\theta}{1+\cos\theta} \\ &= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\bar{\theta}}{1-\cos\bar{\theta}} \\ &\approx \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2}\end{aligned}$$

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So, we rewrite $d\sigma$ in collinear limit as

$$d\sigma = \sigma_0 \sum_{\text{jets}} \frac{d\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} C_F \frac{1+(1-z)^2}{z^2} dz$$

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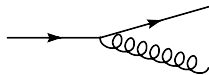
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with DGLAP splitting function $P(z)$.

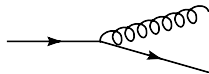
Collinear limit

Universal DGLAP splitting kernels for collinear limit:

$$d\sigma = \sigma_0 \sum_{\text{jets}} \frac{d\theta^2}{\theta^2} \frac{\alpha_s}{2\pi} P(z) dz$$



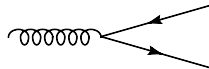
$$P_{q \rightarrow qg}(z) = C_F \frac{1+z^2}{1-z}$$



$$P_{q \rightarrow gq}(z) = C_F \frac{1+(1-z)^2}{z}$$



$$P_{g \rightarrow gg}(z) = C_A \frac{(1-z(1-z))^2}{z(1-z)}$$



$$P_{g \rightarrow qq}(z) = T_R (1 - 2z(1-z))$$

Collinear limit

Universal DGLAP splitting kernels for collinear limit:

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Note: Other variables may equally well characterize the collinear limit:

$$\frac{d\theta^2}{\theta^2} \sim \frac{dQ^2}{Q^2} \sim \frac{dp_{\perp}^2}{p_{\perp}^2} \sim \frac{d\tilde{q}^2}{\tilde{q}^2} \sim \frac{dt}{t}$$

whenever $Q^2, p_{\perp}^2, t \rightarrow 0$ means “collinear”.

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- θ : HERWIG
- Q^2 : PYTHIA ≤ 6.3 , SHERPA.
- $p_{\perp}$: PYTHIA ≥ 6.4 , ARIADNE, Catani–Seymour showers.
- $\tilde{q}$: Herwig++.

Resolution

Need to introduce **resolution** t_0 , e.g. a cutoff in $p_\perp$. Prevent us from the singularity at $\theta \rightarrow 0$.

Emissions below t_0 are **unresolvable**.

Finite result due to virtual corrections:

unresolvable + virtual emissions are included in Sudakov form factor via unitarity (see below!).

Towards multiple emissions

Starting point: factorisation in collinear limit, single emission.

$$\sigma_{2+1}(t_0) = \sigma_2(t_0) \int_{t_0}^t \frac{dt'}{t'} \int_{z_-}^{z_+} dz \frac{\alpha_S}{2\pi} \hat{P}(z) = \sigma_2(t_0) \int_{t_0}^t dt W(t) .$$

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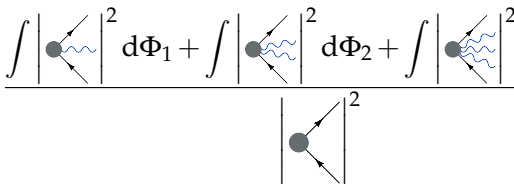
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Simple example:

Multiple photon emissions, strongly ordered in t .

We want

$$W_{\text{sum}} = \sum_{n=1} W_{2+n} = \frac{\int \left| \text{diagram}_1 \right|^2 d\Phi_1 + \int \left| \text{diagram}_2 \right|^2 d\Phi_2 + \int \left| \text{diagram}_3 \right|^2 d\Phi_3 + \dots}{\left| \text{diagram}_0 \right|^2}$$


for any number of emissions.

Towards multiple emissions

$$(n=1) \bullet \begin{array}{c} \nearrow \\ \text{---} \\ \searrow \end{array}$$

$$W_{2+1} = \left(\int \left| \begin{array}{c} \nearrow \\ \text{---} \\ \searrow \end{array} \right|^2 + \left| \begin{array}{c} \nearrow \\ \text{---} \\ \searrow \end{array} \right|^2 d\Phi_1 \right) / \left| \begin{array}{c} \nearrow \\ \text{---} \\ \searrow \end{array} \right|^2 = \frac{2}{1!} \int_{t_0}^t dt W(t) .$$

Towards multiple emissions

$(n = 1)$ 

$$W_{2+1} = \left(\int \left| \text{diagram}_1 \right|^2 + \left| \text{diagram}_2 \right|^2 d\Phi_1 \right) / \left| \text{diagram}_3 \right|^2 = \frac{2}{1!} \int_{t_0}^t dt W(t) .$$

$(n = 2)$ 

$$W_{2+2} = \left(\int \left| \text{diagram}_1 \right|^2 + \left| \text{diagram}_2 \right|^2 + \left| \text{diagram}_3 \right|^2 + \left| \text{diagram}_4 \right|^2 d\Phi_2 \right) / \left| \text{diagram}_5 \right|^2$$

$$= 2^2 \int_{t_0}^t dt' \int_{t_0}^{t'} dt'' W(t') W(t'') = \frac{2^2}{2!} \left(\int_{t_0}^t dt W(t) \right)^2 .$$

We used

$$\int_{t_0}^t dt_1 \dots \int_{t_0}^{t_{n-1}} dt_n W(t_1) \dots W(t_n) = \frac{1}{n!} \left(\int_{t_0}^t dt W(t) \right)^n .$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

$$W_{2+n} = \frac{2^n}{n!} \left(\int_{t_0}^t dt W(t) \right)^n$$

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So, in total we get

$$\sigma_{>2}(t_0) = \sigma_2(t_0) \sum_{k=1}^{\infty} \frac{2^k}{k!} \left(\int_{t_0}^t dt W(t) \right)^k = \sigma_2(t_0) \left(e^{2 \int_{t_0}^t dt W(t)} - 1 \right)$$

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Sudakov Form Factor

$$\Delta(t_0, t) = \exp \left[- \int_{t_0}^t dt W(t) \right]$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

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Sudakov Form Factor in QCD

$$\Delta(t_0, t) = \exp \left[- \int_{t_0}^t dt W(t) \right] = \exp \left[- \int_{t_0}^t \frac{dt}{t} \int_{z_-}^{z_+} \frac{\alpha_S(z, t)}{2\pi} \hat{P}(z, t) dz \right]$$

Sudakov form factor

Note that

$$\sigma_{\text{all}} = \sigma_2 + \sigma_{>2} = \sigma_2 + \sigma_2 \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right) ,$$
$$\Rightarrow \Delta^2(t_0, t) = \frac{\sigma_2}{\sigma_{\text{all}}} .$$

Two jet rate $= \Delta^2 = P^2(\text{No emission in the range } t \rightarrow t_0) .$

Sudakov form factor = No emission probability .

Often $\Delta(t_0, t) \equiv \Delta(t)$.

- Hard scale t , typically CM energy or $p_{\perp}$ of hard process.
- Resolution t_0 , two partons are resolved as two entities if inv mass or relative $p_{\perp}$ above t_0 .
- P^2 (not P), as we have two legs that evolve independently.

Sudakov form factor from Markov property

Unitarity

$$\begin{aligned} P(\text{"some emission"}) + P(\text{"no emission"}) \\ = P(0 < t \leq T) + \bar{P}(0 < t \leq T) = 1 . \end{aligned}$$

Multiplication law (no memory)

$$\bar{P}(0 < t \leq T) = \bar{P}(0 < t \leq t_1) \bar{P}(t_1 < t \leq T)$$

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Multiplication law (no memory)

$$\bar{P}(0 < t \leq T) = \bar{P}(0 < t \leq t_1) \bar{P}(t_1 < t \leq T)$$

Then subdivide into n pieces: $t_i = \frac{i}{n}T, 0 \leq i \leq n$.

$$\begin{aligned} \bar{P}(0 < t \leq T) &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} \bar{P}(t_i < t \leq t_{i+1}) = \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} (1 - P(t_i < t \leq t_{i+1})) \\ &= \exp \left(- \lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} P(t_i < t \leq t_{i+1}) \right) = \exp \left(- \int_0^T \frac{dP(t)}{dt} dt \right) . \end{aligned}$$

Sudakov form factor

Again, no-emission probability!

$$\bar{P}(0 < t \leq T) = \exp \left(- \int_0^T \frac{dP(t)}{dt} dt \right)$$

So,

$$\begin{aligned} dP(\text{first emission at } T) &= dP(T) \bar{P}(0 < t \leq T) \\ &= dP(T) \exp \left(- \int_0^T \frac{dP(t)}{dt} dt \right) \end{aligned}$$

That's what we need for our parton shower! Probability density for next emission at t :

$$dP(\text{next emission at } t) = \frac{dt}{t} \int_{z_-}^{z_+} \frac{\alpha_S(z, t)}{2\pi} \hat{P}(z, t) dz \exp \left[- \int_{t_0}^t \frac{dt}{t} \int_{z_-}^{z_+} \frac{\alpha_S(z, t)}{2\pi} \hat{P}(z, t) dz \right]$$

Parton shower Monte Carlo

Probability density:

$dP(\text{next emission at } t) =$

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Conveniently, the probability distribution is $\Delta(t)$ itself.

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Conveniently, the probability distribution is $\Delta(t)$ itself.

Hence, parton shower very roughly from (HERWIG):

- ① Choose flat random number $0 \leq \rho \leq 1$.
- ② If $\rho < \Delta(t_{\max})$: no resolvable emission, stop this branch.
- ③ Else solve $\rho = \Delta(t_{\max})/\Delta(t)$
(= no emission between $t_{\max}$ and t) for t .
Reset $t_{\max} = t$ and goto 1.

Determine z essentially according to integrand in front of exp.

Parton shower Monte Carlo

Probability density:

$dP(\text{next emission at } t) =$

$$\frac{dt}{t} \int_{z_-}^{z_+} \frac{\alpha_S(z, t)}{2\pi} \hat{P}(z, t) dz \exp \left[- \int_{t_0}^t \frac{dt}{t} \int_{z_-}^{z_+} \frac{\alpha_S(z, t)}{2\pi} \hat{P}(z, t) dz \right]$$

Conveniently, the probability distribution is $\Delta(t)$ itself.

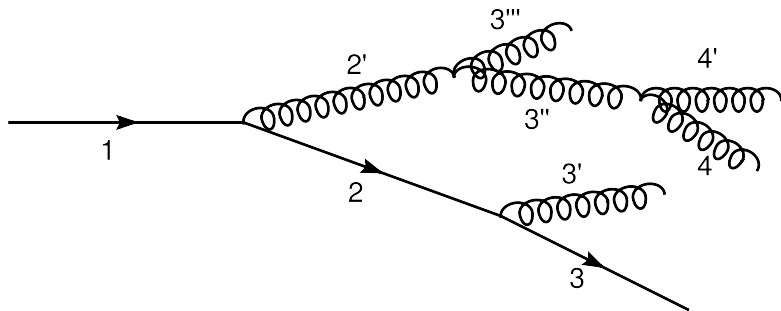
- That was old HERWIG variant. Relies on (numerical) integration/tabulation for $\Delta(t)$.
- Pythia, now also Herwig++, use the **Veto Algorithm**.
- Method to sample x from distribution of the type

$$dP = F(x) \exp \left[- \int^x dx' F(x') \right] dx .$$

Simpler, more flexible, but slightly slower.

Parton cascade

Get tree structure, ordered in evolution variable t :

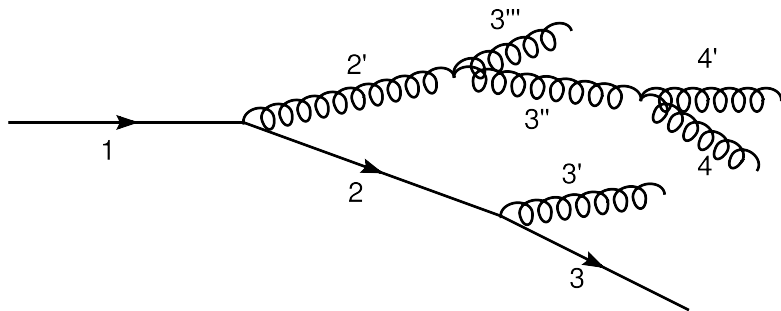


Here: $t_1 > t_2 > t_3; t_2 > t_{3'}$ etc.

Construct four momenta from (t_i, z_i) and (random) azimuth ϕ .

Parton cascade

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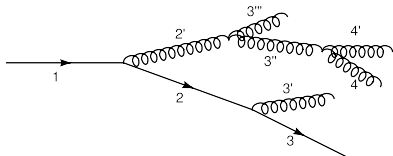
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ .

Not at all unique!

Many (more or less clever) choices still to be made.

Parton cascade

Get tree structure, ordered in evolution variable t :



- t can be $\theta, Q^2, p_{\perp}, \dots$
- Choice of hard scale $t_{\max}$ not fixed. “Some hard scale”.
- z can be light cone momentum fraction, energy fraction, $\dots$
- Available parton shower phase space.
- Integration limits.
- Regularisation of soft singularities.
- $\dots$

Good choices needed here to describe wealth of data!

Soft emissions

- Only *collinear* emissions so far.
- Including *collinear+soft*.
- *Large angle+soft* also important.

Soft emissions

- Only *collinear* emissions so far.
- Including *collinear+soft*.
- *Large angle+soft* also important.

Soft emission: consider *eikonal factors*,
here for $q(p+q) \rightarrow q(p)g(q)$, soft g :

$$u(p) \not{\epsilon} \frac{\not{p} + \not{q} + m}{(p+q)^2 - m^2} \longrightarrow u(p) \frac{p \cdot \epsilon}{p \cdot q}$$

soft factorisation. Universal, *i.e.* independent of emitter.

In general:

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{\alpha_S}{2\pi} \sum_{ij} C_{ij} W_{ij} \quad (\text{"QCD-Antenna"})$$

with

$$W_{ij} = \frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{iq})(1 - \cos \theta_{jq})} .$$

Soft emissions

We define

$$W_{ij} = \frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{iq})(1 - \cos \theta_{qj})} \equiv W_{ij}^{(i)} + W_{ij}^{(j)}$$

with

$$W_{ij}^{(i)} = \frac{1}{2} \left(W_{ij} + \frac{1}{1 - \cos \theta_{iq}} - \frac{1}{1 - \cos \theta_{qj}} \right) .$$

$W_{ij}^{(i)}$ is only collinear divergent if $q \parallel i$ etc .

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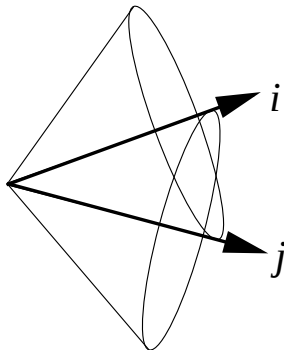
After integrating out the azimuthal angles, we find

$$\int \frac{d\phi_{iq}}{2\pi} W_{ij}^{(i)} = \begin{cases} \frac{1}{1 - \cos \theta_{iq}} & (\theta_{iq} < \theta_{ij}) \\ 0 & \text{otherwise} \end{cases}$$

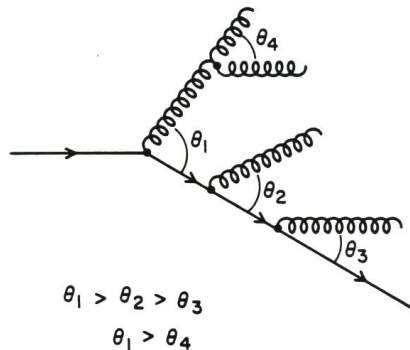
That's angular ordering.

Angular ordering

Radiation from parton i is bound to a cone, given by the colour partner parton j .



Results in angular ordered parton shower and suppresses soft gluons viz. hadrons in a jet.



Colour coherence from CDF

Events with 2 hard (> 100 GeV) jets and a soft 3rd jet (~ 10 GeV)

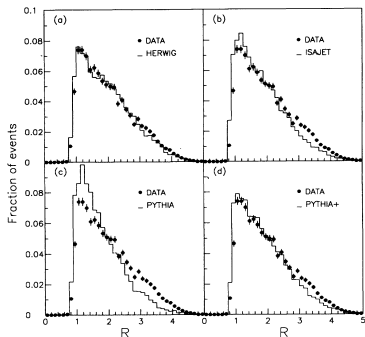


FIG. 14. Observed R distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

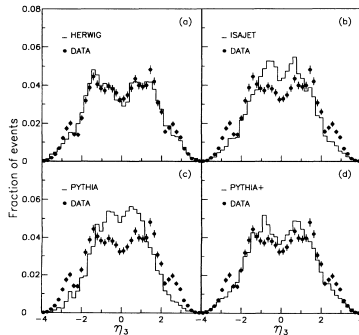


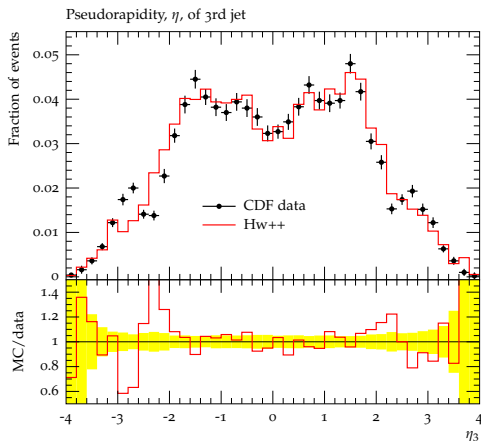
FIG. 13. Observed η_3 distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **50** (1994) 5562.

Best description with angular ordering.

Colour coherence from CDF

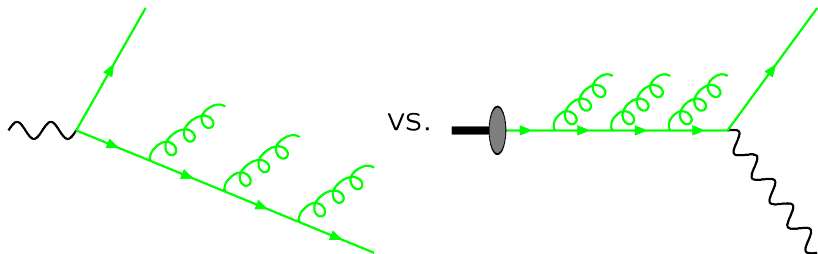
Events with 2 hard (> 100 GeV) jets and a soft 3rd jet (~ 10 GeV)



F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **50** (1994) 5562.

Best description with angular ordering.

Initial state radiation



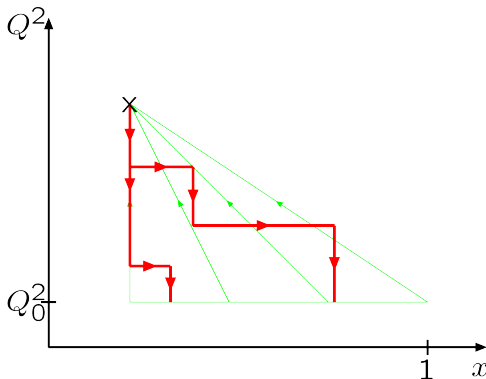
Similar to final state radiation. Sudakov form factor ($x' = x/z$)

$$\Delta(t, t_{\max}) = \exp \left[- \sum_b \int_t^{t_{\max}} \frac{dt}{t} \int_{z_-}^{z_+} dz \frac{\alpha_S(z, t)}{2\pi} \frac{x' f_b(x', t)}{x f_a(x, t)} \hat{P}_{ba}(z, t) \right]$$

Have to **divide out the pdfs**.

Initial state radiation

Evolve backwards from hard scale Q^2 *down* towards cutoff scale Q_0^2 . Thereby increase x .

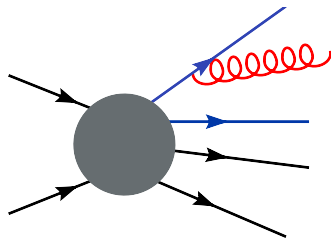


With parton shower we *undo* the DGLAP evolution of the pdfs.

Dipoles

Exact kinematics when recoil is taken by `spectator(s)`.

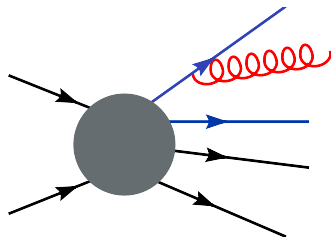
- Dipole showers.
- Ariadne.
- Recoils in Pythia.



Dipoles

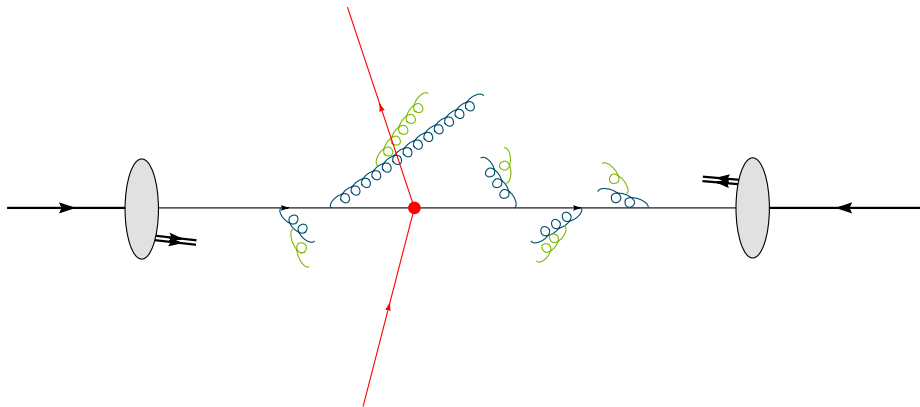
Exact kinematics when recoil is taken by **spectator(s)**.

- Dipole showers.
- Ariadne.
- Recoils in Pythia.
- New dipole showers, based on
 - Catani Seymour dipoles.
 - QCD Antennae.
 - Goal: matching with NLO.
- Generalized to IS–IS, IS–FS.

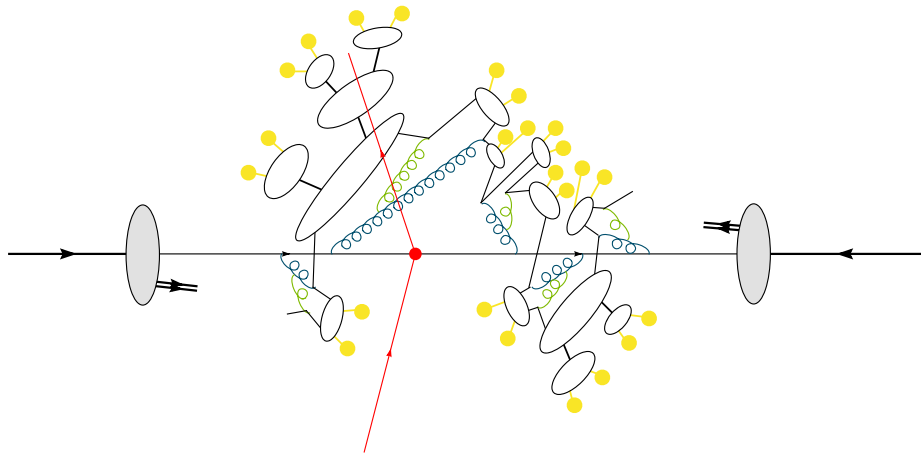


Hadronization

Parton shower



Parton shower $\longrightarrow$ hadrons



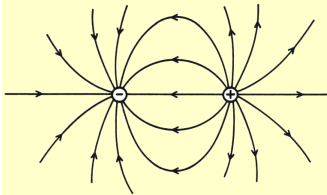
Parton shower $\longrightarrow$ hadrons

- Parton shower terminated at t_0 = lower end of PT.
- Can't measure quarks and gluons.
- Degrees of freedom in the detector are **hadrons**.
- Need a description of **confinement**.

Physical input

Self coupling of gluons
 $\leftrightarrow$ “attractive field lines”

QED FIELD LINES

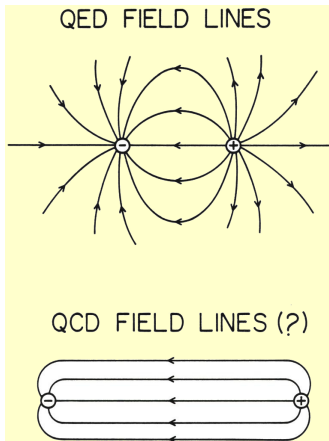


QCD FIELD LINES (?)

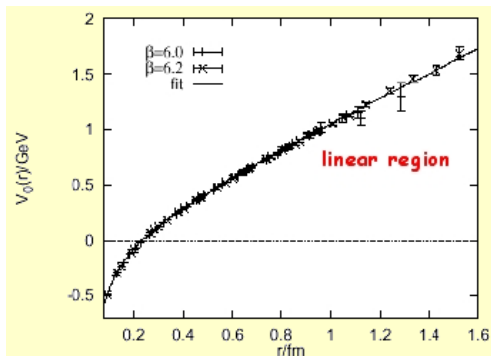


Physical input

Self coupling of gluons
↔ “attractive field lines”



Linear static potential $V(r) \approx \kappa r$.



Supported by lattice QCD,
hadron spectroscopy.

Hadronization models

Older models:

- Flux tube model.
- Independent fragmentation.

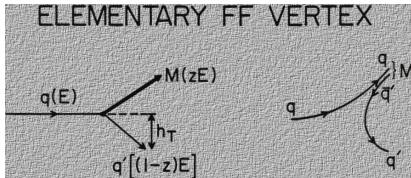
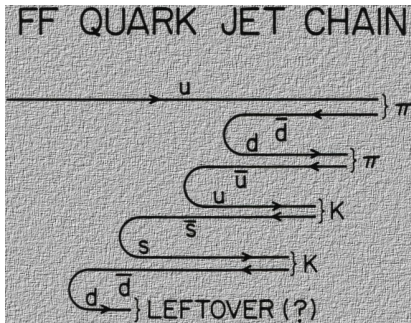
Today's models.

- Lund string model (Pythia).
- Cluster model (Herwig).

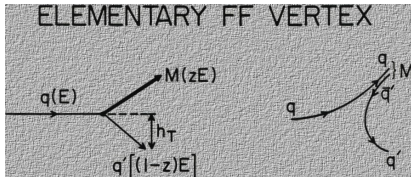
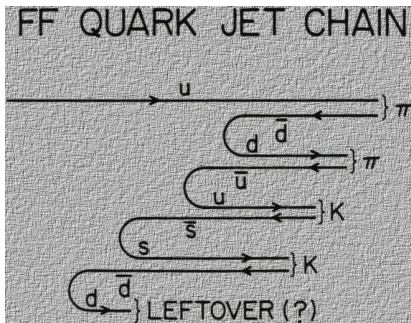
Independent fragmentation

Feynman–Field fragmentation ('78).

- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q \rightarrow h}(z)$ = density of momentum fraction z carried away by hadron h from quark q .
- Gaussian $p_{\perp}$ distribution.



Independent fragmentation



Feynman–Field fragmentation ('78).

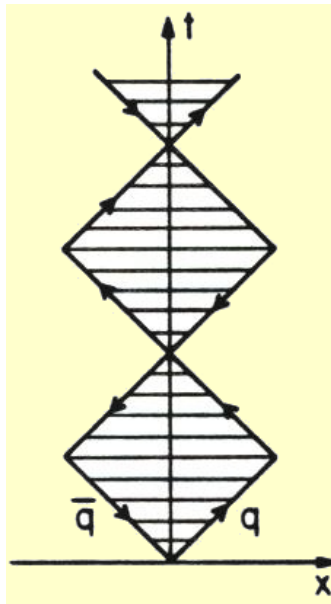
- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q \rightarrow h}(z)$ = density of momentum fraction z carried away by hadron h from quark q .
- Gaussian $p_{\perp}$ distribution.
- Problems:
 - “last quark”.
 - not Lorentz invariant.
 - infrared safety.
 - ...
- Good at that time.
- Still usefull for inclusive descriptions.

Lund string model

String model of mesons.

$L = 0$ mesons move in yoyo modes.

Area law: $m^2 \sim \text{area}$.



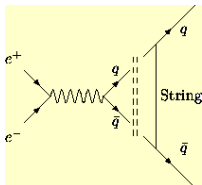
Lund string model

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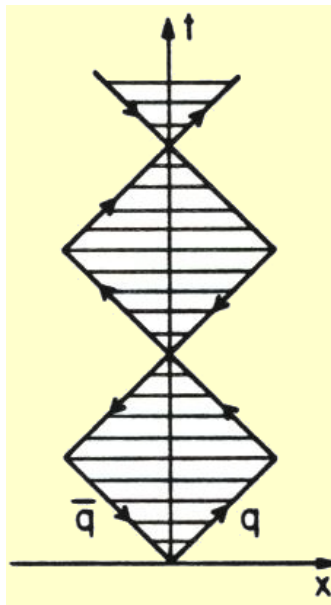
$L = 0$ mesons move in yoyo modes.

Area law: $m^2 \sim \text{area}$.

Simple model for particle production
in e^+e^- annihilation:



$q\bar{q}$ pair as pointlike source of string.

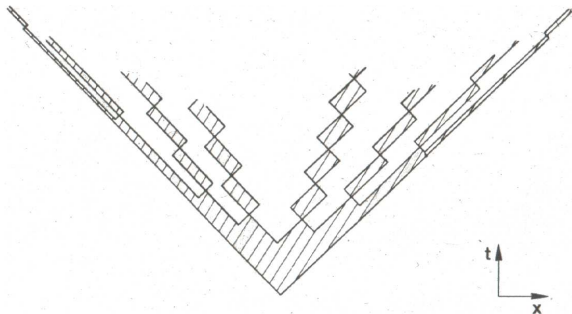


Lund string model

String energy $\sim$ intense chromomagnetic field.

→ Additional $q\bar{q}$ pairs created by QM tunneling.

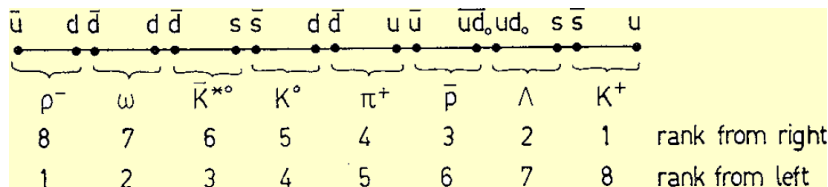
$$\frac{d\text{Prob}}{dxdt} \sim \exp\left(-\pi m_q^2/\kappa\right) \quad \kappa \sim 1\text{ GeV}.$$



String breaking expected long before yoyo point.

Lund string model

Ajacent breaks form hadrons.



Works in both directions (symmetry).

Lund symmetric fragmentation function

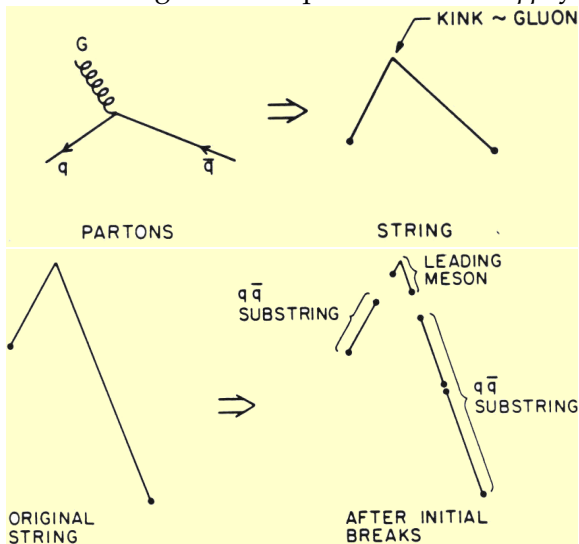
$$f(z, p_{\perp}) \sim \frac{1}{z} (1-z)^a \exp \left(-\frac{b(m_h^2 + p_{\perp}^2)}{z} \right)$$

a, b, m_h^2 main adjustable parameters.

Note: diquarks $\rightarrow$ baryons.

Lund string model

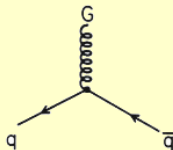
gluon = kink on string = motion pushed into the $q\bar{q}$ system.



Lund string model

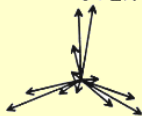
gluon = kink on string = motion pushed into the $q\bar{q}$ system.

SYMMETRIC PARTON CONFIGURATION



HADRONIZATION

INDEPENDENT FRAGMENTATION

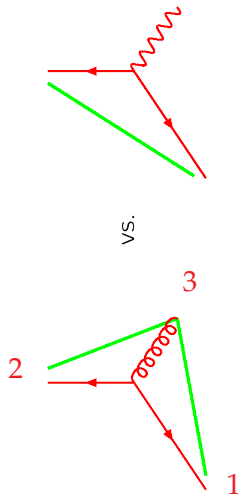


LUND PICTURE

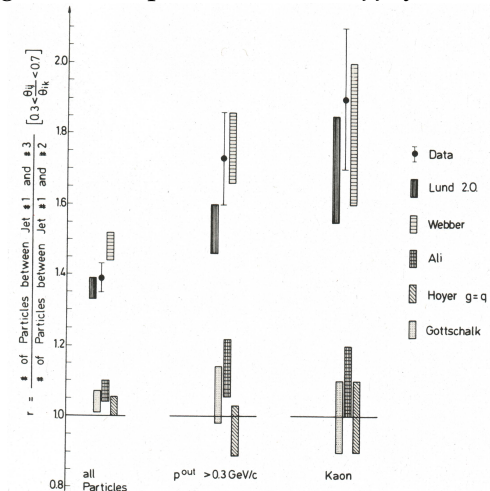


Lund string model

gluon = kink on string = motion pushed into the $q\bar{q}$ system.



"String effect"



Lund string model

Some remarks:

- Originally invented without parton showers in mind.

Lund string model

Some remarks:

- Originally invented without parton showers in mind.
- Strong physical motivation.
- Very successful description of data.
- Universal description of data
(fit at e^+e^- , transfer to hadron-hadron).
- Many parameters, ~ 1 per hadron.
- Too easy to hide errors in perturbative description?

Lund string model

Some remarks:

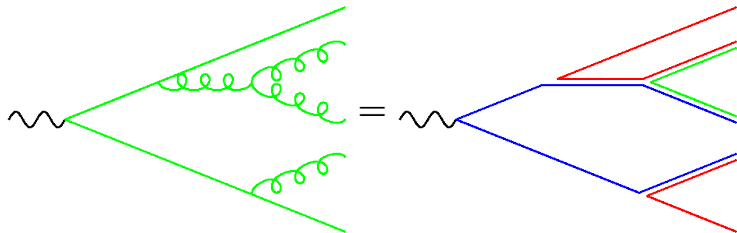
- Originally invented without parton showers in mind.
- Strong physical motivation.
- Very successful description of data.
- Universal description of data
(fit at e^+e^- , transfer to hadron-hadron).
- Many parameters, ~ 1 per hadron.
- Too easy to hide errors in perturbative description?

→ try to use more QCD information/intuition.

Colour preconfinement

Large N_C limit $\rightarrow$ planar graphs dominate.

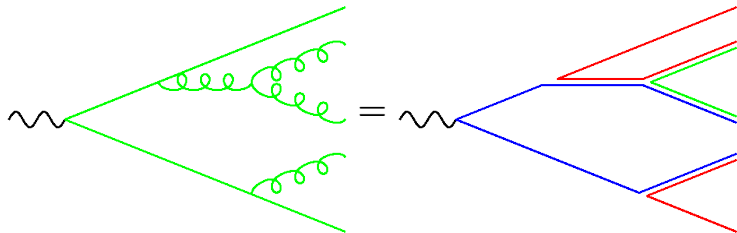
Gluon = colour — anticolourpair



Colour preconfinement

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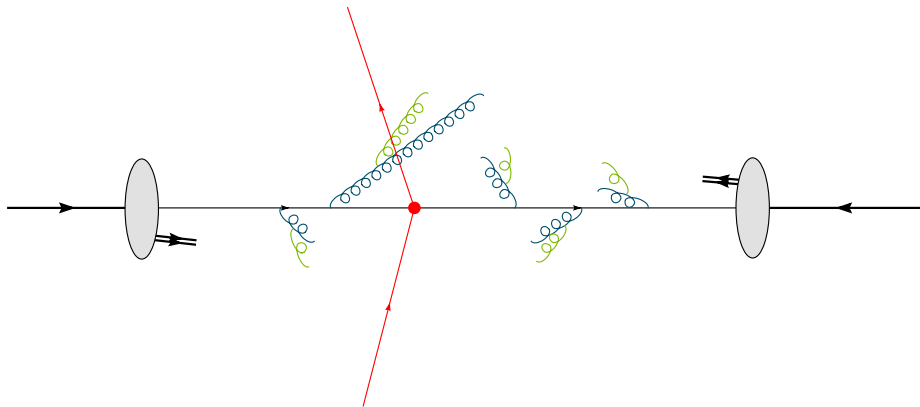
Gluon = colour — anticolourpair



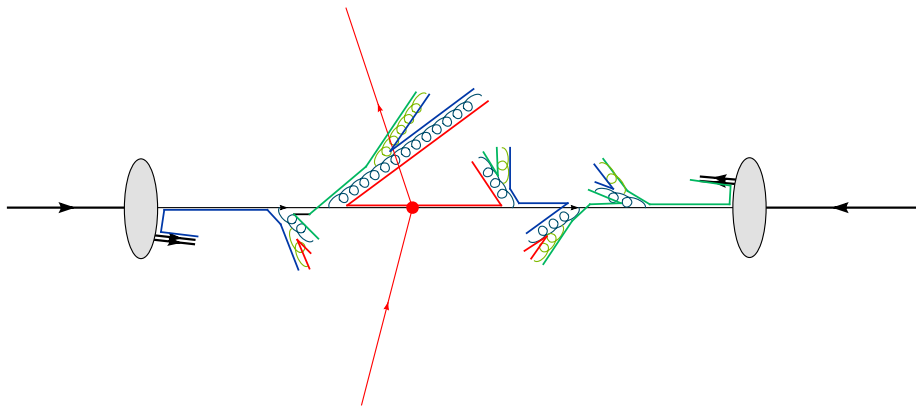
Parton shower organises partons in colour space. Colour partners (=colour singlet pairs) end up close in phase space.

$\rightarrow$ Cluster hadronization model

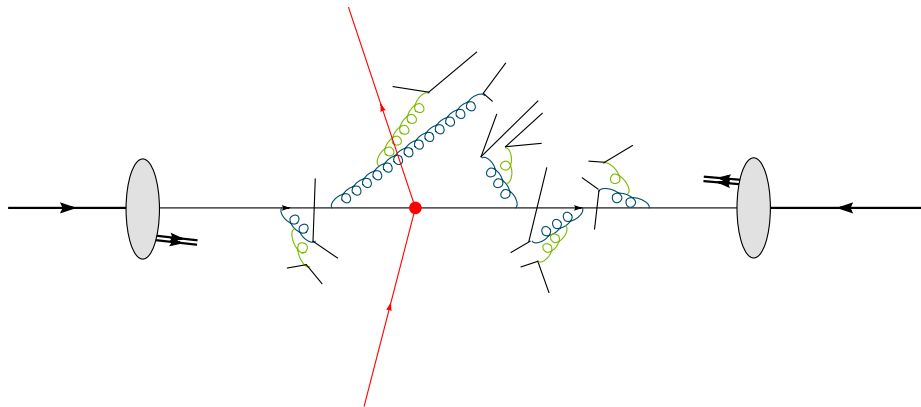
Cluster hadronization



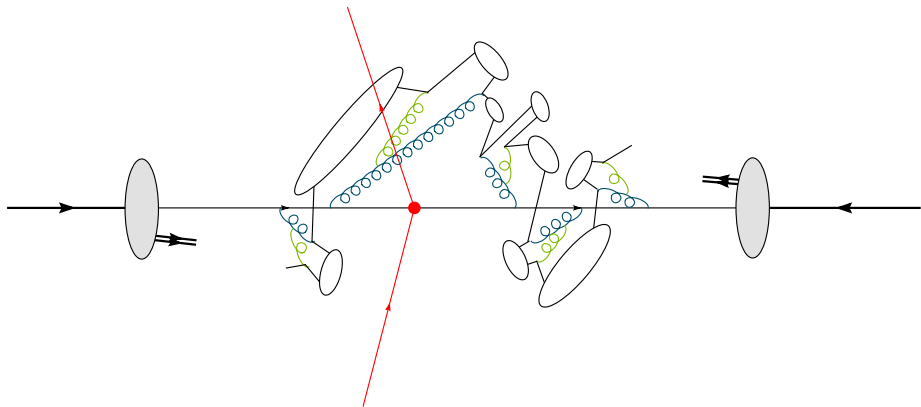
Cluster hadronization



Cluster hadronization

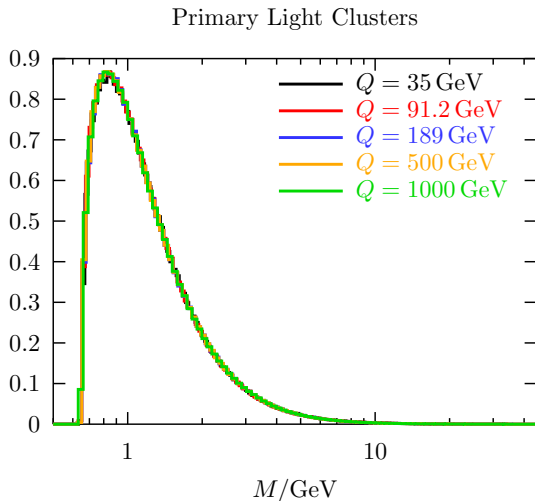


Cluster hadronization



Cluster hadronization

Primary cluster mass spectrum independent of production mechanism. Peaked at some low mass.



Cluster hadronization

Primary cluster mass spectrum independent of production mechanism. Peaked at some low mass.

Cluster = continuum of high mass resonances.

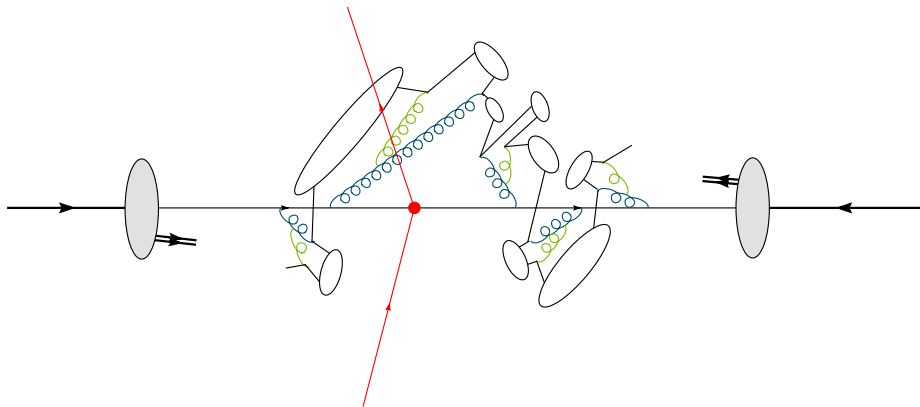
Decay into well-known lighter mass resonances
= discrete spectrum of hadrons.

No spin information carried over, i.e. only phase space.

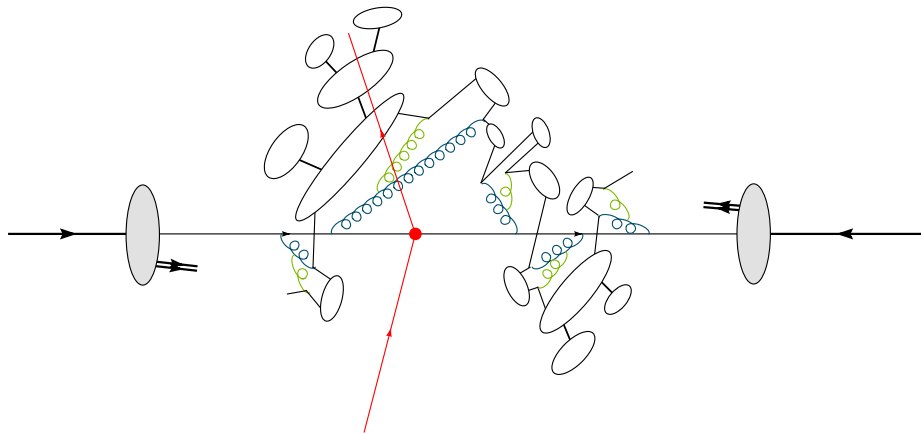
Suppression of heavier particles
(particularly baryons, can be problematic).

Cluster spectrum determined entirely by parton shower,
i.e. perturbation theory. Hence, t_0 crucial parameter.

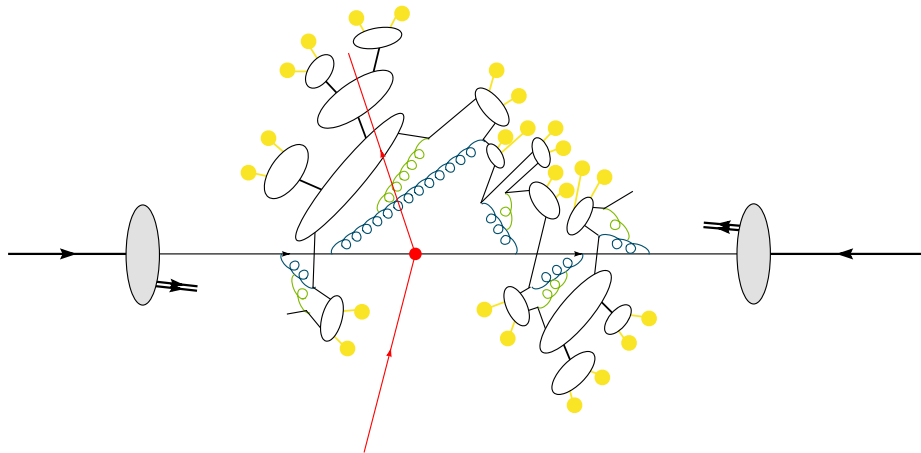
Cluster hadronization



Cluster hadronization



Cluster hadronization



Cluster hadronization in a nutshell

- **Nonperturbative $g \rightarrow q\bar{q}$ splitting** ($q = uds$) isotropically. Here, $m_g \approx 750\text{MeV} > 2m_q$.
- **Cluster formation**, universal spectrum (see below)
- **Cluster fission**, until

$$M^p < M_{\text{max}}^p + (m_1 + m_2)^p$$

where masses are chosen from

$$M_i = \left[\left(M^p - (m_i + m_3)^p \right) r_i + (m_i + m_3)^p \right]^{1/p},$$

with additional phase space constraints. Constituents keep moving in their original direction.

- **Cluster Decay**

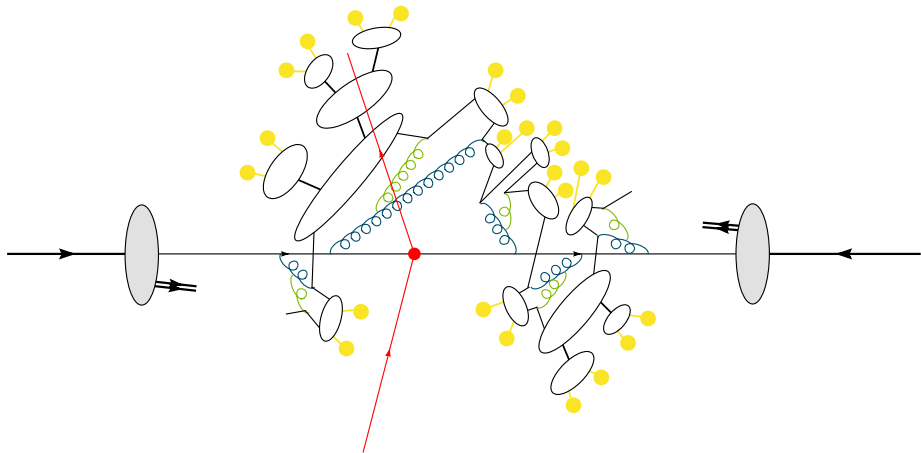
$$P(a_{i,q}, b_{q,j} | i, j) = \frac{W(a_{i,q}, b_{q,j} | i, j)}{\sum_{\mathbf{M/B}} W(c_{i,q'}, d_{q',j} | i, j)}.$$

Hadronization

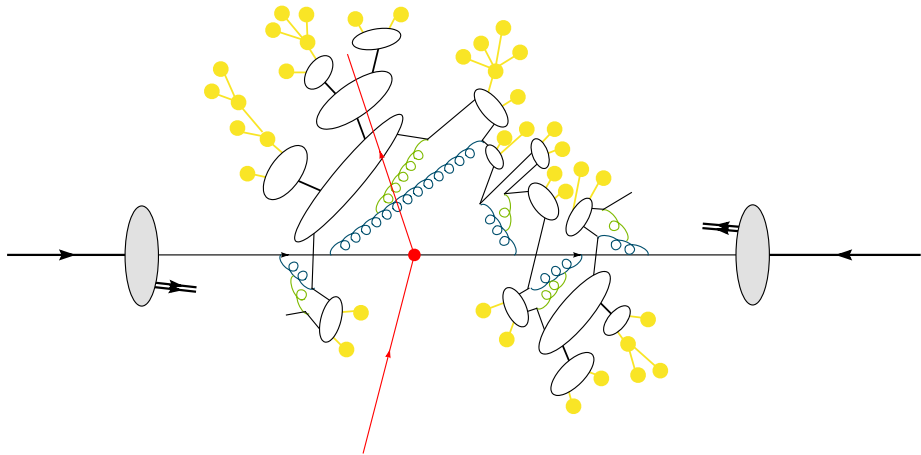
- Only string and cluster models used in recent MC programs.
Independent fragmentation only for inclusive observables.
- Strings started non-perturbatively, improved by parton shower.
- Cluster model started mostly on perturbative side, improved by string like cluster fission.

Hadronic Decays

Hadronic decays



Hadronic decays



Hadronic decays

Many aspects:

$$B^{*0} \rightarrow \gamma B^0$$

$$\hookrightarrow \bar{B}^0$$

$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$

$$\hookrightarrow \pi^+ D^0$$

$$\hookrightarrow K^- \rho^+$$

$$\hookrightarrow \pi^+ \pi^0$$

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Hadronic decays

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EM decay.

Hadronic decays

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Weak mixing.

Hadronic decays

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Strong decay.

Hadronic decays

Many aspects:

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$$\hookrightarrow \pi^+ D^0$$

$$\hookrightarrow K^- \rho^+$$

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$$\hookrightarrow e^+ e^- \gamma$$

Weak decay, ρ^+ mass smeared.

Hadronic decays

Many aspects:

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$$\hookrightarrow \pi^+ D^0$$

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$$\hookrightarrow \pi^+ \pi^0$$

$$\hookrightarrow e^+ e^- \gamma$$

ρ^+ polarized, angular correlations.

Hadronic decays

Many aspects:

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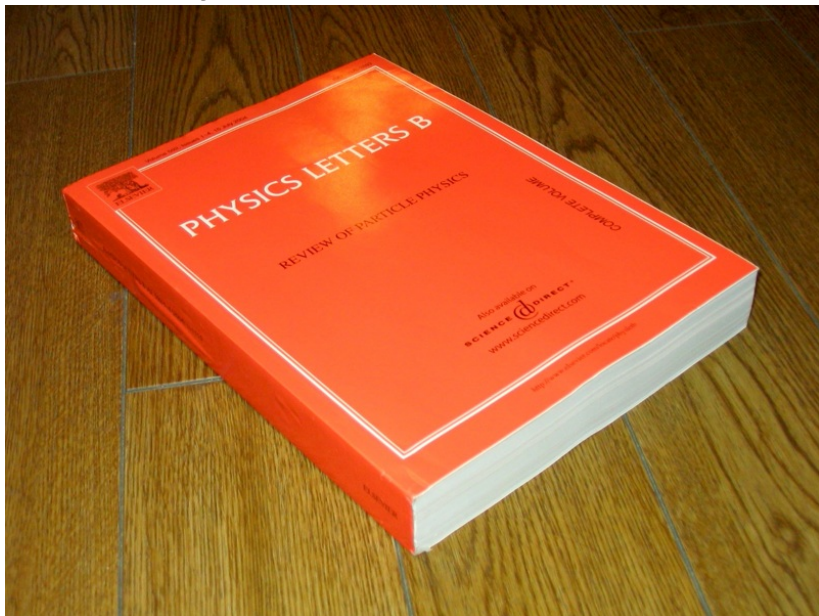
Dalitz decay, m_{ee} peaked.

Hadronic decays

Tedious.

100s of different particles, 1000s of decay modes,
phenomenological matrix elements with parametrized form
factors...

Hadronic decays



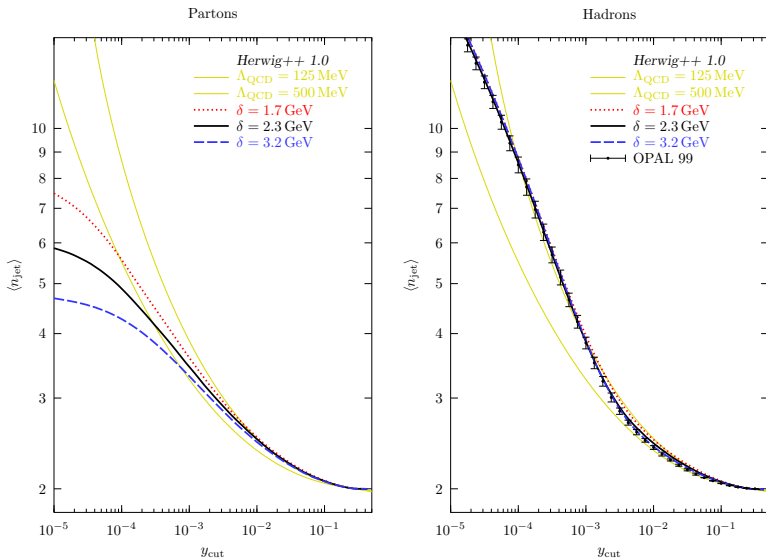
A few plots

How well does it work?

- $e^+e^- \rightarrow$ hadrons, mostly at LEP.
- Jet shapes, jet rates, event shapes, identified particles...
- 'Tuning' of parameters.
- Use *all* analyses available in Rivet.
- Want to get *everything* right with *one* parameter set.
- Compare to literally ≈ 20000 plots.

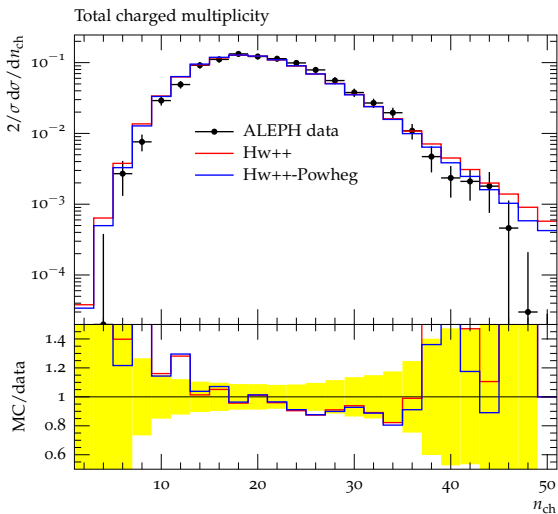
How well does it work?

Smooth interplay between shower and hadronization.



How well does it work?

N_{ch} at LEP. Crucial for t_0 (Herwig++ 2.5.2)



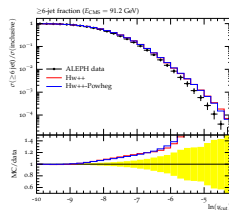
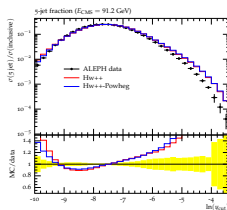
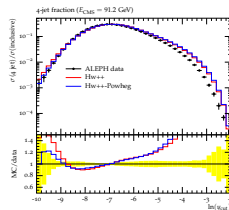
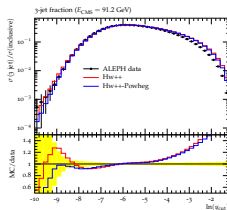
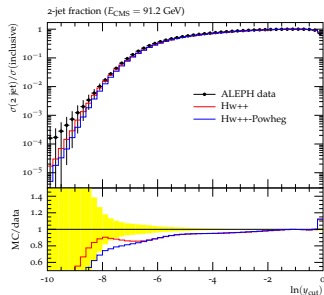
How well does it work?

Jet rates at LEP.

$$R_n = \sigma(n\text{-jets})/\sigma(\text{jets})$$

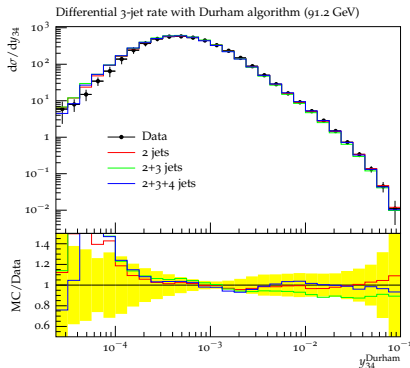
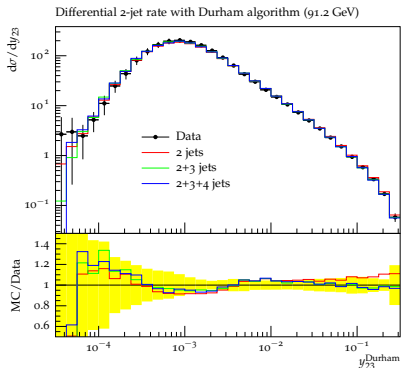
$$R_6 = \sigma(> 5\text{-jets})/\sigma(\text{jets})$$

(Herwig++ 2.5.2)



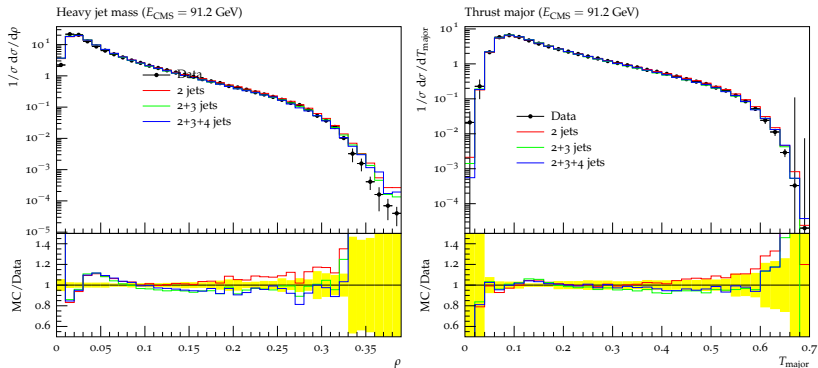
How well does it work?

Differential Jet Rates at LEP (Herwig++ pre-3.0). Dipole shower + some merging



How well does it work?

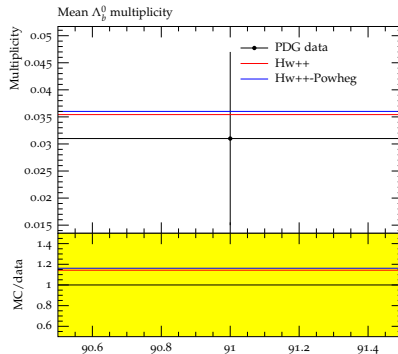
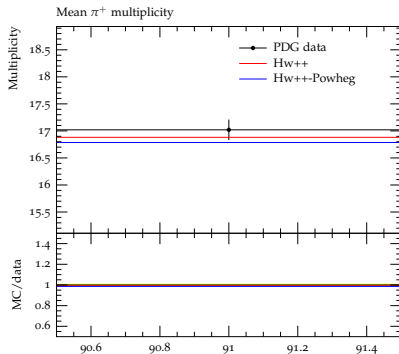
Event Shapes at LEP (Herwig++ pre-3.0).
Dipole shower + some merging



Parton showers do very well, today!

How well does it work?

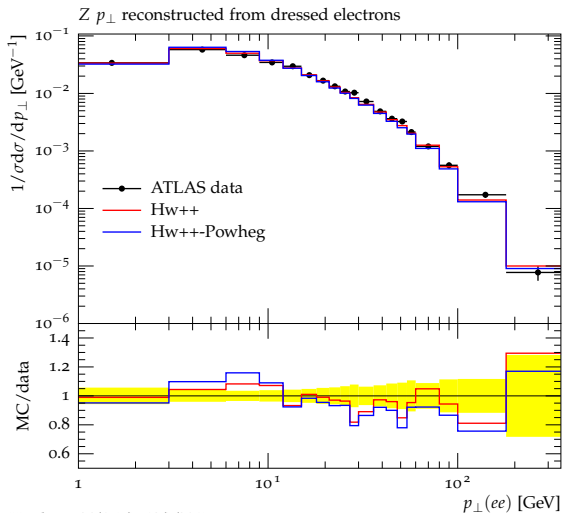
Hadron Multiplicities at LEP (e.g. π^+ , Λ_b^0).



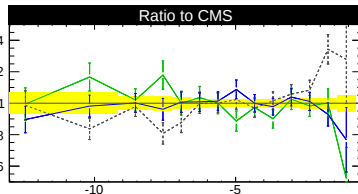
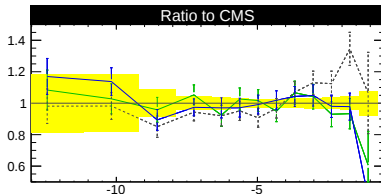
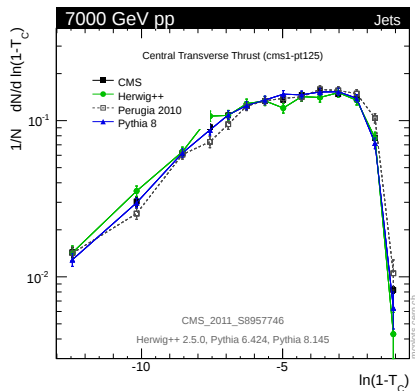
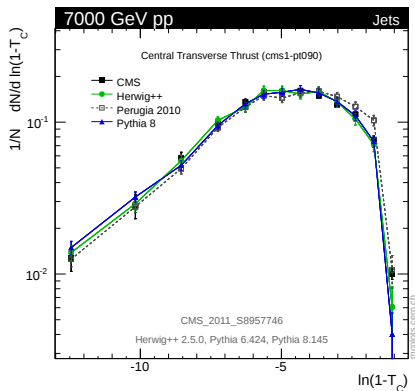
How well does it work?

$p_{\perp}(Z^0) \rightarrow \text{intrinsic } k_{\perp}$ (LHC 7 TeV).

See also in context of matching/marging.

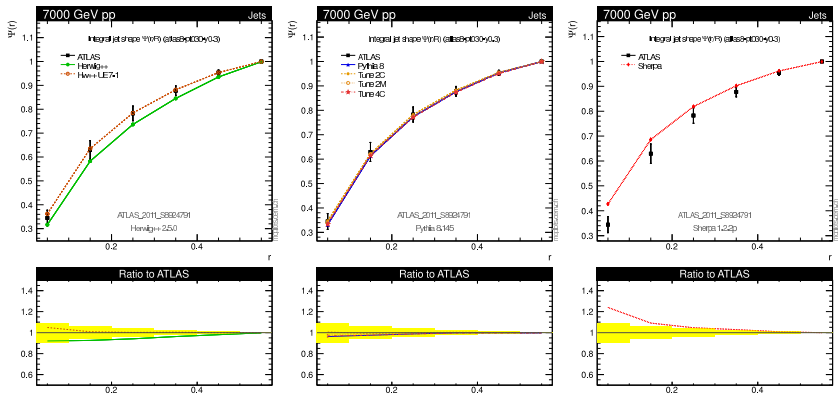


Transverse thrust



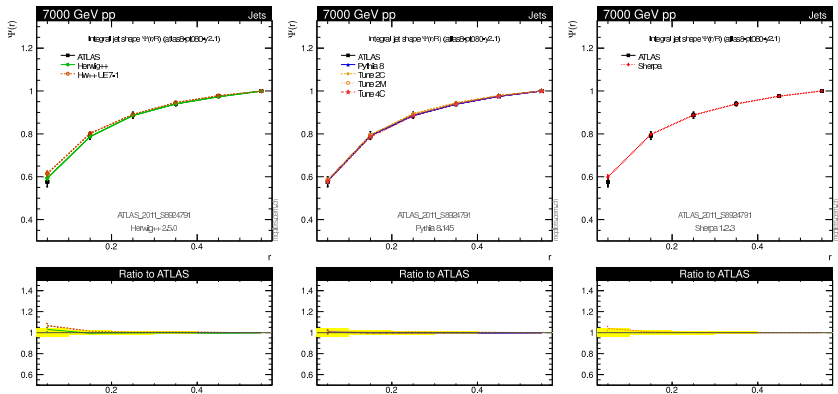
Integral jet shapes

not too hard, central ($30 < p_T/\text{GeV} < 40; 0 < |y| < 0.3$)



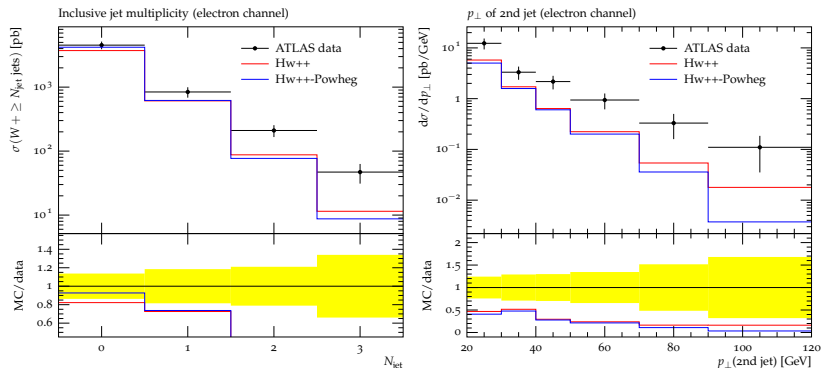
Integral jet shapes

harder, more forward ($80 < p_T/\text{GeV} < 110; 1.2 < |y| < 2.1$)



Limits of parton shower

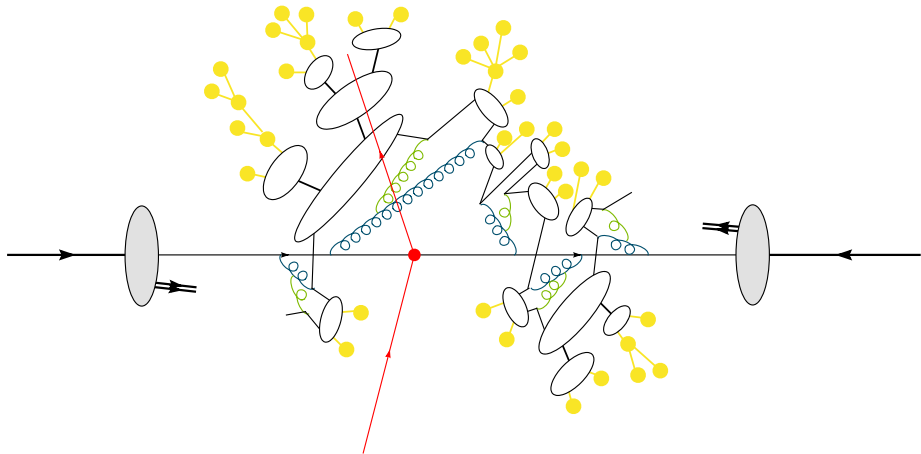
$W + \text{jets}$, LHC 7 TeV.



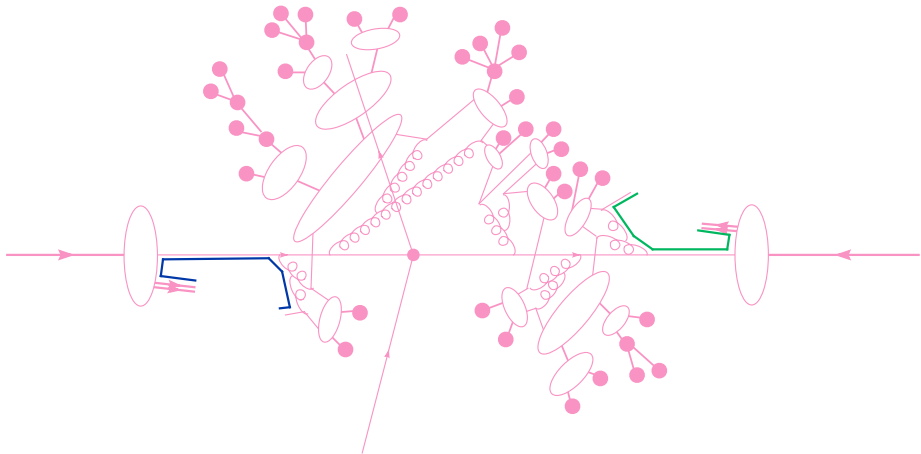
Higher jets not covered by parton shower only $\rightarrow$ merging.

Min Bias/Underlying event in data

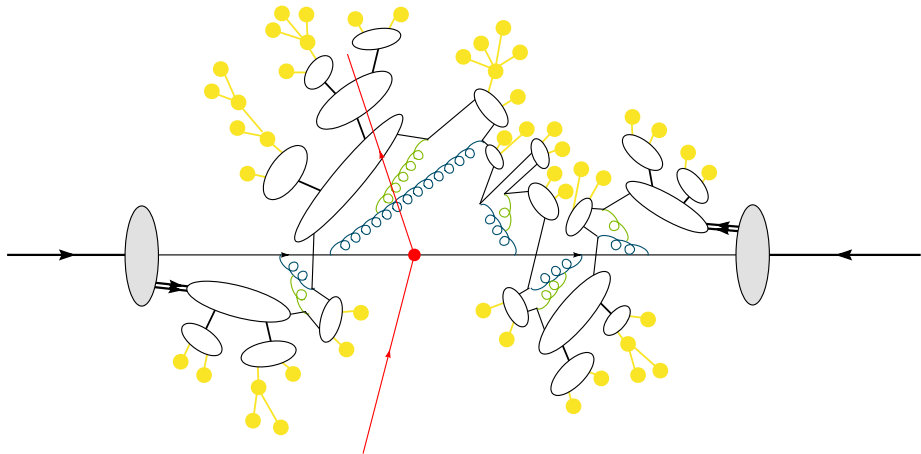
pp Event Generator



pp Event Generator



pp Event Generator

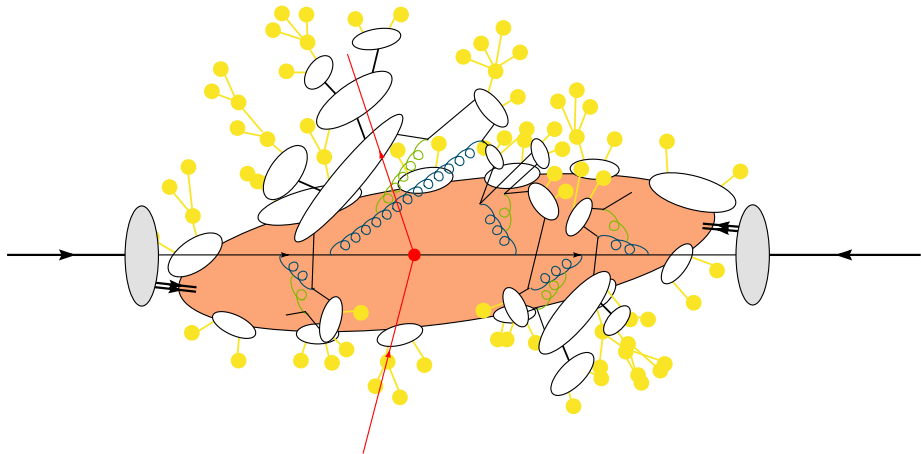


No UE model

Just remnant clusters

- Simplest model?
- Connects loose colour ends and produces some N_{ch} .
- No extra transverse energy.
- Fails.

pp Event Generator



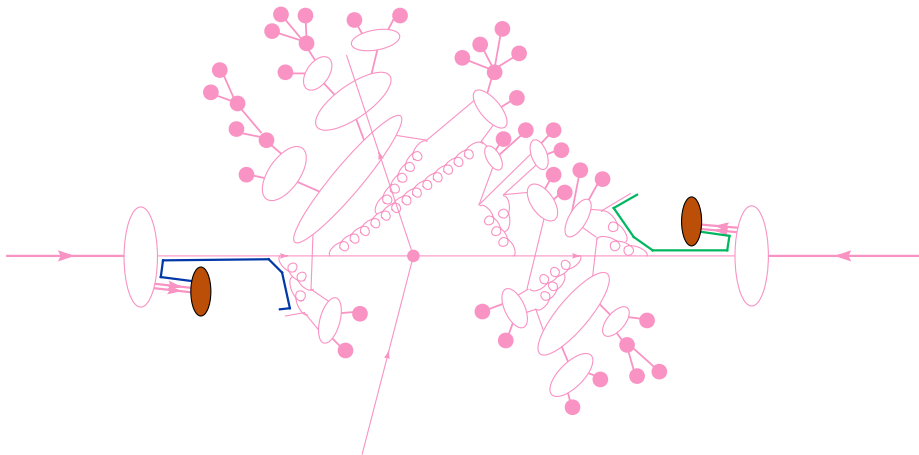
UA5 model

UA5 model

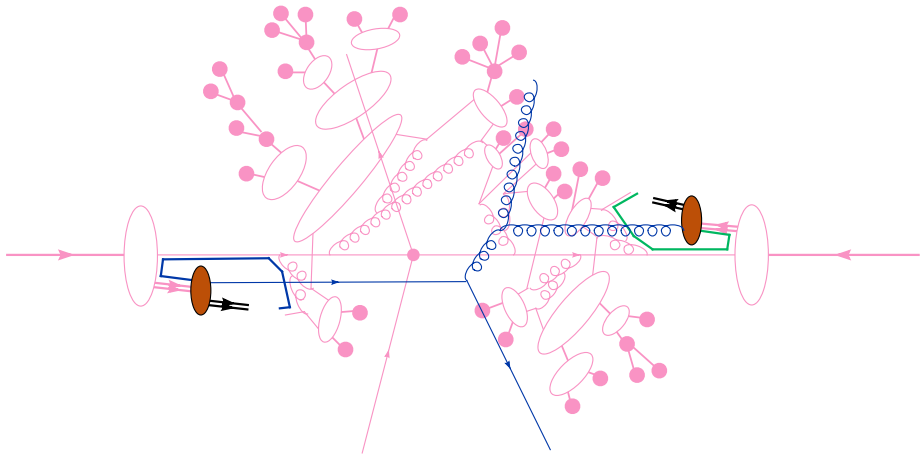
- Produce $\langle n \rangle$ extra clusters, flat in y , with soft $p_{\perp}$ spectrum.
- Included from Herwig++ 2.0. [Herwig++, hep-ph/0609306]
- Little predictive power.
- Only gets averages right, not large (and interesting!) fluctuations $\rightarrow$ mini jets.
- Was default in fHerwig. Superseded by JIMMY.

[JM Butterworth, JR Forshaw, MH Seymour, ZP C72 637 (1996)]

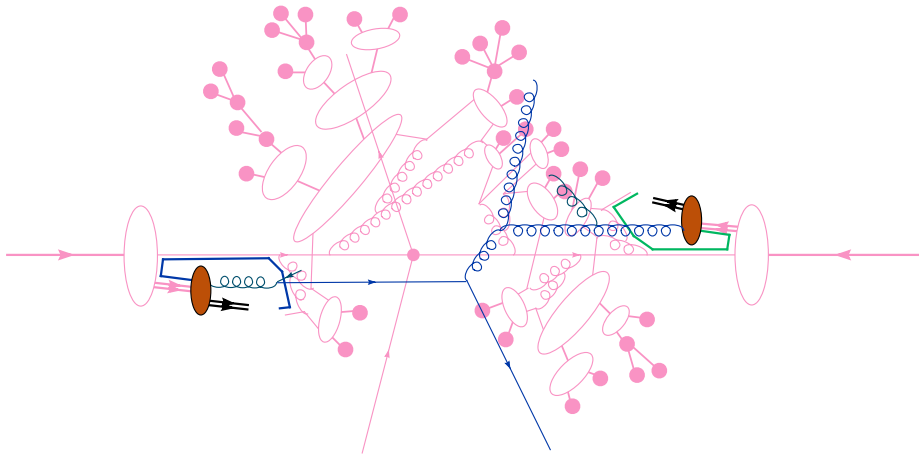
pp Event Generator



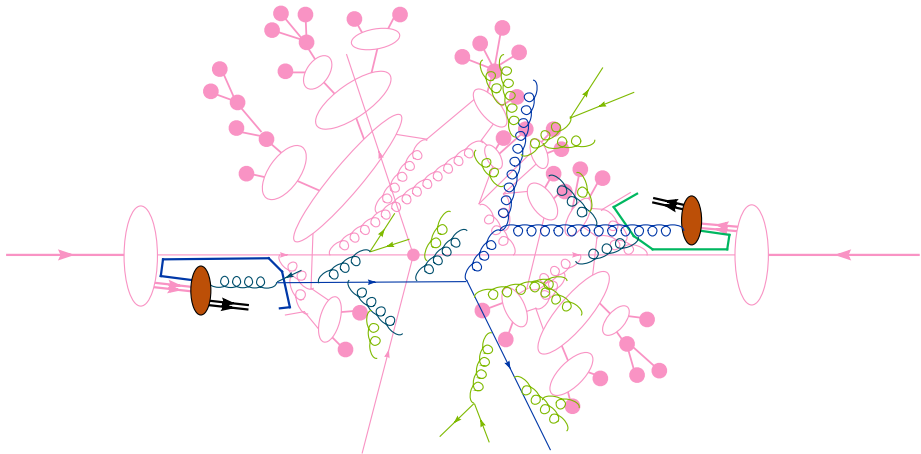
pp Event Generator



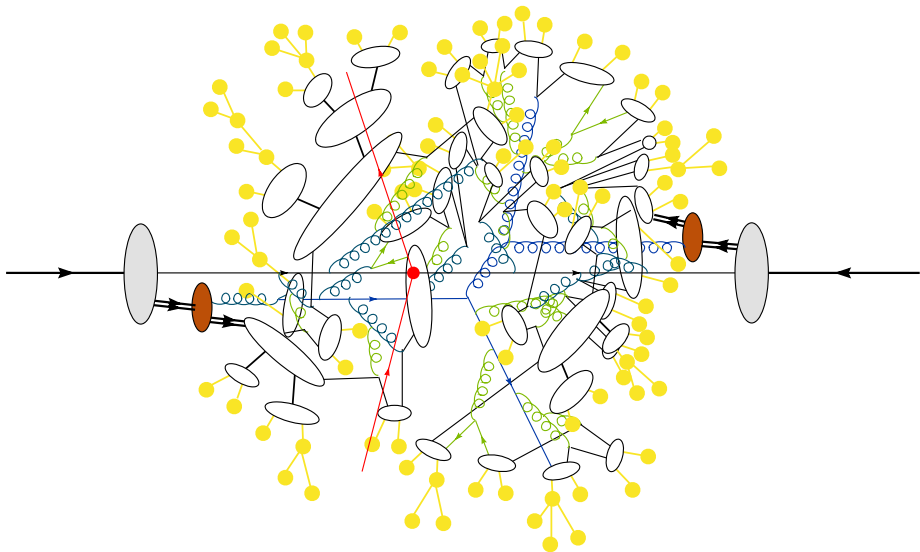
pp Event Generator



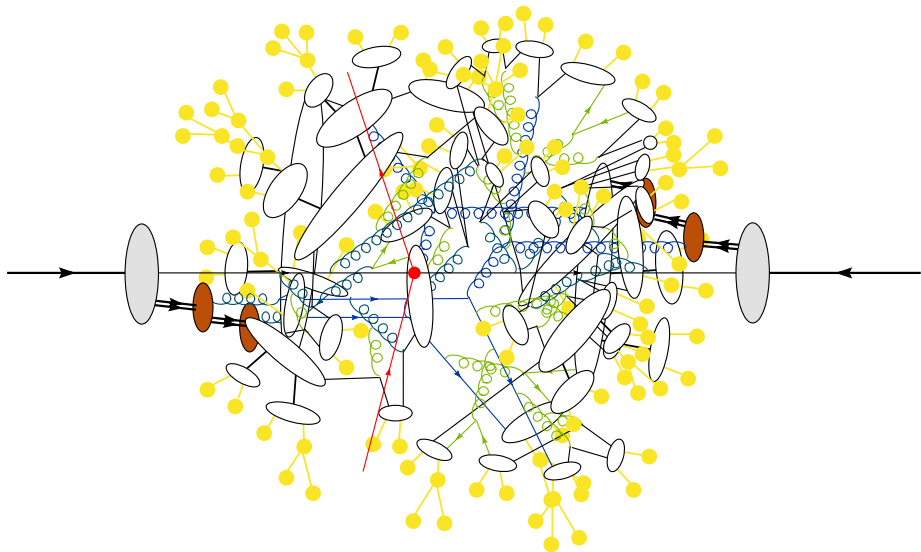
pp Event Generator



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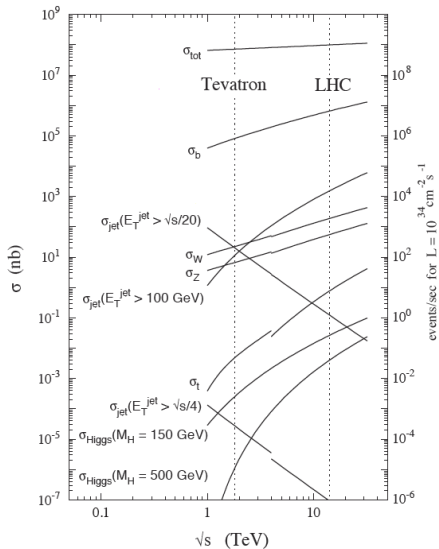


Collider cross sections

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \underbrace{\sigma_{\text{SD}} + \sigma_{\text{DD}}}_{\sigma_{\text{Diff}}} + \overbrace{\sigma_{\text{soft}} + \sigma_{\text{hard}}}^{\sigma_{\text{NSD}}}$$

σ_{ND}

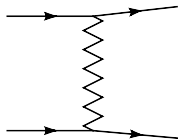
Collider cross sections



What is the Underlying event?

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{SD}} + \overbrace{\sigma_{\text{DD}} + (\sigma_{\text{soft}} + \sigma_{\text{hard}})}^{\sigma_{\text{NSD}}}$$

σ_{ND}

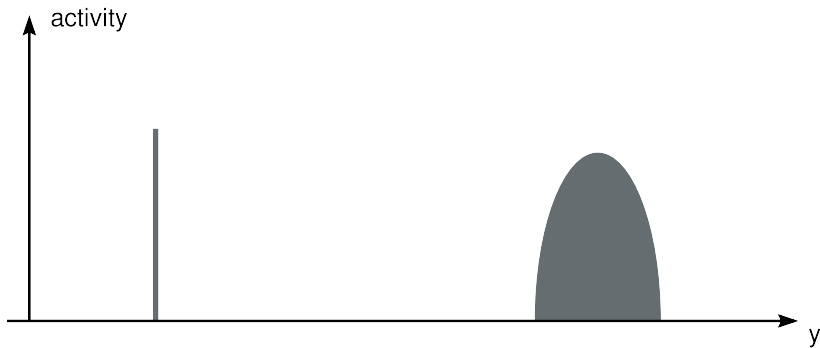
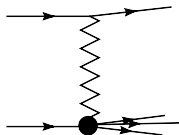


elastic

What is the Underlying event?

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{SD}} + \overbrace{\sigma_{\text{DD}} + (\sigma_{\text{soft}} + \sigma_{\text{hard}})}^{\sigma_{\text{NSD}}}$$

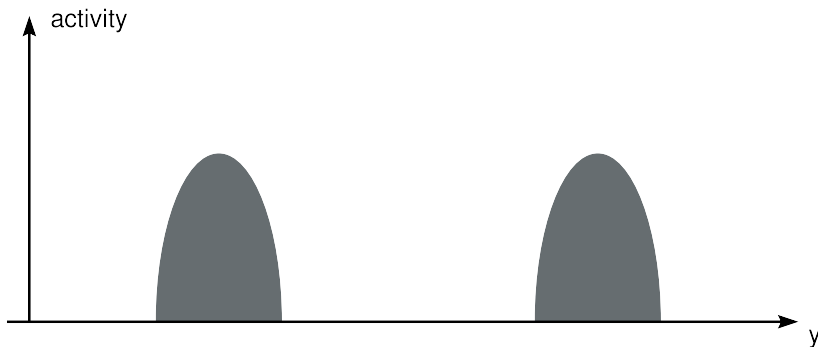
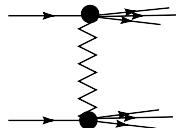
σ_{ND}



single diffractive

What is the Underlying event?

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{SD}} + \underbrace{\sigma_{\text{DD}} + (\sigma_{\text{soft}} + \sigma_{\text{hard}})}_{\sigma_{\text{ND}}} \underbrace{\sigma_{\text{NSD}}}_{\sigma_{\text{ND}}}$$

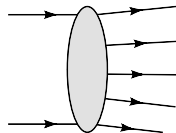


double diffractive

What is the Underlying event?

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σ_{ND}

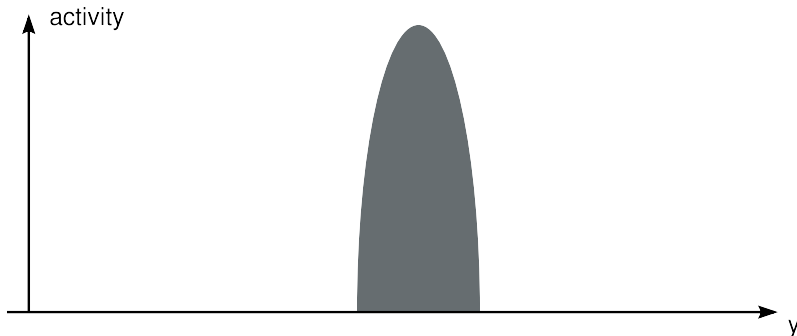
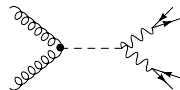


(multiple/soft) interactions

What is the Underlying event?

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{SD}} + \overbrace{\sigma_{\text{DD}} + (\sigma_{\text{soft}} + \sigma_{\text{hard}})}^{\sigma_{\text{NSD}}}$$

σ_{ND}

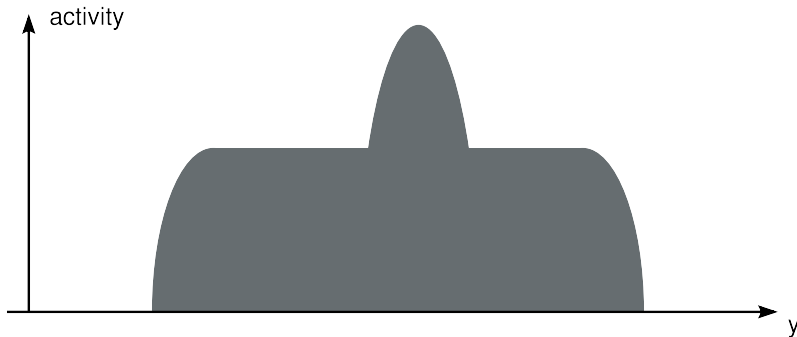


hard scattering

What is the Underlying event?

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{SD}} + \overbrace{\sigma_{\text{DD}} + (\sigma_{\text{soft}} + \sigma_{\text{hard}})}^{\sigma_{\text{NSD}}}$$

σ_{ND}



hard scattering + underlying event

What is the Underlying event?

“Everything except the process of interest.”

- Experimentalist: “includes parton showers etc.”
- MC author: “everything on top of primary hard process.”

The Underlying event (UE) is everywhere in the detector.

- Cannot select UE
- May spoil measurements.
- What characteristics?
- Hard?
- Soft?

Why should I learn about it?

- UE comes with every event.
- Can't trigger/select it away.
- Gives additional tracks and calorimeter hits, in the same cells as your signal.
- Jet energy scale determination.
- Important systematic error.
- Jets where your signal shouldn't give any (VBF).

Triggers

- Zero bias
 - *Every* event in a perfect 4π detector.

Triggers

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 - *Every* event in a perfect 4π detector.
- Minimum bias (MB)
 - Require “some activity”
 - At least have to distinguish from noise/cosmics.
 - small number of tracks of charged tracks (e.g. 1, 2, 6),
 - forward calorimeter hits,
 - $\rightarrow$ with some minimum $p_{\perp}$.
 - Often want non-single-diffractive

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 - Very selective trigger
 - BUT accompanied by soft stuff $\rightarrow$ underlying event.

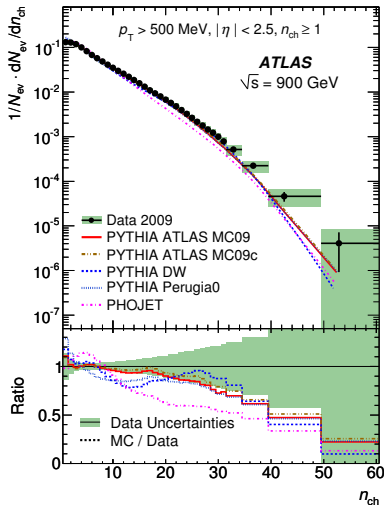
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 - BUT accompanied by soft stuff $\rightarrow$ underlying event.

Physics in MB and UE very similar.

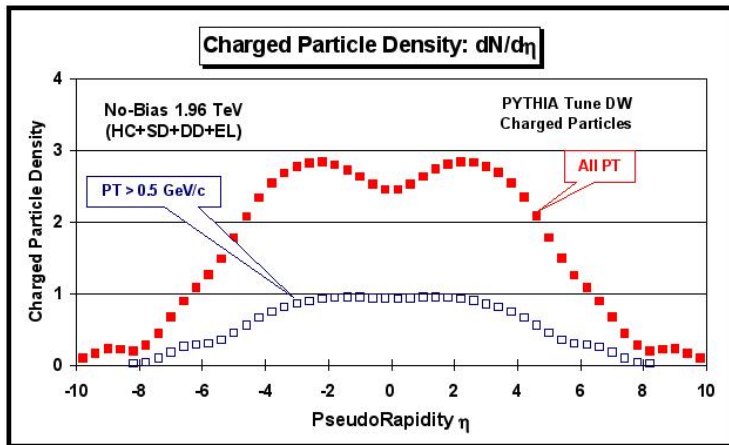
Characteristics of MB events

N_{ch}



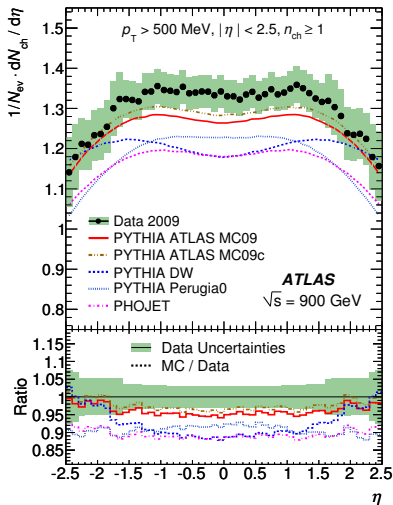
Charakteristics of MB events

$dN/d\eta$ Zero bias vs min bias (Tevatron)



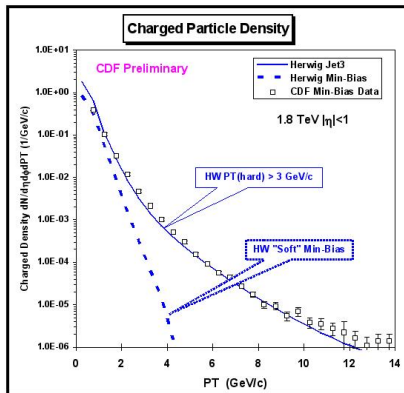
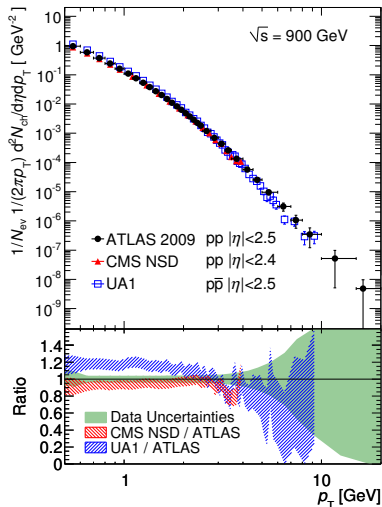
Characteristics of MB events

$dN/d\eta$ ATLAS



Characteristics of MB events

$p_{\perp}$ spectra of all particles



Charakteristics of MB events

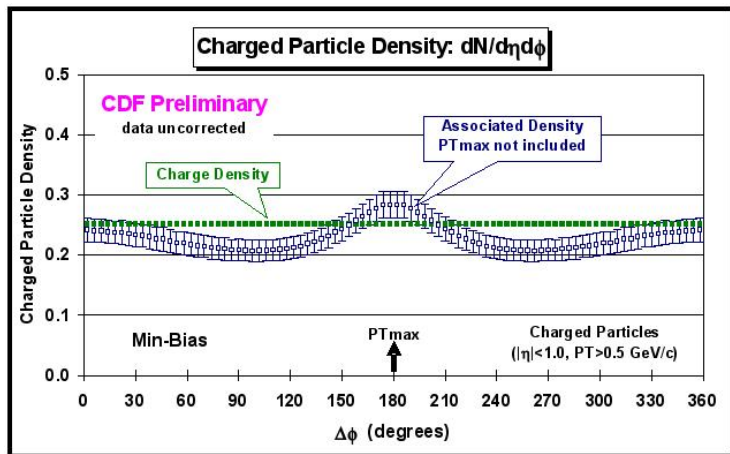
- Inclusive quantities have to be correct, of course.
- Already show, that soft component is important in modelling.

Characteristics of MB events

- Inclusive quantities have to be correct, of course.
- Already show, that soft component is important in modelling.
- Don't tell much about morphology of event.
- → look at distributions inside detector.
- → leading particles.

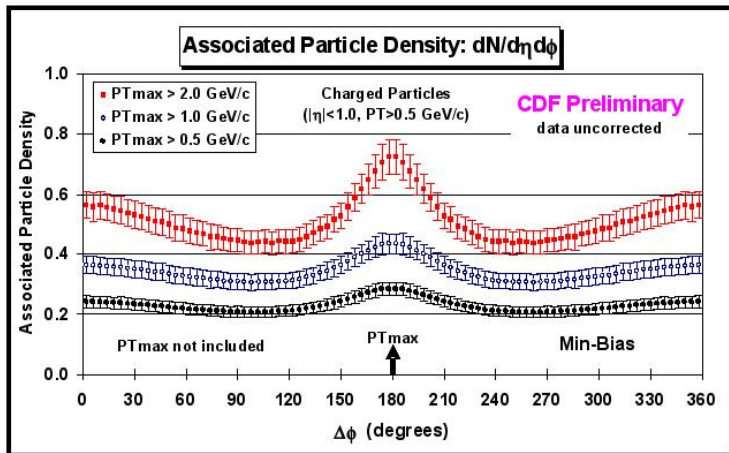
Azimuthal distributions

Measure $\Delta\phi$ relative to leading particle/jet/track.



Azimuthal distributions

Measure $\Delta\phi$ relative to leading particle/jet/track.



Azimuthal distributions

Observation:

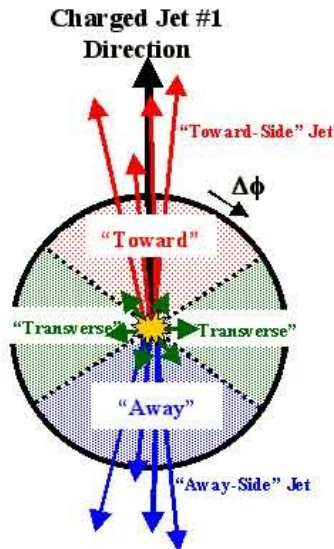
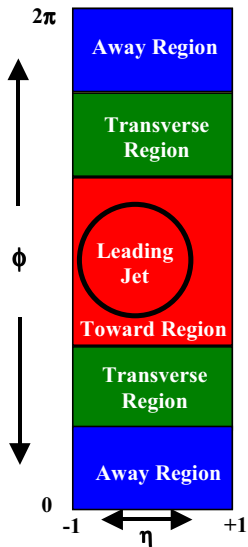
- Events not flat. Have 'leading object'.
- Harder leading object:
 - harder recoil.
 - more activity everywhere, also transverse.

Trigger: The harder leading object, the more jets are inclusively just below this threshold (pedestal effect).

Closer look at transverse region!

“Rick Field analysis”.

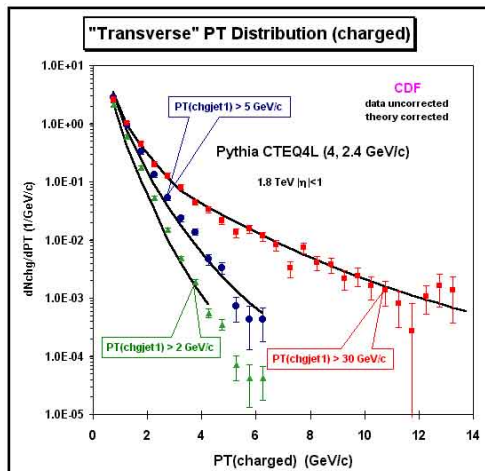
Towards, away, transverse



Measurements of the UE: separate from hard bit of event.

- How big is the 'activity' in the different regions?
- How does it depend on the leading object?
- If UE is really *underlying*,
should decouple from leading event.

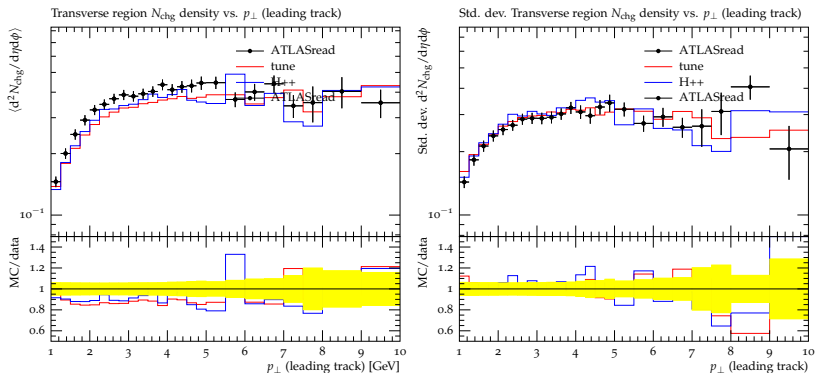
Spectrum in transverse region



Not only average important. The UE has a jetty substructure!

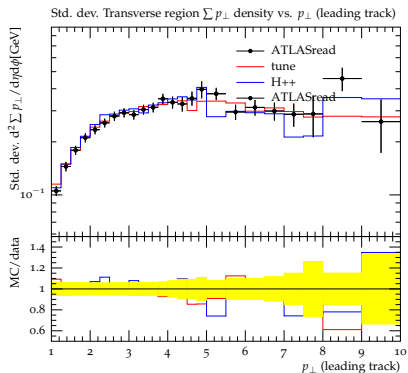
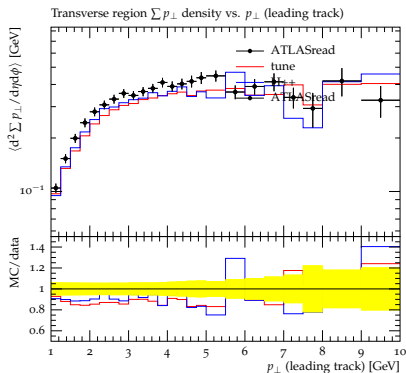
Underlying Event (ATLAS 900 GeV)

⟨“activity”⟩ and 1σ deviation



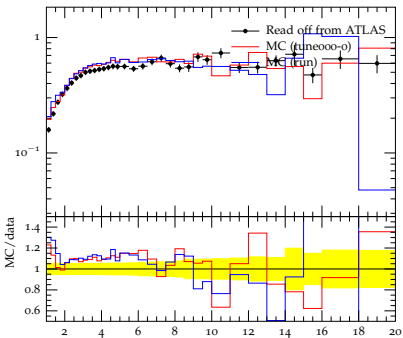
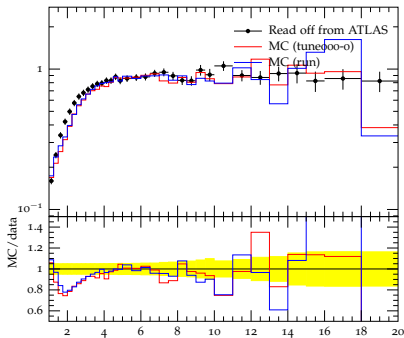
Underlying Event (ATLAS 900 GeV)

⟨“activity”⟩ and 1σ deviation



Underlying Event (ATLAS 7 TeV)

$N_{\text{ch}}/\text{StdDev}$ transverse vs $p_t^{\text{lead}}/\text{GeV}$.

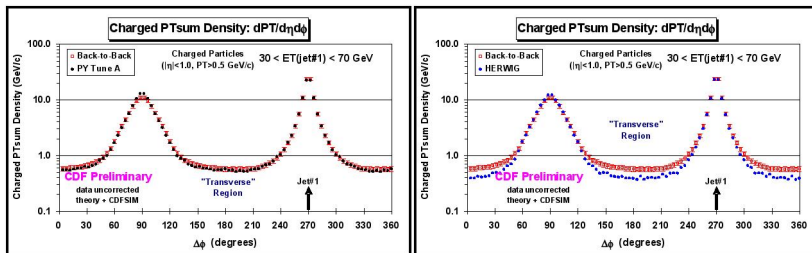


So far

- Idea of decoupling UE from hard event seems to hold.
- UE has jetty structure.
- Must contain hard physics as well.

More azimuthal distributions

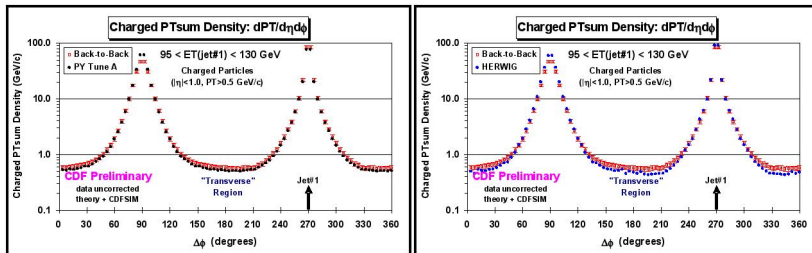
Require at least two nearly b2b jets.
Dominated by hard physics.



Old Herwig soft model not sufficient.

More azimuthal distributions

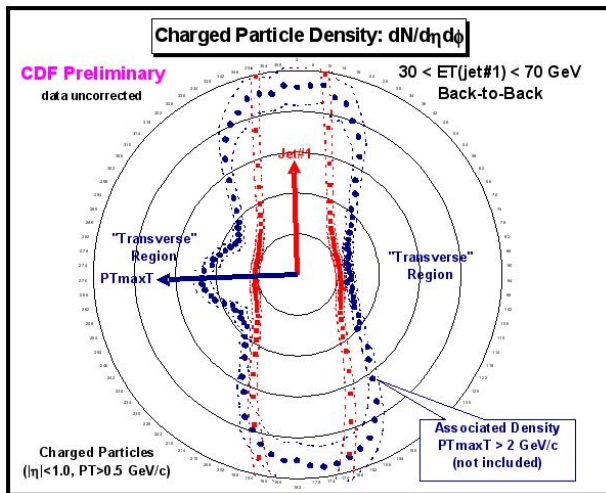
Require at least two nearly b2b jets.
Dominated by hard physics.



Better with harder jets.

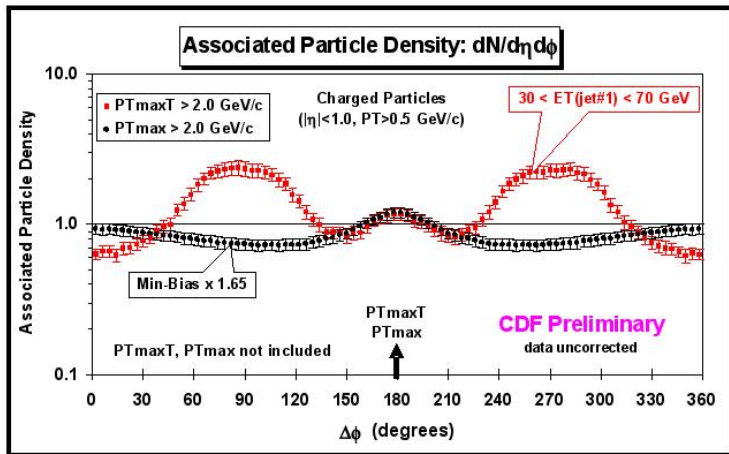
More azimuthal distributions

Now select the hardest of the two transverse regions only (TransMAX): associated distribution:



More azimuthal distributions

Now select the hardest of the two transverse regions only (TransMAX): associated distribution:



Birth of 3rd jet $\sim$ leading jet in MinBias

Towards modelling

- Leading jet in Minimum bias $\sim$ 3rd jet in back-to-back sample.
- UE and MB really seem to reflect the same physics.
- Hard component important.
- Hard jets not sufficient
(but well described $\rightarrow$ D0 dijet angular decorrelation).

Hard jets in the UE via multiple interactions?

- Additional Partonic $2 \rightarrow 2$ interactions (MPI).
- No correlation with hard event.

Indirect evidence for MPI

N_{ch} distribution (vs UA5; Sjöstrand, van Zijl (1987))

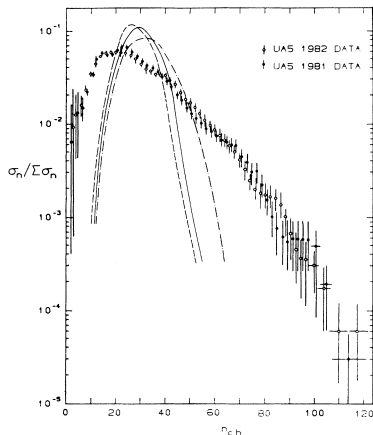


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low p_T only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

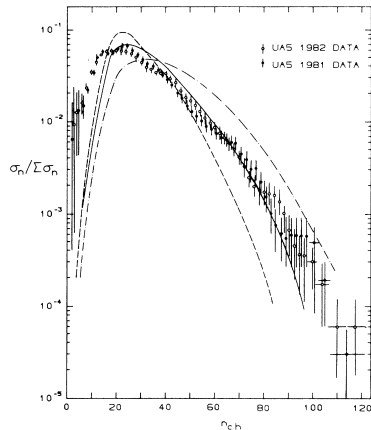


FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line, $p_{T\text{min}} = 2.0$ GeV; solid line, $p_{T\text{min}} = 1.6$ GeV; dashed-dotted line, $p_{T\text{min}} = 1.2$ GeV.

no MPI (left)/MPI (right).

Indirect evidence for MPI

FB correlation in η bins (vs UA5; Sjöstrand, van Zijl (1987))

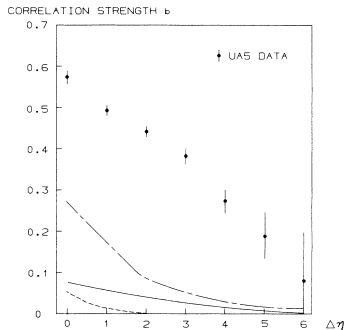


FIG. 4. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs simple models; the latter models with notation as in Fig. 3.

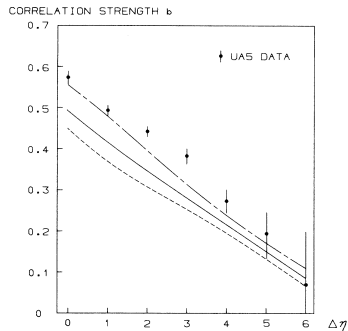
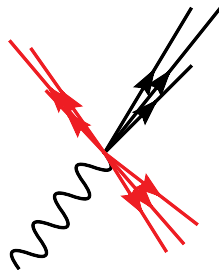
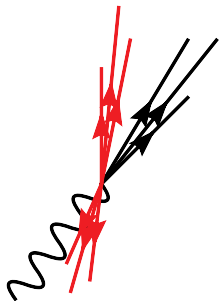


FIG. 6. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs impact-parameter-independent multiple-interaction model; the latter with notation as in Fig. 5.

no MPI (left)/MPI (right).

Evidence for MPI

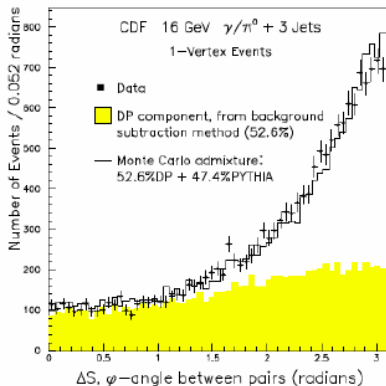
Angle ϕ from 4 final state objects (jets, γ).



Evidence for MPI

Angle ϕ from 4 final state objects (jets, γ). Latest: CDF ('97).

$$\phi = \angle(\vec{p}_1 \pm \vec{p}_2, \vec{p}_3 \pm p_4)$$

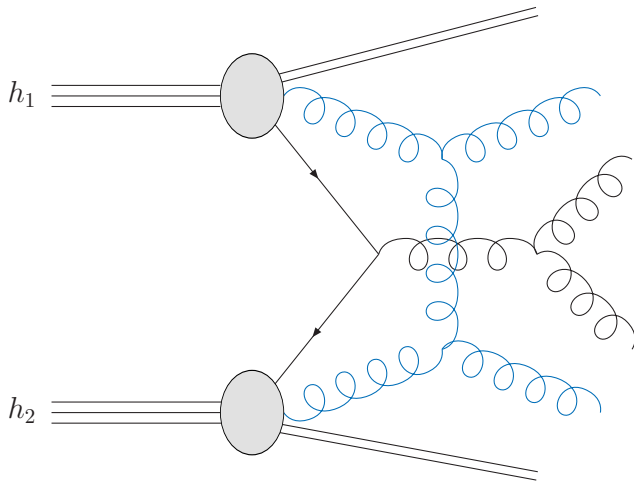


53% double parton scattering needed!

Modelling MPI (in Herwig)

Eikonal model basics

Multitple hard interactions



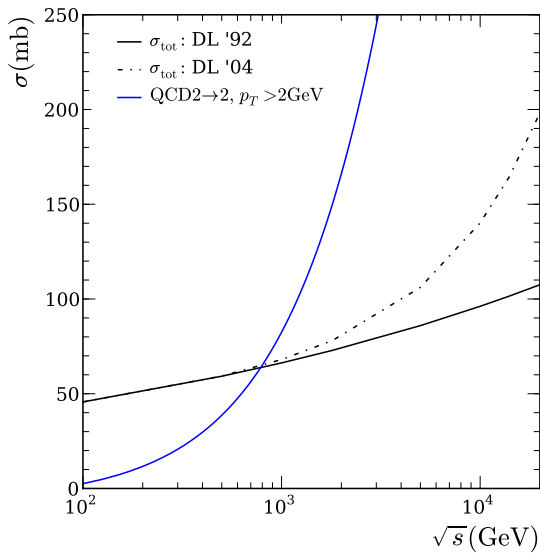
Eikonal model basics

Starting point: hard inclusive jet cross section.

$$\sigma^{\text{inc}}(s; p_t^{\text{min}}) = \sum_{i,j} \int_{p_t^{\text{min}2}} dp_t^2 f_{i/h_1}(x_1, \mu^2) \otimes \frac{d\hat{\sigma}_{i,j}}{dp_t^2} \otimes f_{j/h_2}(x_2, \mu^2),$$

$\sigma^{\text{inc}} > \sigma_{\text{tot}}$ eventually (for moderately small p_t^{min}).

Eikonal model basics



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$\sigma^{\text{inc}} > \sigma_{\text{tot}}$ eventually (for moderately small p_t^{min}).

Interpretation: σ^{inc} counts *all* partonic scatters that happen during a single pp collision $\Rightarrow$ more than a single interaction.

$$\sigma^{\text{inc}} = \bar{n} \sigma_{\text{inel}}.$$

Eikonal model basics

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number m of additional scatters,

$$P_m(\vec{b}, s) = \frac{\bar{n}(\vec{b}, s)^m}{m!} e^{-\bar{n}(\vec{b}, s)} .$$

Then we get σ_{inel} :

$$\sigma_{\text{inel}} = \int d^2\vec{b} \sum_{m=1}^{\infty} P_m(\vec{b}, s) = \int d^2\vec{b} \left(1 - e^{-\bar{n}(\vec{b}, s)} \right) .$$

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Cf. σ_{inel} from scattering theory in eikonal approx. with scattering amplitude $a(\vec{b}, s) = \frac{1}{2i} (e^{-\chi(\vec{b}, s)} - 1)$

$$\sigma_{\text{inel}} = \int d^2\vec{b} \left(1 - e^{-2\chi(\vec{b}, s)} \right) \quad \Rightarrow \quad \chi(\vec{b}, s) = \frac{1}{2} \bar{n}(\vec{b}, s) .$$

$\chi(\vec{b}, s)$ is called *eikonal* function.

Eikonal model basics

Calculation of $\bar{n}(\vec{b}, s)$ from parton model assumptions:

$$\begin{aligned}\bar{n}(\vec{b}, s) &= L_{\text{partons}}(x_1, x_2, \vec{b}) \otimes \sum_{ij} \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} \\ &= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2\vec{b}' \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} \\ &\quad \times D_{i/A}(x_1, p_t^2, |\vec{b}'|) D_{j/B}(x_2, p_t^2, |\vec{b} - \vec{b}'|)\end{aligned}$$

Eikonal model basics

Calculation of $\bar{n}(\vec{b}, s)$ from parton model assumptions:

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Eikonal model basics

Calculation of $\bar{n}(\vec{b}, s)$ from parton model assumptions:

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$$\Rightarrow \quad \chi(\vec{b}, s) = \frac{1}{2} \bar{n}(\vec{b}, s) = \frac{1}{2} A(\vec{b}) \sigma^{\text{inc}}(s; p_t^{\text{min}}) .$$

Overlap function

$$A(b) = \int d^2\vec{b}' G_A(|\vec{b}'|) G_B(|\vec{b} - \vec{b}'|)$$

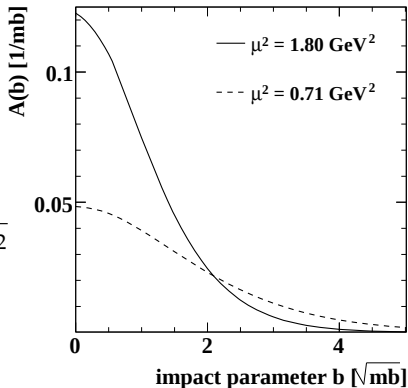
$G(\vec{b})$ from electromagnetic FF:

$$G_p(\vec{b}) = G_{\bar{p}}(\vec{b}) = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{e^{i\vec{k}\cdot\vec{b}}}{(1 + \vec{k}^2/\mu^2)^2}$$

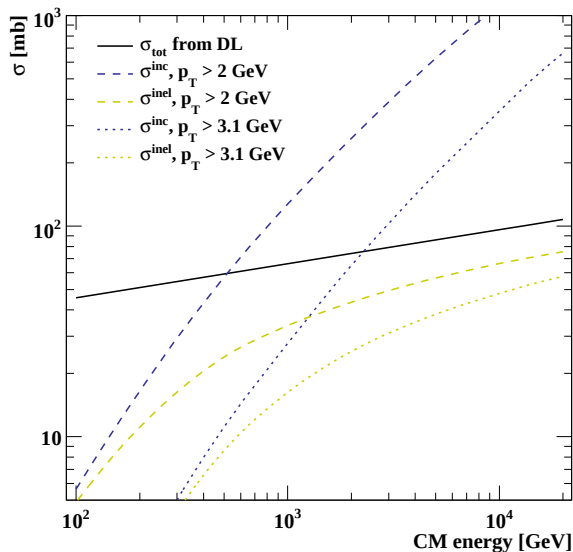
But μ^2 *not fixed* to the
electromagnetic 0.71 GeV^2 .

Free for colour charges.

$\Rightarrow$ Two main parameters: $\mu^2, p_t^{\min}$.

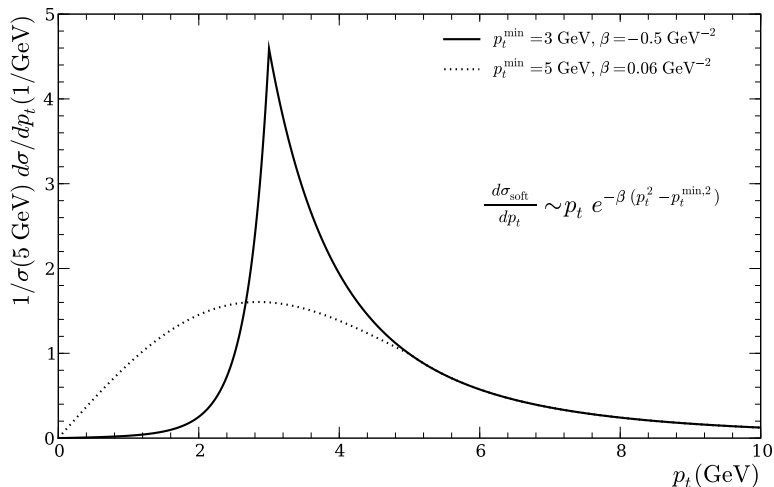


Unitarized cross sections



Extending into the soft region

Continuation of the differential cross section into the soft region $p_t < p_t^{\min}$ (here: p_t integral kept fixed)



Hot Spot model

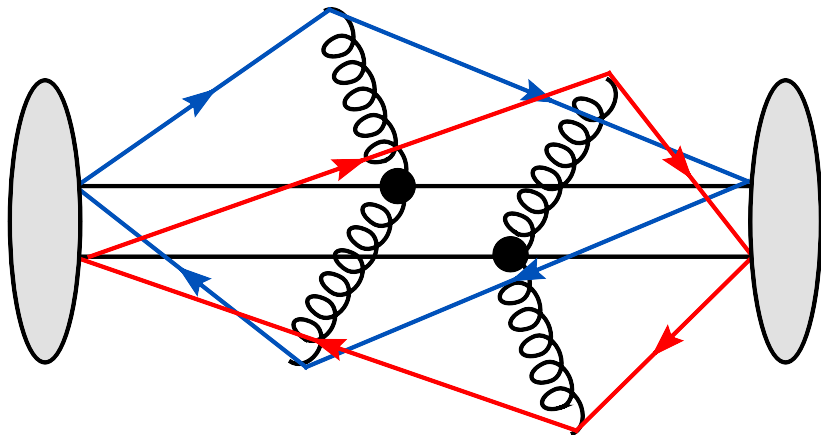
Fix the two parameters μ_{soft} and $\sigma_{\text{soft}}^{\text{inc}}$ in

$$\chi_{\text{tot}}(\vec{b}, s) = \frac{1}{2} \left(A(\vec{b}; \mu) \sigma^{\text{inc}}_{\text{hard}}(s; p_t^{\text{min}}) + A(\vec{b}; \mu_{\text{soft}}) \sigma_{\text{soft}}^{\text{inc}} \right)$$

from two constraints. Require simultaneous description of σ_{tot} and b_{el} (measured/well predicted),

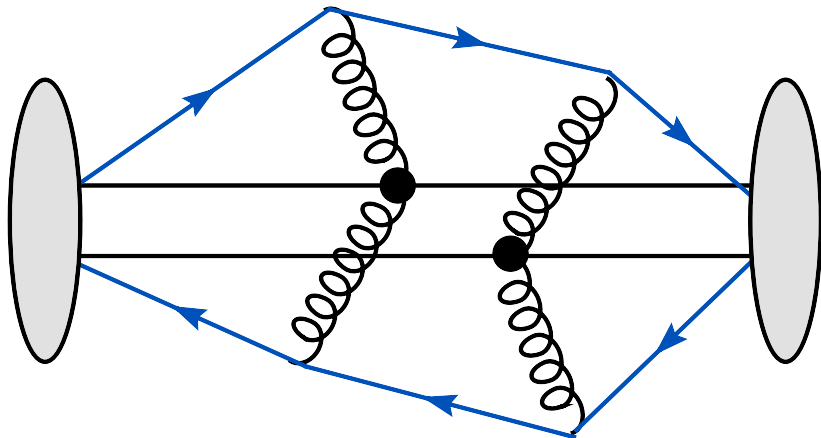
$$\begin{aligned} \sigma_{\text{tot}}(s) &\stackrel{!}{=} 2 \int d^2\vec{b} \left(1 - e^{-\chi_{\text{tot}}(\vec{b}, s)} \right) , \\ b_{\text{el}}(s) &\stackrel{!}{=} \int d^2\vec{b} \frac{b^2}{\sigma_{\text{tot}}} \left(1 - e^{-\chi_{\text{tot}}(\vec{b}, s)} \right) . \end{aligned}$$

Colour reconnection at hadron colliders



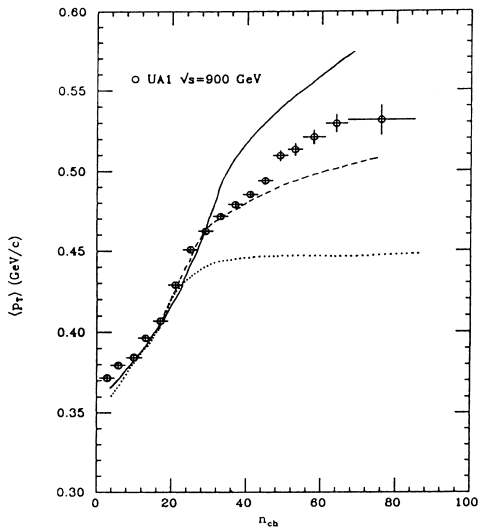
- Colour preconfinement
- Shorten colour string/lower mass clusters.

Colour reconnection at hadron colliders



- Colour preconfinement
- Shorten colour string/lower mass clusters.

Colour reconnections



- Sensitivity to CR already known since UA1.
- (From Sjöstrand/van Zijl)

Old implementation of soft scattering

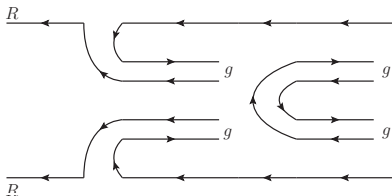
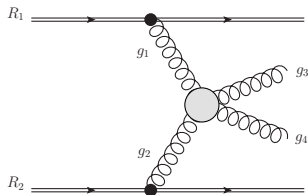
Soft gluon production with soft $p_t < p_t^{\min}$ spectrum.

Colour structure important. Two extreme cases possible.

Sensitivity to parameter

$$\text{colourDisrupt} = P(\text{disrupt colour lines})$$

Long colour lines appear when swapping outgoing gluons.

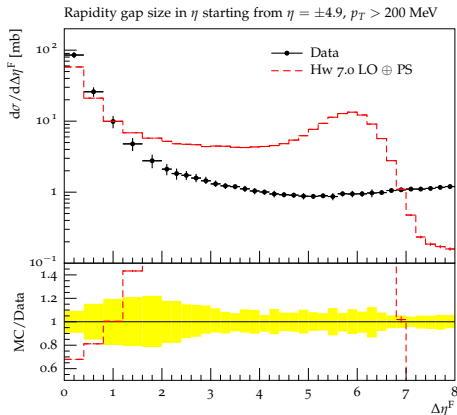


Colour reconnections applied!

The bump

A clear case of abusing a model for the hard UE in forward/diffractive final states...

[ATLAS, Eur.Phys.J. C72 (2012) 1926]



Bump is artefact. No Diffraction. Poor modeling of soft interactions. Colour assignment ad hoc.

Diffraction as part of minimum bias simulation

Diffractive final states directly modeled.

Not embedded in MPI approach via cuts through triple pomeron vertices. Therefore change in constraint

$$\textcolor{red}{x}\sigma_{\text{tot}}(s) \stackrel{!}{=} 2 \int d^2\vec{b} \left(1 - e^{-\chi_{\text{tot}}(\vec{b},s)} \right) ,$$

where

$$x \approx 1 - \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \approx 75\% .$$

In min-bias simulation: every event is either

- diffractive, directly modeled from pp initial state.
- non-diffractive, modeled in the MPI picture, parton level.

Diffraction final states

Strictly low mass diffraction only. Allow M^2 large nonetheless.

M^2 power-like, t exponential (Regge).

$$pp \rightarrow (\text{baryonic cluster}) + p .$$

Hadronic content from cluster fission/decay $C \rightarrow hh \dots$

Cluster may be quite light. If very light, use directly

$$pp \rightarrow \Delta + p .$$

Also double diffraction implemented.

$$pp \rightarrow (\text{cluster}) + (\text{cluster}) \quad pp \rightarrow \Delta + \Delta .$$

Technically: new MEs for diffractive processes set up.

Model for soft particle production in Herwig

Reproduce core properties of soft particle production.

“flat in rapidity”, “narrow in p_t ”.

Main idea: “soft interaction = cut pomeron = particle ladder”.

N_{soft} from MPI model = #ladders.

Clusters produced via colour connected quarks and gluons.

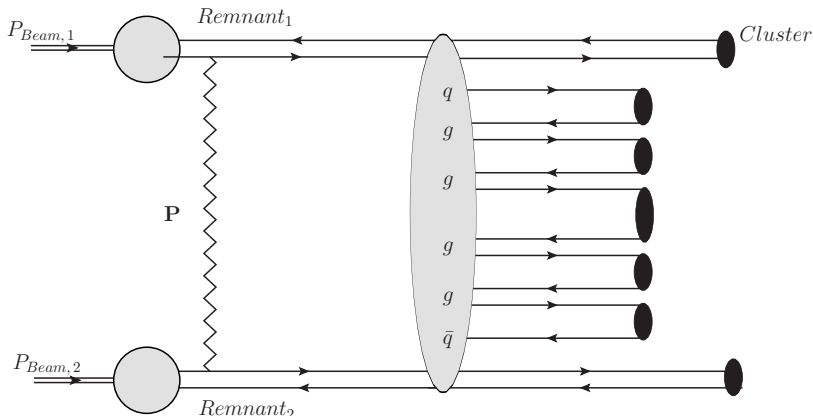
Adopt to soft interactions in Herwig via remnant decays.

Soft particle production model in Herwig

- $\#ladders = N_{\text{soft}}$ (MPI).
- N particles from Poissonian, width $\langle N \rangle$.
Model parameter $1/\ln C \equiv n_{\text{ladder}} \rightarrow$ tuned.
- x_i smeared around $\langle x \rangle$ (calculated).
- $p_{\perp}$ from Gaussian acc to soft MPI model.
- particles are q, g , see figure.
Symmetrically produced from both remnants.
- Colour connections between neighboured particles.

Soft particle production model in Herwig

Single soft ladder with MinBias initiating process.

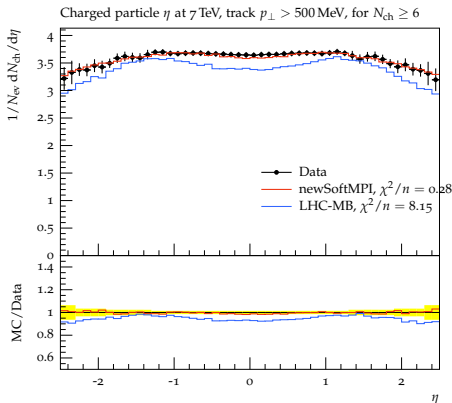


Further hard/soft MPI scatters possible.

Tuned results

ATLAS Min Bias 7 TeV.

[ATLAS, New.J.Phys. 13 (2011) 053033]

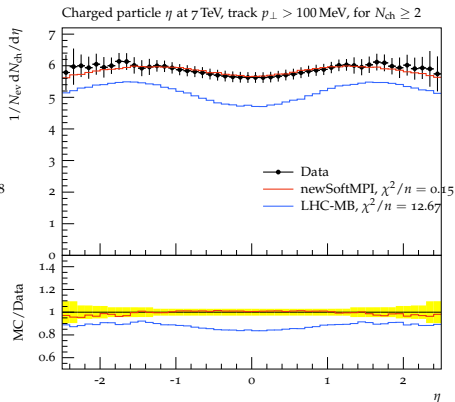
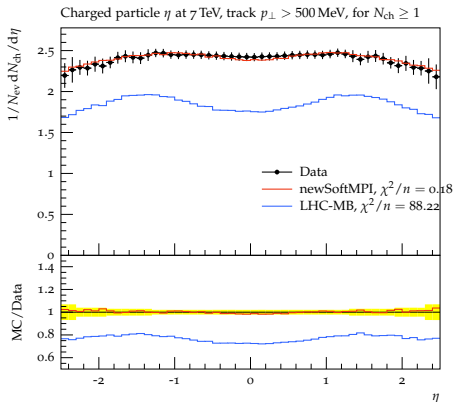


Similar to previous results, “harder part of Min Bias”.

Tuned results

ATLAS Min Bias 7 TeV.

[ATLAS, New.J.Phys. 13 (2011) 053033]

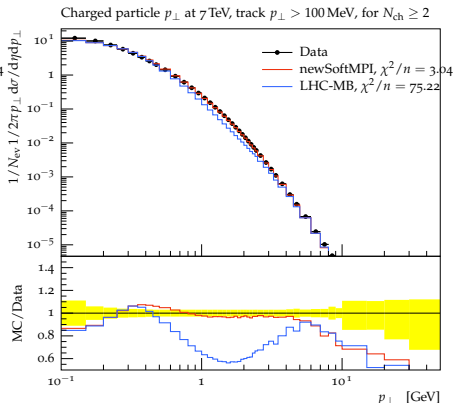
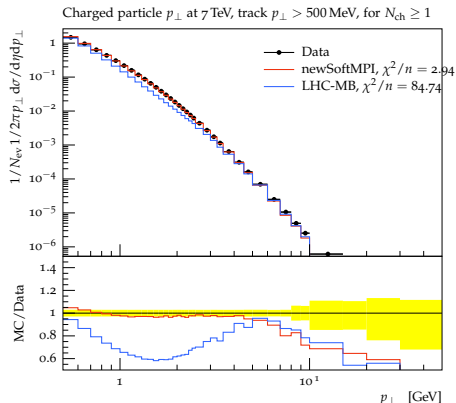


Also soft rates well described.

Tuned results

ATLAS Min Bias 7 TeV.

[ATLAS, New.J.Phys. 13 (2011) 053033]

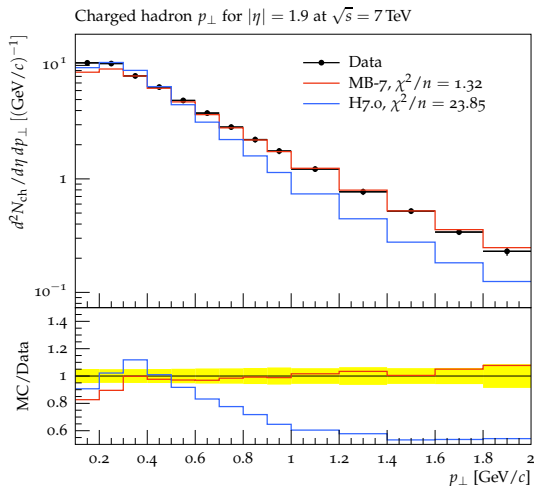


Tails? Still within 1σ .

More results

CMS, NSD analysis 7 TeV

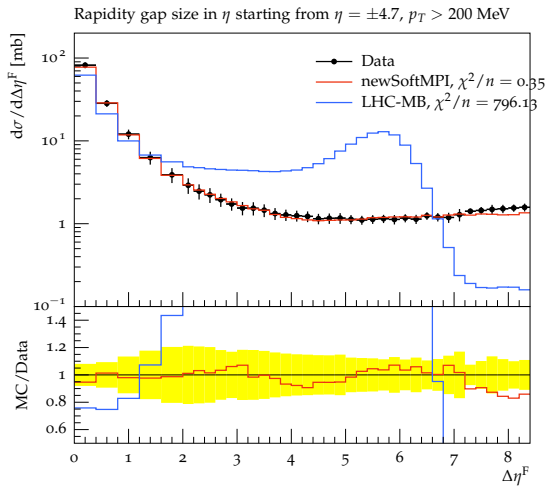
[CMS, PRL 105 (2010) 022002]



Lowest bin $\rightarrow$ potential to be tunable.

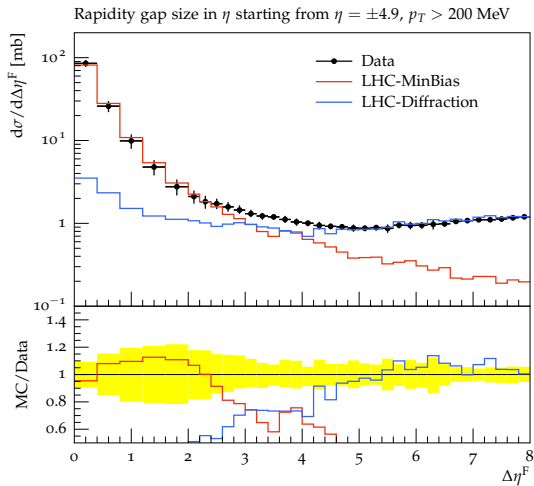
The bump plot, $\Delta\eta_F$

[CMS, PRD 92 (2015) 012003]



Individual contributions to $\Delta\eta_F$

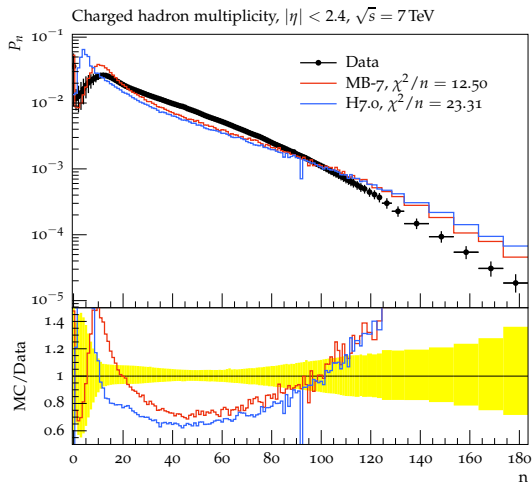
[ATLAS, Eur.Phys.J. C72 (2012) 1926]



Charged particle multiplicity

CMS, NSD analysis 7 TeV

[CMS, PRL 105 (2010) 022002]

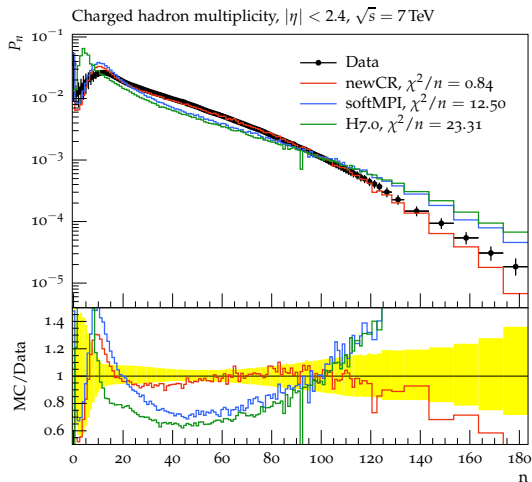


Large discrepancies, tail in particular. Low $n \rightarrow$ “NSD”?

Charged particle multiplicity

CMS, NSD analysis 7 TeV

[CMS, PRL 105 (2010) 022002]

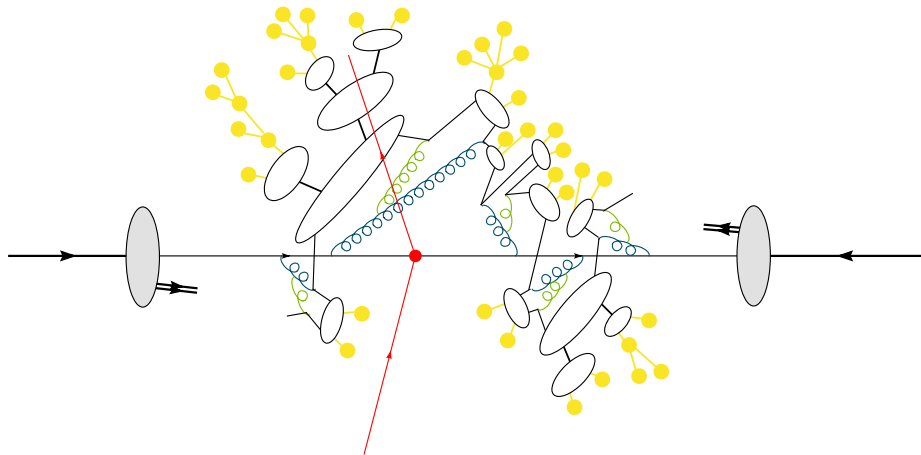


CR model with Baryons (preliminary teaser).

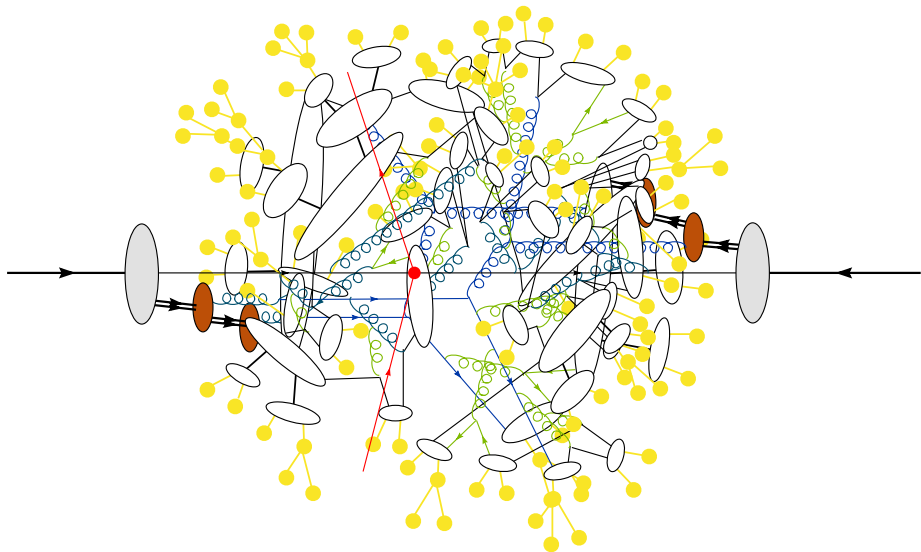
MPI Summary

- MPI (with colour reconnections) currently model of choice.
- Describes averages *and* fluctuations.
- Not always universal, but all models tunable.
- soft component needed for MB modelling.
- Constraints from inclusive cross sections.
- Different emphasis on hard/soft modelling between generators.
- Many details still only models.

Brief graphical summary



Brief graphical summary



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