Introduction to Free-Electron Laser Theory

Tutorial lecture at ARD-ST3 Annual Workshop
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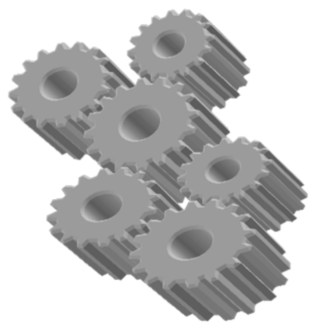
- Low-gain FEL
- High-gain FEL
- Experimental results on FEL performance →

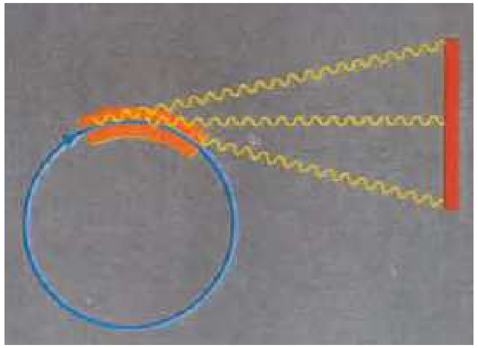
Do we understand the machinery ?



- Some notes on key components

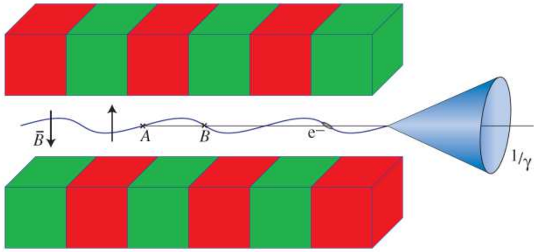
Electron Accelerators as Light Sources

a)



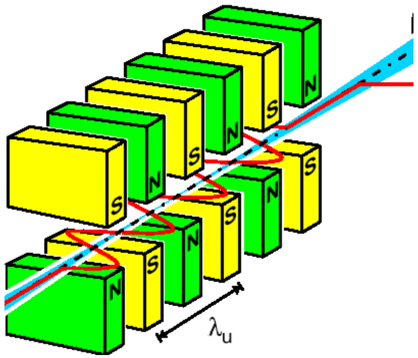
Synchrotron radiation from electron storage ring with bending magnets:
- continuous spectrum
- wide angular distribution

b)



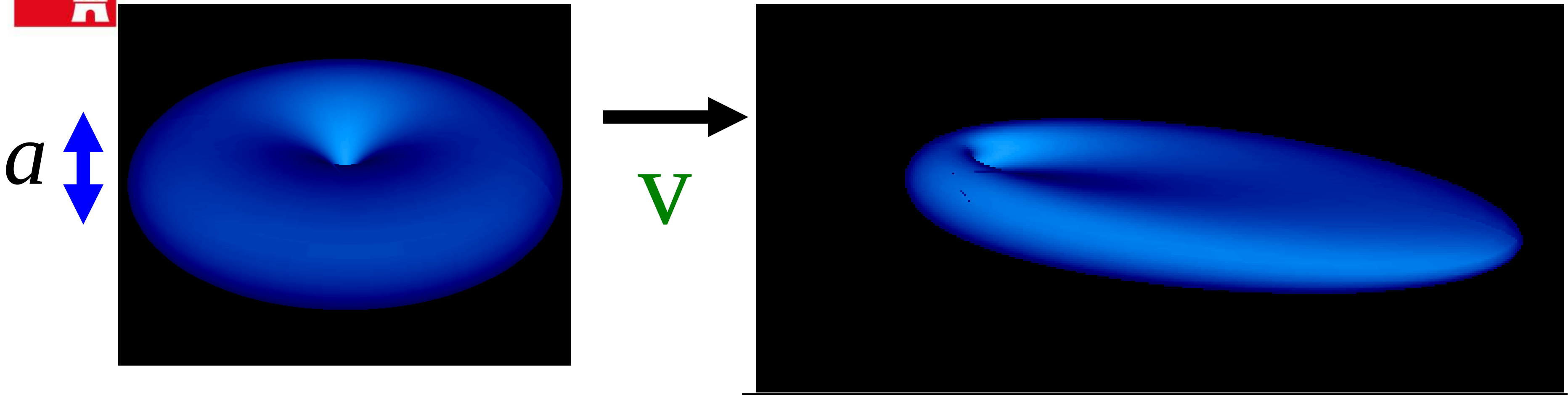
Undulator radiation:
- (almost) monochromatic
- narrow angular distribution

c)



Free-Electron Laser (FEL):
- narrow spectral line
- transverse coherence
- powerful: $I_N = N^2 \cdot I_1$

Basics: Radiation of a moving oscillating dipole



v = 0

local oscillator

$$\dot{v}_\perp = a\omega^2 \sin(\omega t)$$

$$P = \frac{Q^2 a^2}{4\pi\varepsilon_0 3c^3}\omega^4$$

$$(a < \lambda)$$

v = 0.9 c

moving oscillator

$$\gamma = \frac{E}{m_0 c^2}$$

$$P = \frac{Q^2 a^2}{4\pi\varepsilon_0 3c^3}\gamma^4\omega^4$$

We gain a factor of γ^4 in power!

note the quadratic dependence on charge!

NOTE: $P = \frac{Q^2 a^2}{4\pi\varepsilon_0 3c^3}\gamma^4\omega^4$ assumes point-like charge Q!

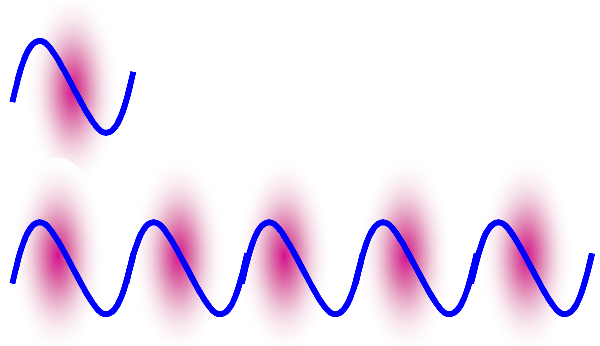
If Q consists of many particles, this requires that all charges are concentrated within distance λ !

→ FREE-ELECTRON LASER

→ desired: bunch length < wavelength

OR (even better)

Density modulation at desired wavelength



→ Potential gain in power: $N_e \sim 10^6$!!

FEL Basics

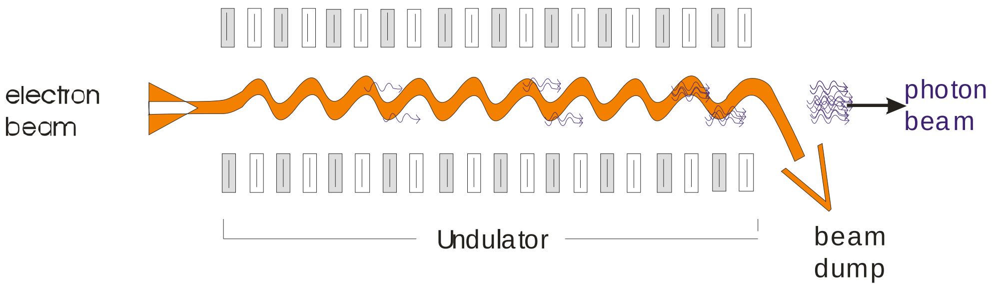
Idea:

Start with an electron bunch much longer than the desired wavelength
and find a mechanism that cuts the beam into equally spaced pieces automatically

Free-Electron Laser (Motz 1950, Phillips ~1960, Madey 1970)

Special version: starting from noise (no input needed)
Single pass saturation (no mirrors needed)

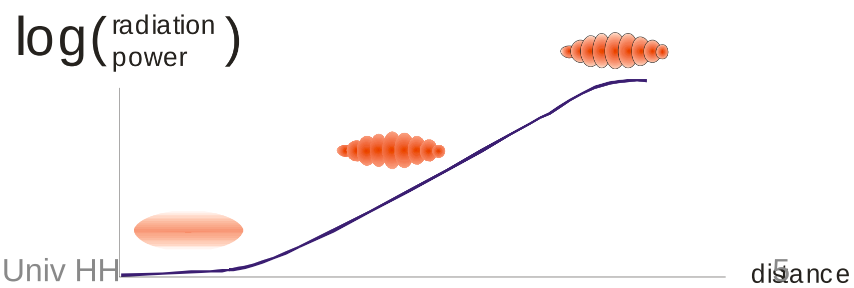
Self-Amplified Spontaneous Emission (SASE)
(Kondratenko, Saldin 1980)
(Bonifacio, Pellegrini 1984)



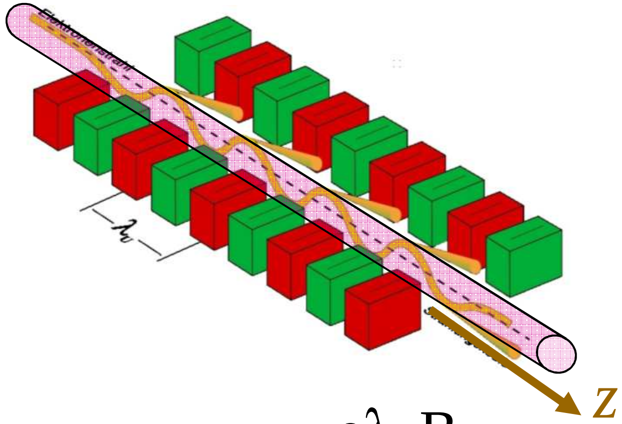
Resonance wavelength:

$$\lambda_{ph} = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

Undulator parameter ≈ 1



Basic theory of FELs



Step 1: Energy modulation

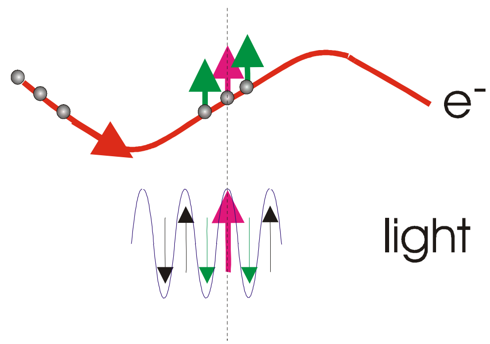
A: Electron travels on sine-like trajectory

$$v_x(z) = c\frac{K}{\gamma}\cos\left(\frac{2\pi}{\lambda_u}z\right)$$, with undulator parameter: $$K = \frac{e\lambda_u B}{2\pi m_e c}$$

B: External electromagnetic wave moving parallel to electron beam:

$$E_x(z,t) = E_0\cos(k_L z - \omega_L t)$$

Change of energy W in presence of electric field:



$$\frac{dW}{dz} = \frac{q}{v_z}\vec{v}\vec{E} = -\frac{qE_0K}{\gamma\beta_z}\sin\Psi,$$

with the ponderomotive phase: $\boxed{\Psi = (k_u + k_L)z - \omega_L t + \varphi_0}$

Note: $\cos\alpha\cdot\cos\beta = \tfrac{1}{2}\cos(\alpha+\beta) + \cancel{\tfrac{1}{2}\cos(\alpha-\beta)} = \tfrac{1}{2}\sin(\alpha+\beta+\pi/2)$

Basic FEL theory

$$\boxed{\frac{dW}{dz} = -\frac{qE_0 K}{\gamma\beta_z}\sin\Psi}$$

$$\Psi = (k_u + k_L)z - \omega_L t + \varphi_0$$

The energy *dW* is taken from or transferred to the radiation field.
For most frequencies, *dW/dz* oscillates very rapidly.

Continuous energy transfer ?

Yes, if Ψ constant. $\rightarrow \quad \frac{d\Psi}{dz} = 0\ !$ $\quad \frac{d\Psi}{dz} = k_u + k_L - \omega_L \frac{dt}{dz} \rightarrow$

$$\rightarrow \quad \mathrm{k_u} + k_L - \frac{k_L}{\beta_z} = 0 \qquad \rightarrow \quad \mathrm{k_u} = k_L \frac{1-\beta_z}{\beta_z} = k_L \frac{1}{2\gamma_z^2 \beta_z} = k_L \frac{1+K^2/2}{2\gamma^2 \beta_z}$$

→ Resonance condition: $\lambda_\mathrm{L} = \frac{\lambda_\mathrm{u}}{2\gamma^2}\left(1+\frac{\mathrm{K}^2}{2}\right)$

Note: Same equation as for wavelength of undulator radiation.

→ Energy modulation inside electron bunch <u>at optical wavelength !</u>

Basic FEL theory

<u>**Step 2:** Current modulation</u>

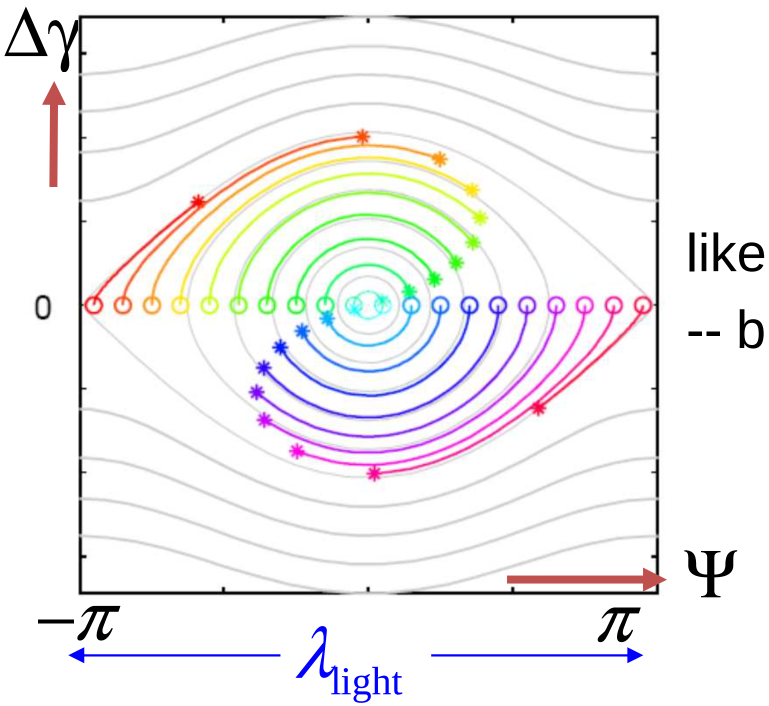
Energy modulation by $\Delta\gamma$ leads to change of Phase Ψ:

$$\boxed{\frac{d\Psi}{dz} = k_u \frac{2}{\gamma_{res}} \Delta\gamma}$$

Combination with Step 1: $\dfrac{dW}{dz} = -\dfrac{qE_0K}{\gamma\beta_z} \sin\Psi$ yields

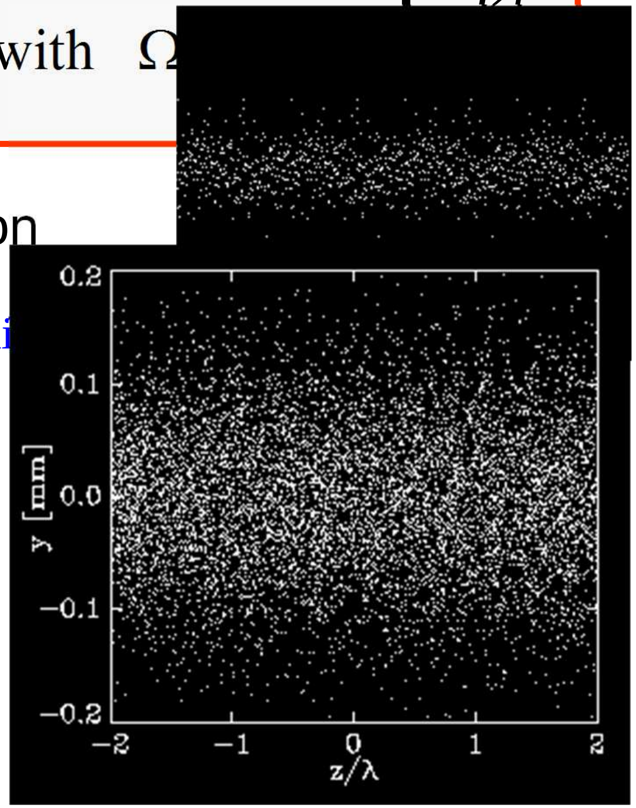
$$\frac{d^2\Psi}{dz^2} = -\Omega^2 \sin\Psi$$

with Ω



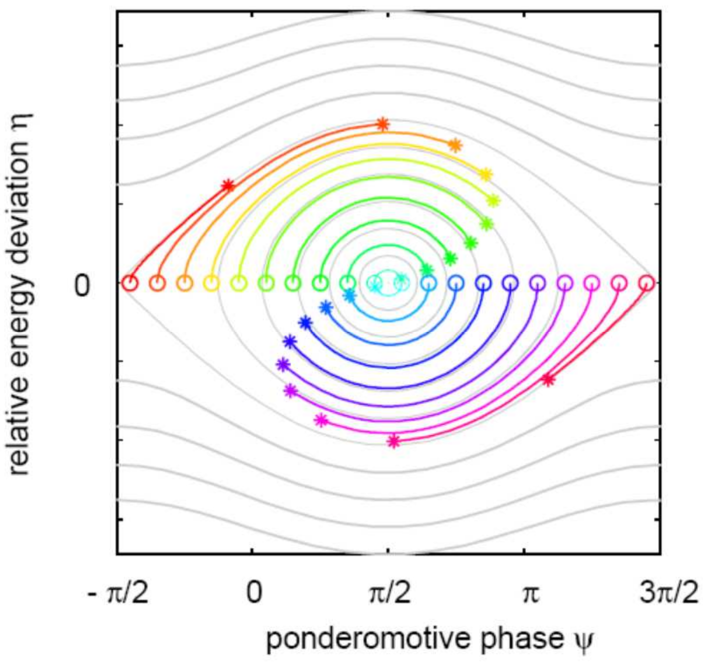
like synchrotron oscillation

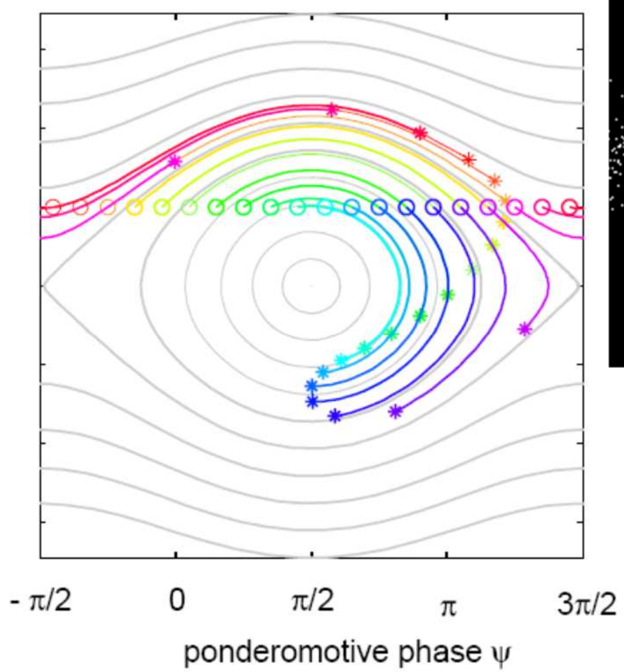
-- but at spatial period λ_{li}



→ current modulation !!

Gain (or loss) in field energy per undulator passage, depending on where to start in phase space :





Phase space simulation of low gain FEL
slightly above resonance →
See electron bunch losing energy in average

$$G_i = \frac{\text{gain of field energy produced by electron } i}{\text{total field energy}} \rightarrow \text{ requires solution of pendulum equation for } \gamma(z).$$

Real beam may have well defined energy, but all phases are equally probable!

→ Need to average gain for fixed energy $\Delta\gamma$ over all phases

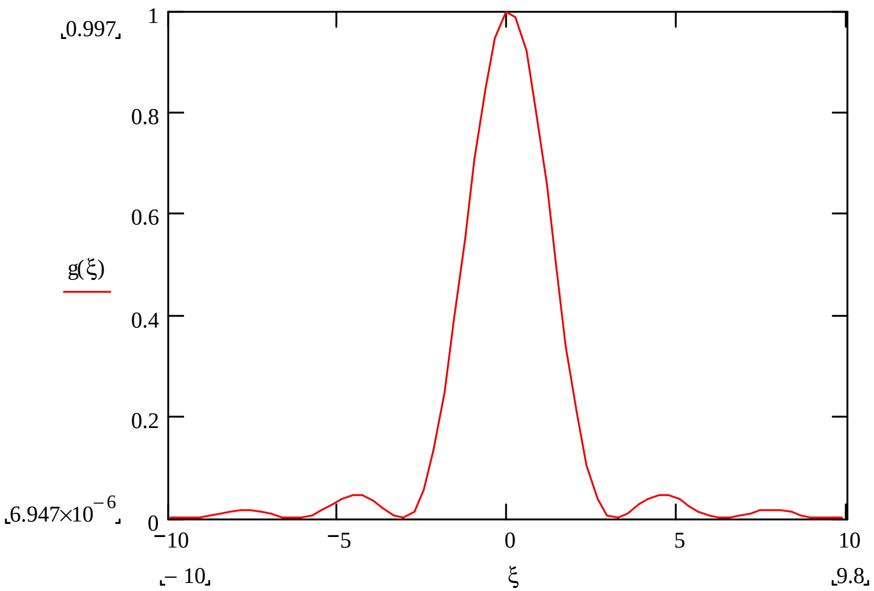
FEL Theory: Madey-Theorem

$$\text{Gain} \propto -\frac{d}{d\omega}\frac{\sin^2\left(\pi N_u \frac{\Delta\omega}{\omega_{res}}\right)}{(\Delta\omega)^2}$$

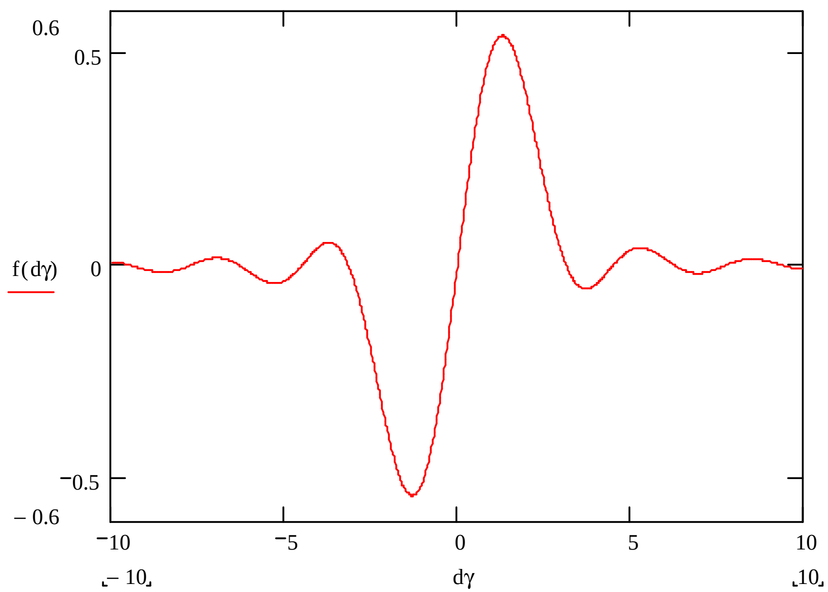
The FEL gain curve is the derivative of the line shape of spontaneous undulator radiation

Madey-Theorem

$$\frac{\sin^2\xi}{\xi^2}$$

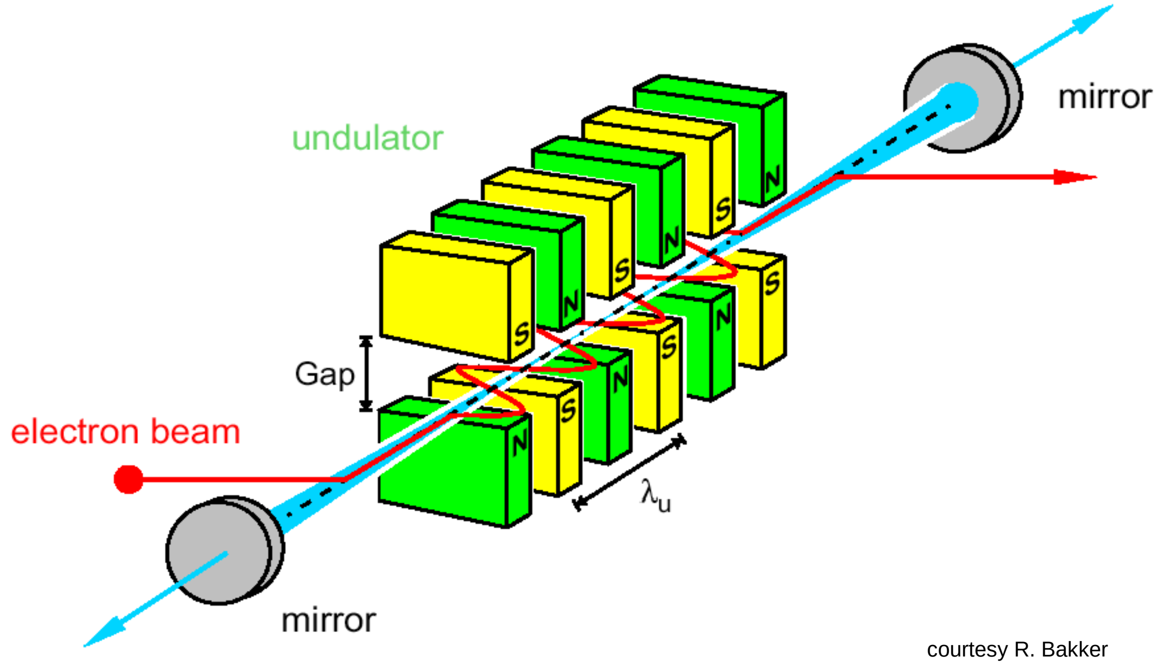


$$-\frac{d}{d\xi}\frac{\sin^2\xi}{\xi^2} = \frac{1}{\xi^3}\left(1-\cos(2\xi)-\xi\sin(2\xi)\right)$$



The “low gain” FEL

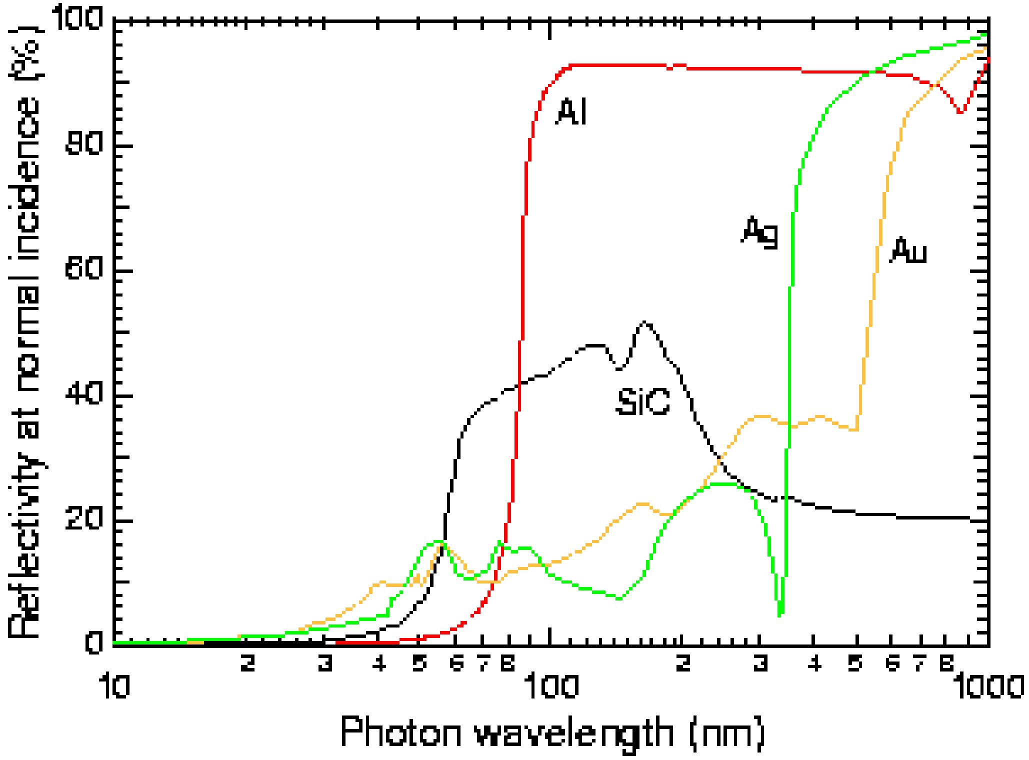
For many FELs, it is sufficient to have only a few % power gain (low gain FEL).
Using a pair of mirrors, one can multiply the gain, if on each round trip of radiation there is a fresh electron bunch available.
After N round trips, $G_{total} = G^N$, which can be a very big number.



courtesy R. Bakker

Only few % of radiation intensity is extracted per electron passage (mirror reflectivity) to keep stored field high

Very nice scheme.
But what if we want wavelength < approx. 100nm where no good mirrors exist?

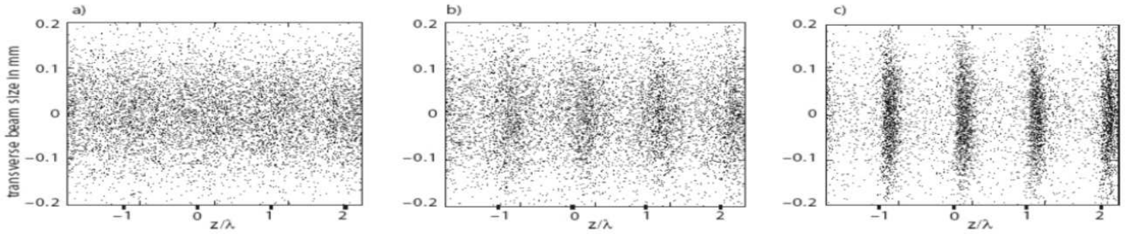


Reflectivity of most surfaces at normal incidence drops drastically at wavelengths below 100 – 200 nm.

NOTE challenge: Use Bragg crystals instead of normal incidence mirrors!
→ optical resonator for X-rays!
Idea by K.-J. Kim; investigated for European XFEL at Univ.HH

High gain FEL =

we take into account that the initial, external e.m. field changes during FEL process



Step 3: Radiation

Current modulation j_{Light} drives radiation of light:

$$\frac{\partial^2 E_\perp}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_\perp}{\partial t^2} = \mu_0 \frac{\partial j_\perp}{\partial t}$$

Approximation: Field growth slow compared to $1/\omega_{Light}$

$$\boxed{\frac{dE_{Light}}{dz} \approx \text{const} \cdot j_{modulation}}$$

System of Diff. Eqs. defines
High Gain FEL:
(To be solved numerically for given distribution of electrons)

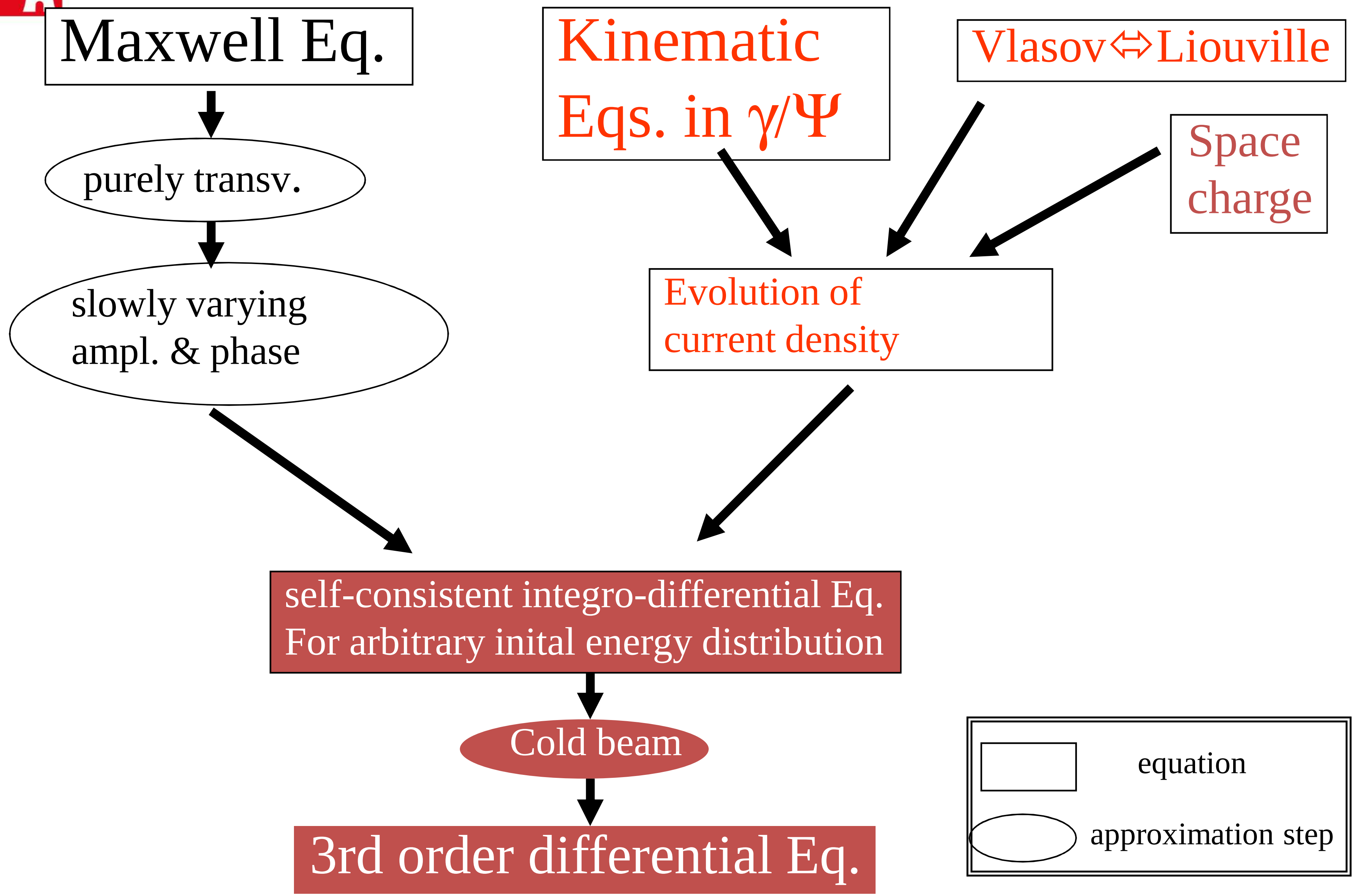
$$\frac{d}{dz}\begin{pmatrix}\Delta\gamma \\ \psi \\ E_{Light}\end{pmatrix} = \begin{pmatrix} -\frac{qE_{Light}K}{m_e c^2 \gamma_{res}^2}\,\mathbf{sin}\,\Psi \\ k_u \frac{2}{\gamma_{res}}\Delta\gamma \\ \text{const}\cdot j_{modulation}\end{pmatrix}$$

Energy modulation

Density modulation

Radiation

Major steps to derive the 3rd order Diff. Eq. for High Gain FEL



Theory: High-gain FEL

Ansatz: $\mathrm{j}(\mathrm{z}) = \mathrm{j}_0 + j_1(z)\cos(\Psi + \psi_0)$

i.e. we assume a density modulation at the optical wavelength

Maxwell Eq. combined with Vlasov Eq. results in a linear integro-differential equation for the electric field amplitude **E** growing with z (in a way to be calculated).

Can be translated into an ordinary differential equation of 3rd order:

$$\boxed{\frac{\mathrm{d}^3\mathbf{E}}{dz^3} + 2iC\,\frac{\mathrm{d}^2\mathbf{E}}{dz^2} - C^2\frac{\mathrm{d}\mathbf{E}}{dz} = i\Gamma^3\mathbf{E}}\quad . \quad \text{Ansatz:}\quad \mathbf{E} = A\,\mathbf{exp}(\Lambda z) \quad\rightarrow\quad \Lambda(\Lambda + \mathrm{i}C)^2 = i\Gamma^3$$

Abbreviations

Gain Factor: $\Gamma = \left(\frac{\pi j_0 K^2 (1 + K^2)\omega_L}{I_A c \gamma^5}\right)^{1/3}$

Detuning parameter: $\mathrm{C} = k_u + k_L - \frac{\omega_L}{v_z}$

Alven current: $\mathrm{I_A} = 17\ \mathrm{kA}$

Analytical Theory of High-gain FEL

Most simple case: All electrons on resonance energy →

$$\boxed{\frac{d^3\mathbf{E}}{dz^3} = i\Gamma^3\mathbf{E}} \quad . \quad \text{Ansatz:} \quad \mathbf{E} = A\,\mathbf{exp}(\Lambda z) \quad \rightarrow \quad \Lambda^3 = i\Gamma^3$$

("characteristic equation")

$$\Rightarrow \Lambda_1 = -i\Gamma; \quad \Lambda_2 = \frac{i+\sqrt{3}}{2}\Gamma; \quad \Lambda_3 = \frac{i-\sqrt{3}}{2}\Gamma$$

The general solution is: $\mathbf{E}(z) = A_1\,\mathbf{exp}(-i\Gamma z) + A_2\,\mathbf{exp}(\frac{i+\sqrt{3}}{2}\Gamma z) + A_3(\mathbf{exp}\frac{i-\sqrt{3}}{2}\Gamma z)$

All contributions to solution oscillate or vanish, except for:

For an undulator much longer than $1/\Gamma$, this part of solution dominates.
Coefficients $A_{1,2,3}$ need to be determined by initial conditions:

With the initial condition, the amplitudes $A_{1,2,3,}$ can be calculated as follows:

We write $\tilde{\mathbf{E}}(z) = A_1 \,\mathbf{exp}(\Lambda_1 z) + A_2 \,\mathbf{exp}(\Lambda_2 z) + A_3 \,\mathbf{exp}(\Lambda_3 z)$ in the form

$\tilde{\mathbf{E}}(z) = A_1 \tilde{\mathbf{E}}_1(z) + A_2 \tilde{\mathbf{E}}_2(z) + A_3 \tilde{\mathbf{E}}_3(z)$ with $\tilde{\mathbf{E}}_1(z) = \mathbf{exp}(\Lambda_1 z)$, etc. and write $\frac{d}{dz}\tilde{\mathbf{E}} = \tilde{\mathbf{E}}'$, etc.

The general solution is written in the form

$$\begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{E}}' \\ \tilde{\mathbf{E}}'' \end{pmatrix}_z = \begin{pmatrix} \tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3 \\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3' \\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3'' \end{pmatrix}_z \cdot \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$$

Since $\Lambda_{1,2,3}$ are known from characteristic Eq., all matrix elements are known..

Using initial conditions $\begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{E}}' \\ \tilde{\mathbf{E}}'' \end{pmatrix}_{z=0}$, we can determine $\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3 \\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3' \\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3'' \end{pmatrix}_{z=0}^{-1} \cdot \begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{E}}' \\ \tilde{\mathbf{E}}'' \end{pmatrix}_{z=0}$. Thus

$$\begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{E}}' \\ \tilde{\mathbf{E}}'' \end{pmatrix}_z = \begin{pmatrix} \tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3 \\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3' \\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3'' \end{pmatrix}_z \cdot \begin{pmatrix} \tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3 \\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3' \\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3'' \end{pmatrix}_{z=0}^{-1} \cdot \begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{E}}' \\ \tilde{\mathbf{E}}'' \end{pmatrix}_{z=0}$$

, or using $\tilde{\mathbf{E}}_1(z) = \mathbf{exp}(\Lambda_1 z)$, $\tilde{\mathbf{E}}_1'(z) = \Lambda_1 \,\mathbf{exp}(\Lambda_1 z)$, etc.

$$\begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{E}}' \\ \tilde{\mathbf{E}}'' \end{pmatrix}_z = \begin{pmatrix} \tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3 \\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3' \\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3'' \end{pmatrix}_z \cdot \begin{pmatrix} 1 & 1 & 1 \\ \Lambda_1 & \Lambda_2 & \Lambda_3 \\ \Lambda_1^2 & \Lambda_2^2 & \Lambda_3^2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{E}}' \\ \tilde{\mathbf{E}}'' \end{pmatrix}_{z=0}$$

Thus the General Solution is

$$\begin{pmatrix}\tilde{\mathbf{E}}\\ \tilde{\mathbf{E}}'\\ \tilde{\mathbf{E}}''\end{pmatrix}_z = \begin{pmatrix}\tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3\\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3'\\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3''\end{pmatrix}_z \cdot \overbrace{\begin{pmatrix} \dfrac{\Lambda_2\Lambda_3}{(\Lambda_1-\Lambda_2)(\Lambda_1-\Lambda_3)} & -\dfrac{\Lambda_2+\Lambda_3}{(\Lambda_1-\Lambda_2)(\Lambda_1-\Lambda_3)} & \dfrac{1}{(\Lambda_1-\Lambda_2)(\Lambda_1-\Lambda_3)}\\ \dfrac{\Lambda_1\Lambda_3}{(\Lambda_2-\Lambda_1)(\Lambda_2-\Lambda_3)} & -\dfrac{\Lambda_1+\Lambda_3}{(\Lambda_2-\Lambda_1)(\Lambda_2-\Lambda_3)} & \dfrac{1}{(\Lambda_2-\Lambda_1)(\Lambda_2-\Lambda_3)}\\ \dfrac{\Lambda_2\Lambda_1}{(\Lambda_3-\Lambda_2)(\Lambda_3-\Lambda_1)} & -\dfrac{\Lambda_2+\Lambda_1}{(\Lambda_3-\Lambda_2)(\Lambda_3-\Lambda_1)} & \dfrac{1}{(\Lambda_3-\Lambda_2)(\Lambda_3-\Lambda_1)} \end{pmatrix}}^{\begin{pmatrix}1 & 1 & 1\\ \Lambda_1 & \Lambda_2 & \Lambda_3\\ \Lambda_1^2 & \Lambda_2^2 & \Lambda_3^2\end{pmatrix}^{-1}} \cdot \begin{pmatrix}\tilde{\mathbf{E}}\\ \tilde{\mathbf{E}}'\\ \tilde{\mathbf{E}}''\end{pmatrix}_{z=0}$$

For the simple case C=0, $k_p = 0$ we got $\Lambda_1 = -i\Gamma; \Lambda_2 = \dfrac{i+\sqrt{3}}{2}\Gamma; \Lambda_3 = \dfrac{i-\sqrt{3}}{2}\Gamma$, thus

$$\begin{pmatrix}\tilde{\mathbf{E}}\\ \tilde{\mathbf{E}}'\\ \tilde{\mathbf{E}}''\end{pmatrix}_z = \begin{pmatrix}\tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3\\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3'\\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3''\end{pmatrix}_z \cdot \begin{pmatrix} \dfrac{1}{3} & \dfrac{i}{3\Gamma} & -\dfrac{1}{3\Gamma^2}\\ \dfrac{1}{3} & \dfrac{1}{3\Gamma}\,\mathbf{exp}\left(-i\dfrac{\pi}{6}\right) & \dfrac{1}{3\Gamma^2}\,\mathbf{exp}\left(-i\dfrac{\pi}{3}\right)\\ \dfrac{1}{3} & \dfrac{-1}{3\Gamma}\,\mathbf{exp}\left(i\dfrac{\pi}{6}\right) & \dfrac{1}{3\Gamma^2}\,\mathbf{exp}\left(i\dfrac{\pi}{3}\right) \end{pmatrix} \cdot \begin{pmatrix}\tilde{\mathbf{E}}\\ \tilde{\mathbf{E}}'\\ \tilde{\mathbf{E}}''\end{pmatrix}_{z=0}$$

$\boxed{(\text{use } 1+i\sqrt{3} = 2\,\mathbf{exp}\, i\frac{\pi}{3})}$

First example: Seeding by input e.m. wave

In this case: $\tilde{\mathbf{E}}(z=0)=\mathbf{E}_{ext}$, $\tilde{j}_1(z=0)=0$, $\frac{d}{dz}\tilde{j}_1(z=0)=0$ (i.e. no current modulation at the beginning) $\rightarrow \begin{pmatrix}\tilde{\mathbf{E}}\\ \tilde{\mathbf{E}}'\\ \tilde{\mathbf{E}}''\end{pmatrix}_{z=0}=\begin{pmatrix}\mathbf{E}_{ext}\\0\\0\end{pmatrix}$

Thus
$$\begin{pmatrix}\tilde{\mathbf{E}}\\ \tilde{\mathbf{E}}'\\ \tilde{\mathbf{E}}''\end{pmatrix}_z=\begin{pmatrix}\tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3\\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3'\\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3''\end{pmatrix}_z\cdot\begin{pmatrix}\frac{1}{3} & \frac{i}{3\Gamma} & -\frac{1}{3\Gamma^2}\\ \frac{1}{3} & \frac{1}{3\Gamma}\mathbf{exp}\left(-i\frac{\pi}{6}\right) & \frac{1}{3\Gamma^2}\mathbf{exp}\left(-i\frac{\pi}{3}\right)\\ \frac{1}{3} & \frac{-1}{3\Gamma}\mathbf{exp}\left(i\frac{\pi}{6}\right) & \frac{1}{3\Gamma^2}\mathbf{exp}\left(i\frac{\pi}{3}\right)\end{pmatrix}\cdot\begin{pmatrix}\mathbf{E}_{ext}\\0\\0\end{pmatrix}=\begin{pmatrix}\tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3\\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3'\\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3''\end{pmatrix}_z\cdot\begin{pmatrix}\frac{1}{3}\mathbf{E}_{ext}\\ \frac{1}{3}\mathbf{E}_{ext}\\ \frac{1}{3}\mathbf{E}_{ext}\end{pmatrix}=\frac{1}{3}\mathbf{E}_{ext}\begin{pmatrix}\tilde{\mathbf{E}}_1+\tilde{\mathbf{E}}_2+\tilde{\mathbf{E}}_3\\ \tilde{\mathbf{E}}_1'+\tilde{\mathbf{E}}_2'+\tilde{\mathbf{E}}_3'\\ \tilde{\mathbf{E}}_1''+\tilde{\mathbf{E}}_2''+\tilde{\mathbf{E}}_3''\end{pmatrix}=$$

$$\frac{1}{3}\mathbf{E}_{ext}\begin{pmatrix}\mathbf{exp}(\Lambda_1 z)+\mathbf{exp}(\Lambda_2 z)+\mathbf{exp}(\Lambda_3 z)\\ \Lambda_1\,\mathbf{exp}(\Lambda_1 z)+\Lambda_2\,\mathbf{exp}(\Lambda_2 z)+\Lambda_3\,\mathbf{exp}(\Lambda_3 z)\\ \Lambda_1^2\,\mathbf{exp}(\Lambda_1 z)+\Lambda_2^2\,\mathbf{exp}(\Lambda_2 z)+\Lambda_3^2\,\mathbf{exp}(\Lambda_3 z)\end{pmatrix};\ \text{explicitly: } \boxed{\tilde{\mathbf{E}}(z)=\frac{1}{3}\mathbf{E}_{ext}\left[\mathbf{exp}(-i\Gamma z)+\mathbf{exp}\left(\frac{i+\sqrt{3}}{2}\Gamma z\right)+\mathbf{exp}\left(\frac{i-\sqrt{3}}{2}\Gamma z\right)\right]}$$

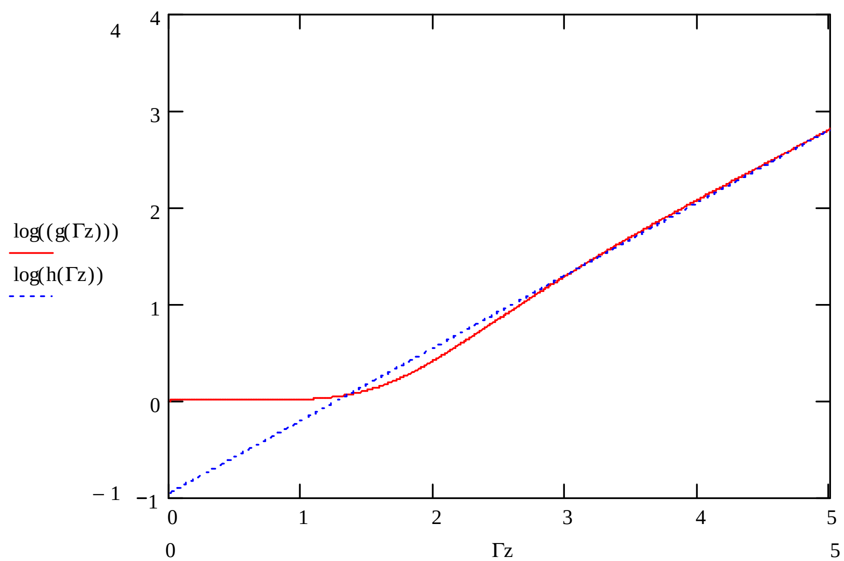
for z>>1/Γ : $\boxed{\tilde{\mathbf{E}}(z)=\frac{1}{3}\mathbf{E}_{ext}\,\mathbf{exp}\left(\frac{i+\sqrt{3}}{2}\Gamma z\right)}$

The power gain is given by (prove it!)

$$G=\frac{\left|\tilde{\mathbf{E}}\right|^2}{\mathbf{E}_{ext}^2}=\frac{1}{9}\left[1+4\,\mathbf{cosh}\frac{\sqrt{3}}{2}\Gamma z\left(\mathbf{cosh}\frac{\sqrt{3}}{2}\Gamma z+\mathbf{cos}\frac{3}{2}\Gamma z\right)\right]$$

→ (for z>>1/Γ): $\boxed{G=\frac{1}{9}\mathbf{exp}\sqrt{3}\Gamma z}$

The factor 1/9 describes the coupling of the incoming e.m. field to FEL gain process



2nd example: Initial long. density modulation

Initial condition: $\tilde{\mathbf{E}}(z=0)=0,\ \ \tilde{j}_1(z=0)\neq 0\ ,\ \ \frac{d}{dz}\tilde{j}_1(z=0)=0$ requires $\eta=0$

Thus: $\tilde{\mathbf{E}}'(z=0)=i\mu_0\frac{cK}{2\gamma_0}\tilde{j}_1(z=0),\ \tilde{\mathbf{E}}''(z=0)=0$ and $\begin{pmatrix}\tilde{\mathbf{E}}\\ \tilde{\mathbf{E}}'\\ \tilde{\mathbf{E}}''\end{pmatrix}_{z=0}=\begin{pmatrix}0\\ i\mu_0\frac{cK}{2\gamma_0}\tilde{j}_1\\ 0\end{pmatrix}_{z=0}=\begin{pmatrix}0\\ \tilde{\mathbf{E}}_0'\\ 0\end{pmatrix}$

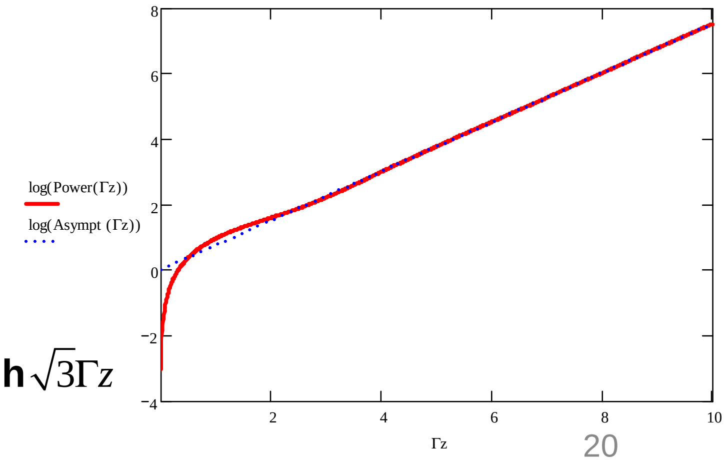
$$\rightarrow\ \begin{pmatrix}\tilde{\mathbf{E}}\\ \tilde{\mathbf{E}}'\\ \tilde{\mathbf{E}}''\end{pmatrix}_z=\begin{pmatrix}\tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3\\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3'\\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3''\end{pmatrix}_z\cdot\begin{pmatrix}\frac{1}{3} & \frac{i}{3\Gamma} & -\frac{1}{3\Gamma^2}\\ \frac{1}{3} & \frac{1}{3\Gamma}\mathbf{exp}\left(-i\frac{\pi}{6}\right) & \frac{1}{3\Gamma^2}\mathbf{exp}\left(-i\frac{\pi}{3}\right)\\ \frac{1}{3} & \frac{-1}{3\Gamma}\mathbf{exp}\left(i\frac{\pi}{6}\right) & \frac{1}{3\Gamma^2}\mathbf{exp}\left(i\frac{\pi}{3}\right)\end{pmatrix}\cdot\begin{pmatrix}0\\ \tilde{\mathbf{E}}_0'\\ 0\end{pmatrix}_{z=0}=\begin{pmatrix}\tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3\\ \tilde{\mathbf{E}}_1' & \tilde{\mathbf{E}}_2' & \tilde{\mathbf{E}}_3'\\ \tilde{\mathbf{E}}_1'' & \tilde{\mathbf{E}}_2'' & \tilde{\mathbf{E}}_3''\end{pmatrix}_z\begin{pmatrix}\frac{i}{3\Gamma}\tilde{\mathbf{E}}_0'\\ \frac{1}{3\Gamma}\mathbf{exp}\left(-i\frac{\pi}{6}\right)\tilde{\mathbf{E}}_0'\\ -\frac{1}{3\Gamma}\mathbf{exp}\left(i\frac{\pi}{6}\right)\tilde{\mathbf{E}}_0'\end{pmatrix}$$

Explicitly, the solution for $\tilde{\mathbf{E}}(z)$ is:

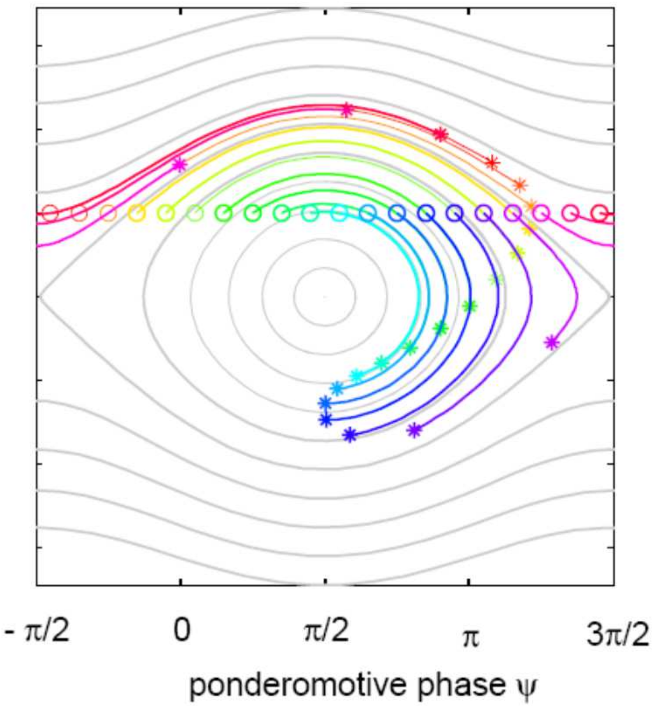
$$\tilde{\mathbf{E}}(z)=\frac{1}{3\Gamma}\tilde{\mathbf{E}}_0'\left[i\,\mathbf{exp}(\Lambda_1 z)+\mathbf{exp}\left(-i\frac{\pi}{6}\right)\mathbf{exp}(\Lambda_2 z)-\mathbf{exp}\left(i\frac{\pi}{6}\right)\mathbf{exp}(\Lambda_3 z)\right]$$

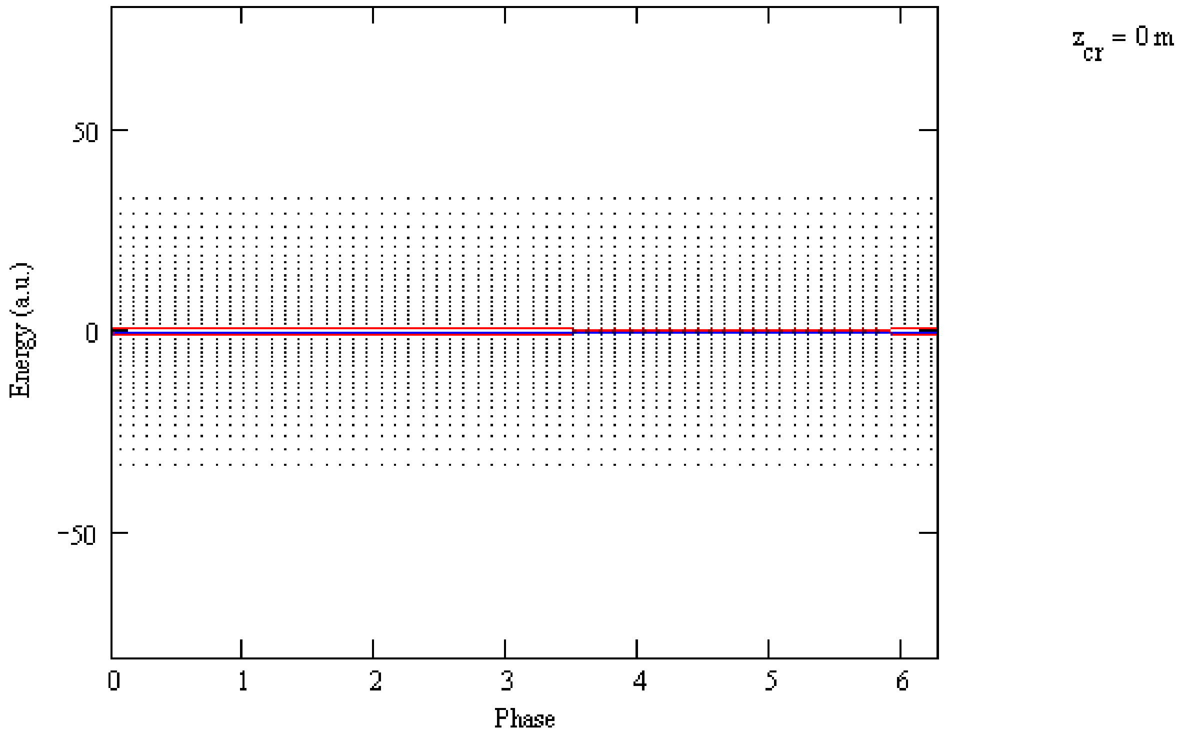
Radiation power:

$$P(z)\propto\left|\tilde{\mathbf{E}}(z)\right|^2\propto-\mathbf{cos}\frac{3}{2}\Gamma z\cdot\mathbf{cosh}\frac{\sqrt{3}}{2}\Gamma z+\sqrt{3}\,\mathbf{sin}\frac{3}{2}\Gamma z\cdot\mathbf{sinh}\frac{\sqrt{3}}{2}\Gamma z+\mathbf{cosh}\sqrt{3}\Gamma z$$



Evolution of FEL bucket and capture of electrons inside buckets





Evolution of phases

Phases of electric field and of current modulation slip w.r.t. ponderomotive phase:

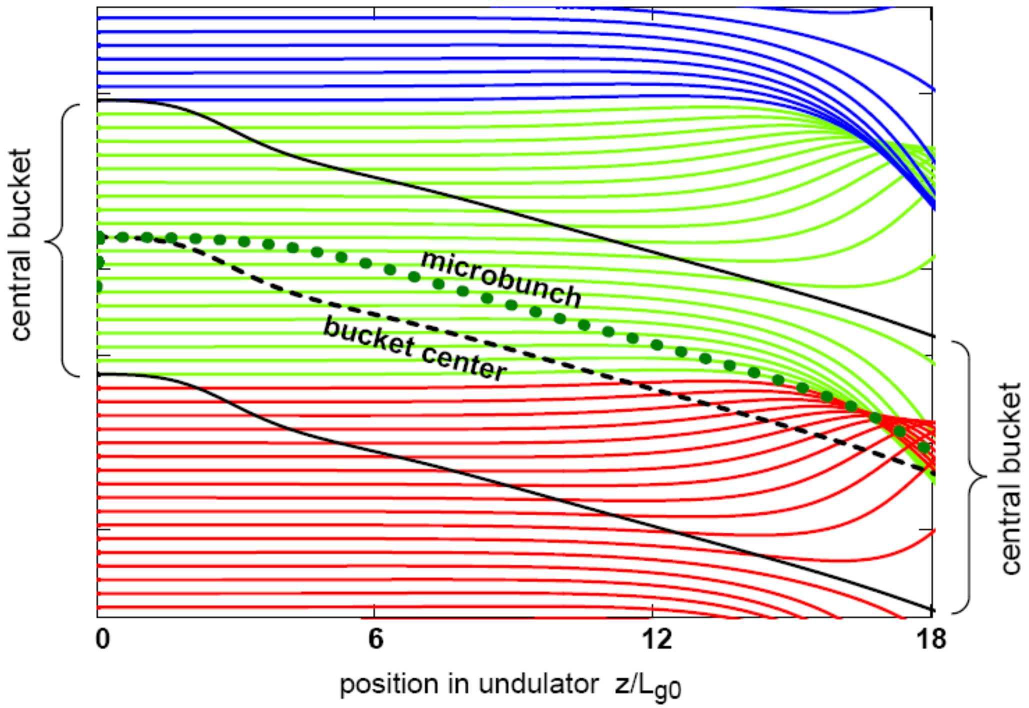
Don’t get confused:

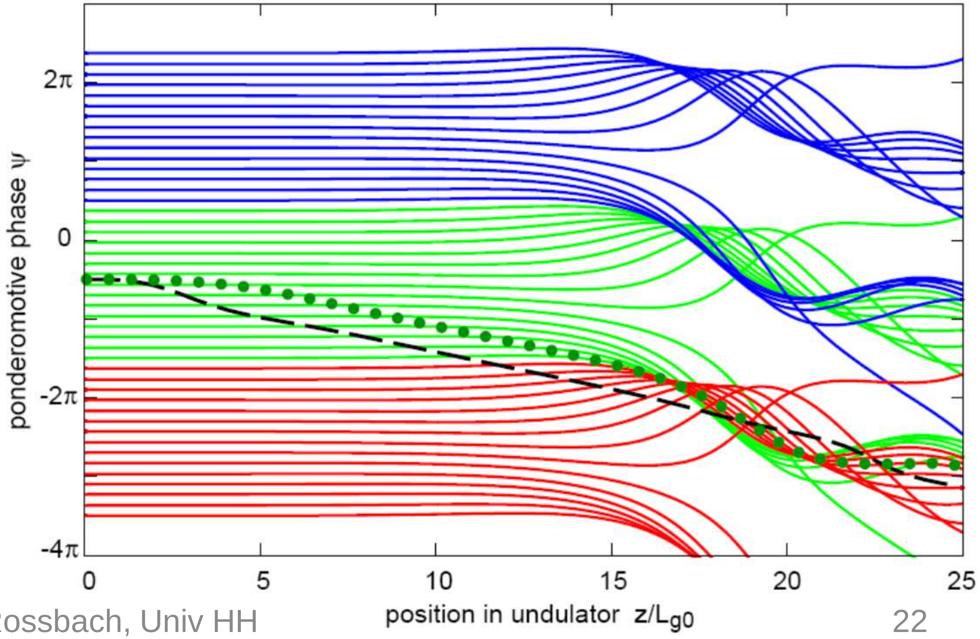
Electrons do NOT move that quickly ! (Only in saturation regime)

Bucket center slips together with E-field.

→Center of microbunch remains in that (r.h.s.) side of bucket where particles lose energy

(i.e. they keep on pumping energy into E-field)





$$P_{rad} = \frac{1}{9} P_{in} \exp\left(\sqrt{3}\Gamma z\right) = \frac{1}{9} P_{in} \exp\left(z/L_G\right) \quad .$$

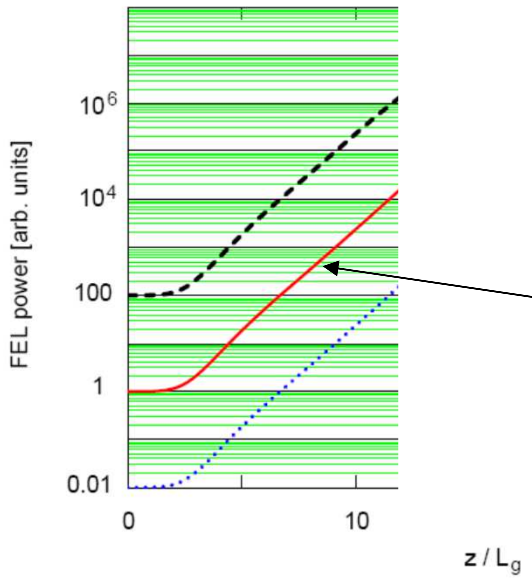
$$L_G = \frac{1}{\sqrt{3}} \left(\frac{I_A c \gamma^5}{\pi j_0 K^2 (1+K^2) \omega_L} \right)^{1/3} \quad \text{or, using} \quad \omega_L = \frac{4\pi c \gamma^2}{\lambda_u (1+K^2)} \quad \text{and } j_0 \approx \frac{\hat{I}}{\pi \sigma_r^2} \;,$$

$$L_G = \frac{1}{\sqrt{3}} \left(\frac{I_A \gamma^3 \sigma_r^2 \lambda_u}{4\pi \hat{I} K^2} \right)^{1/3} \quad \text{is called power gain length.}$$

Note:

This is 1D FEL theory, assuming a perfect electron beam with zero emittance and zero momentum spread.

Theory: High-gain FEL



Evolution of radiation power:

$z >> \Gamma^{-1}$: exponential growth:

$$\boxed{P_{rad} = \frac{1}{9} P_{in} \exp\left(\frac{z}{L_G}\right)} \qquad L_G = \frac{1}{\sqrt{3}} \left(\frac{I_A \gamma^3 \sigma_r^2 \lambda_u}{4\pi \cdot \hat{I} \cdot K^2} \right)^{1/3}$$

$$L_G \propto (\text{current density})^{-1/3}$$

Also widely used:FEL-parameter: $\rho_{FEL} = \frac{1}{4\pi\sqrt{3}} \frac{\lambda_u}{L_G} \approx 10^{-4} \ldots 10^{-2}$

1. Expect exponential gain with e-folding length L_G

Major additional assumption: Orbit is perfectly straight

2. Gain should saturate when modulation is complete → Happens after approx. 20 gain lengths

Start-up from noise

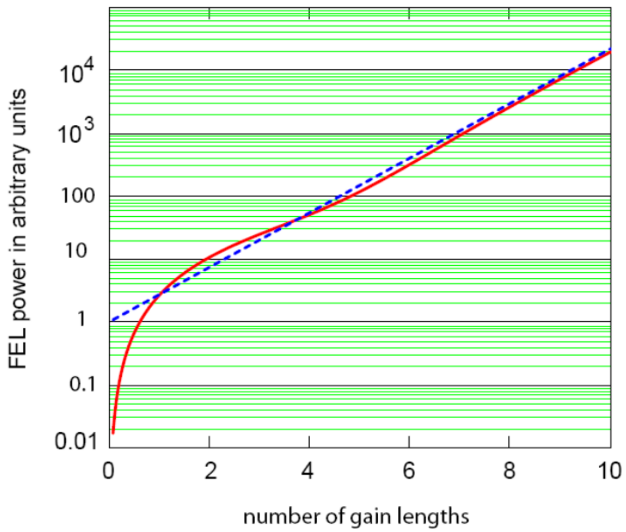
FEL can also start from initial density modulation given by noise.

Equivalent: starting from spontaneous undulator radiation.

Self-Amplified Spontaneous Radiation

SASE

Very robust mode of operation !



Theory must model shot noise.

Predicts effectiv “initial conditions”

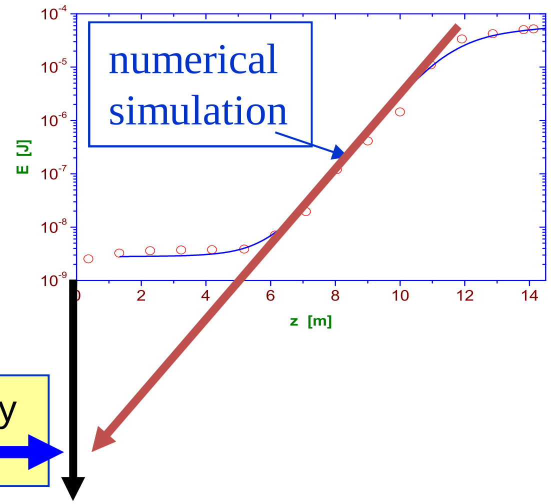
Critical bench mark test for numerical FEL codes, e.g.

GENESIS (Reiche)

GINGER (Fawley)

SIMPLEX (Tanaka)

FAST (Yurkov)



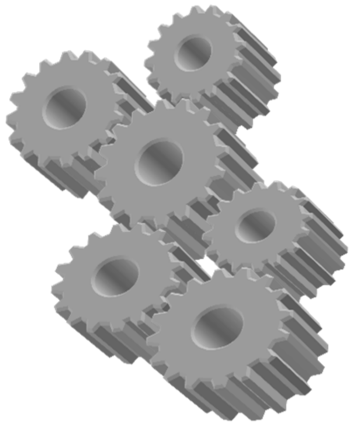
Equivalent input energy by shot noise:0.3 pJ

Theorie vs. Experiment

INPUT (electrons)

- Momentum
- Momentum spread/chirp
- Slice emittance/ phase space distribution
- Total charge
- Long. charge profile
- Peak current
- Orbit control

FEL machinery



OUTPUT (photons)

- Gain length
- Saturation behaviour
- Spectrum
- Harmonics
- Transverse coherence
- Pulse length
- Effective input power
- Fluctuations

Do we understand the machinery ?

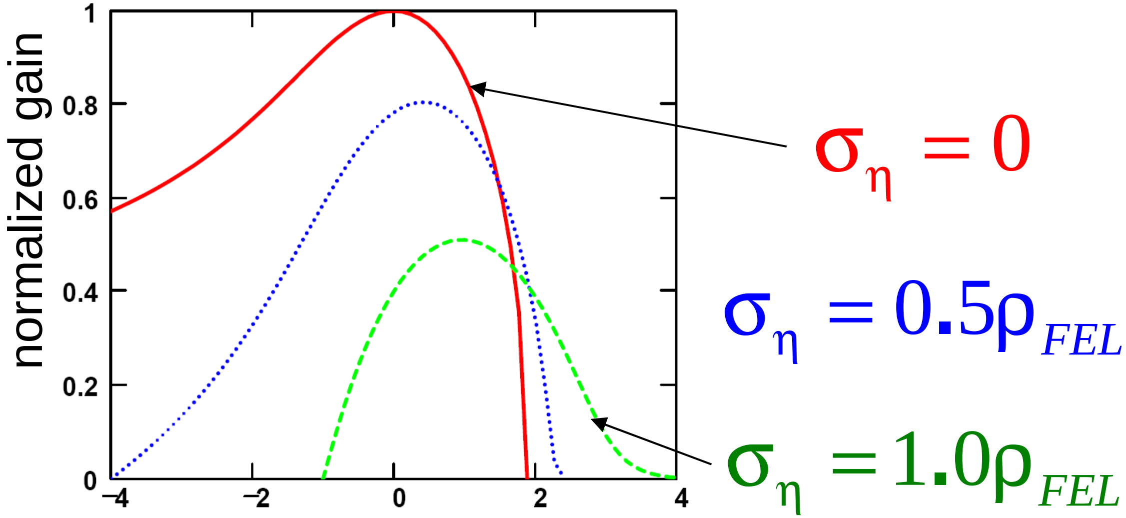
Exponential growth ? ✔

Reasonable gain length ? ✔

Achieve full density modulation ? ✔

Bandwidth

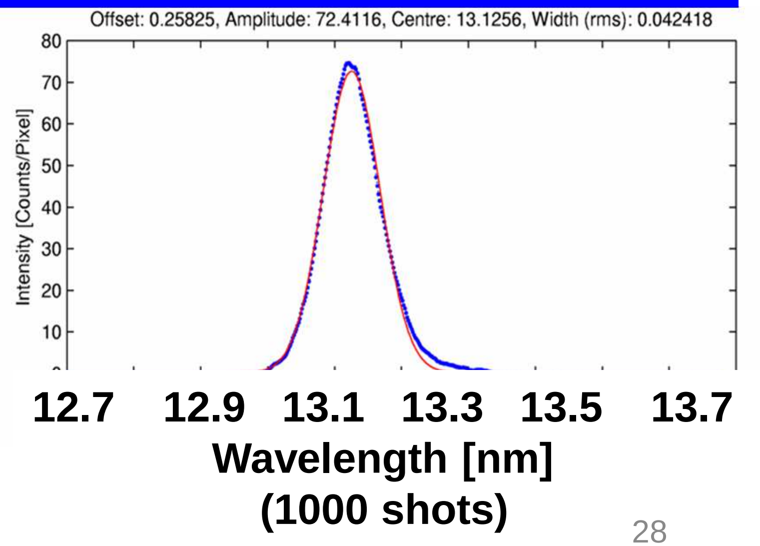
Gain vs.
momentum error η=dp/p
(momentum spread σ_η)
Note:
Emittance effect similar



FEL is a narrow band amplifier; bandwidth related to gain length
Note: Cannot produce few-cycle pulses !

FLASH experiment:

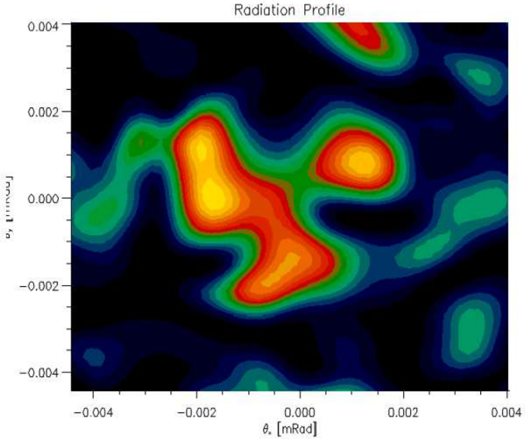
Bandwidth ? ✔



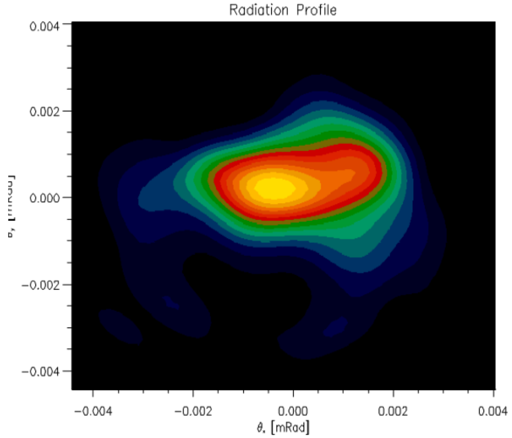
Transverse coherence

S. Reiche

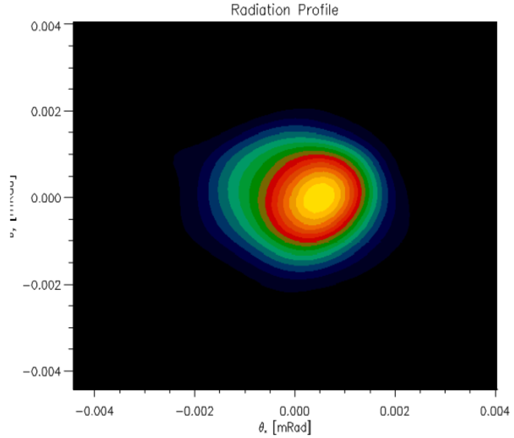
Z=25 m



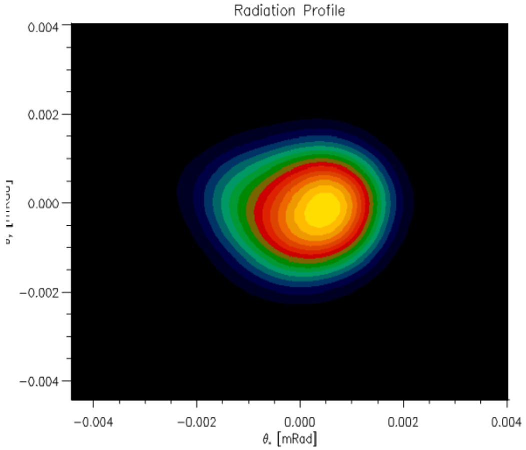
Z=37.5 m



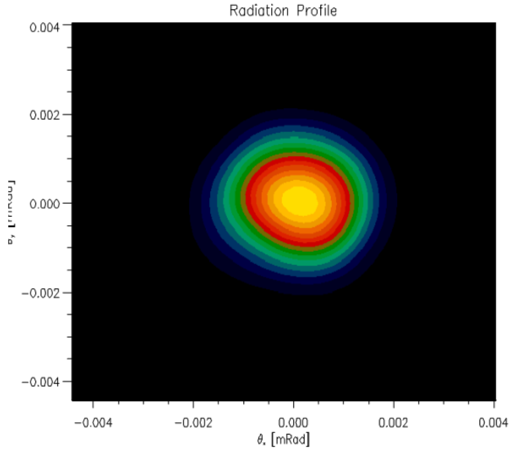
Z=50 m



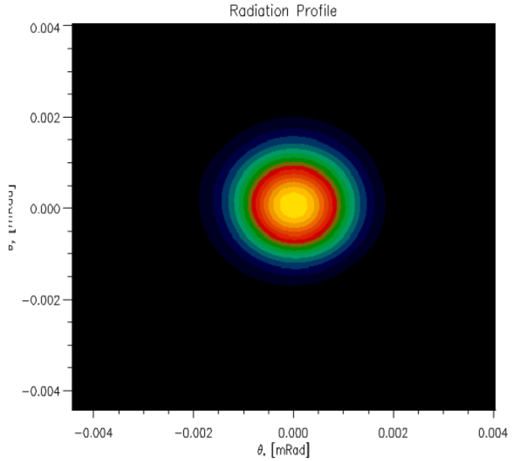
Z=62.5 m



Z=75 m



Z=87.5



Single mode dominates ➔ close to 100% transverse coherence

Transverse Coherence

Emittance of a perfectly coherent ("gaussian") light beam:

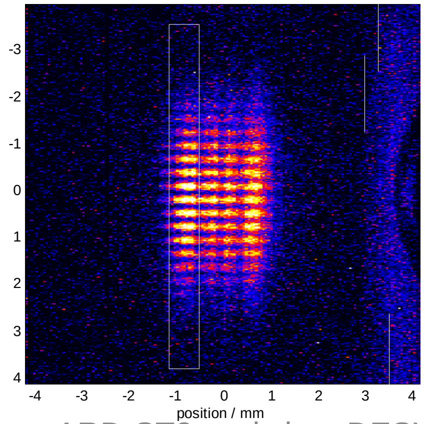
$$\varepsilon_{Light} = \sigma_r \cdot \sigma_\theta = \frac{\lambda_{Light}}{4\pi}$$

→ FEL theory predicts high transverse coherence of photon beam, if electron beam emittance:

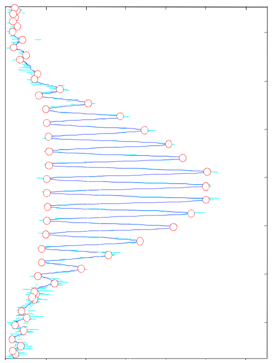
$$\boxed{\varepsilon_{electrons} < \approx \frac{\lambda_{Light}}{4\pi}}$$

Observation of interference pattern at FLASH:

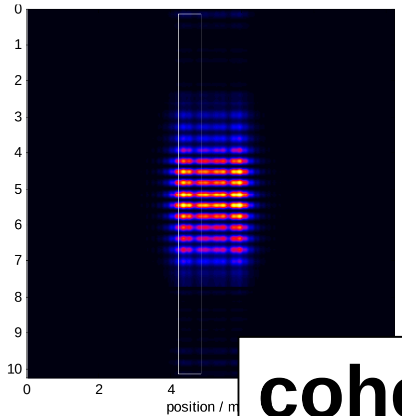
double slit



intensity modulation



FEL simulation



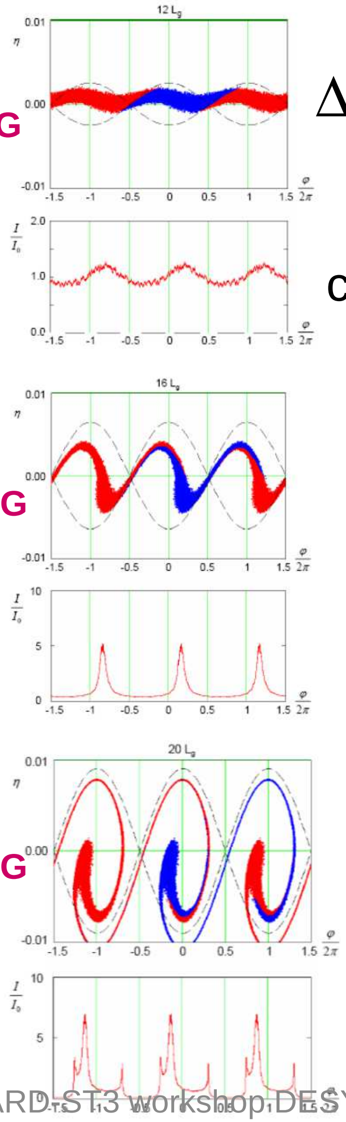
Verified @ 32nm + 13nm

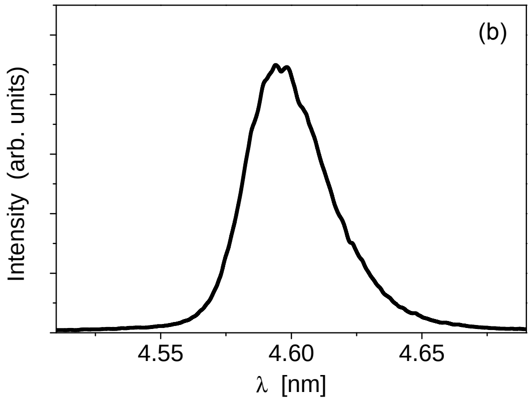
LCLS: Verified @ 1.6nm + 0.14nm

coherence ? ✔

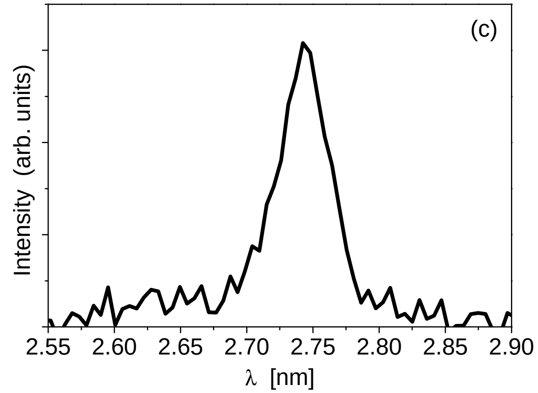
Higher Harmonics

Density modulation becomes anharmonic at high gain:





3rd harmonics
@ 4.8 nm



5th harmonics
@ 2.75 nm

new third harmonic

FLASH typical pulse energies (avg.):

Fundamental (13.8 nm):	40 µJ
3rd harmonics (4.6 nm):	(0.25 ± 0.1) µJ
5th harmonics (2.75 nm):	(10 ± 4) nJ

Harmonics ? ✔

Longitudinal coherence

FEL-equation tells:
Electric field gain depends on frequency E= E(z, ω)

First-order correlation function can be expressed as:

$$C(\tau)_{Light} = \frac{\left\langle \int \left| E(\omega) \right|^2 \exp(i\omega\tau) d\omega \right\rangle}{\left\langle \int \left| E(\omega) \right|^2 d\omega \right\rangle}$$

Longitudinal coherence time:

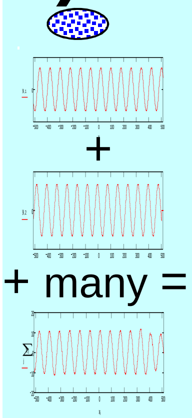
$$\tau_{coh} = \int_{-\infty}^{+\infty} \left(C(\tau) \right)^2 d\tau = 2\sqrt{\pi}\sigma_t$$

with: $\sigma_t = \dfrac{1}{2\sigma_\omega}$

Start-up from noise

Simple 1D model: Superposition of many wavetrains with random phases

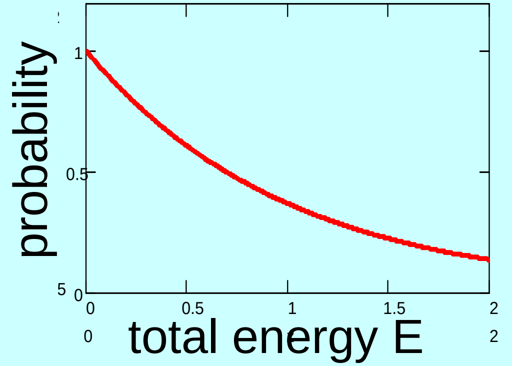
A) Short bunch << wavetrain (coherence length)



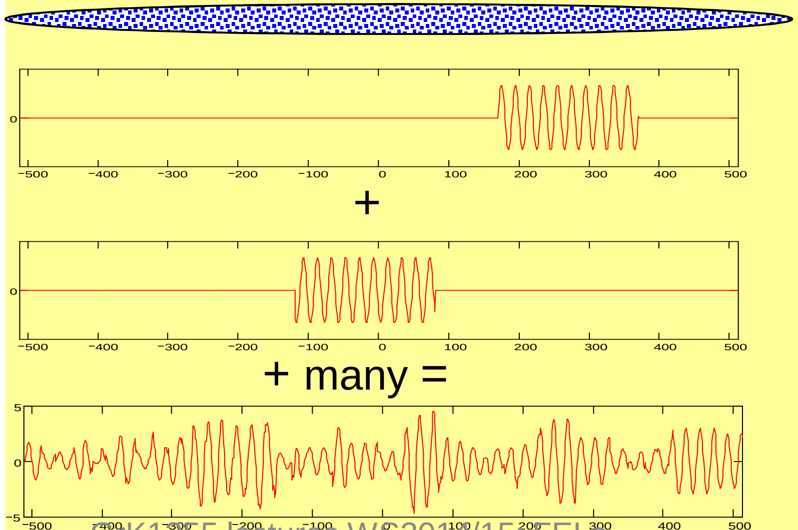
Large probability of destructive interference

"single Mode"

$$P(E)dE = \exp(-E)\,dE$$



B) Bunch length >> wavetrain: ***"many Modes"***

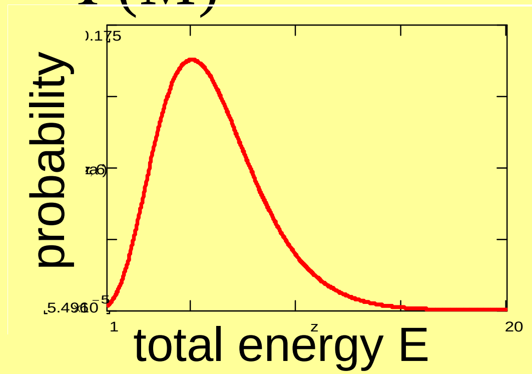


$$g_M(E)\cdot dE = \frac{1}{\Gamma(M)}(E)^{M-1}e^{-E}\cdot dE$$

Extract M from

histrogram

→ pulse length

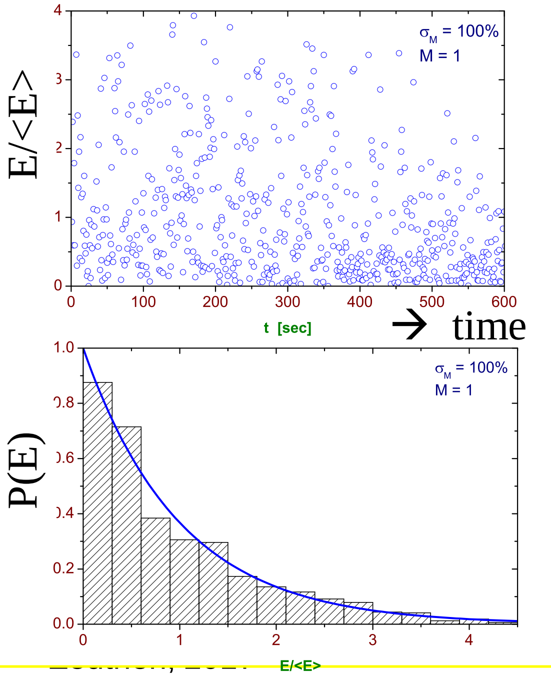


Start-up from noise

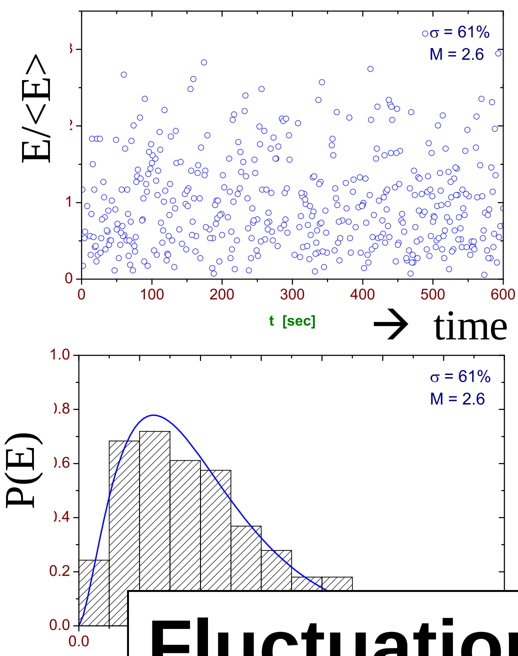
SASE output will fluctuate from pulse to pulse,
-- just as ANY part of spontaneous synchrotron radiation does !
Remember: FEL is just an amplifier !

Photon pulse length = M × (coherence length of single mode)

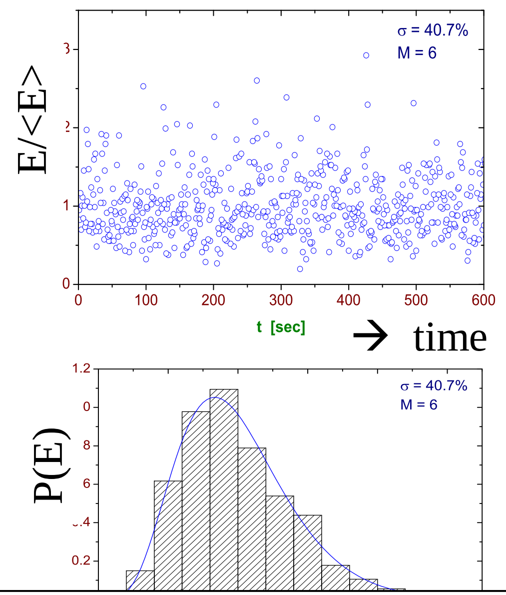
single Mode
(after monochromator slit)



short pulses
M=2.6 modes



long pulses
M=6 modes



Fluctuation properties ? ✔

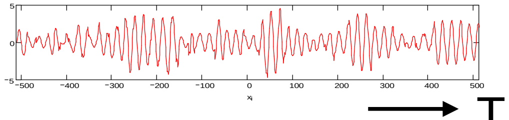
Pulse length

Time-domain measurement of pulse length:

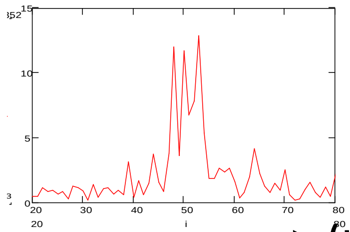
not (yet) regularly available for X-ray (established in the visible, FROG etc.)

Alternative: intensity fluctuation translates into spectral fluctuation:

Width of frequency spikes ↔ length of pulse



Fourier

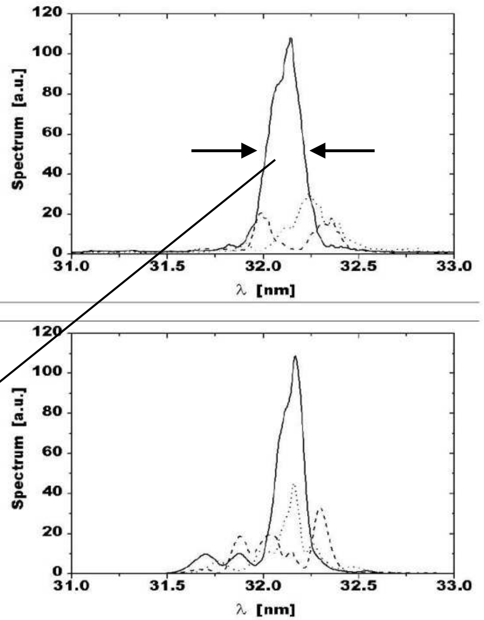


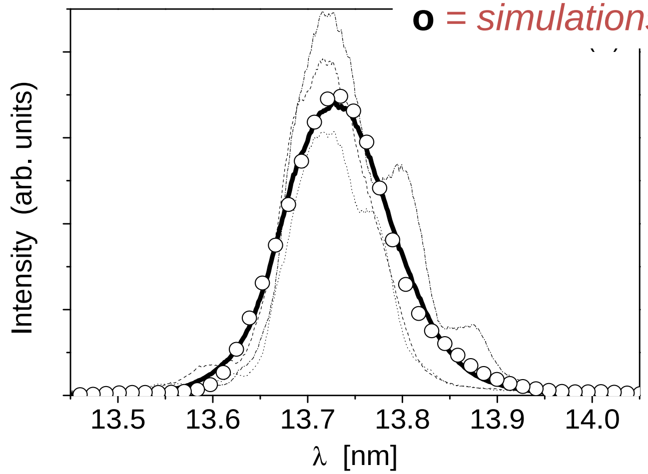
new_fund_harmonic_w.avi

3 single pulse Spectra @FLASH:

measured

predicted





~0.4% → ~25 fs pulse duration @ 32 nm

Pulse length ? ✔

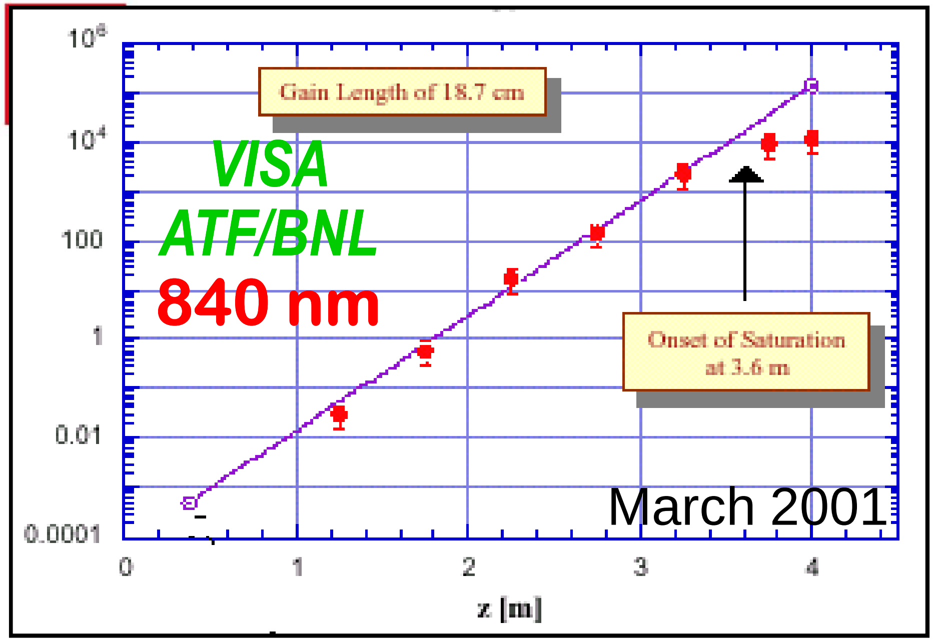
Why is such a device called a laser?

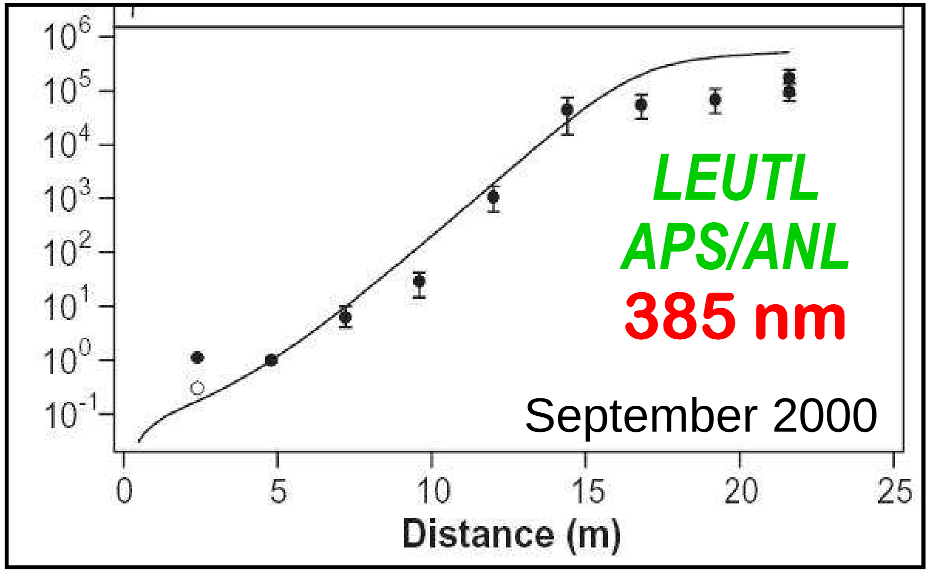
1. Emission of photons is stimulated by the presence of the electromagnetic field inside the undulator

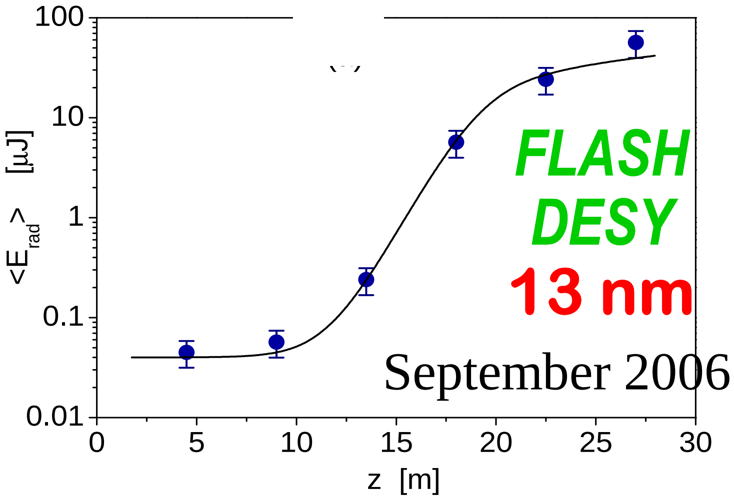
   - electron beam takes the role of active medium

2. Radiation properties are typical for lasers

What do we observe ?

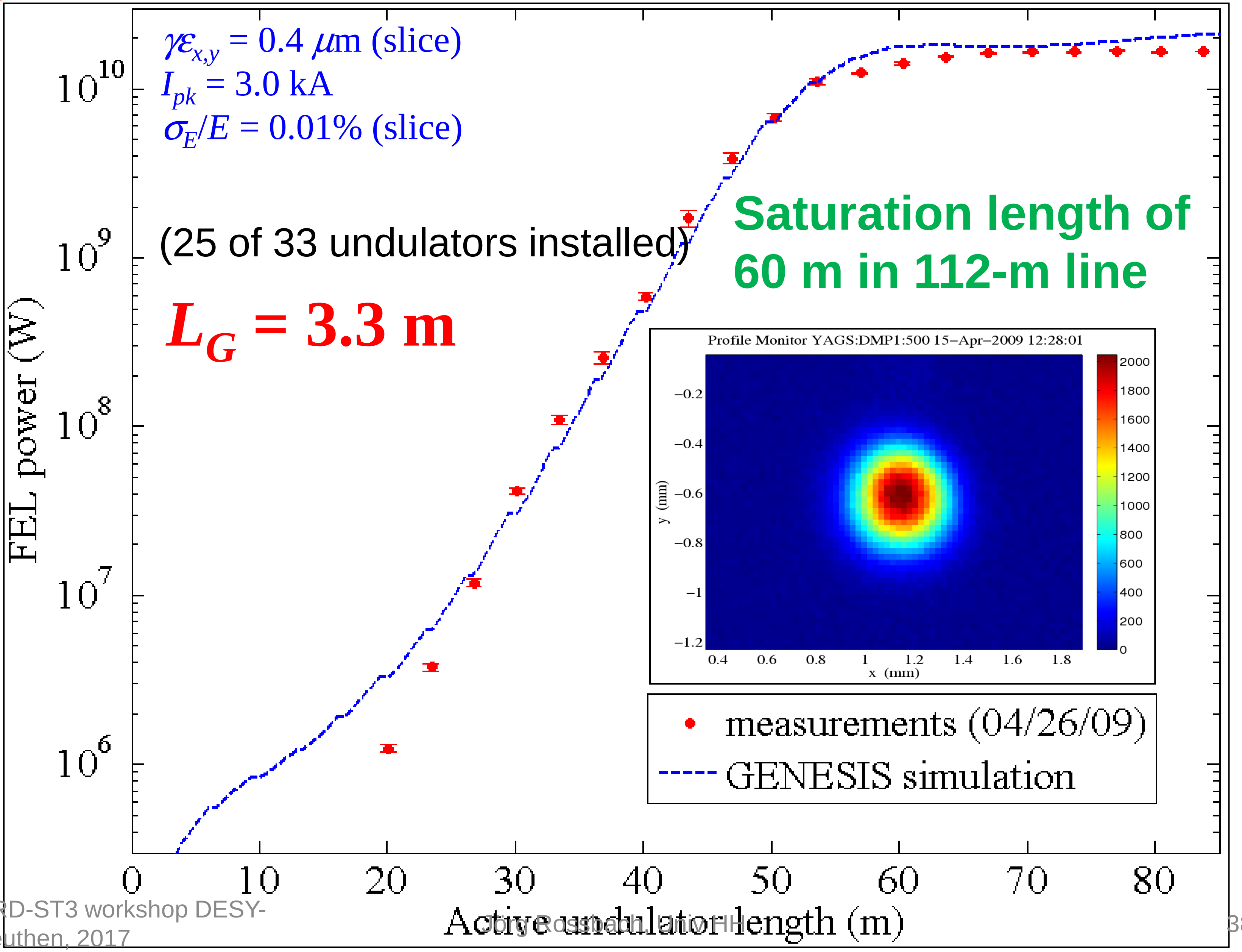




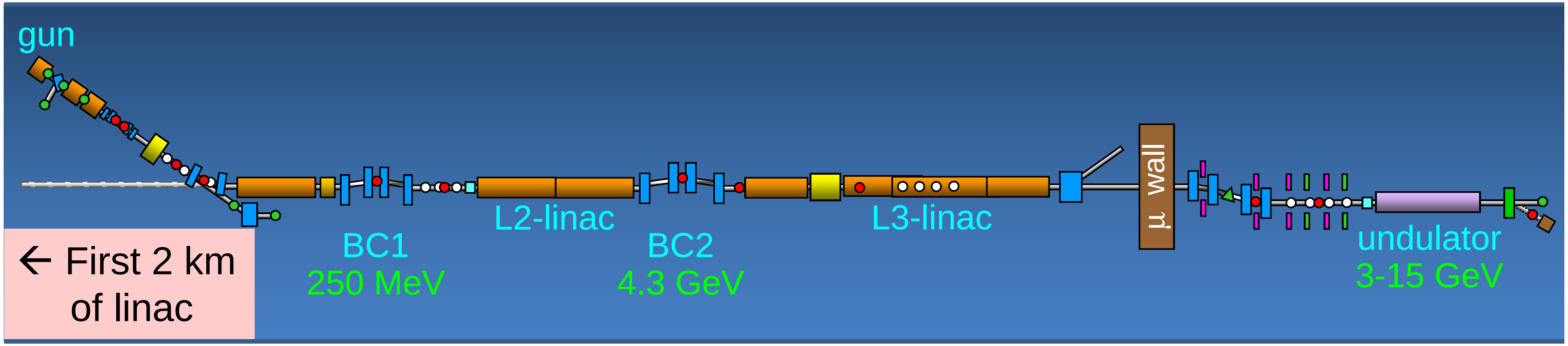


For all experiments there exists a “reasonable” set of electron beam parameters such that gain length and saturation level agree with theoretical expectations.

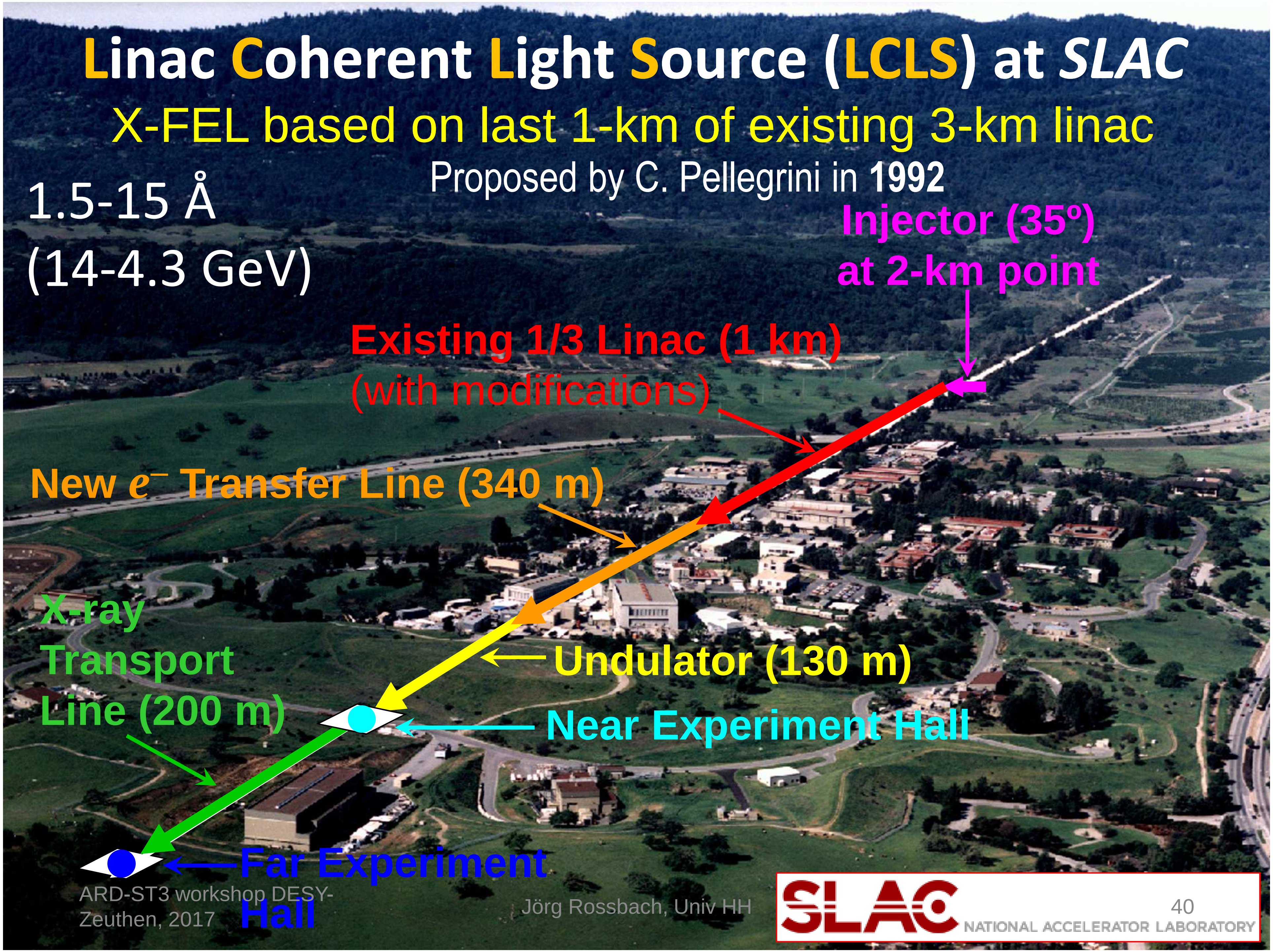
Gain Length at 1.5 A at LCLS



The Linac Coherent Light Source LCLS - the blue print of all SASE FELs



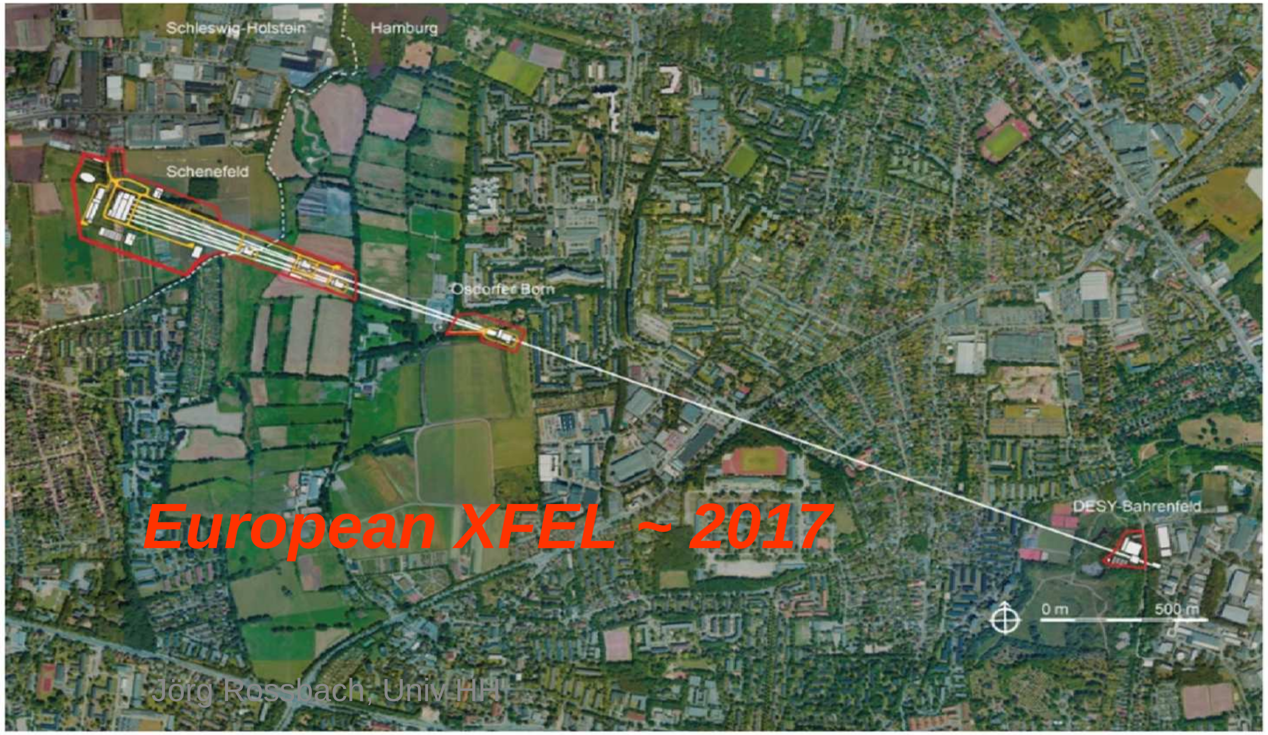
- Proposed 1992 by C. Pellegrini, et al.
- Based on 1/3 of SLAC linac → max. 15 GeV → 1.5 Å
- Normal conducting S-band (3 GHz) linac very well understood


Linac Coherent Light Source (LCLS) at SLAC
X-FEL based on last 1-km of existing 3-km linac
Proposed by C. Pellegrini in 1992
1.5-15 Å
(14-4.3 GeV)
Injector (35º)
at 2-km point
Existing 1/3 Linac (1 km)
(with modifications)
New e⁻ Transfer Line (340 m)
X-ray
Transport
Line (200 m)
Undulator (130 m)
Near Experiment Hall
Far Experiment
Hall
SLAC
NATIONAL ACCELERATOR LABORATORY

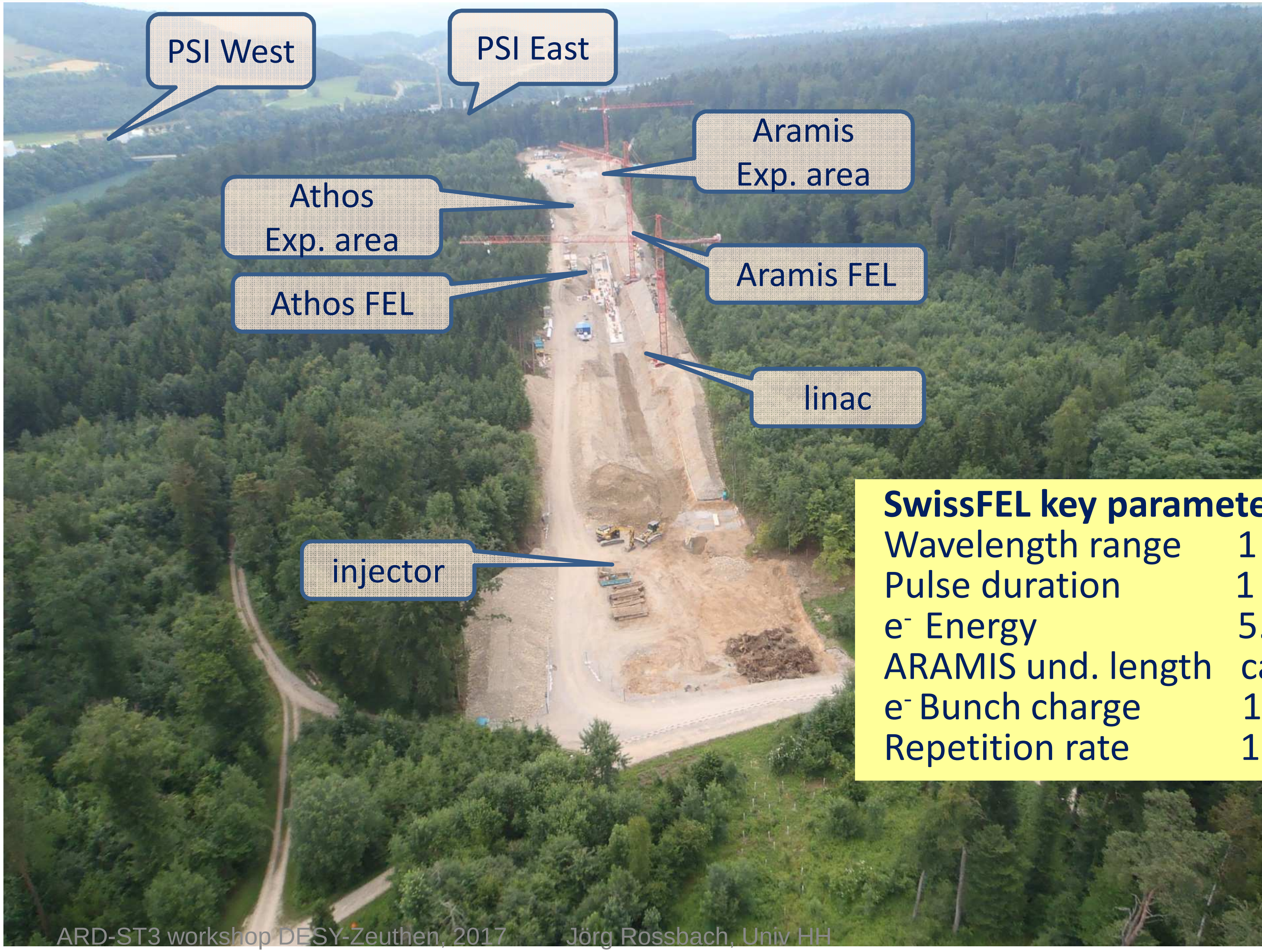
And more XFELs ….

- SASE Wavelength range: **3 – 0.6** Å
- Photon energy range: **4 - 20** keV
- Pulse length (**10** fs FWHM)
- Pulse energy up to **1** mJ

more to come:
Europ. XFEL (2017)
PAL-XFEL (2017)
SwissFEL (2017)
LCLS-II (>2020)
…



SwissFEL construction site 27 June 2013



SwissFEL key parameters

Wavelength range	1 Å - 70 Å
Pulse duration	1 fs - 20 fs
e^- Energy	5.8 GeV
ARAMIS und. length	ca. 60 m
e^- Bunch charge	10-200 pC
Repetition rate	100 Hz

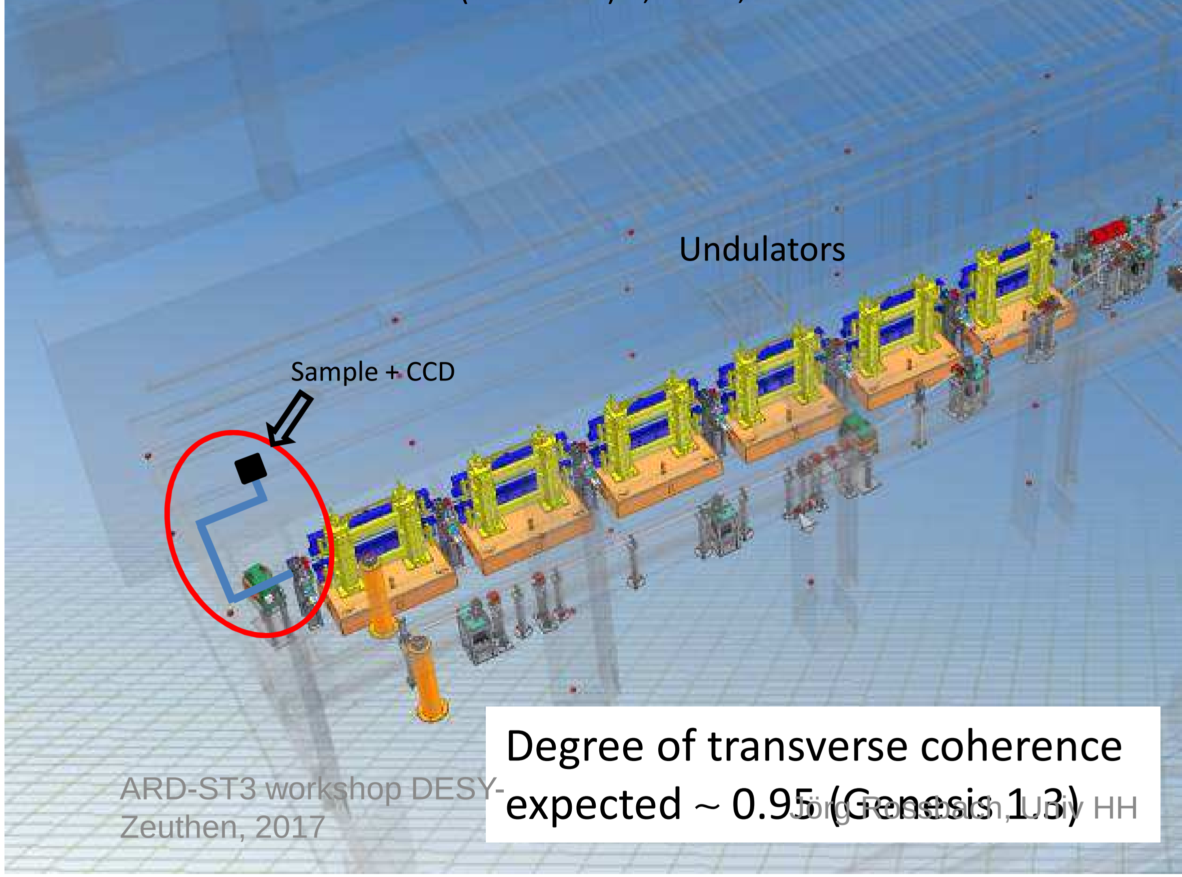
Jan’16 first users of game crossing observed

SPARC @LNF, Frascati: Test bed for many new ideas

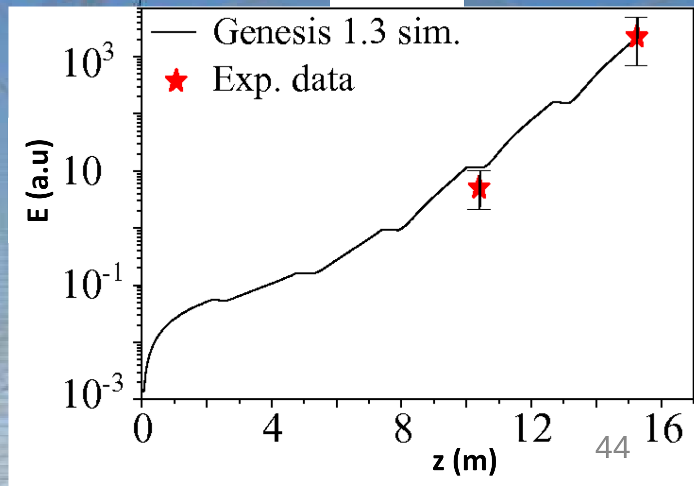
- **6 sections undulator; 77 periods per section**
- Magnetic Length 215 cm
- K factor: from 0.5 and 3
- Beam energy E_B(MeV) 162,5±0,27
- Beam charge(pC) 312 ±16
- Energy Spread (proj: %) 0,2 ± 0,015
- Energy Spread (slice %) 0,050 ±0,005
- Length r.m.s. (ps) 1,65 ± 0,05
- Beam current I_{peak}(A) 75,63 ±3,5
- Vertical Emittance 90% (mm mrad) 1,95 ± 1,0
- Horizontal Emittance 90%(mm mrad) 1,74 ± 1,1

1.6 cells RF injector
UCLA/BNL/SLAC design
120 MV/m

3 SLAC type Focalization
solenoids
(velocity bunching)



SASE



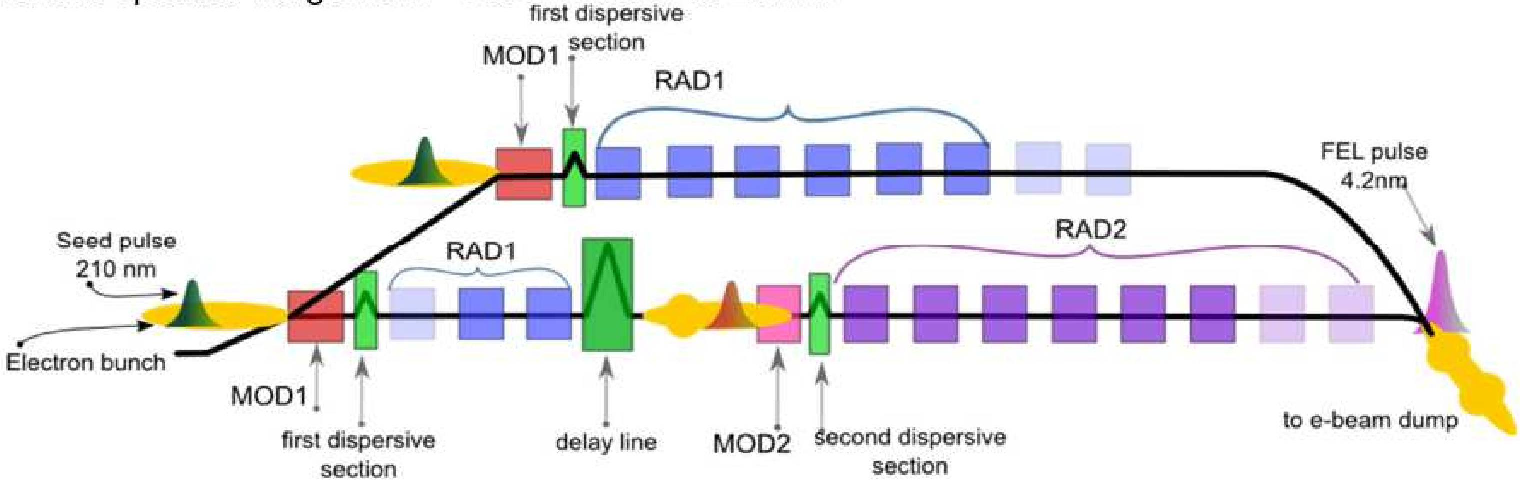
Degree of transverse coherence expected ~ 0.95 (Genesis 1.3)

Also soft X-ray FELs:

HGHG at FERMI: FEL-1 and FEL-2

FERMI's two FELs will cover different spectral regions.

FEL-1, based on a single stage high gain harmonic generation scheme initialized by a UV laser will cover the spectral range from ~100 nm down to 20nm.



SASE FEL Electron Beam Requirement

radiation wavelength

$$\varepsilon_N < \gamma \frac{\lambda_r}{4\pi}$$

transverse emittance: ε^n < 1 mrad mm at 6 nm, 1 GeV

peak current

undulator period

FEL parameter

$$\sigma_\delta < \rho_{FEL} \approx \frac{1}{2}\left[\frac{\hat{I}\lambda_u^2}{4\pi I_A \varepsilon^n \beta}\left(\frac{\hat{K}}{\gamma}\right)^2\right]^{1/3}$$

relative energy spread:

<0.3% at I_{pk} = 2 kA,

$K \approx 1$, $\lambda_u \approx 3$ cm, …

beta function

undulator ‚field' = 0.93·B/T·λ_u/cm

$$L_g \approx \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$

FEL gain length

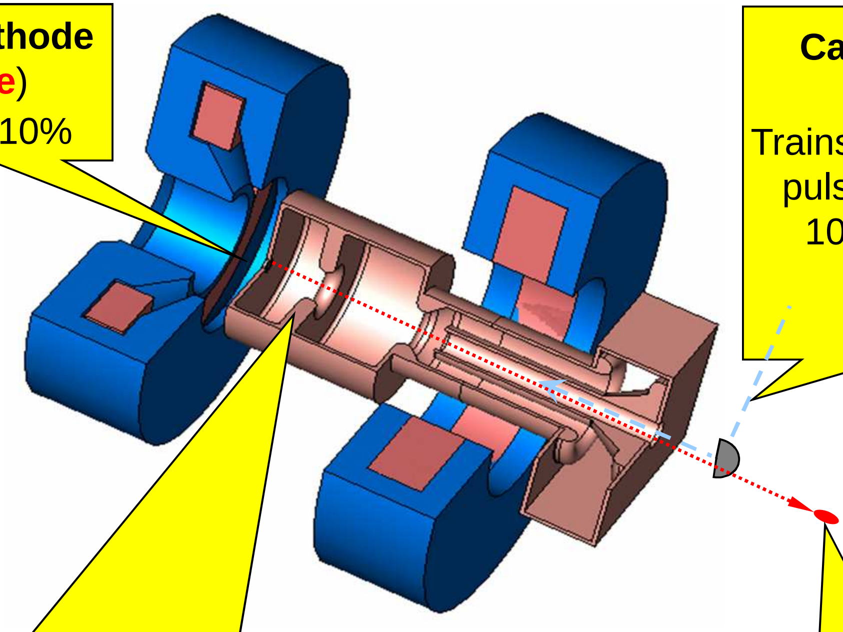
$L_G \approx 0.5$ m for $\varepsilon_N \approx 1$ mrad mm

- We must increase peak current, preserve emittance, and maintain small energy spread so that power grows exponentially with undulator distance, z, $P(z) = P_0 \cdot \exp(z/L_G)$
- FEL power reaches *saturation* at $\sim 18 L_G$
- SASE performance depends *exponentially* on e^- beam quality ! (challenge)

Courtesy: Z. Huang

Key components: electron gun
example: gun for FLASH and XFEL

Photo cathode
(Cs_2Te)
QE~0.5-10%



Cathode laser
λ=257nm
Trains with up to 800 pulses (1MHz) at 10Hz rep.rate.

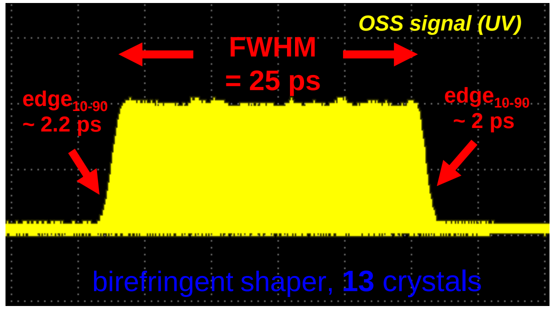
RF-gun:
- L-band (**1.3 GHz**) normal conducting (copper) standing wave 1½-cell cavity
- Peak rf power: ~**7MW** (E_z@cathode: > 60MV/m)
- **850 µs** RF pulse length with a repetition rate of **10 Hz**, duty cycle ~ 1%,
- Dry ice cleaning

  ➔ Dark current < 50 µA at 7.75 MW

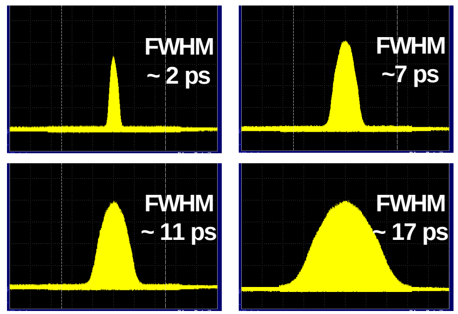
Electron bunches:
- 1nC nominal charge
- ~7MeV/c max. mean momentum
- Pulse trains

Temporal pulse shaping

Flattop (nominal)

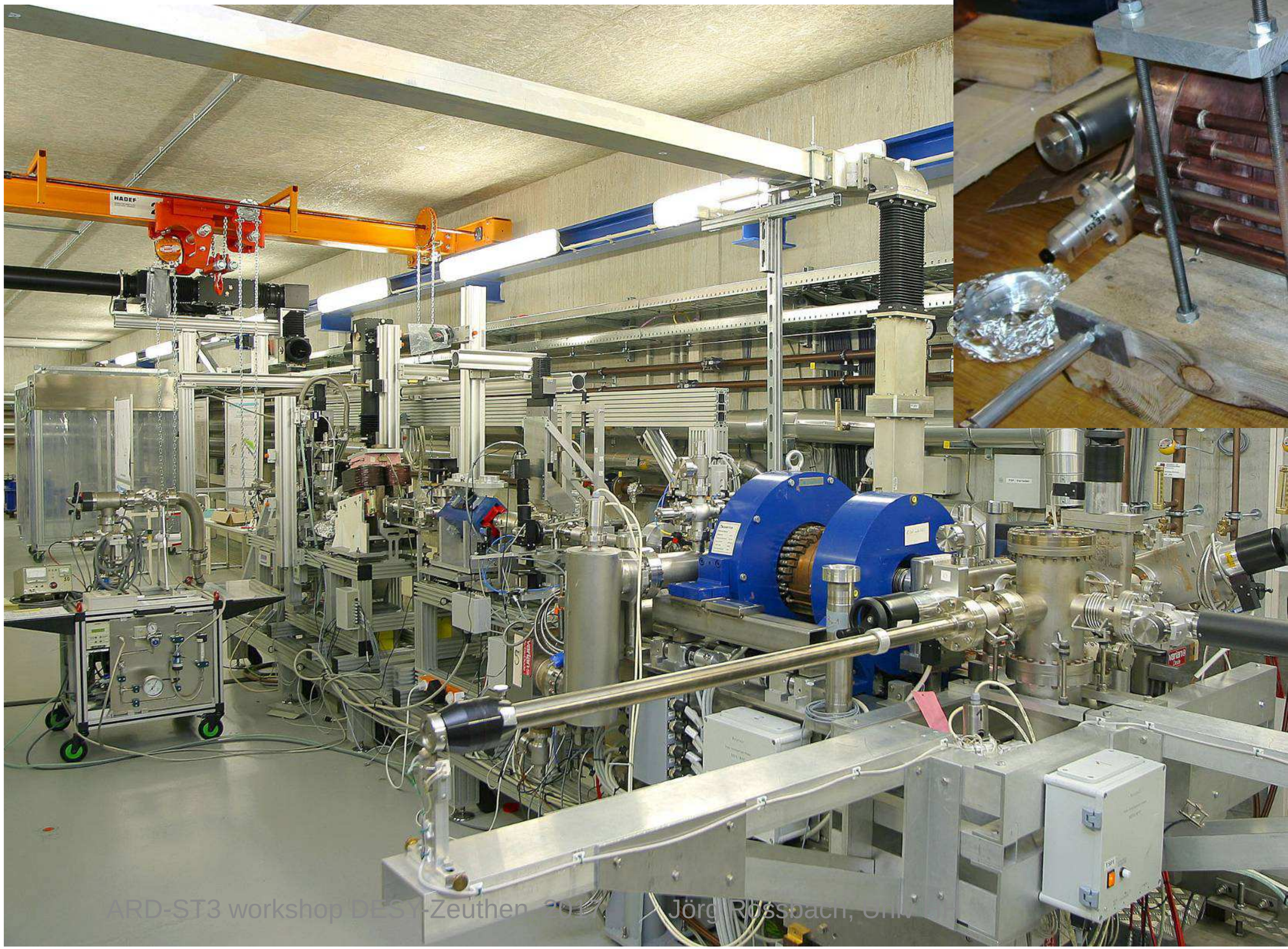


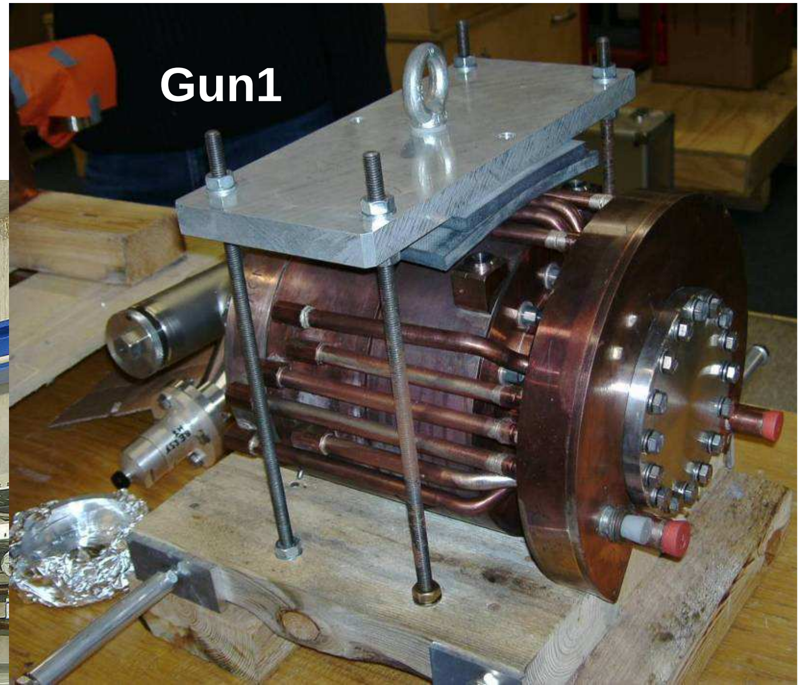
Gaussian:



Courtesy: F. Stephan/PITZ@DESY-Zeuthen

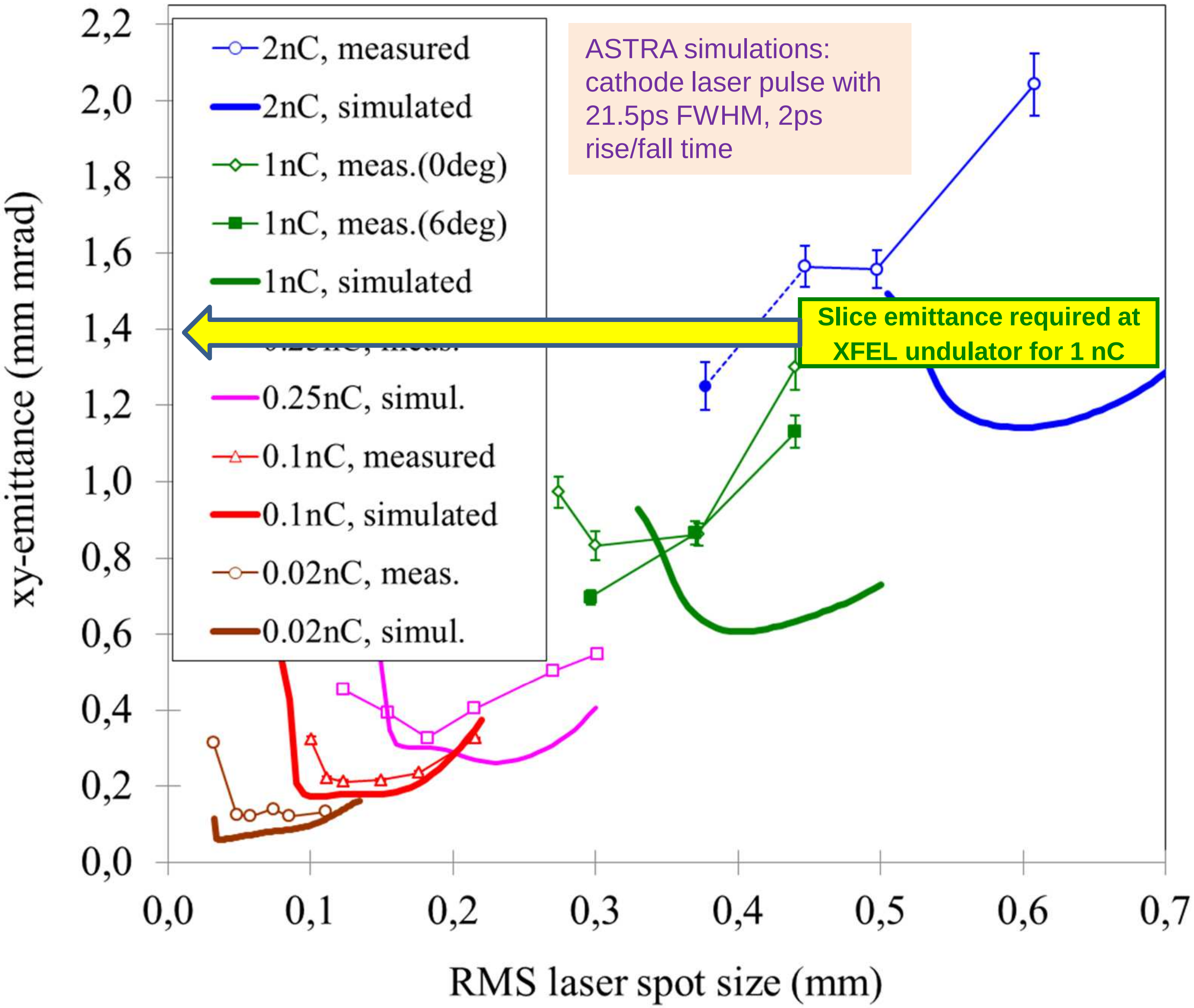
The Photoinjector Test Facility PITZ at DESY-Zeuthen



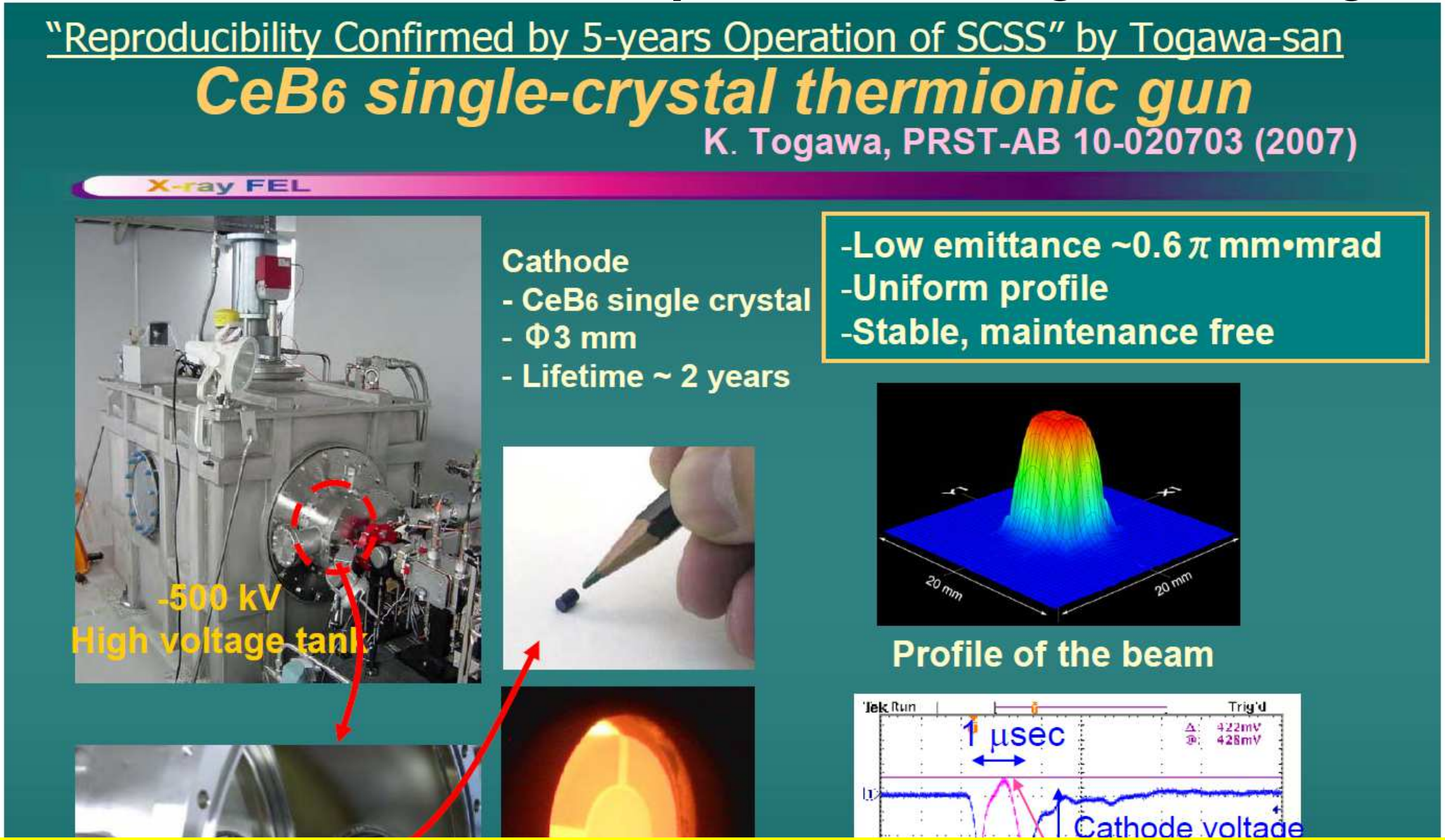


Emittance vs. Laser Spot size for various bunch charges

Note: these are "**100% RMS emittance**" values



T. Shintake/SCSS: You can also use a thermionic cathode with subsequent multi-stage bunching

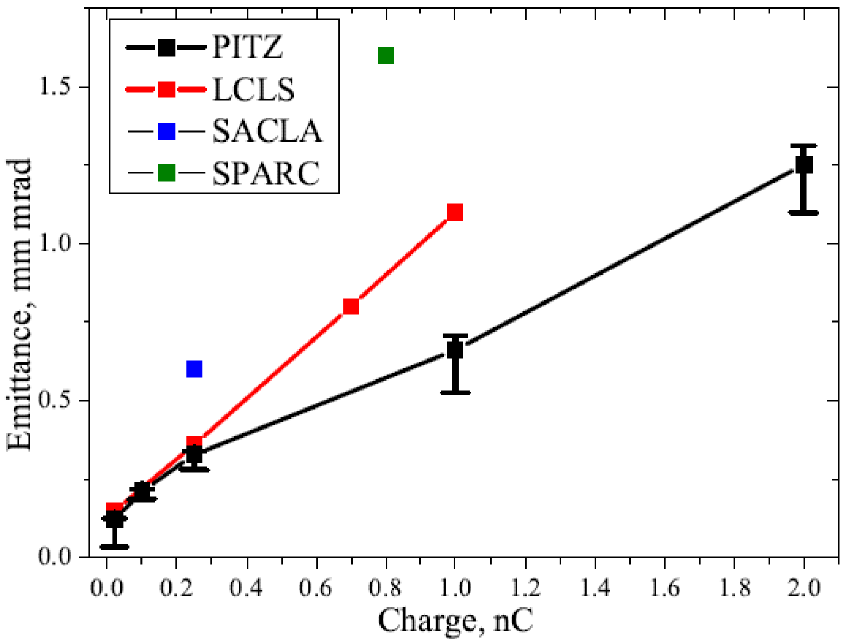


Nice stability, but suffers also from micro-bunching instability.

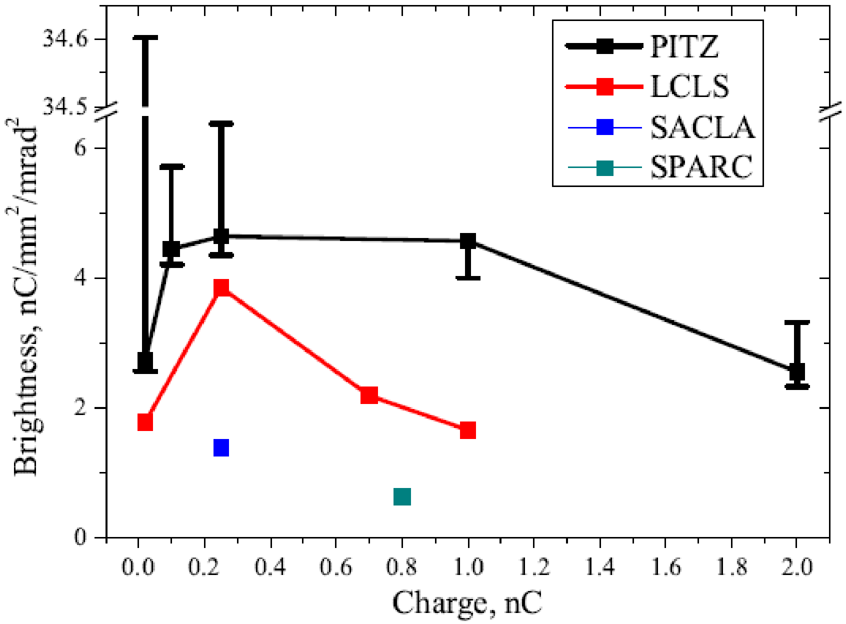
My view: Less flexible than rf gun,

No final conclusion in terms of preference

Electron guns: comparison



(a) Electron beam emittance.



(b) Electron beam brightness.

compiled by G. Vashchenko

Key components: linac

Superconducting vs. normal conducting:

Key issues:
high accelerating field → + escape from space charge forces
+ short device

Normal conducting rf linac prefers high frequency
LCLS: S-band (3 GHz): ~20-25 MV/m
SCSS, SwissFEL: C-band (6 GHz): 35/28 MV/m

BUT: Ohmic losses in copper resonator → heating → rf duty cycle $<10^{-4}$
→ 120 – 400 rf pulses of ~1µs length, only one electron bunch per pulse

Superconducting resonators: L-band 1.3 GHz OK
Max. acc. field fundamentally limited to approx. 45 MV/m,
but in practice (manufacturing) ~25 MV/m
Still dissipative losses: ca. 1 W/m @ G=25 MV/m, scaling $\sim G^2$
→ rf duty cycle ca. 1% @25MV/m or 100% @ <10MV/m
+ large resonators → large stored rf energy, small wake fields
→ Can accelerate several 1000 bunches per second.
-- more expensive (but cheaper per photon!)

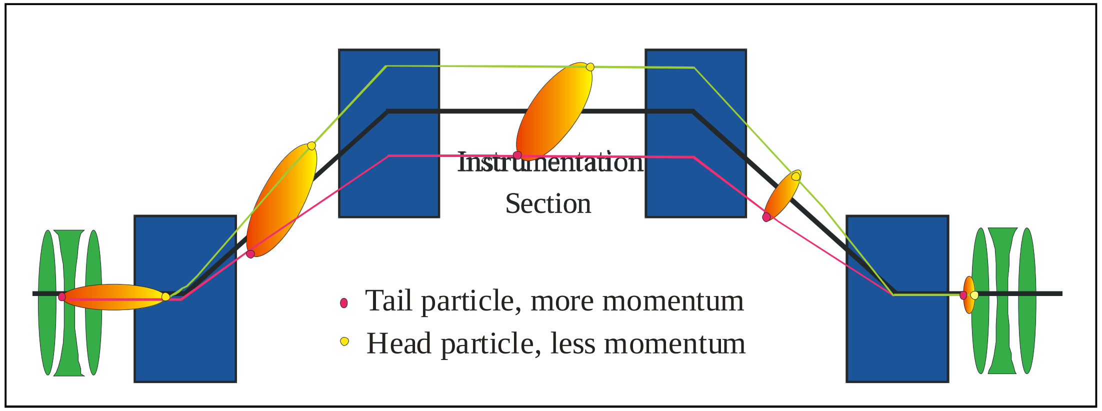
Key components: bunch compressors

ISSUE:
Electron bunches must be generated at few A peak current (to mitigate radial Coulomb forces), but FEL needs kA !

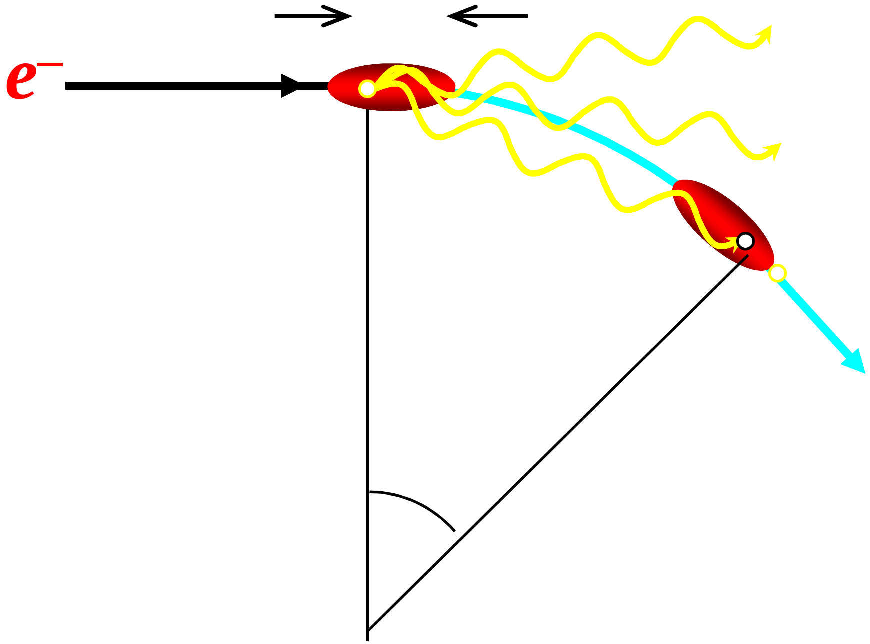
☺ Accelerate to ultrarelativistic energies
and profit from $1/\gamma^3$ scaling of space charge effect.

☹ Must compress longitudinally at ultrarelativistic energy where all particles have same speed, irrespective of momentum
→ „Velocity bunching“ option limited. → need to use magnetic chicane

2) pass a series of deflections



Coherent Synchrotron Radiation (CSR) in Bends

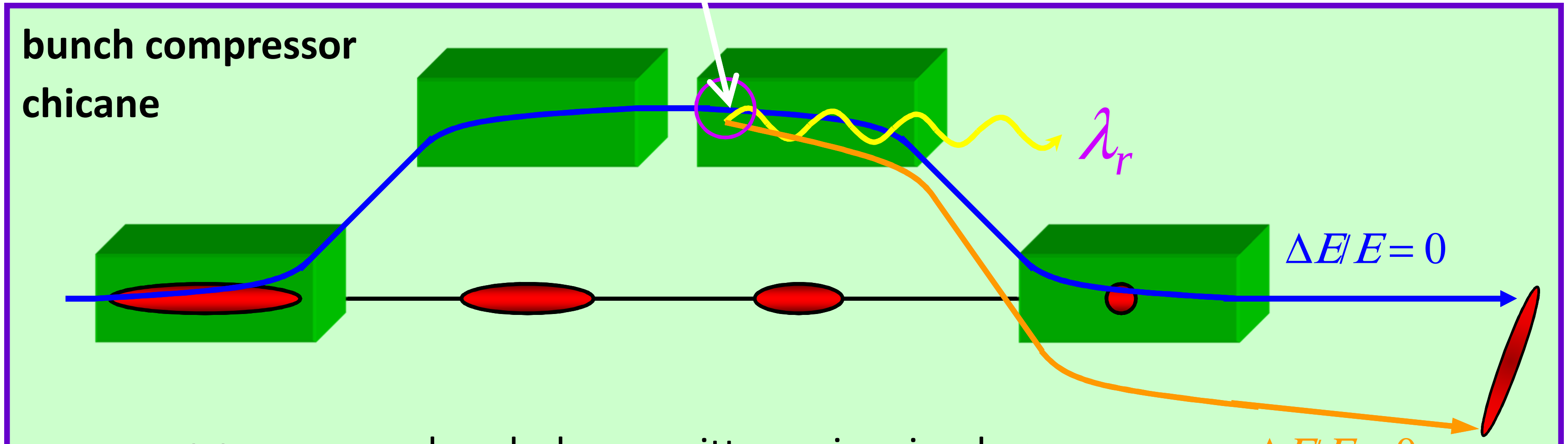


Radiation from tail catches head and is coherent for wavelengths greater than the bunch length ($\lambda_r > 2\pi\sigma_z$)

rms energy spread induced by CSR

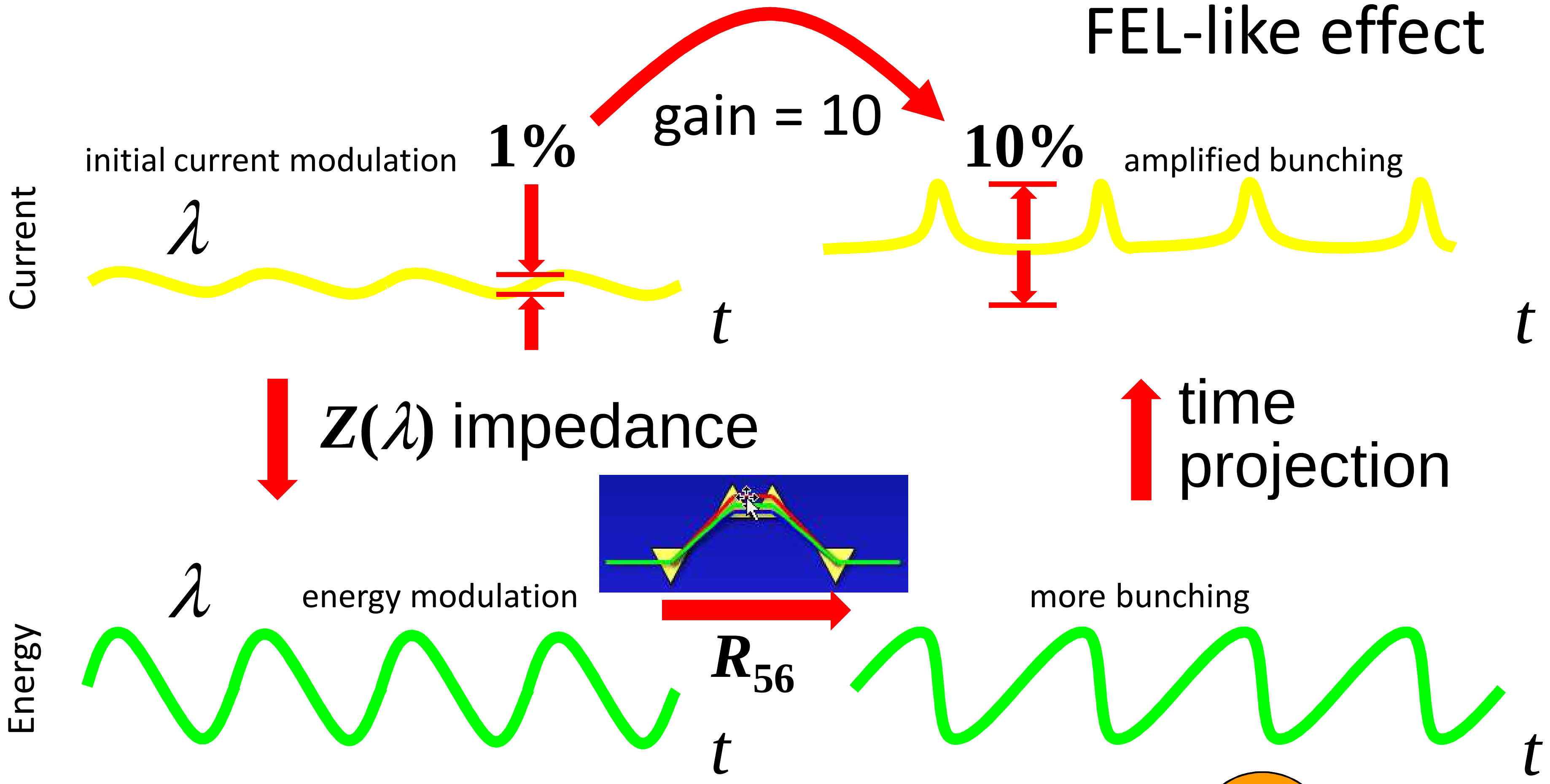
$$\left(\frac{\Delta E}{E}\right)_{rms} \approx 0.22 \frac{r_e N L}{\gamma R^{2/3} \sigma_z^{4/3}}$$

Gaussian bunch



Micro-bunching Instability (can ruin e$^-$ beam)

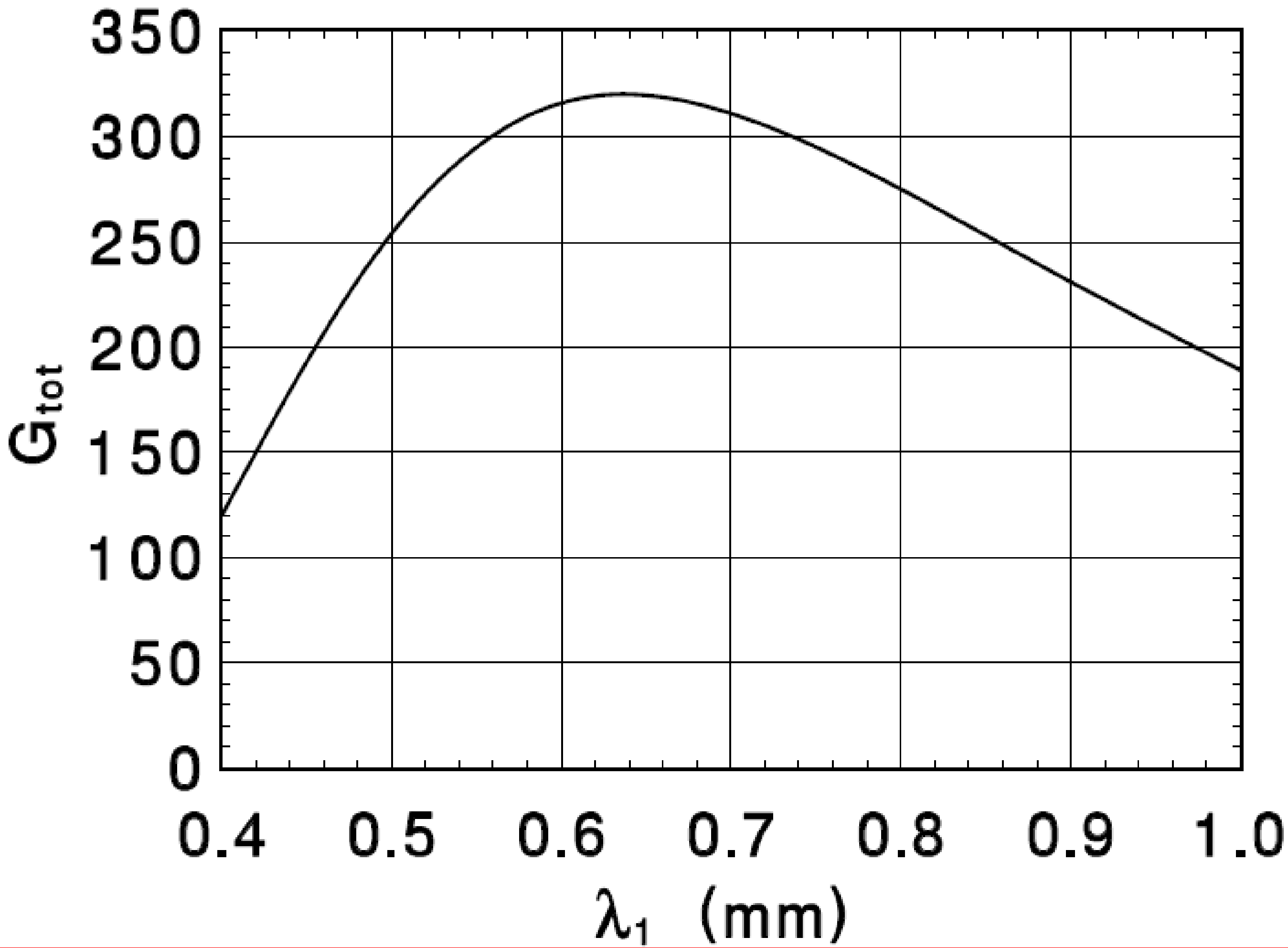
Initial e^- bunch current modulation induces energy modulation through impedance, $Z(\lambda)$, converted to more current modulation by bunch compressor, R_{56}



➔ **growth of slice energy spread (and emittance)**

Z. Huang

Microbunching instability at FLASH

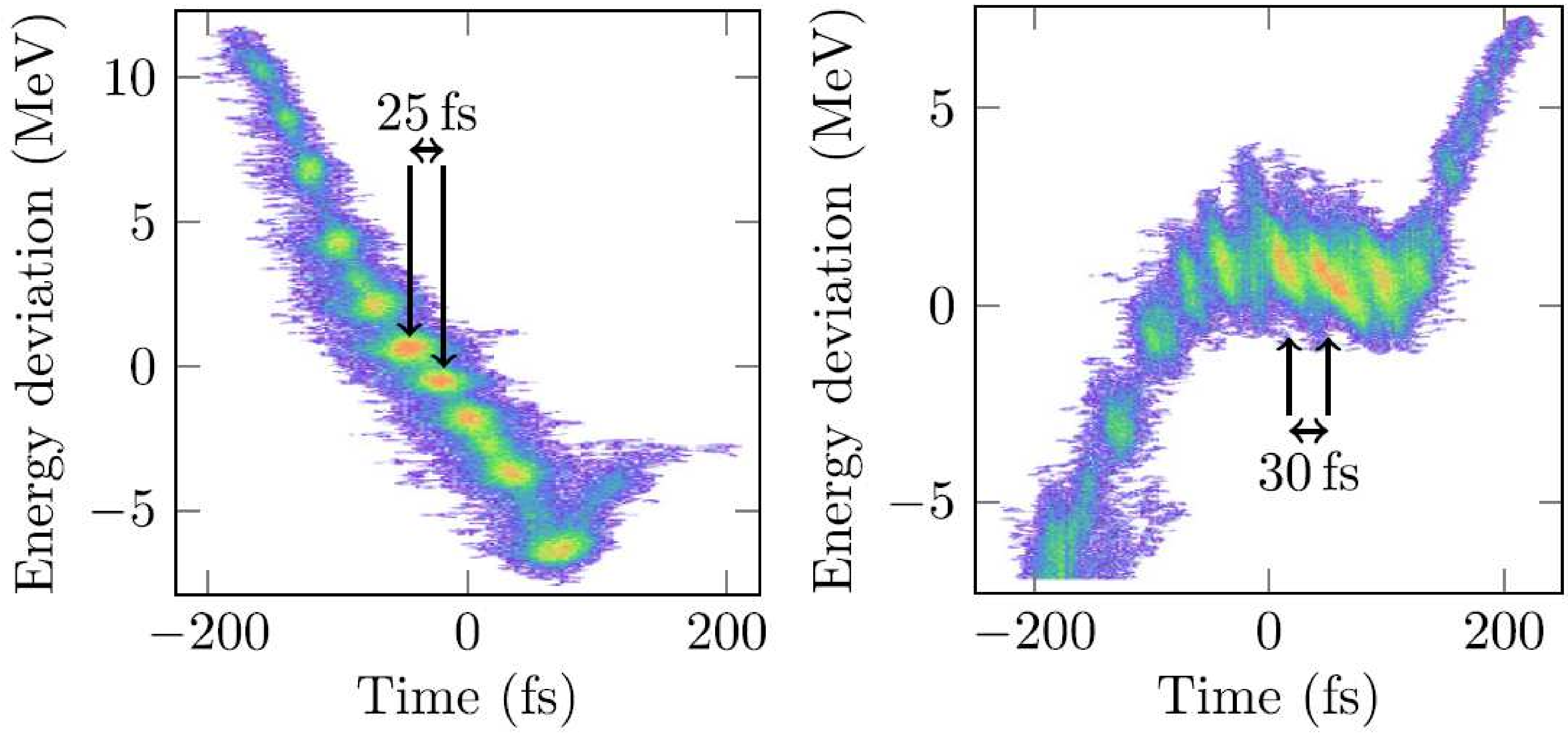


→ Expect modulation at mm scale and below

!! Provoke it and use it to generate coherent radiation ("longitudinal space charge amplifier") !!

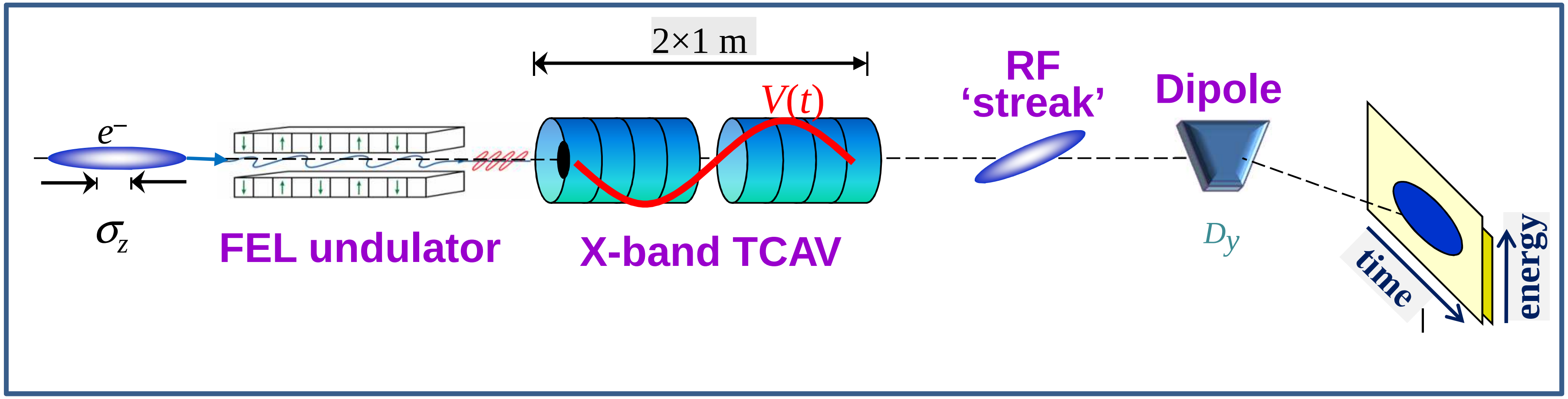
Microbunching instability at FLASH for two different compressor settings

Note: time resolution of device > 20 fs

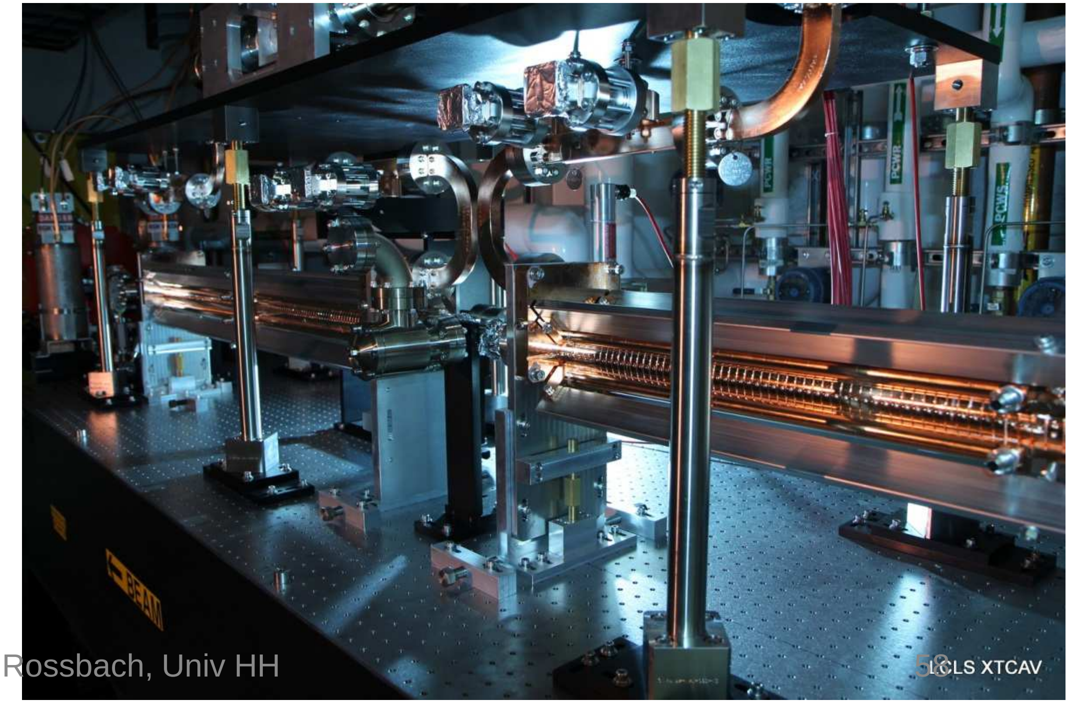


Chr. Behrens et al., PRST-AB 15, 062801 (2012)

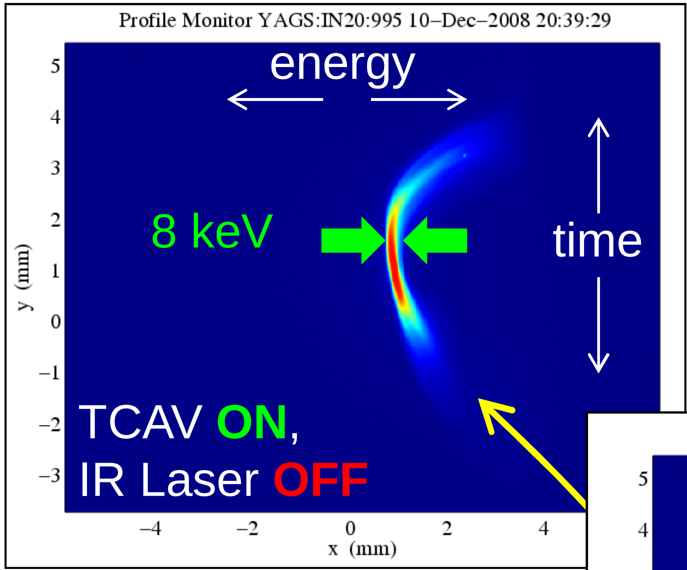
LCLS: TCAV after FEL undulator

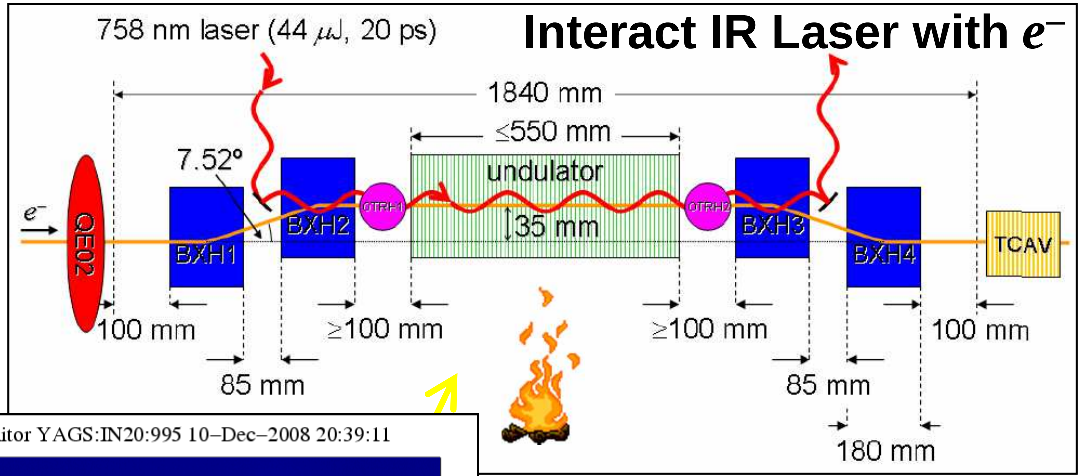


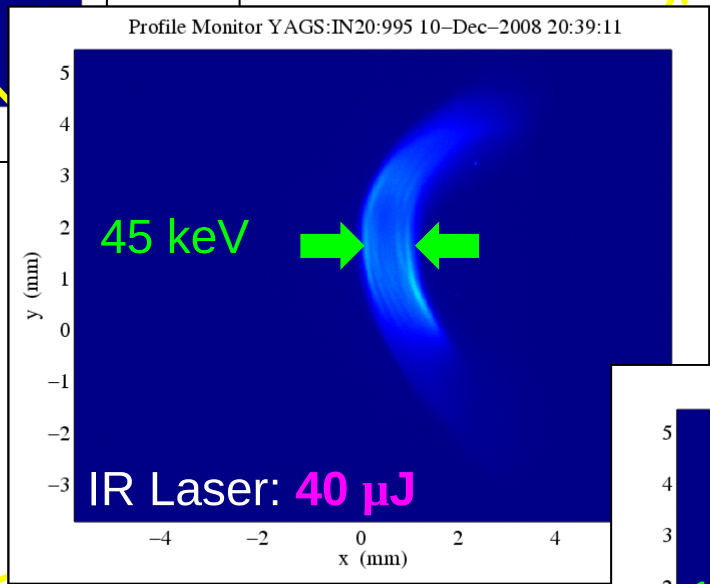
resolution $\propto \frac{\lambda_{rf}}{V_0}\sqrt{E\frac{\varepsilon_{N,x}}{\beta_x(s_0)}}$



Y. Ding, P. Krejcik

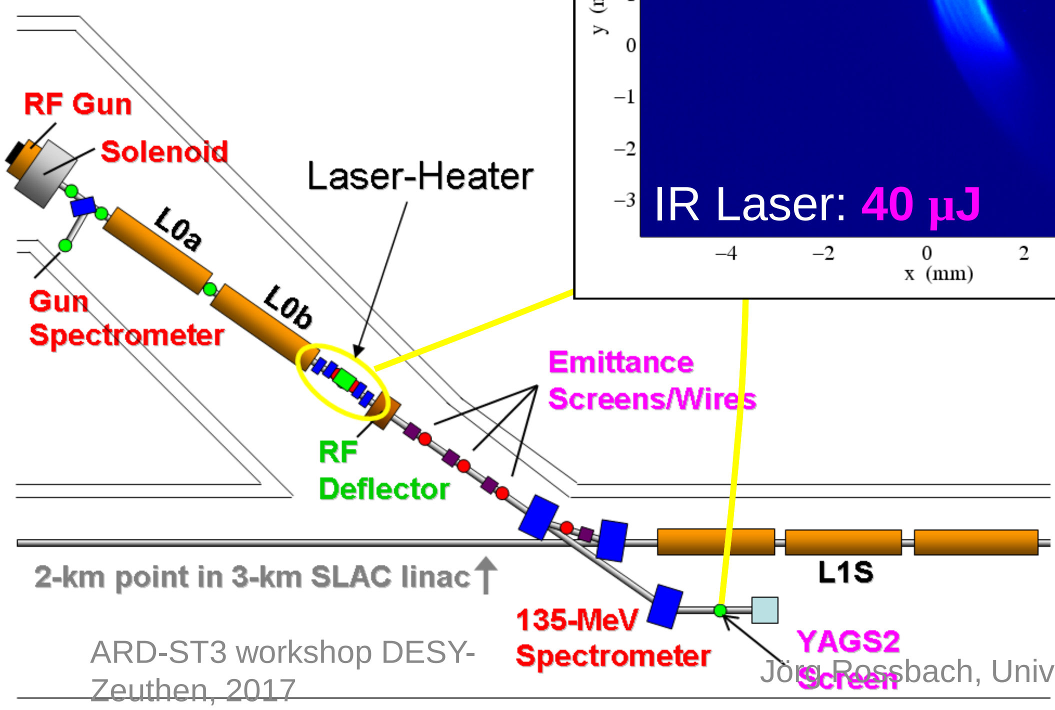


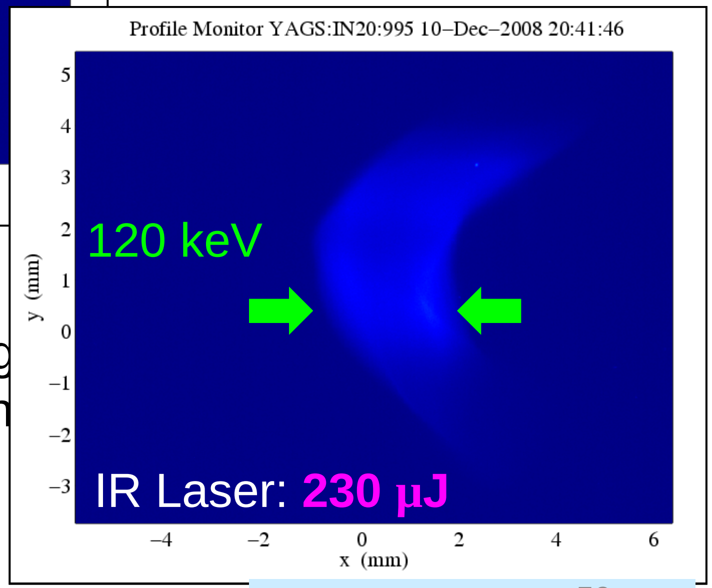




Laser heater adds energy spread which Landau damps any micro-bunching

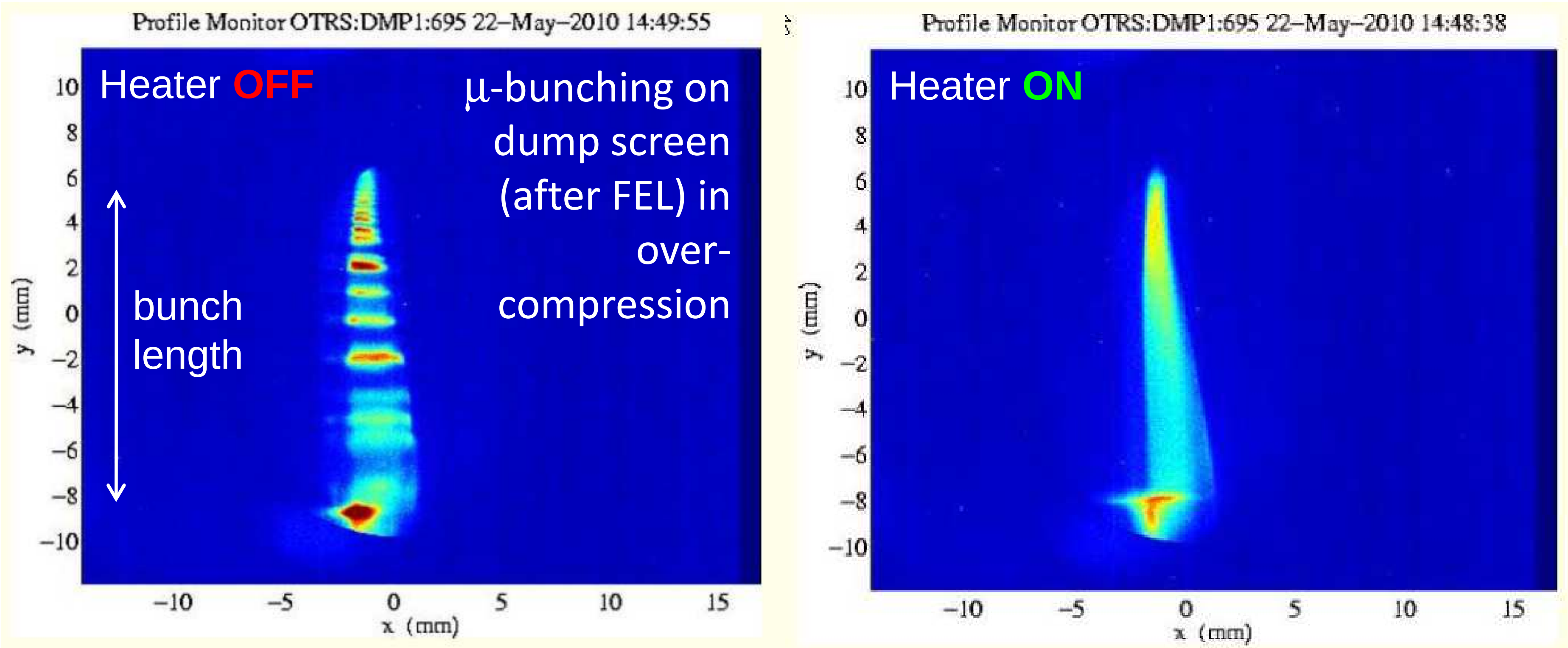
LCLS Injector:





courtesy: P. Emma

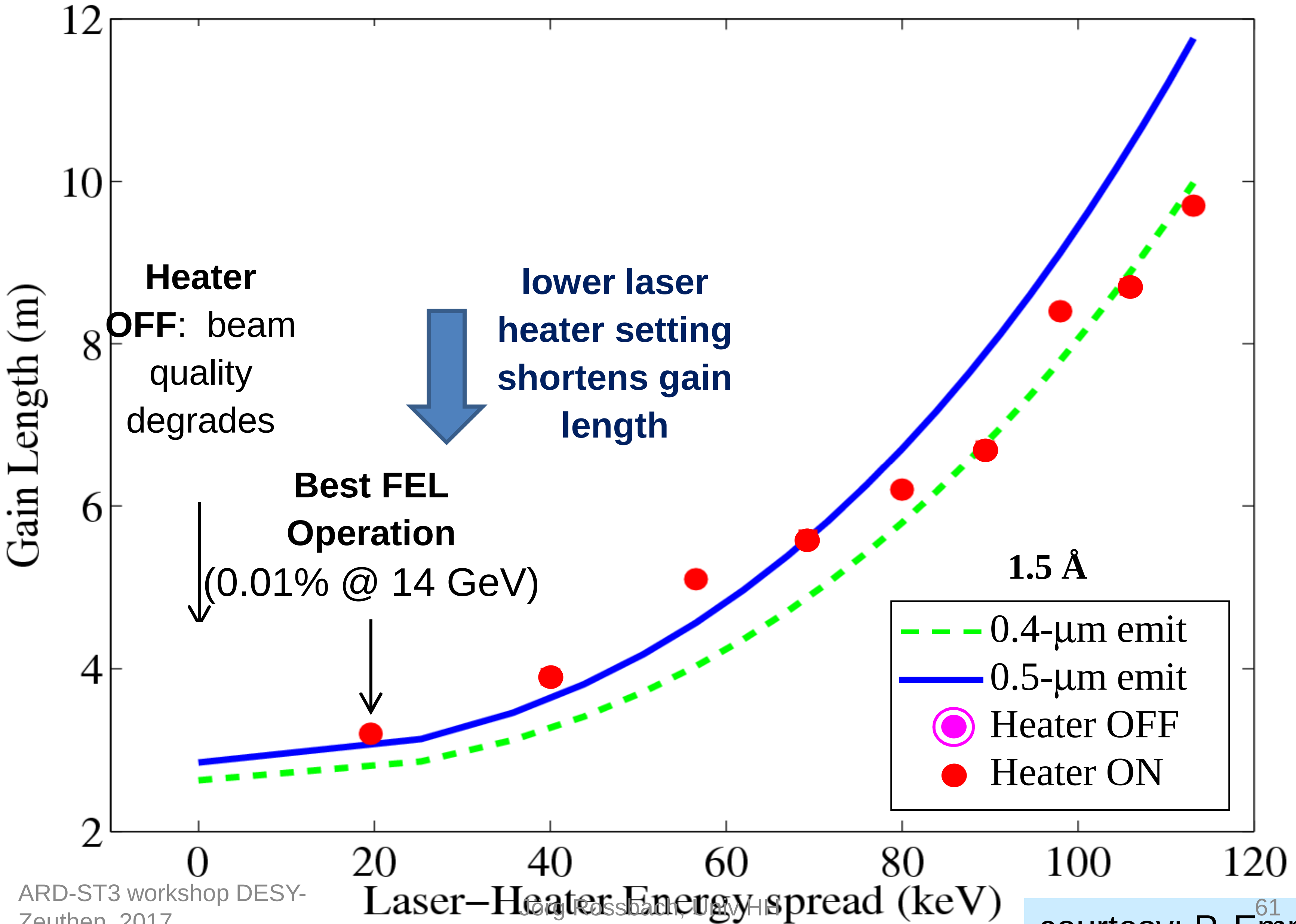
Micro-Bunching on LCLS Electron Beam
measurements with and without laser heater



Heater's energy spread Landau-damps micro-bunching before it can degrade the electron beam brightness (better FEL performance)

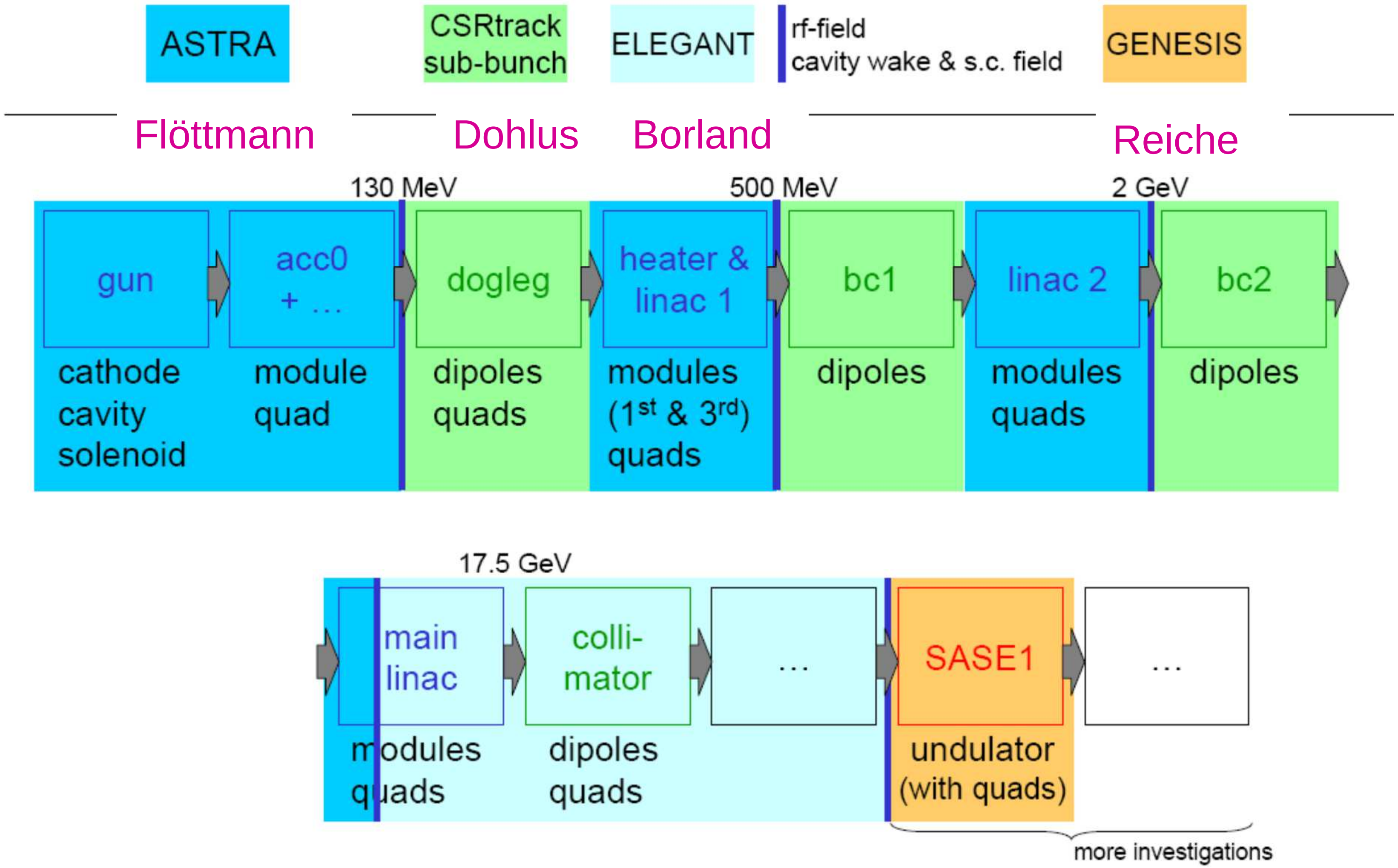
courtesy: P. Emma

Laser Heater Improves FEL Power & Gain Length

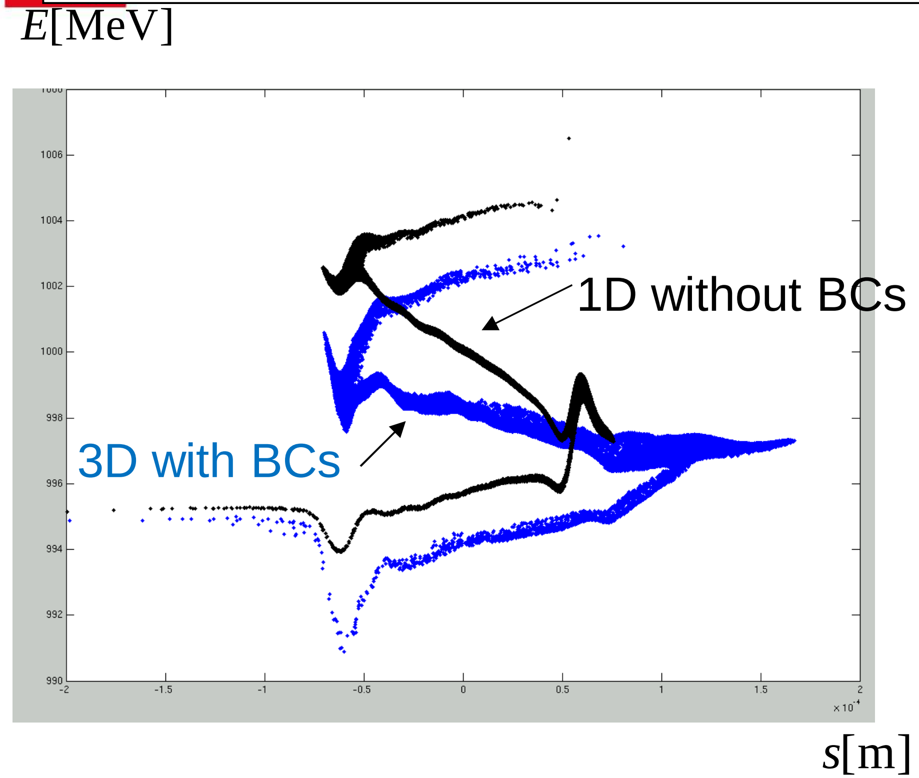


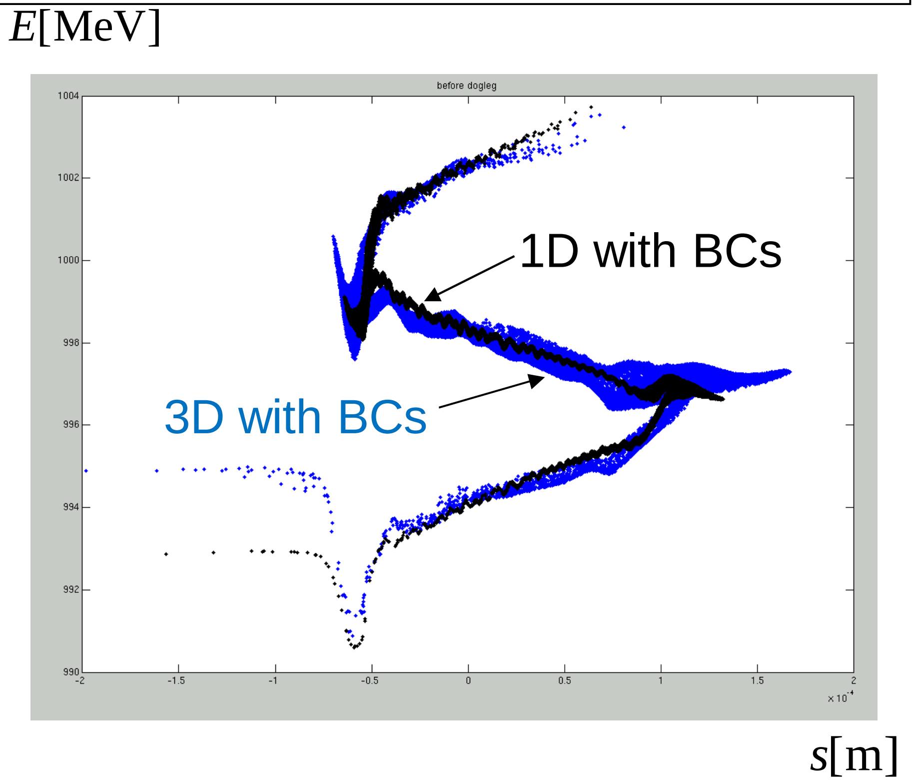
courtesy: P. Emma

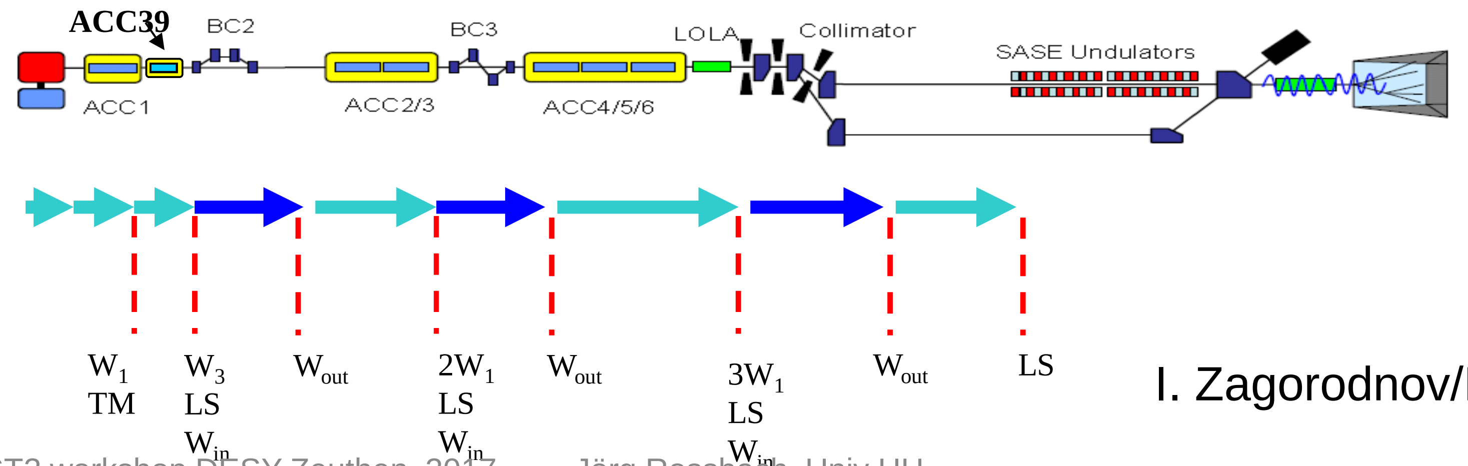
Beam dynamics simulation tools → see Mikhail Krasilnikov's talk



3D simulation with space charge + cavity wakes+self fields in BCs.





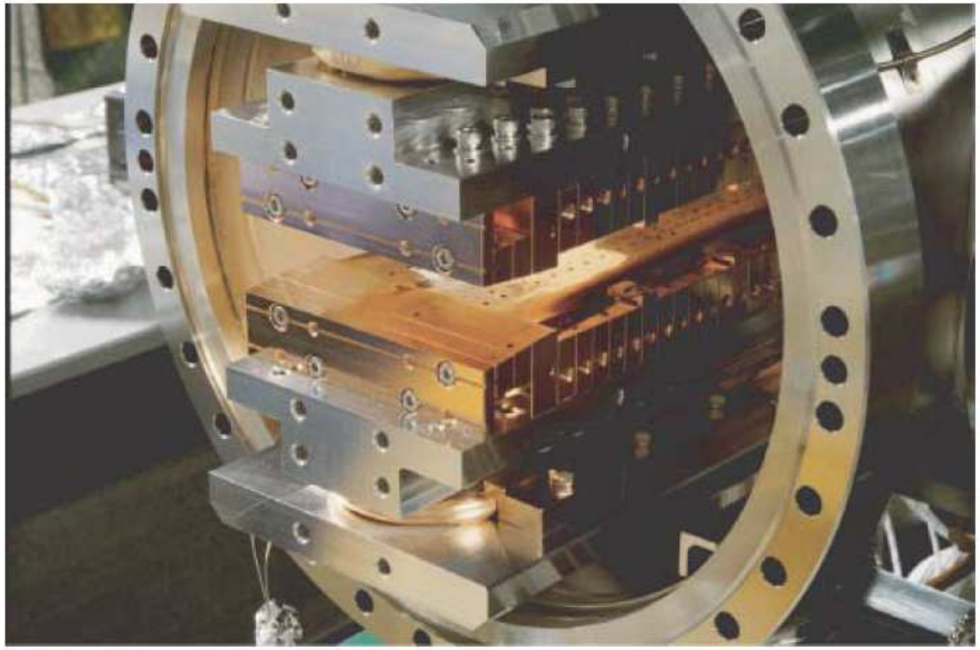


I. Zagorodnov/DESY

Key components: undulators

Must provide
- Periodic magnetic field of ~1T amplitude
- Straight electron trajectory within ~ ±10µm over 10m – 300m (→overlap !)

Basically 3 different design concepts:
- Permanent magnet outside beam vacuum
- Permanent magnet inside vacuum
- superconducting



In-vacuum undulator at Spring-8 (Kitamura et al.)

Magnet array covered with thin Cu sheet for impedance reduction

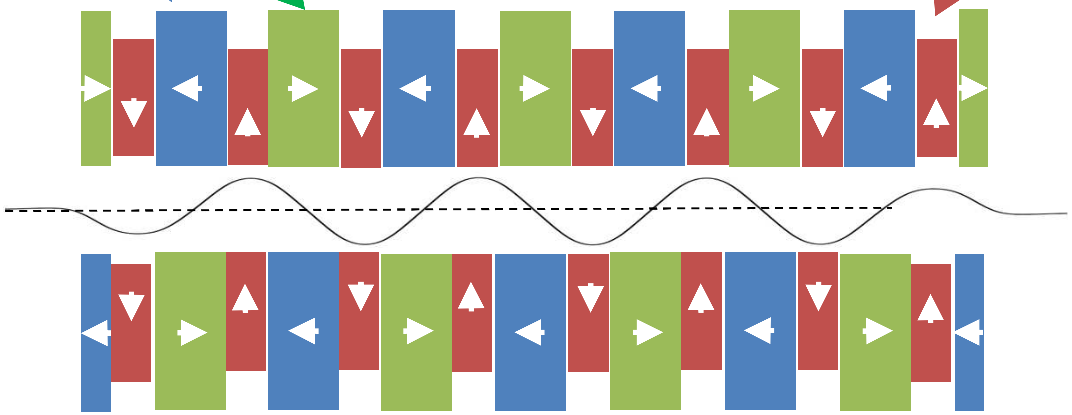
Hybrid magnetic undulator design

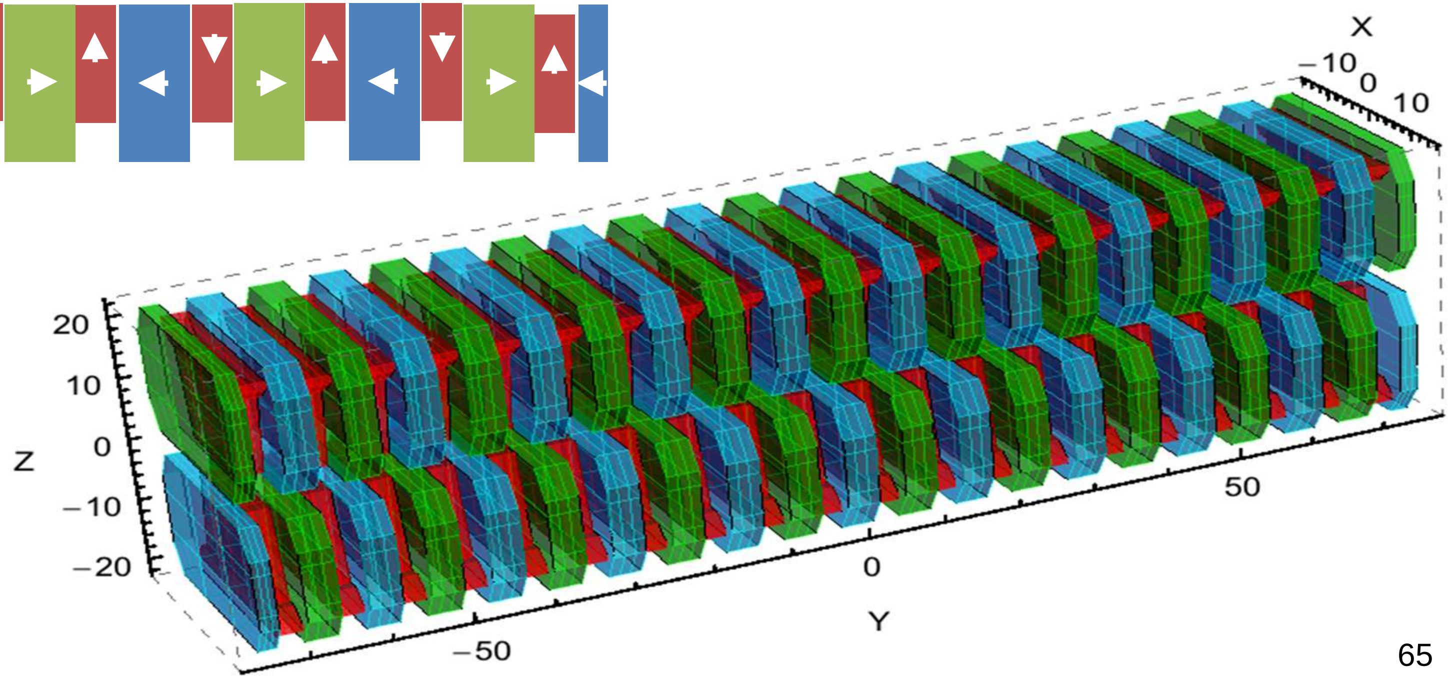
Permanent magnets: NdFeB

Remanent field: 1.25 (T)

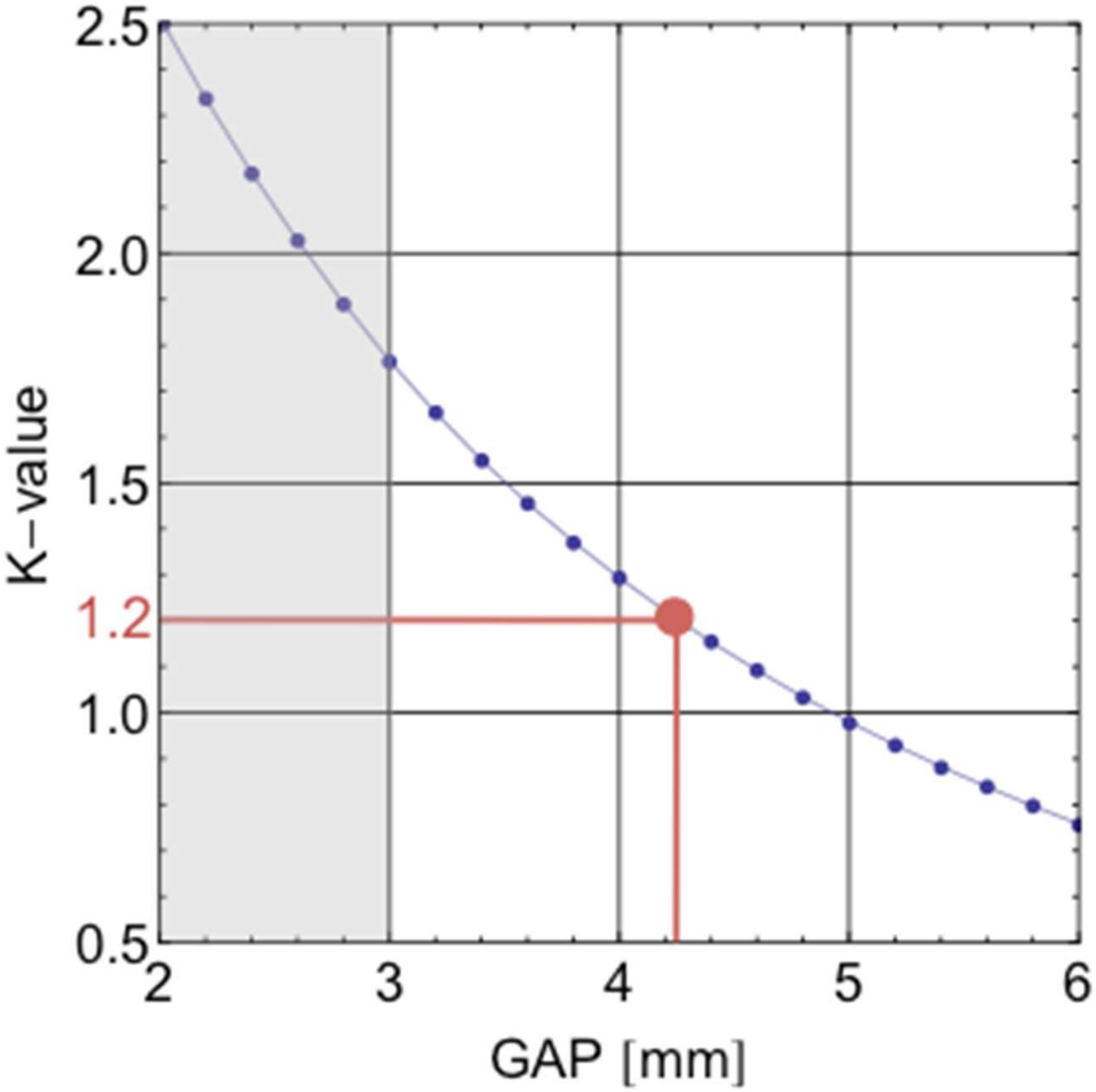
Coercivity >2300 (kA/m)

Poles: Permendur





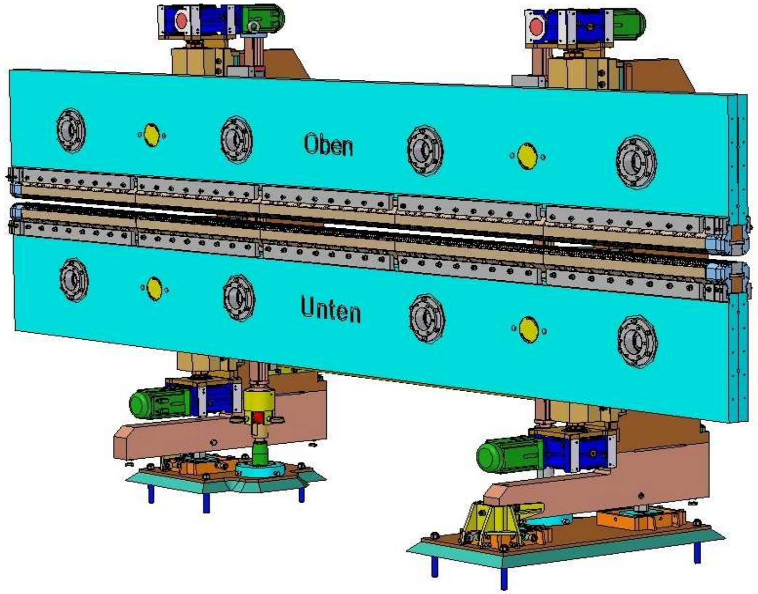
Small gap ↔ large K

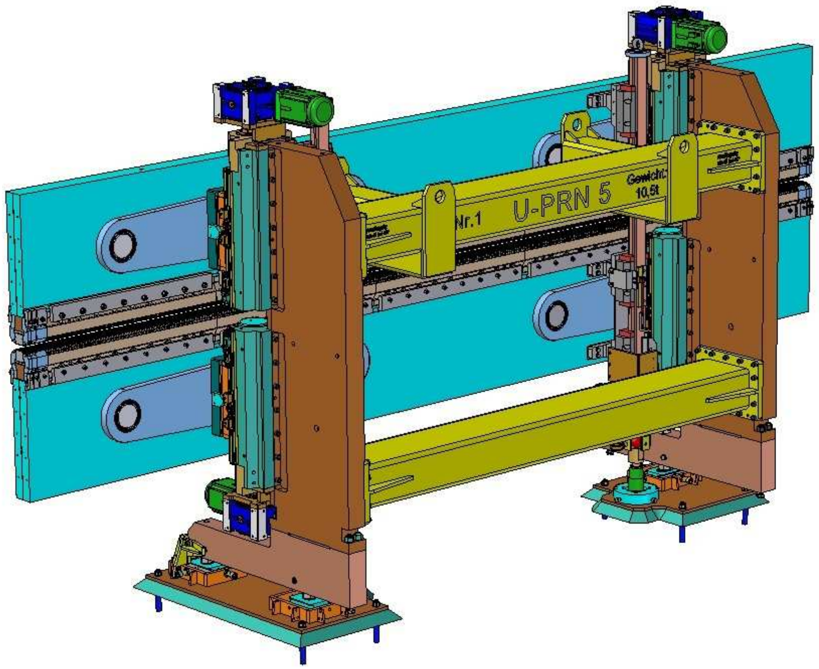


Variable gap undulators

$$\boxed{\lambda_{FEL} = \frac{\lambda_u}{2\gamma^2}\left(1 + K^2/2\right)}$$

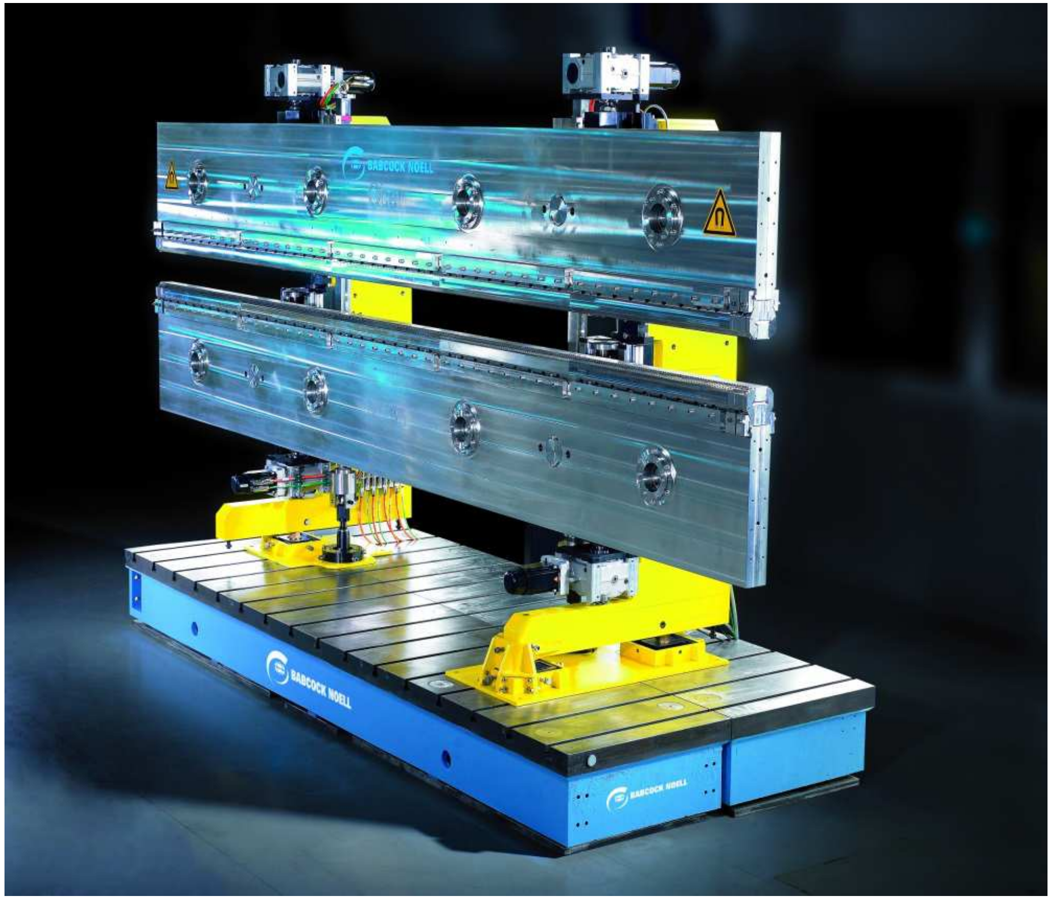
Use K ↔ gap dependence to vary FEL wavelength at constant beam energy





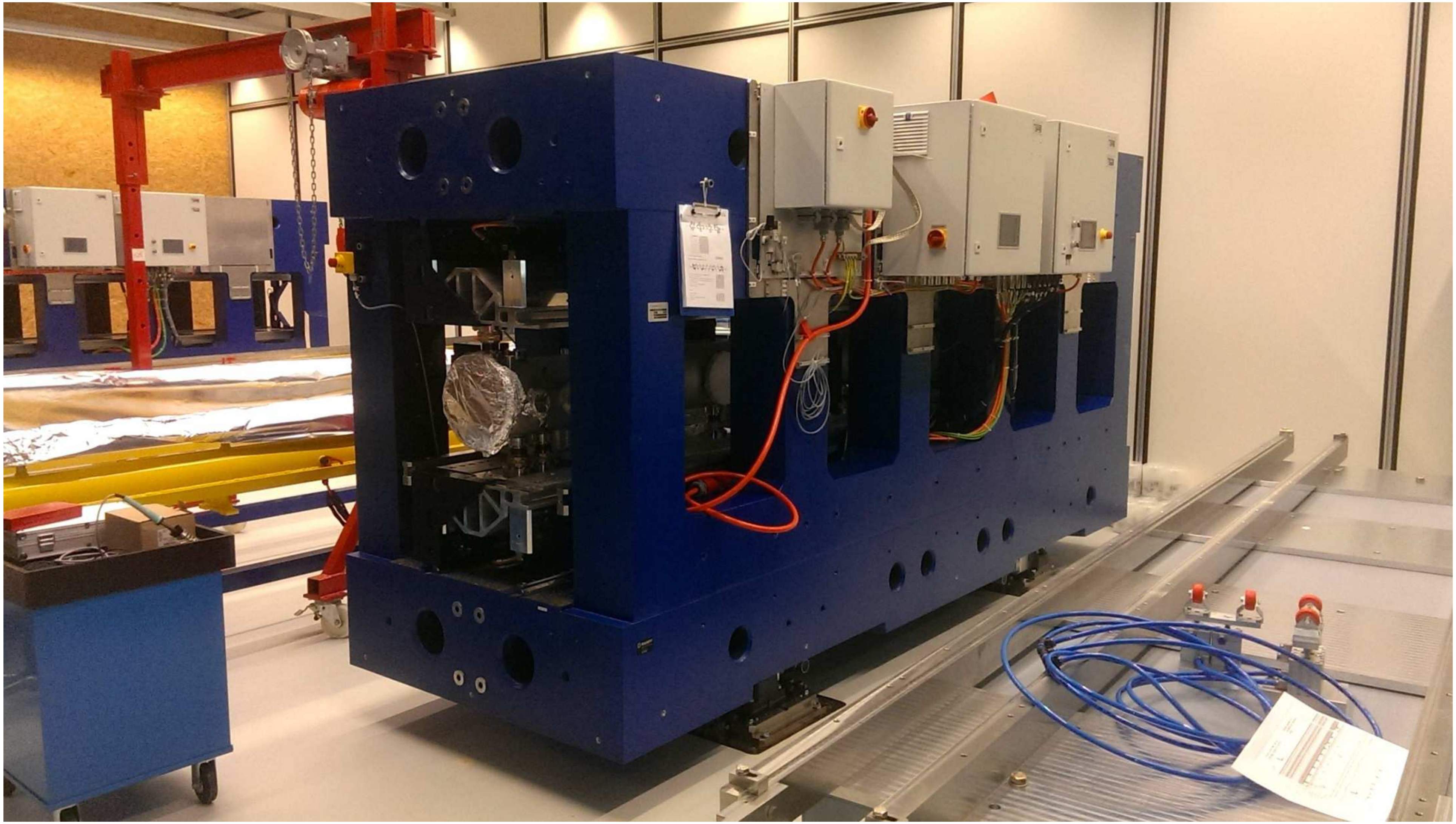
Challenge:
Guarantee reproducibility of gap size at micrometer precision !

Undulators for XFEL

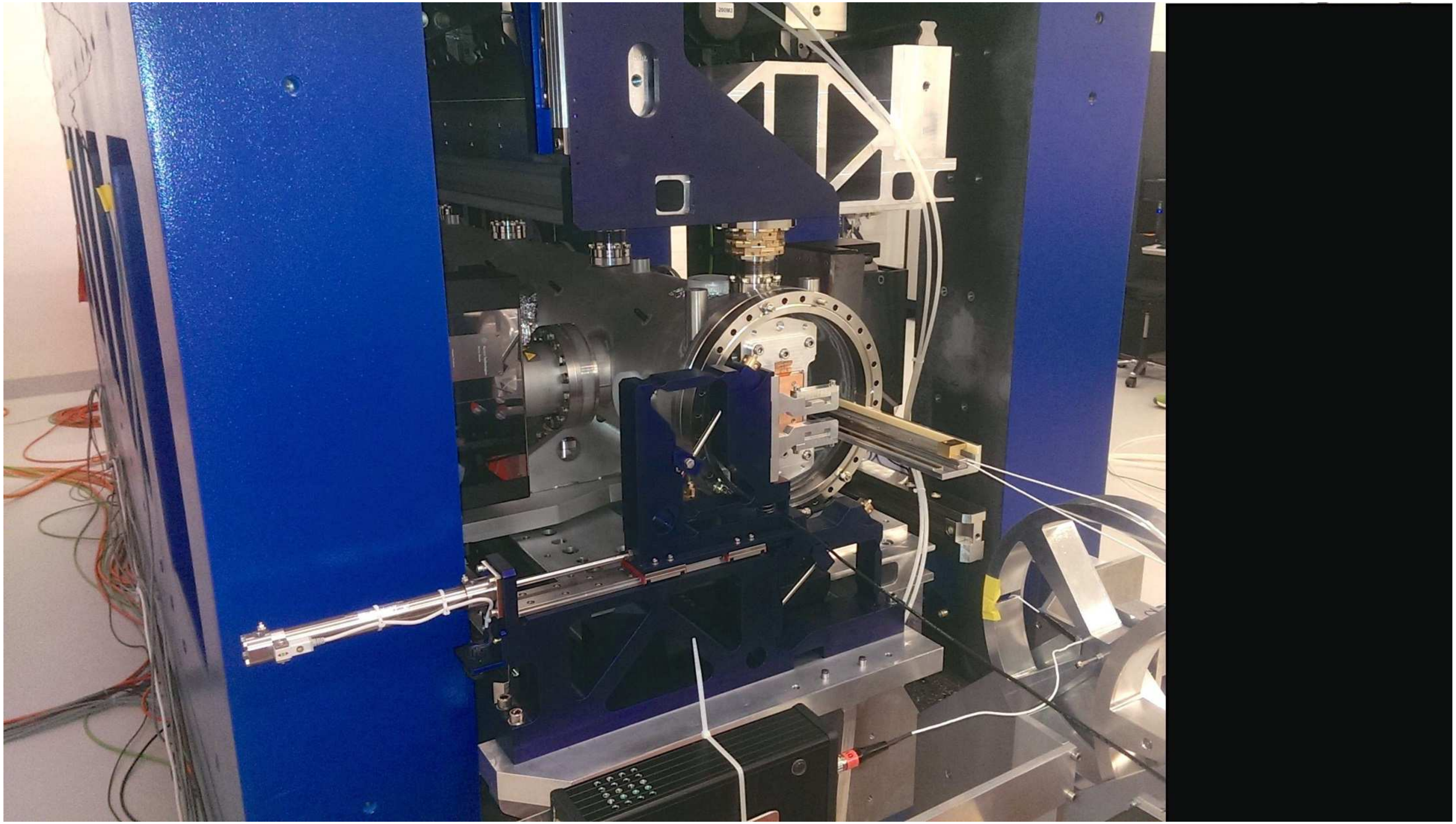


5 m

SwissFEL ARAMIS undulator



SwissFEL ARAMIS undulator



Optimization: trajectory

First field integral:
determines angular deviation

$$I_1(s) = \frac{e}{\gamma mc} \int_{s_0}^{s} B(s')ds'$$

Second field integral:
determines orbit deviation

$$I_2(s) = \frac{e}{\gamma mc} \int_{s_0}^{s} ds' \int_{s_0}^{s'} B(s'')\, ds''$$

→ Iterative modification of individual pole strengths

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Nuclear Instruments and Methods in Physics Research A 429 (1999) 386–391

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

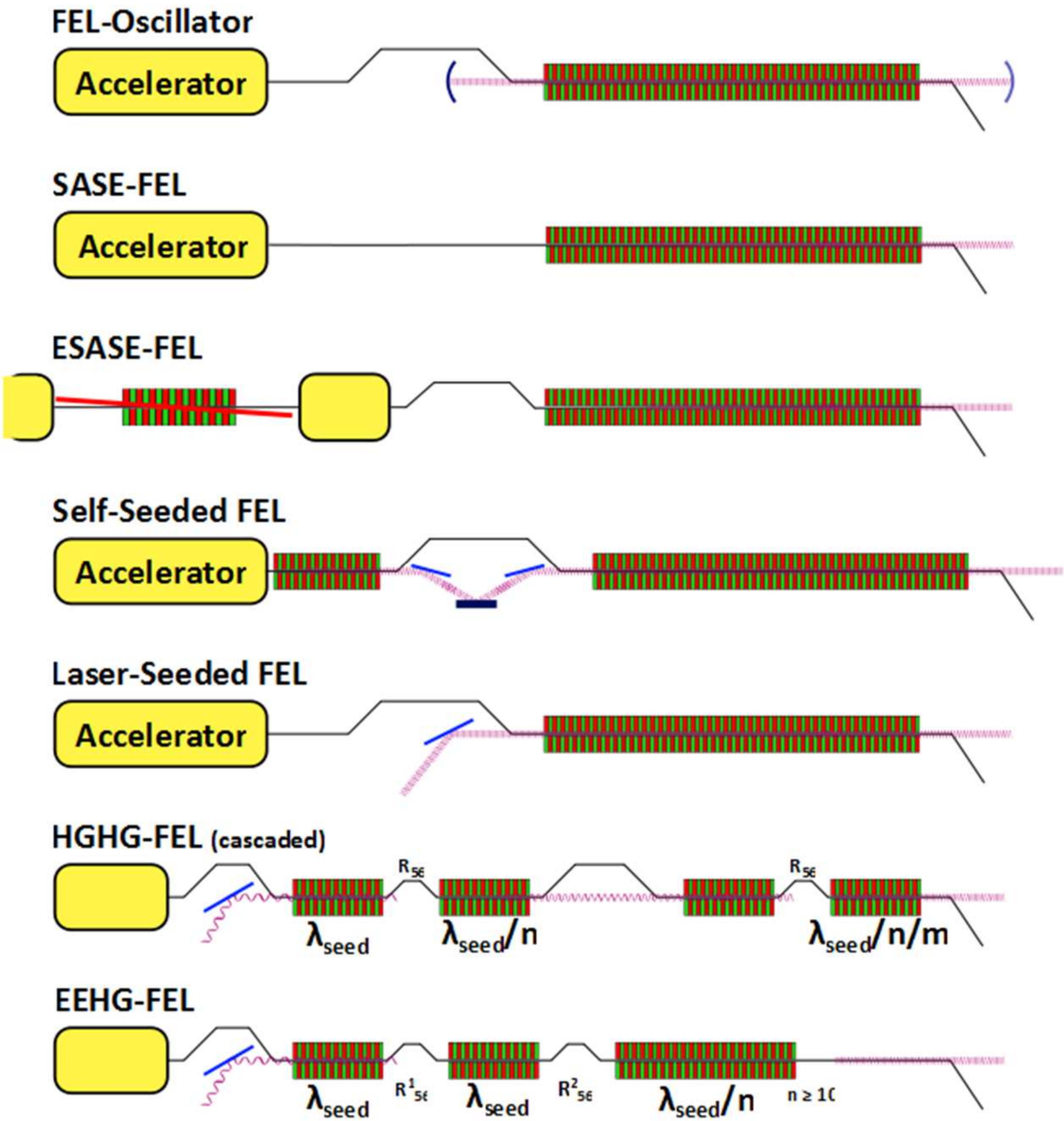
www.elsevier.nl/locate/nima

Field fine tuning by pole height adjustment for the undulator of the TTF–FEL

J. Pflüger*, H. Lu[1], T. Teichmann

Hamburger Synchrotronstrahlungslabor HASYLAB, at Deutsches Elektronen-Synchrotron, DESY, Notkestr 85, 22603 Hamburg, Germany

Seeding Schemes



Requires MHz electron bunch repetition (storage ring or cw linac)
Bandwidth determined by mirror system

„Seeding“ by spontaneous synchrotron radiation, i.e. by shot noise

Increase peak current within mirco-bunches generated through laser modulation and subsequent compression

Cut out monochromatic portion from initial SASE FEL for seeding

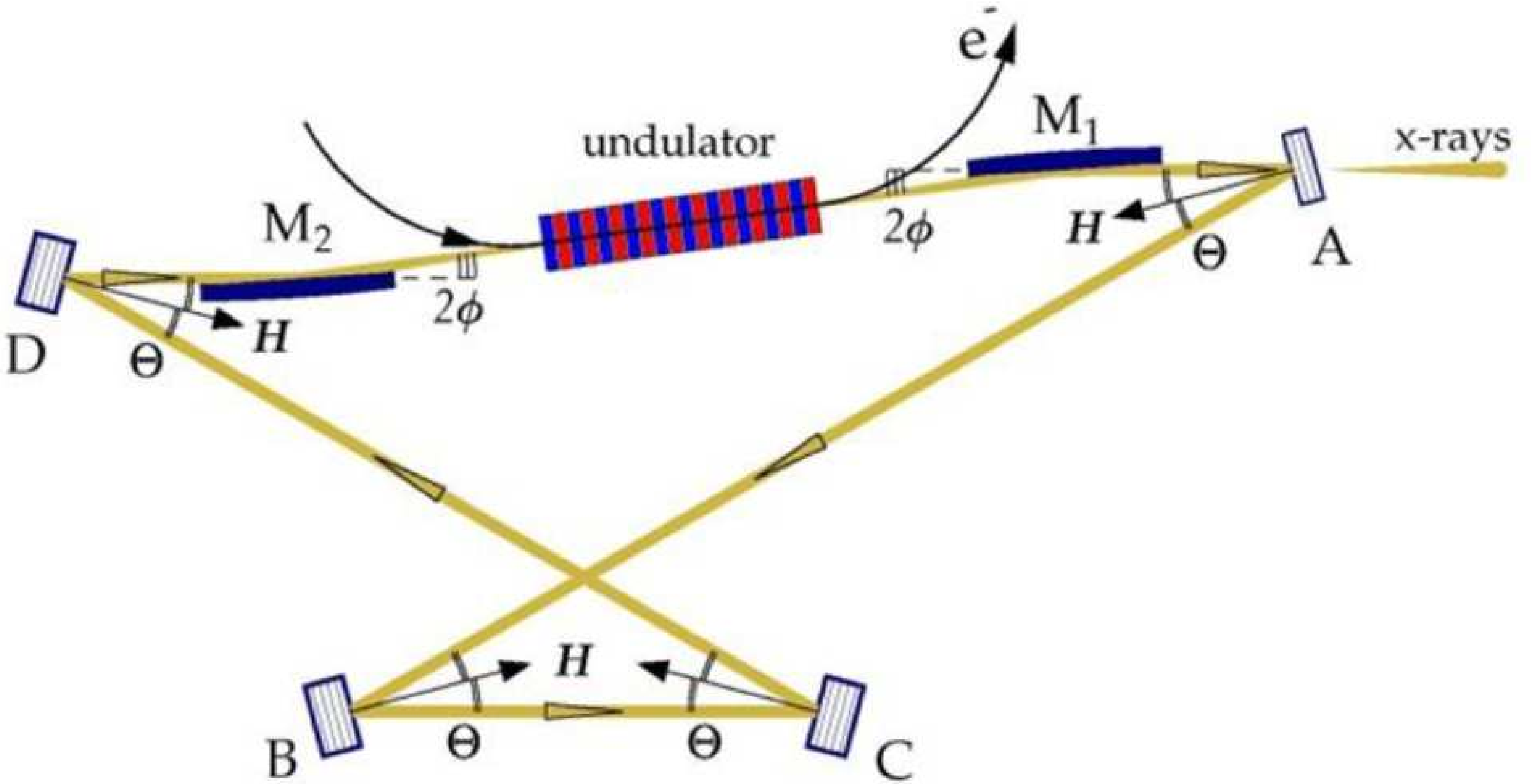
Generate coherent seeding pulse by external laser (synchronized to e-beam!)

Dto., but also produce higher FEL harmonics for further seeding stages.

Like HGHG, but generate very high harmonics by multiple compression and multiple seeding.

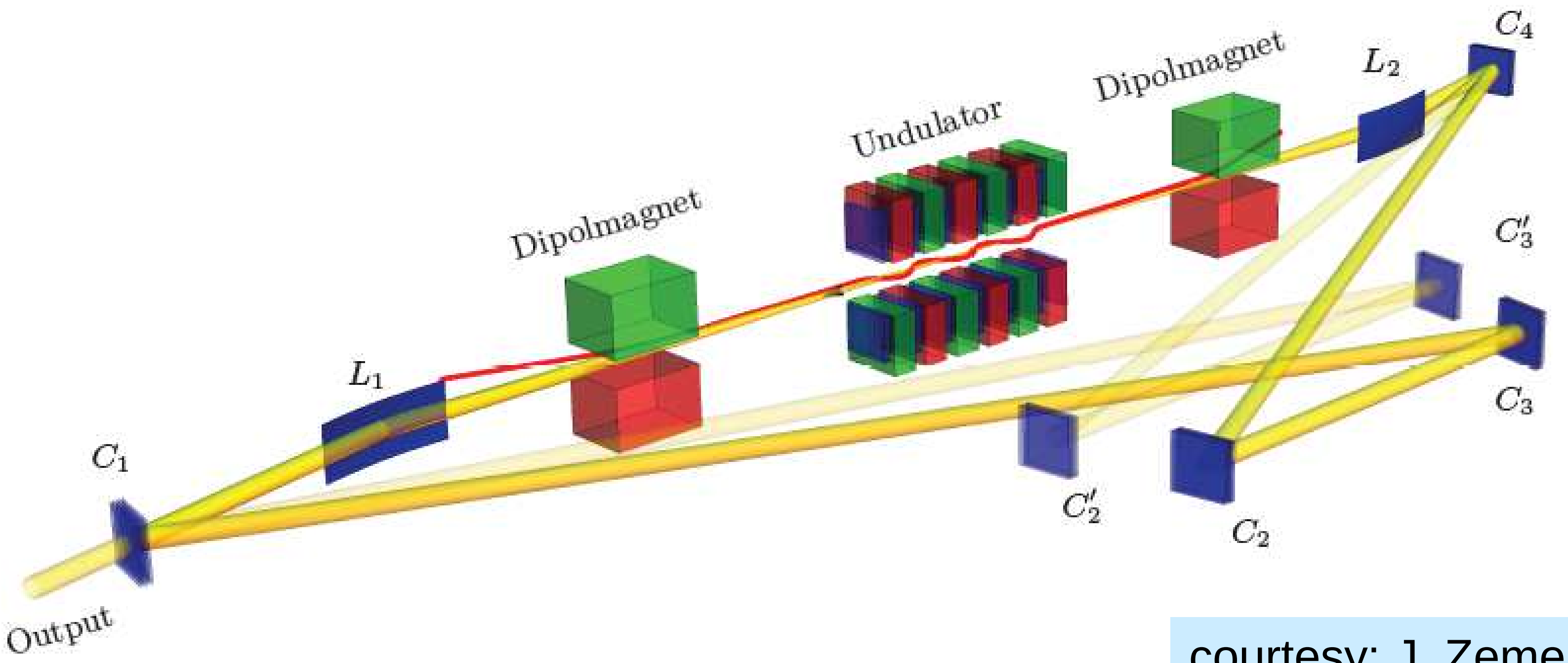
X-ray FEL oscillator (XFELO)

Idea (K-J. Kim/ANL):
Use Bragg crystals to set up an X-ray „resonator“
originally proposed for energy recovery linac (ERL)



X-ray FEL oscillator (XFELO)

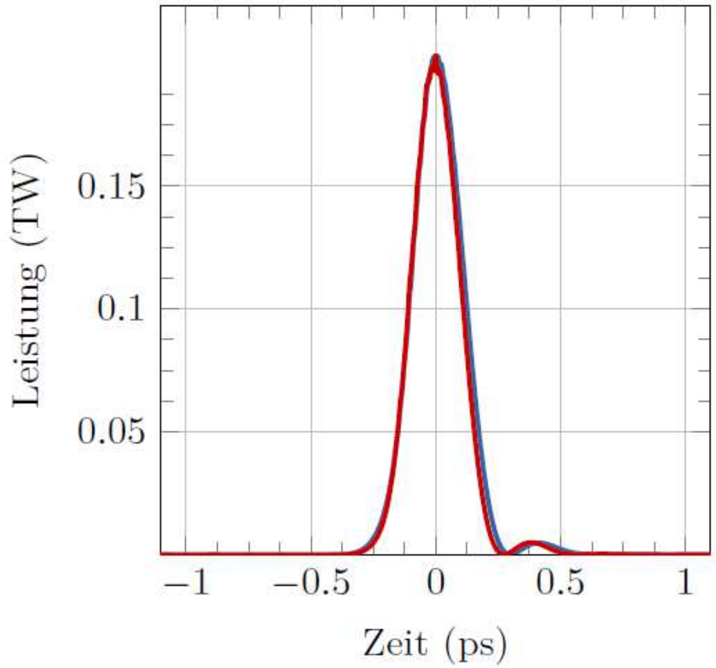
-- adopted for Europ. XFEL, making use of MHz bunch repeptition in bunch trains, investigated at Univ. HH

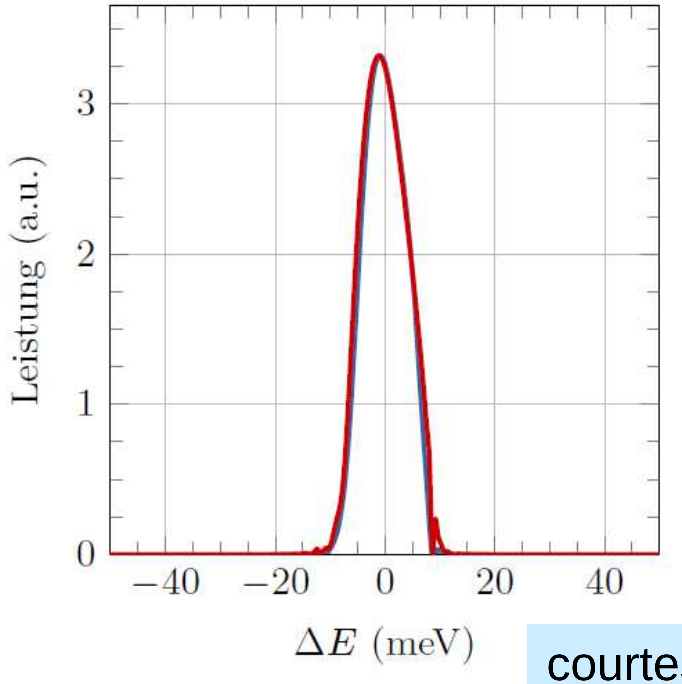


courtesy: J. Zemella

Potentially the first XFELO worldwide

X-ray FEL oscillator (XFELO)



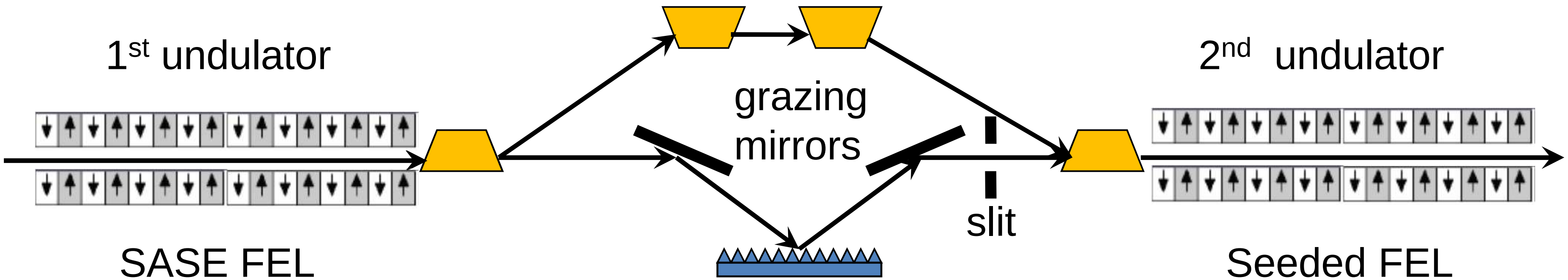


courtesy: J. Zemella

Numerical simulation for Diamond-Bragg-crystals in (4,4,4) geometry (J. Zemella, Univ. HH):
$\lambda = 1$ Å, $\Delta\lambda/\lambda = 10^{-6}$.
Only 4% coupled out → excellent pulse-to-pulse stability
Key issue: heat load on Bragg crystals → investigated by Y. Shvyd‘ko/ANL and J. Zemella, Ch. Maag/Univ. HH

Self-Seeding[1,2]

- First undulator generates SASE
- X-ray monochromator filters SASE and generates seed
- Chicane delays electrons and washes out SASE microbunching
- Second undulator amplifies seed to saturation

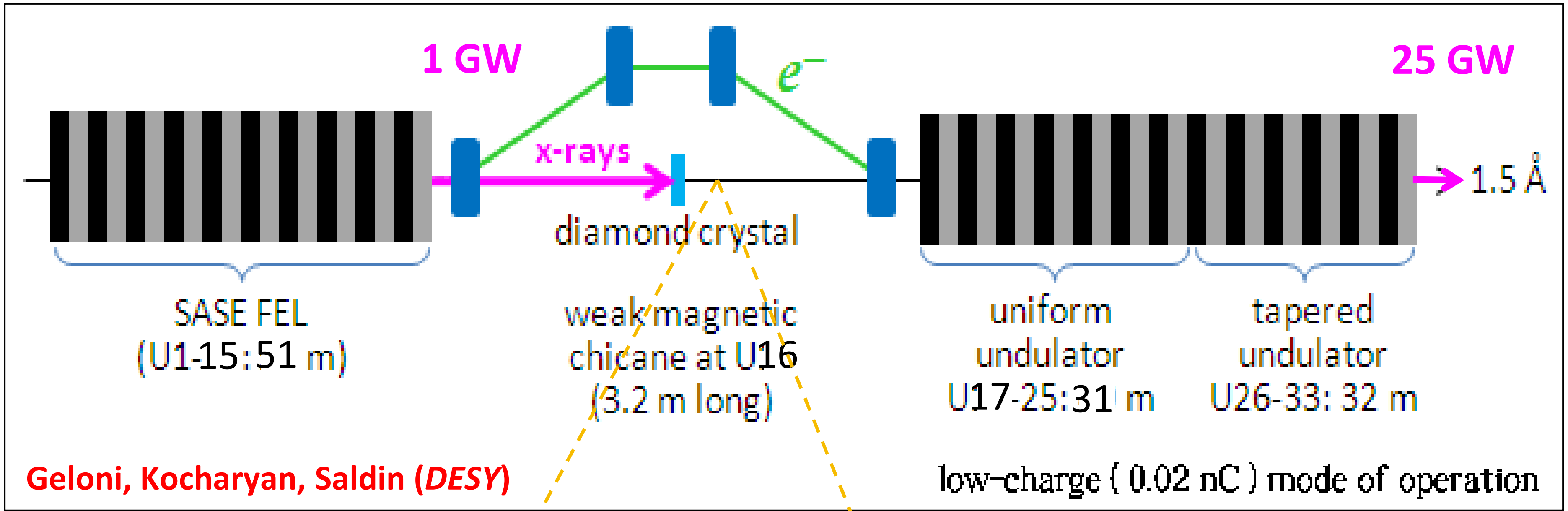


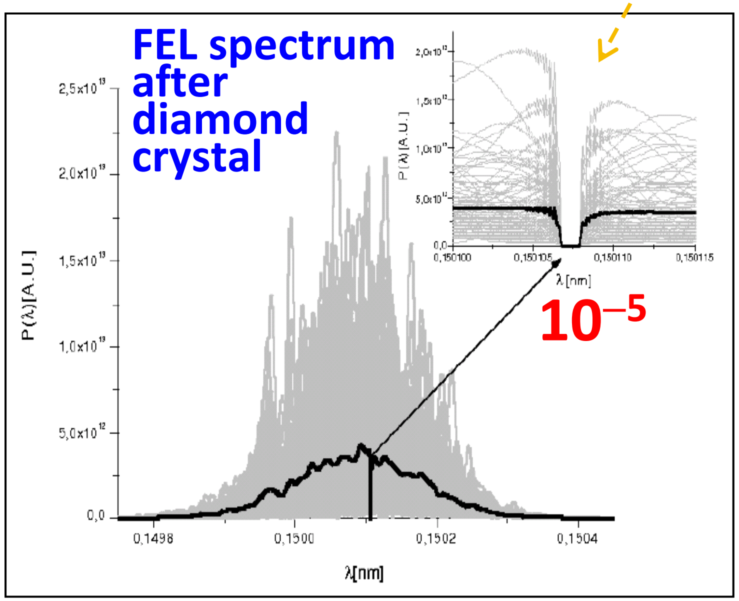
- Long x-ray path delay (~10 ps) requires large chicane that take space and may degrade beam quality
- Reduce chicane size by using two bunches[3] or single-crystal wake monochromator[4].

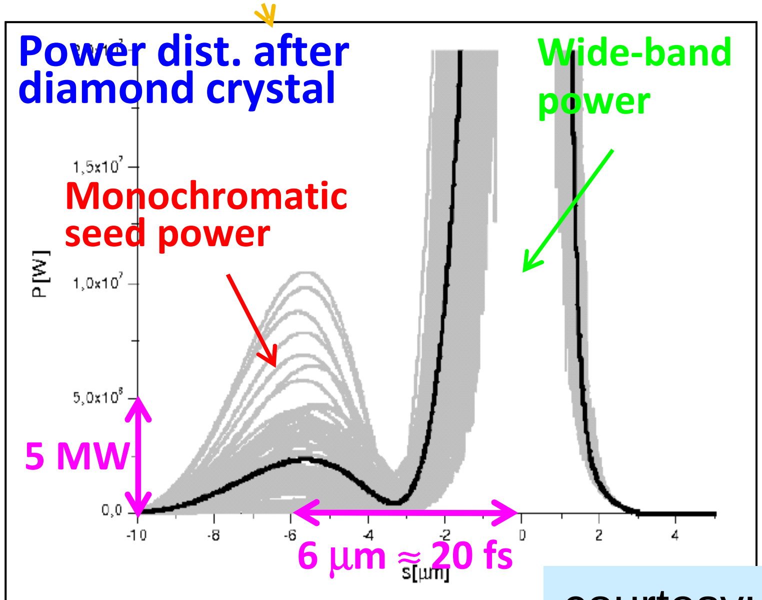
courtesy: Zh. Huang/SLAC

1. J. Feldhaus et al., NIMA, 1997.
2. E. Saldin et al., NIMA, 2001.
3. Y. Ding, Z. Huang, R. Ruth, PRSTAB, 2010.
4. G. Geloni, G. Kocharyan, E. Saldin, DESY 10-133, 2010.

Hard x-ray self-seeding @ *LCLS*



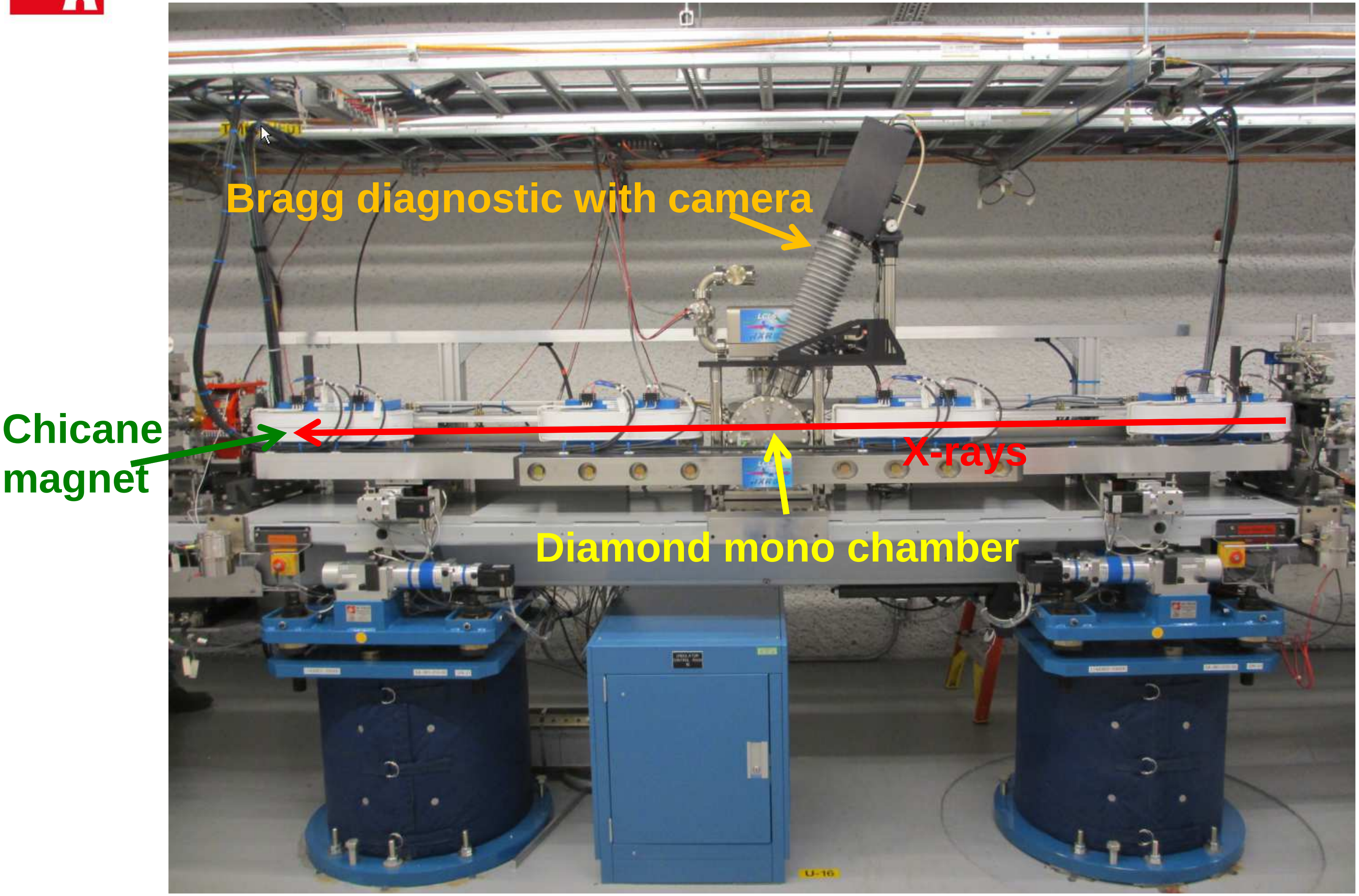




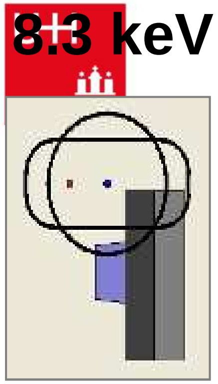
Self-seeding of 1-µm e^- pulse at 1.5 Å yields **10^{-4} BW** with low charge mode

courtesy: P. Emma/SLAC

HXRSS at LCLS (replacing U16)

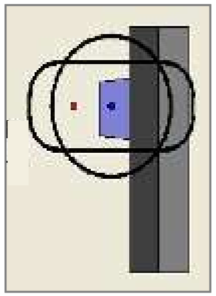


J. Amann, P. Emma (LBL)

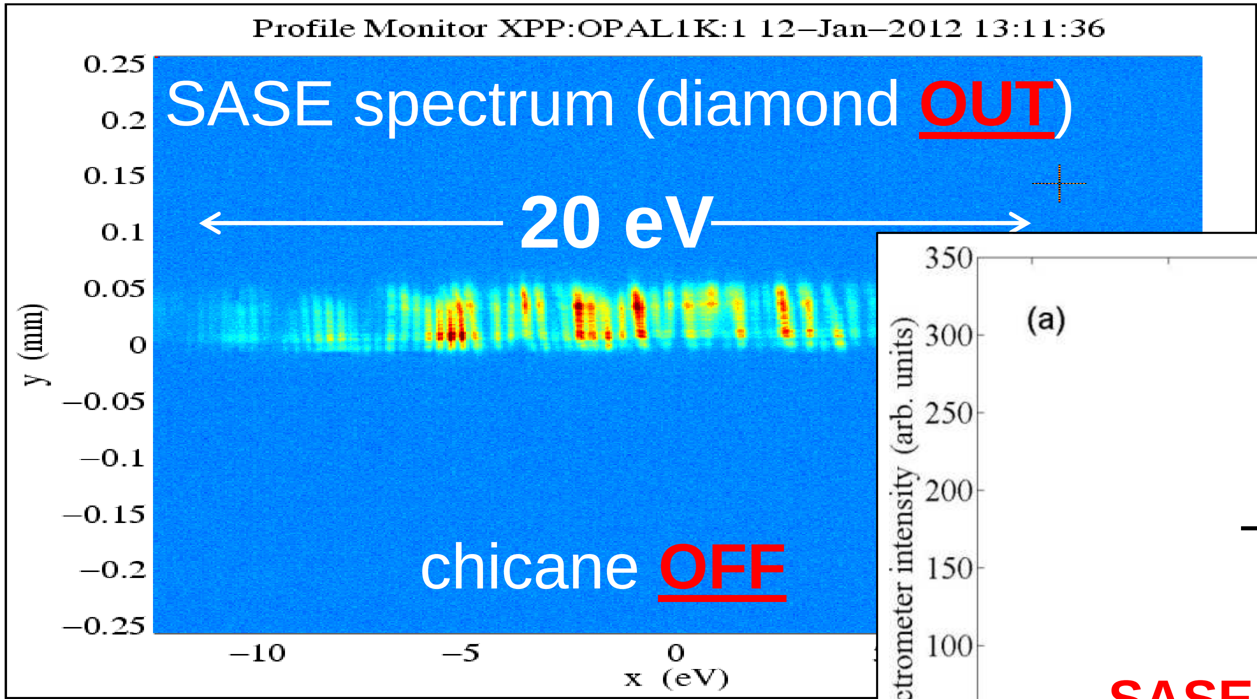


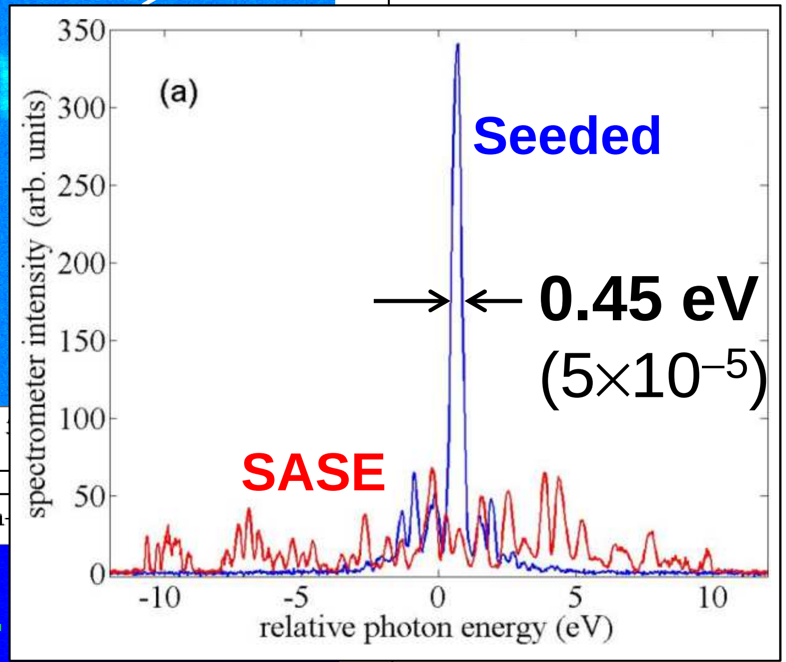
SASE

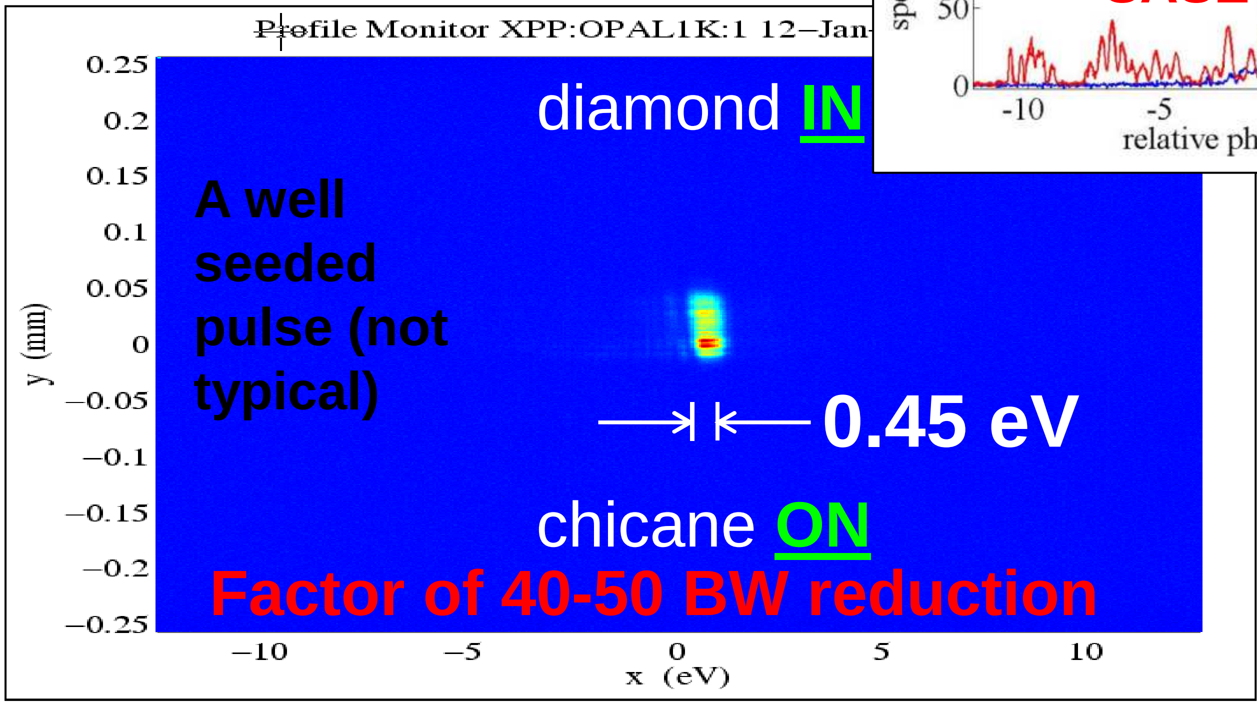
insert diamond & turn on chicane



seeded





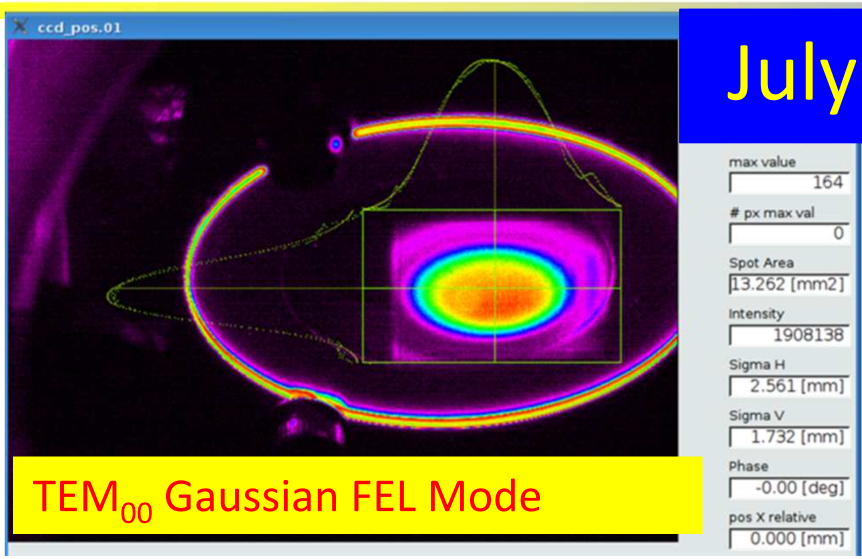


Fourier Transform limit is 5 fs

Nature Photon., 2012

HGHG Seeding at FERMI/ELETTRA

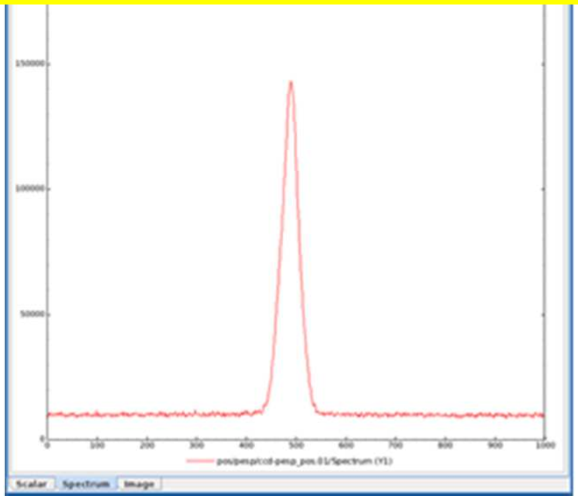
July 2011

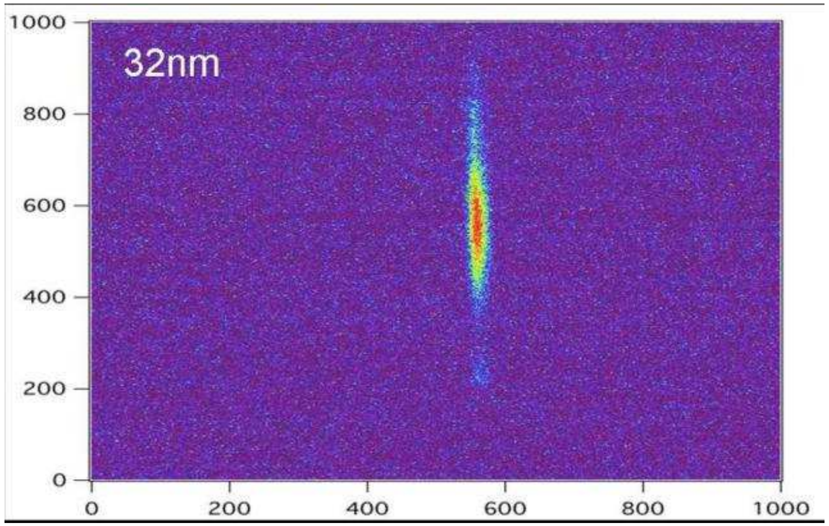


TEM$_{00}$ Gaussian FEL Mode

The Spectrometer on PADReS

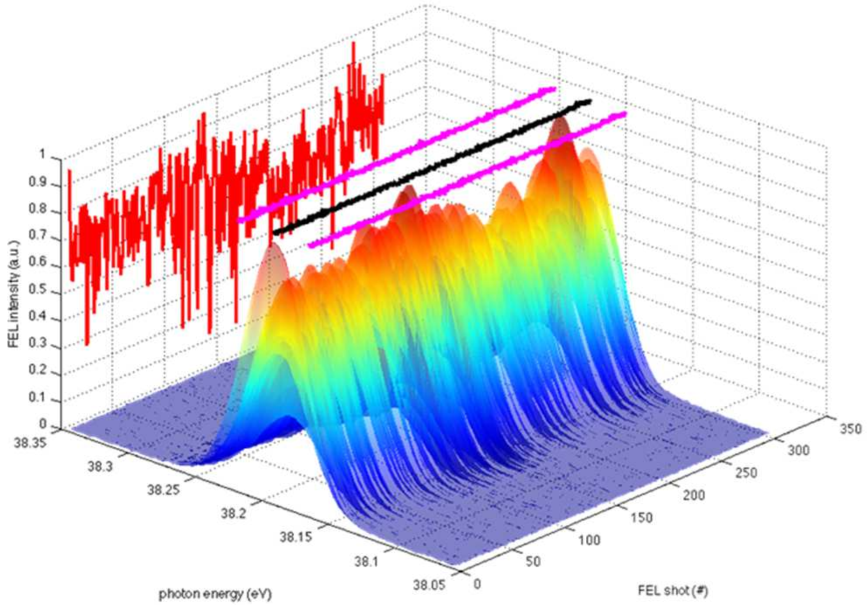
FEL spectrum @ 43 nm



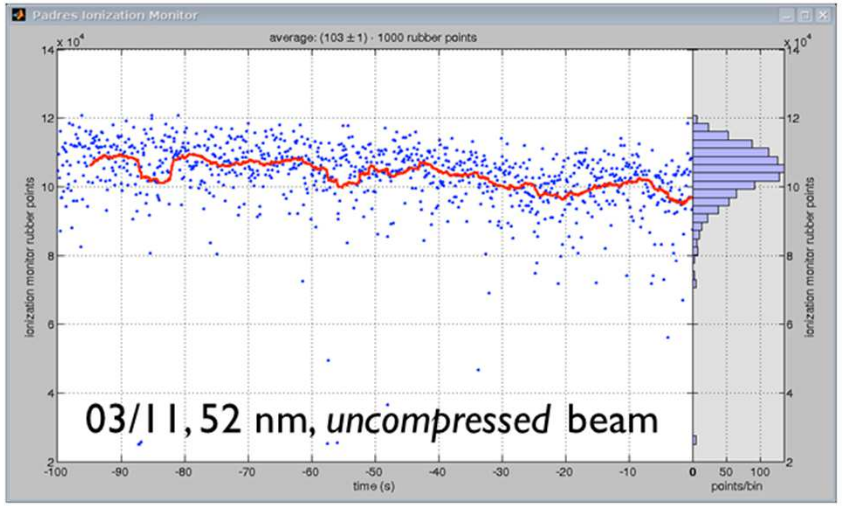


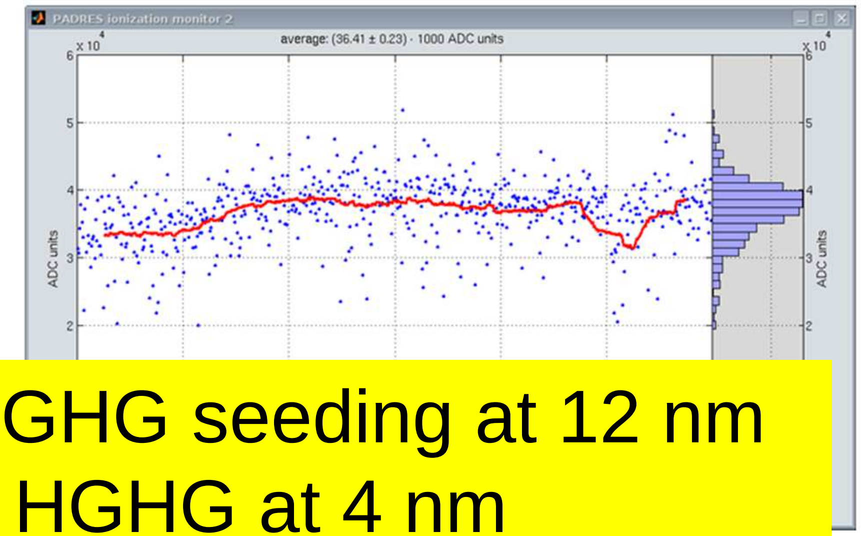
FEL Wavelength Stability

Central Wavelength Stability: ≤ 10^{-4} (RMS)
Spectral BW Stability: ≤ 3% (RMS)



Peak Intensity Stability: ≤ 10% (RMS)



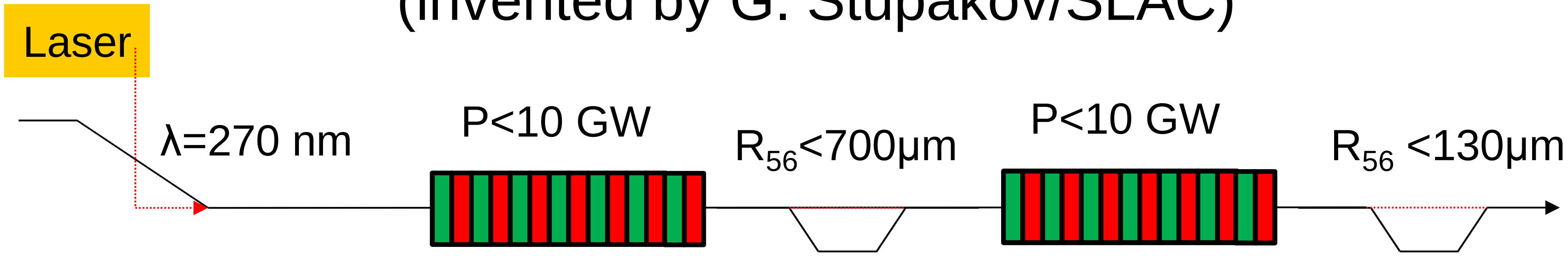


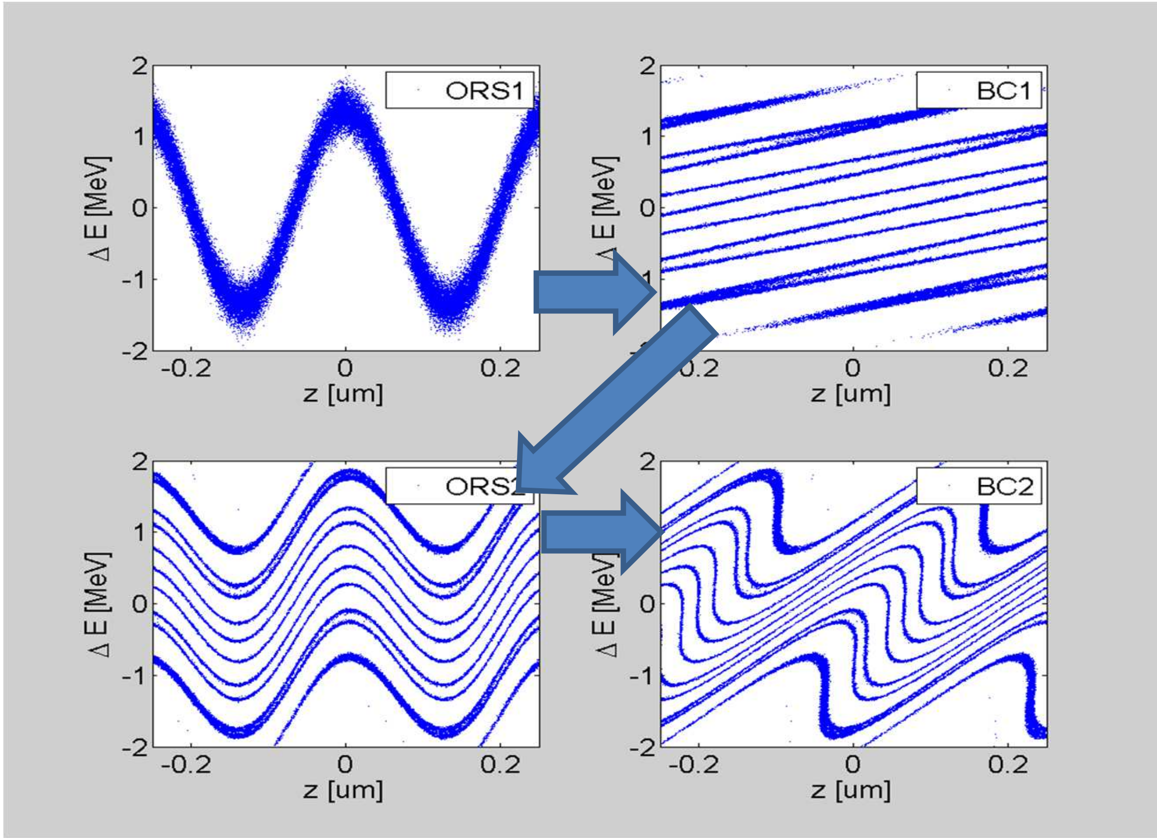
Once FEL operation is optimized, its stability is quite good: the central wavelength stability is below 10^{-4}, the
sp
3%
ab

Have meanwhile reported HGHG seeding at 12 nm and double-stage cascaded HGHG at 4 nm

Efficient Harmonic Generation using Echo-seeding (EEHG)

(invented by G. Stupakov/SLAC)



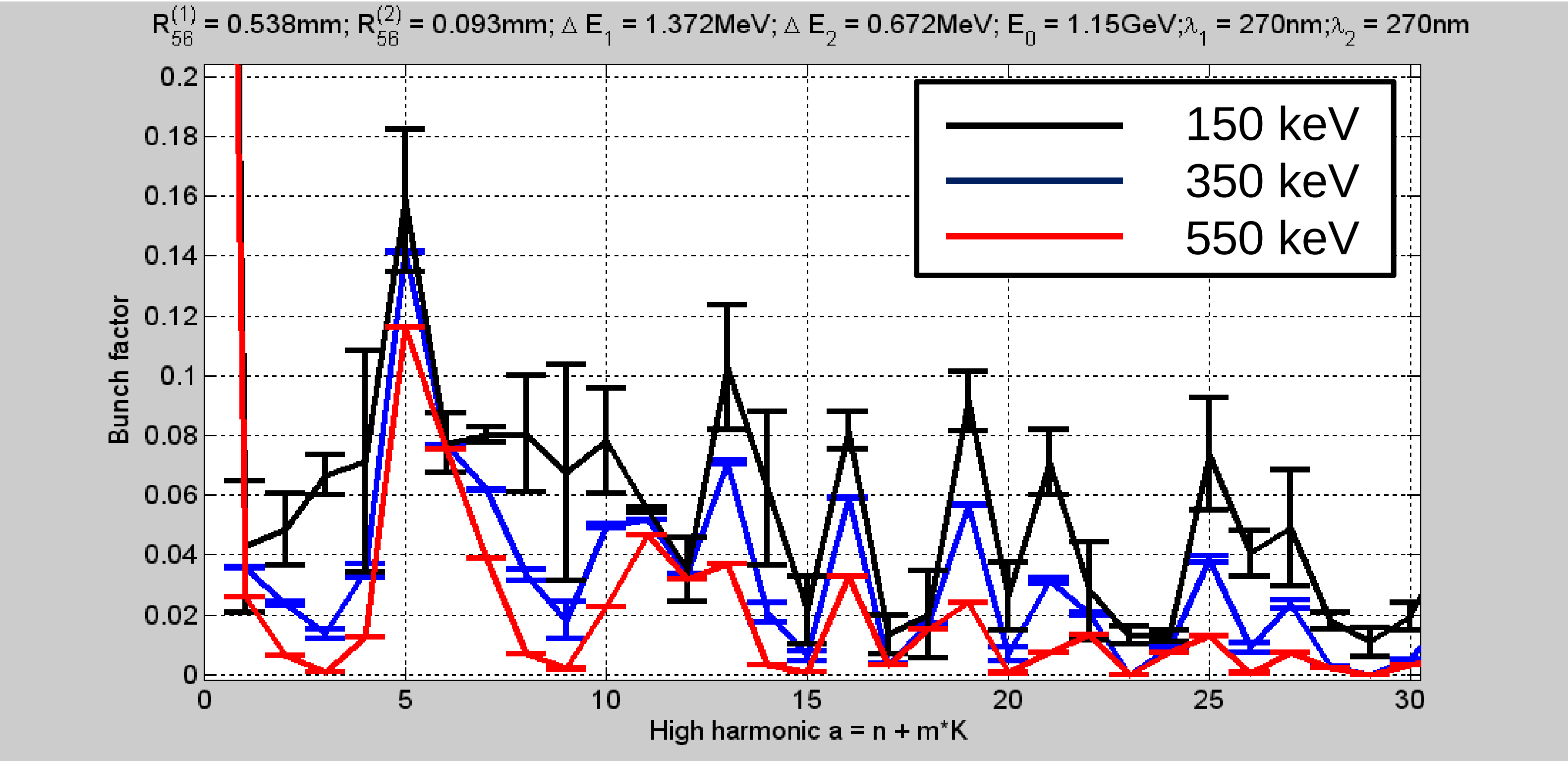


$\Delta E/E < 4 \cdot 10^{-3}$

courtesy
K. Hacker/U Dortmund

Electron density modulation at EEHG

Considerable bunching at very high harmonics – even at large energy spread



Simulation for FLASH by K. Hacker/U Dortmund

Exponential growth ? ✔

Reasonable gain length ? ✔

Achieve full density modulation ? ✔

Fluctuation properties ? ✔

Pulse length ? ✔

coherence ? ✔

Harmonics ? ✔

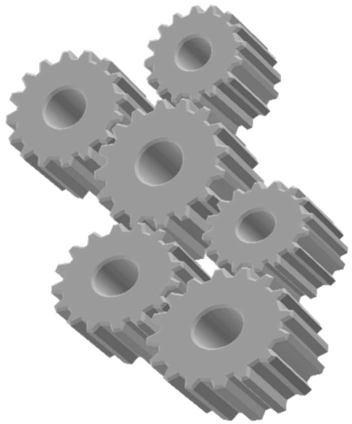
But: measurement of relevant beam parameters is not precise enough to just predict gain length with reasonable precision.

Do we understand the machinery ?

INPUT (electrons)

- Momentum
- Momentum spread/chirp
- Slice emittance/ phase space distribution
- Total charge
- Long. charge profile
- Peak current
- Orbit control

FEL machinery



OUTPUT (photons)

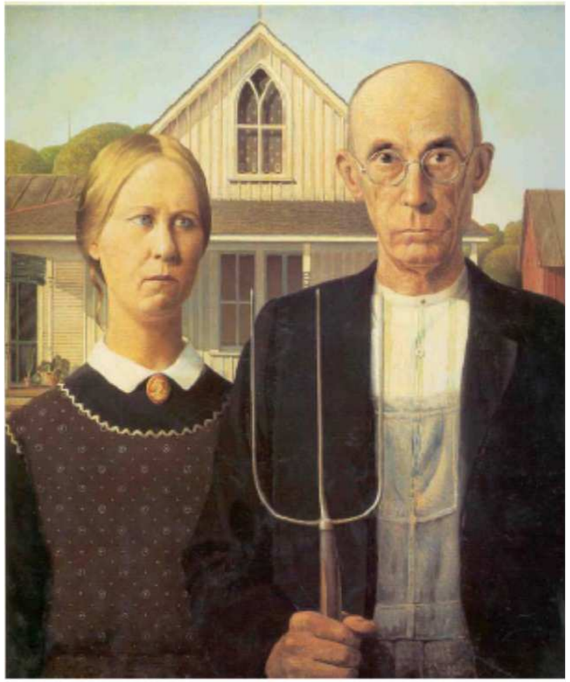
- Gain length
- Saturation behaviour
- Spectrum
- Harmonics
- Transverse coherence
- Pulse length
- Effective input power
- Fluctuations

Most probably yes, but we should know more details about the machinery (electron beam).

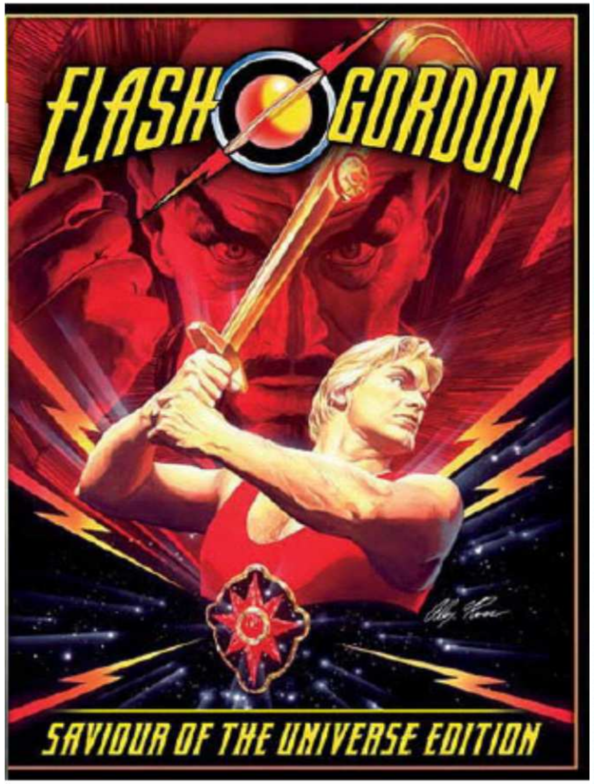
Different FEL generations already visible

Different FELs have complementary characteristics and applications !

FEL 1

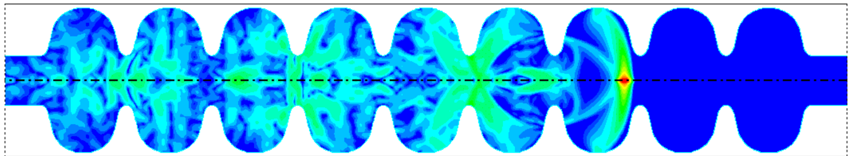


FEL 2

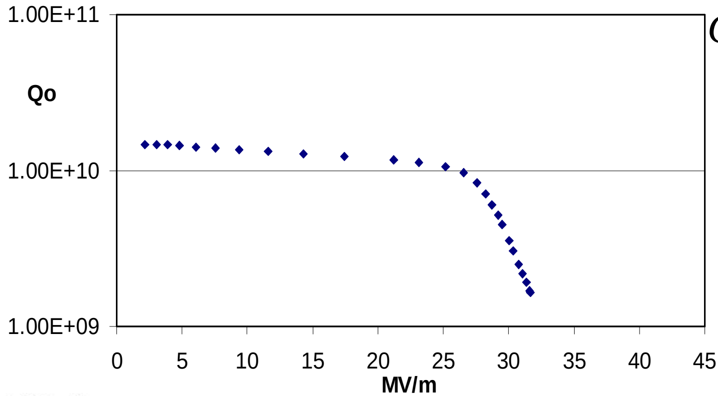


FEL 3

Key components: linac

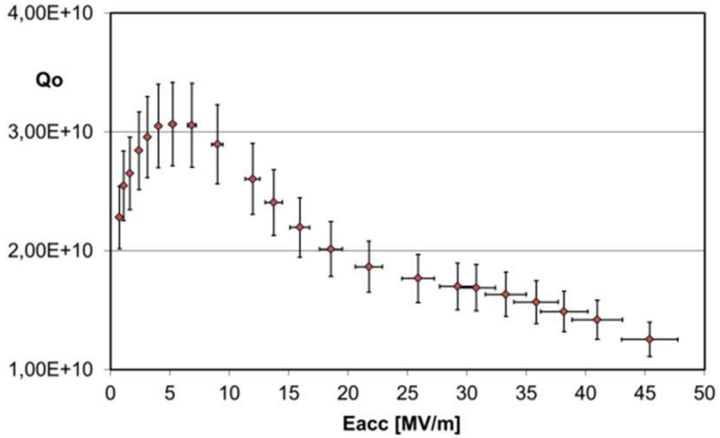


Wake field: trailing particles lose energy w.r.t. head of bunch !



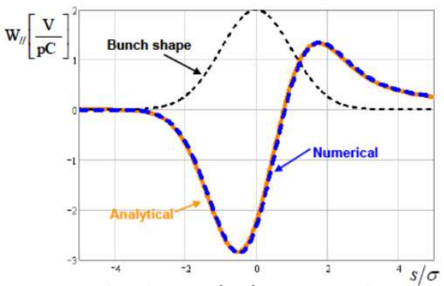
$$Q_0 = \frac{f_0}{\Delta f} \propto \frac{1}{R_{surf}}$$

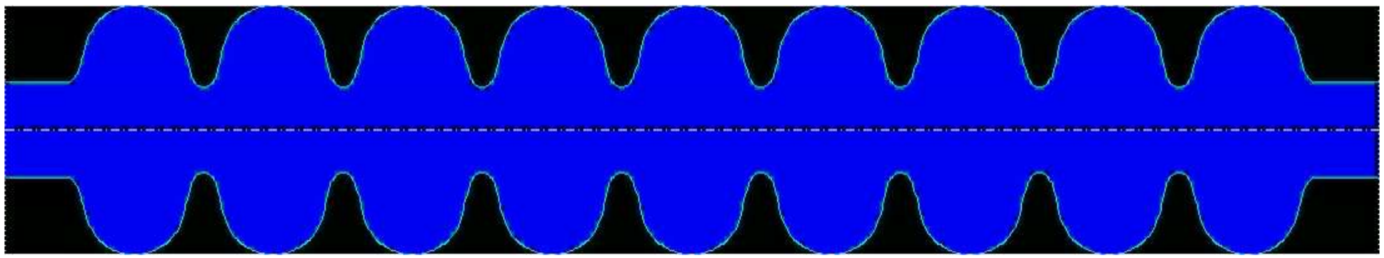
typical



good

Courtesy: D. Reschke/DESY



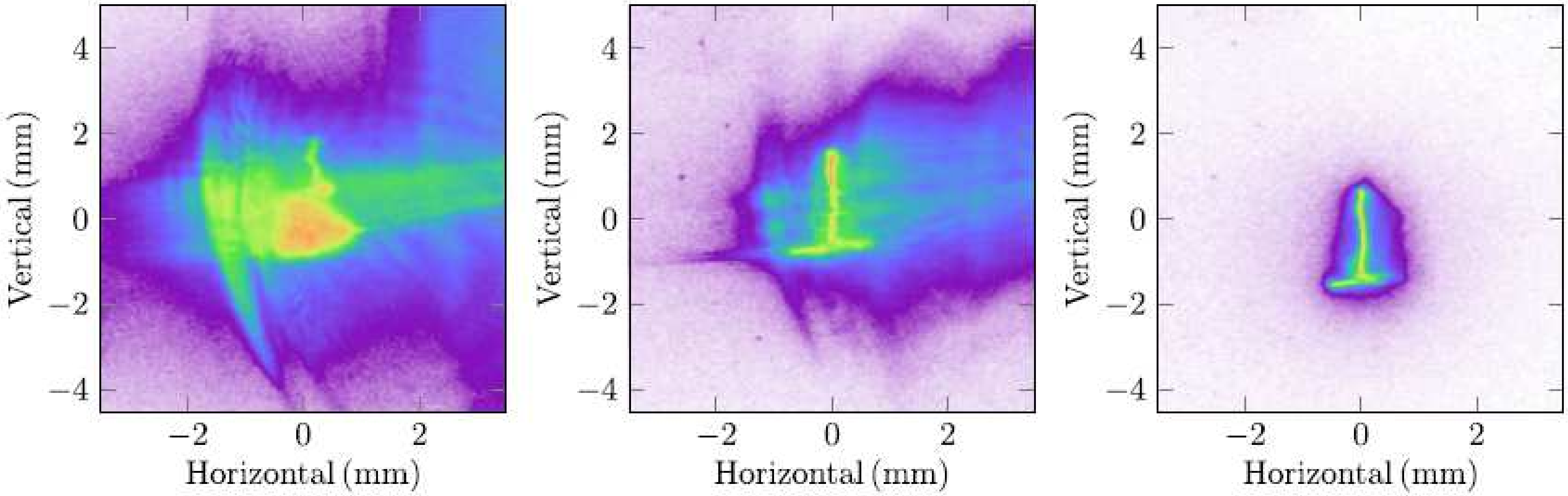


Wake field from a charge passing a FLASH/XFEL cavity

Effect of coherent synchrotron radiation on observation of transverse electron profile

→ Images from screens do NOT reflect charge distribution, but are dominated by charge portions radiating coherently

Idea: observe delayed scintillator light !

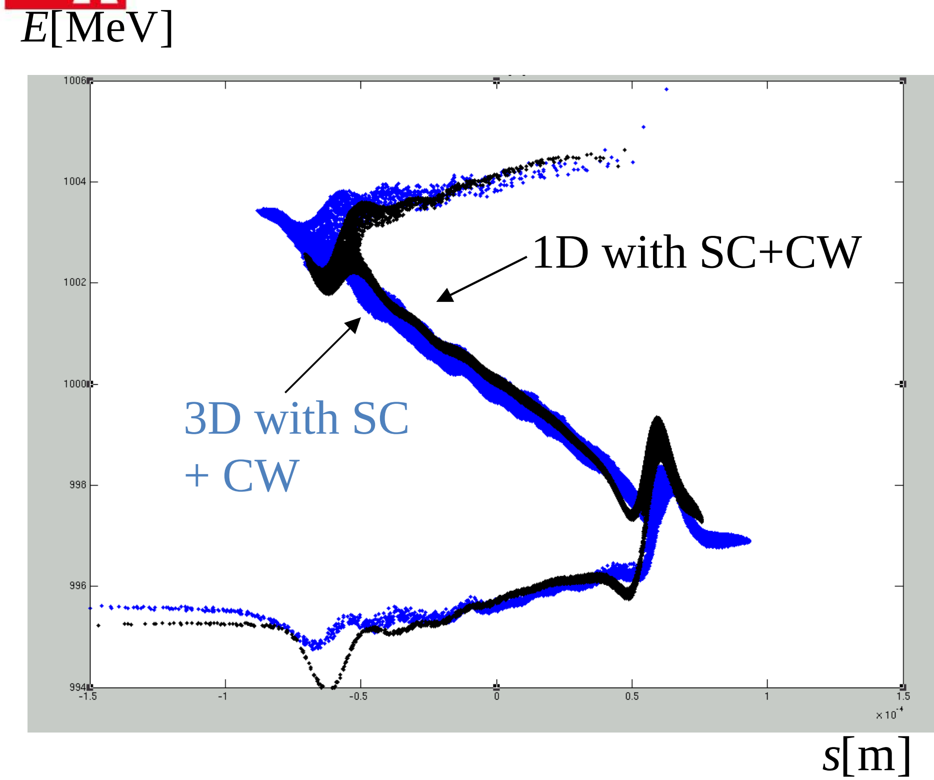


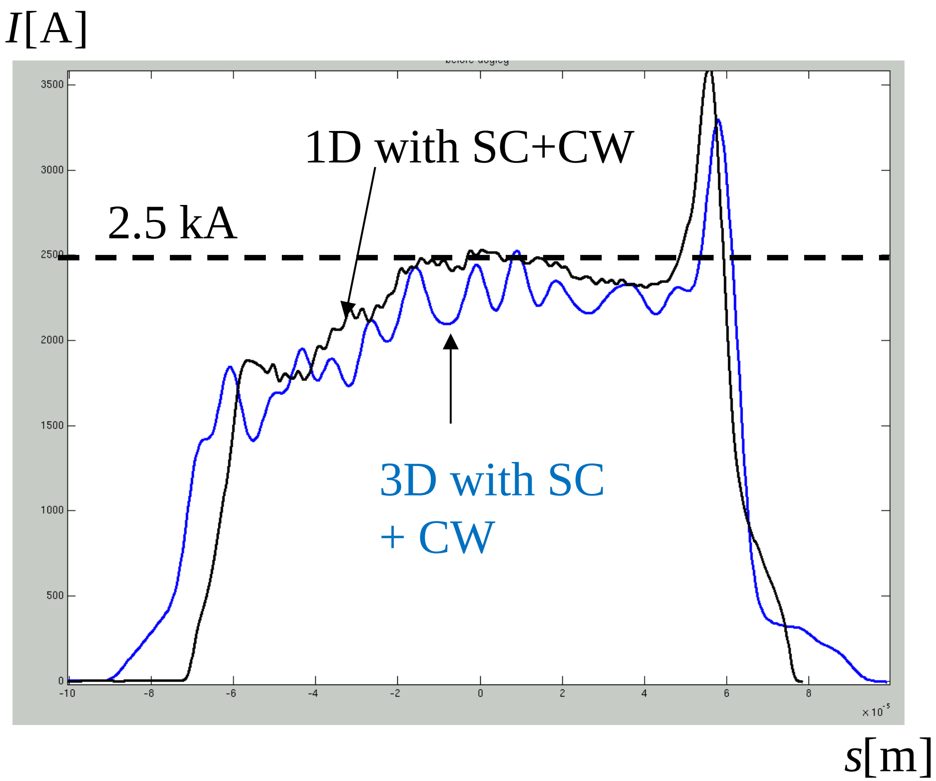
Optical Trans. Rad | LuAG*) scintillator | LuAG delayed by 100ns

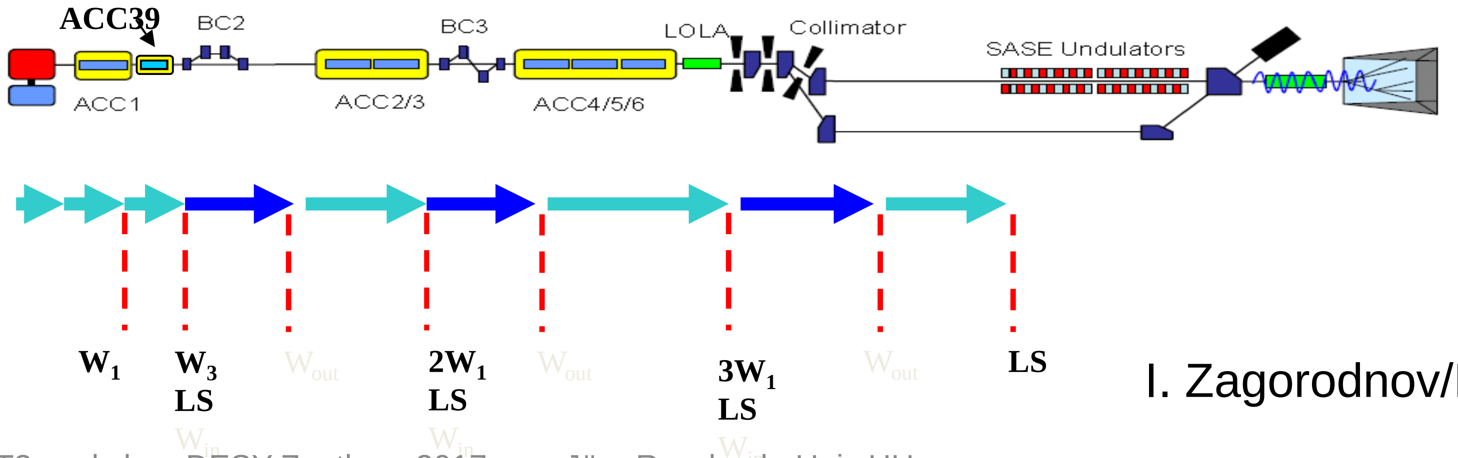
Chr. Behrens et al., PRST-AB 15, 062801 (2012)

*) Cerium-doped lutetium aluminum garnet

3D FLASH simulation with space charge + cavity wakes.

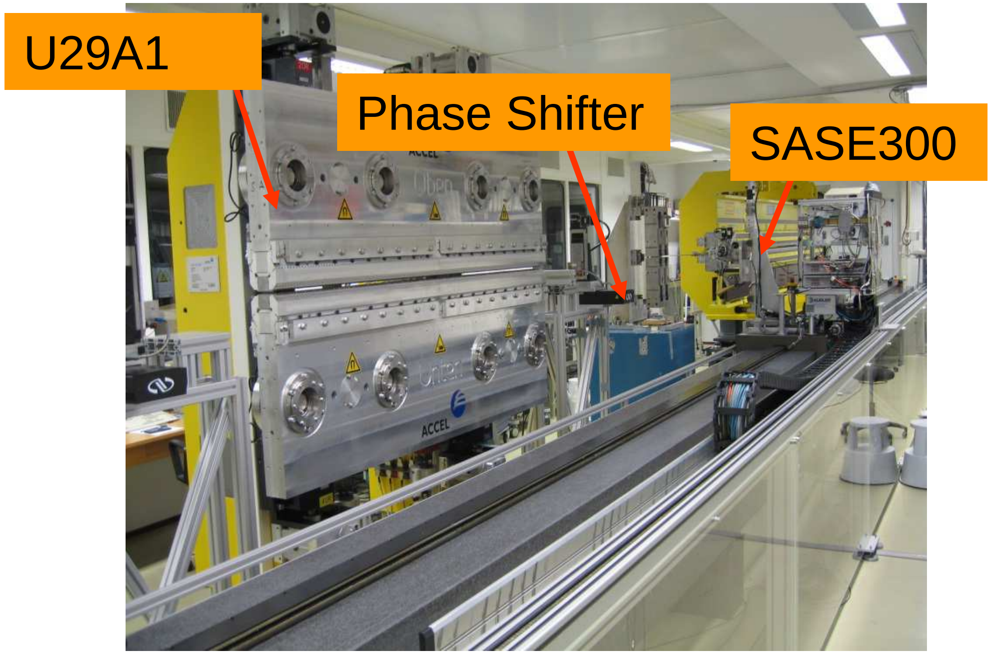






I. Zagorodnov/DESY

Magnetic Measurements for FLASH



12m Bench 22.1.08