Topological superconductors
– from band theory to interacting systems

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The University of Chicago

June 7, 2017

Today's agenda

- Various phases of condensed matter; metal, insulators, superconductivities, magnetisms,...
- In particular quantum phases of matter. Topological phases of condensed matter
- Wide varieties of topological phases; In particular, symmetry protected topological phases of matter
- "Order parameter" of topological phases; Topological invariants; E.g., TKNN integer.
- Contrast between single-particle and many-body formalisms

Varieties of topological phases

Topological systems	Related concepts
Anomalous Hall effect Integer quantum Hall effect Fractional quantum Hall effect	Berry phase Topological invariant Topological order/Anyons
Haldane phase Quantum spin Hall effect Topological insulators	Symmetry-protected topological phases
Topological superconductors	Majorana fermion Topological quantum computation
Weyl/Dirac semimetal	
⋮	⋮

Congratulations!

- 2016 Nobel Prize in Physics awarded to David J. Thouless, F. Duncan M. Haldane and J. Michael Kosterlitz "for theoretical discoveries of topological phase transitions and topological phases of matter".

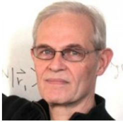
Today's focus: topological invariants

- I will introduce to you the Thouless-Kohmoto-Nightingale-den Nijs (TKNN) formula:

$$\boxed{\sigma_{xy} = \frac{e^2}{\hbar}\frac{1}{2\pi}\int d^2k\,\mathcal{B}(\boldsymbol{k}) = \frac{e^2}{h}\times(\text{integer})}$$

[Thouless-Kohmoto-Nightingale-den Nijs (82), Kohmoto (85)]





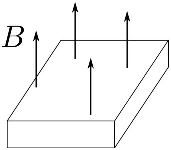

- and its many-body counter part. [Niu-Thouless-Wu (85)]:
- I will then introduce new formulas in the context of topological superconductors. (or More broadly, fermionic symmetry-protected topological phases in general).

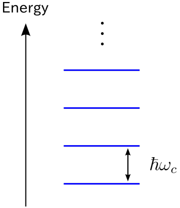
Outline

1. Topological phases of condensed matter
   - The quantum Hall effect
   - Topological superconductor
2. Topology and symmetries
3. Interaction effects
   - Many-body topological invariants
4. Summary

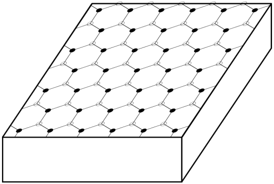
What is it about?

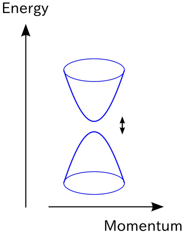
- The TKNN is about the quantum Hall effect realized in 2d electron gas under strong magnetic field





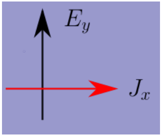
- Or even without uniform magnetic field (often called Chern insulators – e.g., the Haldane model [Haldane (88)])

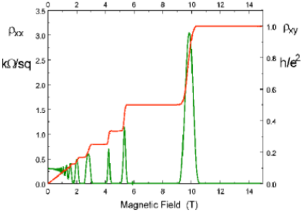




- In these systems, the Hall conductivity/conductance turns out to be quantized to remarkable accuracy!

$$J_x = \sigma_{xy} E_y, \quad \sigma_{xy} = \frac{e^2}{h} \times \text{integer}$$

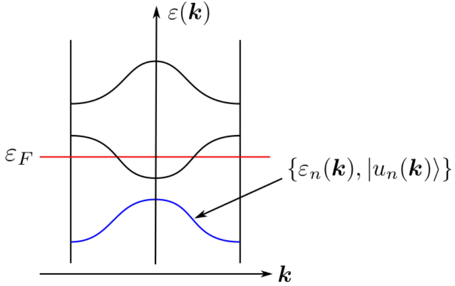




- The TKNN formula explains the quantization.

Ingredients in TKNN

- Electrons in solids $\left[\frac{-\nabla^2}{2m} + V(\boldsymbol{r})\right]\psi = \varepsilon\psi$
- Energy bands $\varepsilon_n(\boldsymbol{k})$ and Bloch wave functions $|u_n(\boldsymbol{k})\rangle$ in solids



- Metal v.s. insulators
- Only energy dispersion ($\varepsilon_n(\boldsymbol{k})$) matters? Role of electron wave functions (Bloch wave functions $|u_n(\boldsymbol{k})\rangle$) ?

- The TKNN formula depends only on wave functions:

$$\sigma_{xy} = \frac{e^2}{\hbar}\frac{1}{2\pi}\int d^2k\,\mathcal{B}(\boldsymbol{k}) = \frac{e^2}{h} \times \text{(integer)}$$

  Here, $\mathcal{B}$ is written in terms of $|u_n(\boldsymbol{k})\rangle$ as

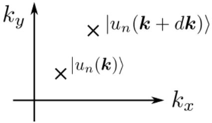
$$\mathcal{A}_j(\boldsymbol{k}) = i\langle u_n(\boldsymbol{k})|\frac{\partial}{\partial k_j}|u_n(\boldsymbol{k})\rangle$$

$$\mathcal{B}(\boldsymbol{k}) = \nabla_{\vec{k}} \times \mathcal{A}(\boldsymbol{k})$$

- $\mathcal{A}$: Non-dynamical $U(1)$ gauge field; The "Berry connection".
- $\mathcal{B}$: the Berry magnetic field or the Berry curvature in momentum space
- Integral of the flux over a closed surface is quantized according to Dirac.

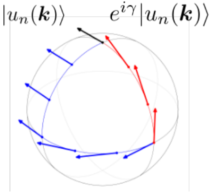
Aharonov-Bohm in momentum space

- How Bloch wave functions at different points in the momentum space are related?



Flat ?

- Consider an adiabatic transport of Bloch wave function in the momentum space. Wave functions may not come back to itself!



Curved !

The Berry phase: $\gamma = \int dS\mathcal{B}(\boldsymbol{k}) = \oint dl \cdot \mathcal{A}(\boldsymbol{k})$

- $\mathcal{A}(\boldsymbol{k})$ and $\mathcal{B}(\boldsymbol{k})$ encodes "geometry" in momentum space.
  Space seen by wave functions may be "curved"!

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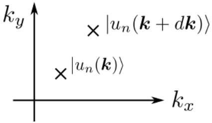

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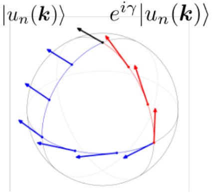

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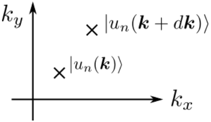

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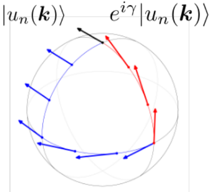

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$$\text{The Berry phase:} \quad \gamma = \int dS\mathcal{B}(\boldsymbol{k}) = \oint dl \cdot \mathcal{A}(\boldsymbol{k})$$

- $\mathcal{A}(\boldsymbol{k})$ and $\mathcal{B}(\boldsymbol{k})$ encodes "geometry" in momentum space. Space seen by wave functions may be "curved"!

Berry phase contribution to Equation of Motion

- Wave packet sees "geometry" of Bloch wave functions:

$$\frac{d\vec{x}(t)}{dt} = \frac{\partial \varepsilon_n(\boldsymbol{k})}{\partial \vec{k}} - \vec{\mathcal{B}}(\boldsymbol{k}) \times \frac{d\vec{k}(t)}{dt}$$

$$\frac{d\vec{k}(t)}{dt} = \frac{\partial V(\boldsymbol{x})}{\partial \vec{x}} - \vec{B}(\boldsymbol{x}) \times \frac{d\vec{x}(t)}{dt}$$

- Extra "kick" by the Berry phase term ("anomalous velocity")

From geometry to topology: The quantum Hall effect

- The Berry curvature enters into the electron transport. In the presence of the external electric field E,

$$\begin{aligned}\vec{J} &= (-e)\int d^d k\, f(\boldsymbol{k})\,\frac{d\vec{x}}{dt} \\ &\sim (-e)\int d^d k\,\Big(\underbrace{(\frac{\tau}{\hbar}\vec{E}\cdot\nabla_k f_0)\frac{\partial\varepsilon}{\hbar\partial\vec{k}}}_{\text{dynamical part}} + \underbrace{f_0\frac{e}{\hbar}\vec{E}\times\vec{\mathcal{B}}}_{\text{``topological`` part}}\Big)\end{aligned}$$

- Extreme case: the quantum Hall effect ($d=2$) and the TKNN formula

$$\vec{J} = (-e)\int d^2 k\,\left(f_0\frac{e}{\hbar}\vec{E}\times\vec{\mathcal{B}}\right)$$

$$\Longrightarrow \quad \sigma_{xy} = \frac{e^2}{\hbar}\frac{1}{2\pi}\int d^2 k\,\mathcal{B}(\boldsymbol{k})$$

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- Extreme case: the quantum Hall effect ($d = 2$) and the TKNN formula

$$\vec{J} = (-e)\int d^2 k\, \left(f_0\frac{e}{\hbar}\vec{E}\times\vec{\mathcal{B}}\right)$$

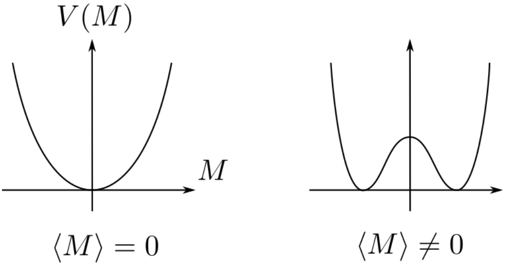
$$\Longrightarrow \quad \sigma_{xy} = \frac{e^2}{\hbar}\frac{1}{2\pi}\int d^2 k\,\mathcal{B}(\boldsymbol{k})$$

Topological phases

- Distinction between quantum phases of matter by their wave functions
- Topology of phase diagram
- Bulk-boundary correspondence (later)

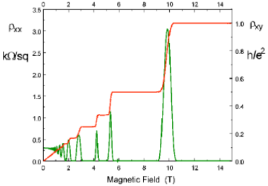
Symmetry breaking phases

- Many phases of matter can be described by *local order parameters* associated to spontaneous symmetry breaking (SSB).
- Example: magnet  Order parameter = magnetization M



Phases beyond symmetry breaking paradigm

- There are, however, phases of matter which defy descriptions in terms of any local order parameter
- Example: the quantum Hall effect

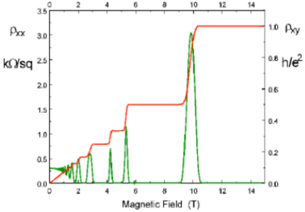


Different plateaus <–> different gapped quantum phases

Topological distinction between phases

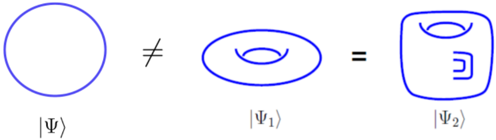
- Distinction between different ground state wave functions (= different quantum Hall phases) by the integrated Berry flux ("wave function topology")

$$\frac{1}{2\pi}\int d^2k\,\mathcal{B}(\boldsymbol{k}) = (\text{integer})$$



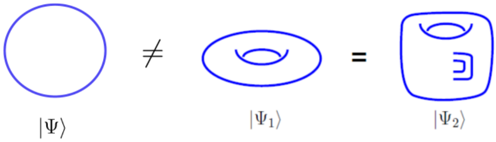
- C.f. Gauss-Bonnet theorem:

$$\frac{1}{4\pi}\int_{2d\ closed\ surf.}(curvature) = (1 - \#\text{ of handles})$$

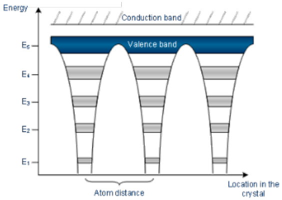


Topological insulators

- Distinction between insulators by their wave functions

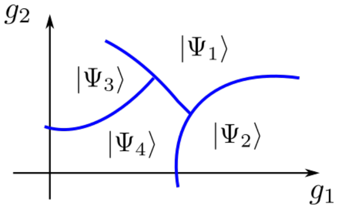


Trivial insulators = deformable to "atomic" insulators



- Topological insulators = not deformable to trivial insulators

- Topology of the phase diagram at zero temperature



- Topologically distinct ground state wave functions must be separated by a quantum phase transition

Topological phenomena in condensed matter

- Very *quantum* (wave function effects)
- *Robust* against disorder and interactions (e.g., quantized Hall conductance)
- No dynamics, *no dissipation*.
- *Controllable* applications:
  - No dissipation, immune to disorder: Transport with low energy cost
  - Cross correlation: spintronics, etc.
  - Non-trivial topological excitations (*anyons*): Decoherence-free quantum computations,
  - etc.

Topological phases beyond the quantum Hall effect?

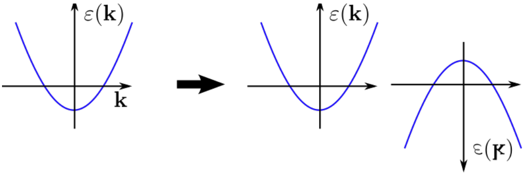
Topological systems	Related concepts
Anomalous Hall effect	Berry phase
Integer quantum Hall effect	Topological invariant
Fractional quantum Hall effect	Topological order/Anyons
Haldane phase	
Quantum spin Hall effect	Symmetry-protected topological phases
Topological insulators	
Topological superconductors	Majorana fermion
	Topological quantum computation
Weyl/Dirac semimetal	
⋮	⋮

Topological phases in superconductors

- Is there a topological distinction between superconductors?
- Superconductor = Cooper pair (boson) + BdG quasi-particle (fermion)
- Bogoliubov-de Genne Hamiltonian:

$$H = \frac{1}{2}\int \Psi^\dagger \mathcal{H}\,\Psi, \quad \mathcal{H} = \begin{pmatrix} \xi & \Delta \\ -\Delta^* & -\xi^T \end{pmatrix}$$

where $\Psi = (\psi_\uparrow^\dagger, \psi_\downarrow^\dagger, \psi_\uparrow, \psi_\downarrow)^T$

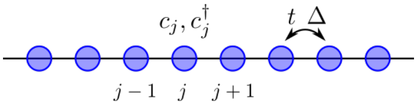


- Fully gapped superconductor = "band insulator" for BdG quasiparticles

The Kitaev chain

- The Kitaev chain (with $\Delta = t$ for simplicity)

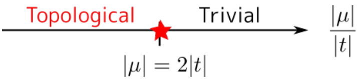
$$H = \sum_j \Big[-t c_j^\dagger c_{j+1} + \Delta c_{j+1}^\dagger c_j^\dagger + h.c. \Big] - \mu \sum_j c_j^\dagger c_j$$



- Proximitized spin-orbit quantum wire [Lutchyn et al (10), Oreg et al (10), Mourik et al (12), ...], magnetic adatomes on the surface of an s-wave superconductor [Nadj-Perge et al (14)]

The Kitaev chain

- Phase diagram: there are only two phases:



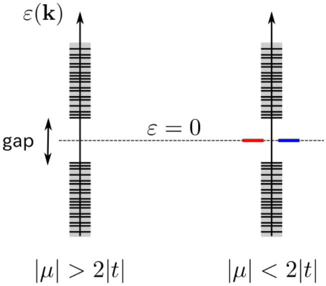
- Topologically non-trivial phase is realized when $2|t| \geq |\mu|$.
- The topological phase is characterized by (i) a bulk $\mathbb{Z}_2$ topological invariant:

$$\exp\left[i\int_{-\pi}^{\pi} dk\, \mathcal{A}_x(k)\right] = \pm 1$$

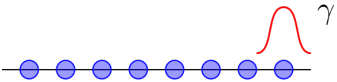
 where $\mathcal{A}(k) = i\langle u(k)|du(k)\rangle$ is the Berry connection.

Majorana end states

- (ii) Majorana end states: In the topological SC phase, there are zero-energy states in the BdG Hamiltonian with open boundary condition, localized at each end.



- The end state is a *Majorana* fermion. $\gamma^\dagger = \gamma$



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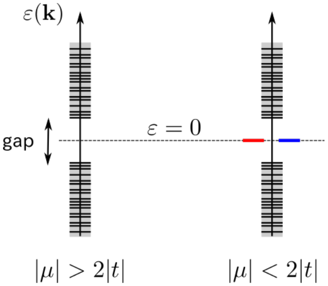

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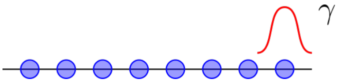


Majorana and quantum computations

- Majorana fermions are useful for topological quantum computation.
- Proximitized spin-orbit quantum wire [Lutchyn et al (10), Oreg et al (10), Mourik et al (12), ...], magnetic adatomes on the surface of an s-wave superconductor [Nadj-Perge et al (14)]

Delft experiments

25 MAY 2012 VOL 336 SCIENCE www.sciencemag.org

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1]†

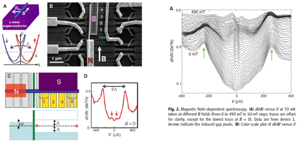


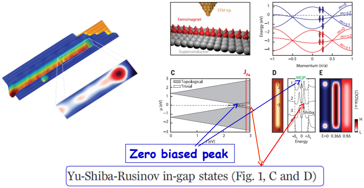
Fig. 2. Magnetic field–dependent spectroscopy. (A) dI/dV versus V at 70 mK taken at different B fields (from 0 to 490 mT in 10-mT steps; traces are offset for clarity, except for the lowest trace at B = 0). Data are from device 1. Arrows indicate the induced gap peaks. (B) Color-scale plot of dI/dV versus V

RESEARCH ARTICLES

TOPOLOGICAL MATTER

Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor

Stevan Nadj-Perge,[1*] Ilya K. Drozdov,[1*] Jian Li,[1*] Hua Chen,[2*] Sangjun Jeon,[1] Jungpil Seo,[1] Allan H. MacDonald,[2] B. Andrei Bernevig,[1] Ali Yazdani[1]†



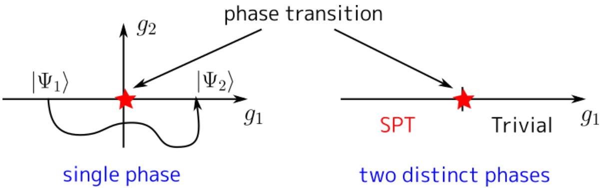
Topology and symmetries

Topological phases with symmetries (preview)

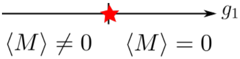
- - Distinction between insulators by their wave functions
  - Topology of phase diagram
  - Bulk-boundary correspondence
- All these can discussed *in the presence of symmetries*
- Interesting interplay between topology and symmetry: symmetry-protected topological phases (SPT phases) and symmetry-enriched topological phases (SET phases)

Symmetry-Protected Topological (SPT) phases

- "Deformable" to a trivial phase (state w/o entanglement) in the absence of symmetries.
- But sharply distinct from trivial state, once symmetries are enforced.



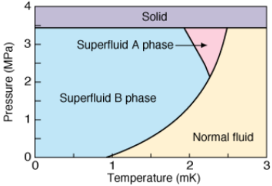
- E.g. quantum spin Hall effect, time-reversal symmetric topological insulators, the Haldane spin chain.
- Need to go beyond "symmetry breaking" paradigm (no local order parameter).



- $\Rightarrow$ Need to consider topological invariants!

^{3}He B is a topological superconductor (superfluid)

- B-phase of ^{3}He



- BdG Hamiltonian

$$H = \int d^3\mathrm{k}\, \Psi^\dagger(\mathrm{k})\mathcal{H}(\mathrm{k})\Psi(\mathrm{k}), \quad \mathcal{H}(\mathrm{k}) = \begin{bmatrix} \frac{k^2}{2m} - \mu & \Delta\sigma\cdot\mathrm{k} \\ \Delta\sigma\cdot\mathrm{k} & -\frac{k^2}{2m} + \mu \end{bmatrix}$$

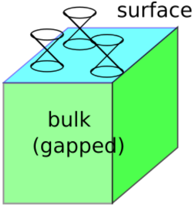
$$\Psi(\mathrm{k}) = (\psi_\uparrow(\mathrm{k}), \psi_\downarrow(\mathrm{k}), \psi_\downarrow^\dagger(-\mathrm{k}), -\psi_\uparrow^\dagger(-\mathrm{k}))^T$$

- Not topological in the absence of symmetries; with time-reversal or inversion symmetry, an SPT phase.

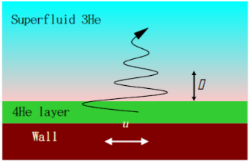
$$I\psi_\sigma^\dagger(\mathrm{r})I^{-1} = i\psi_\sigma^\dagger(-\mathrm{r})$$

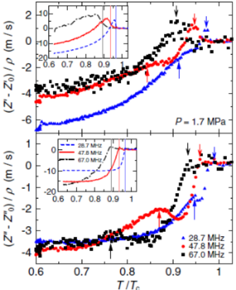
Surface Majorana cone

- Stable surface Majorana cones



- Detected by surface acoustic impedance measurement [Murakawa et al (09)]:





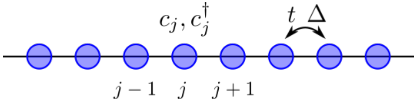
- So far: *single-particle formulation* of topological invariants.
- Tremendous success

How about interactions ?

The Kitaev chain with TR or reflection symmetry

- Recall the Kitaev chain (with $\Delta = t$ for simplicity):

$$H = \sum_j \Big[-tc_j^\dagger c_{j+1} + \Delta c_{j+1}^\dagger c_j^\dagger + h.c. \Big] - \mu \sum_j c_j^\dagger c_j$$



- Characterized by the $\mathbb{Z}_2$ topological invariant $\simeq$ the even/odd Majorana end states.

- The Kitaev chain can be studied in the presence of time-reversal or reflection symmetry.

$$Tc_jT^{-1} = c_j \; (TiT^{-1} = -i) \quad \text{or} \quad Rc_jR^{-1} = ic_{-j}.$$

- Once we impose more symmetries (T or R), we distinguish more phases. Phases are classified by an integer $\mathbb{Z}$.

- There is a topological invariant written in terms of Bloch wave functions.

$$\nu = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk \, \mathcal{A}_x(k) = (\text{integer})$$

- Ex: For N_f copies of the Kitaev chain $H = \sum_{a=1}^{N_f} H^a$ with $|\mu| < 2|t|$, the topological invariant is $\nu = N_f$.

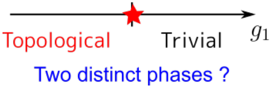
- Add interactions; *Can we destropy the topological phase by interactions?*;

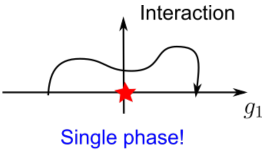
$$H \to H + wV$$

  I.e., Is it possible to go from topological to trivial ?

- Interestingly, the answer is Yes! [Fidkowski-Kitaev(10)]
- You can show there is a (rather complicated) interaction V that destroys the topological case.
- *Only possible when $N_f = 8$ ($N_f \equiv 0$ mod 8)*

Issues and Goal





- Clearly, something is missing in non-interacting classification. We have not explored the phase diagram "hard enough".
- Various other examples in which the non-interacting classification breaks down.
- Non-interacting topological invariants are not enough/spurious.
- Goal: find many-body invariants for (fermionic) SPT phases.

Main results [arXiv:1607.03896 and arXiv:1609.05970]

- We have succeeded in construction many-body topological invariants for SPT phases.
- These invariants do not refer to single particle wave functions (Bloch wave functions). They are written in terms of many-body ground states $|\Psi\rangle$.
- C.f. Many-body Chern number.
- Strategy behind the construction (later). [Hsieh-Sule-Cho-SR-Leigh (14), Kapustin et al (14), Witten (15)]
- Collaborators: Hassan Shapourian (UIUC) and Ken Shiozaki (UIUC)

The Kitaev chain with reflection

- Consider *Partial reflection* operation R_{part}, which acts only a part of the system.

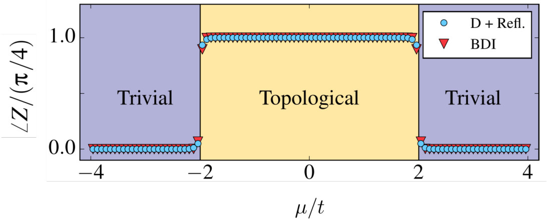
- We claim the phase of the overlap

$$Z = \langle \Psi | R_{part} | \Psi \rangle$$

is quantized, and serves as the many-body topological invariant.

- (Similar but somewhat more complicated invariant for TR symmetric case.)

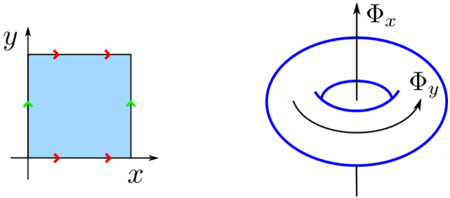
- Numerically check (blue filled circles).



- The phase of Z is the 8th root of unity.

Strategy behind the construction

- In the quantum Hall effect, the many-body Chern number is formulated by putting the system on the spatial torus:



- Introduce the twisting boundary condition by $U(1)$, and measure the response: Ground state on a torus with flux $|\Psi(\Phi_x, \Phi_y)\rangle$
- Berry connection in parameter space

$$A_i = i\langle\Psi(\Phi_x, \Phi_y)|\frac{\partial}{\partial\Phi_i}|\Psi(\Phi_x, \Phi_y)\rangle$$

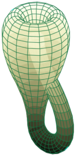
- *Many-body Chern number* [Niu-Thouless-Wu (85)]:

$$Ch = \frac{1}{2\pi}\int_0^{2\pi} d\Phi_x \int_0^{2\pi} d\Phi_y (\partial_{\Phi_x} A_y - \partial_{\Phi_y} A_x)$$

Strategy behind the construction

- A famous saying

- However, new phases of matter requires new kinds of manifolds, *unoriented* manifolds. E.g. Klein bottle



Strategy behind the construction

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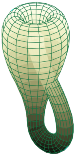
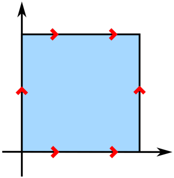
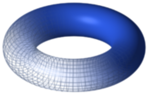
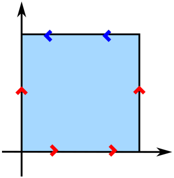


- For topological phases with reflection symmetry, we twist boundary conditions using the symmetry of the problem (reflection)

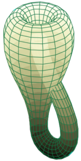


$$\Psi(x+L,y) = e^{i\Phi_x}\Psi(x,y)$$
$$\Psi(x,y+L) = e^{i\Phi_y}\Psi(x,y)$$

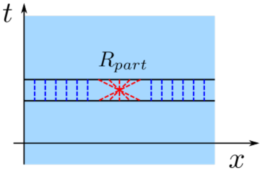


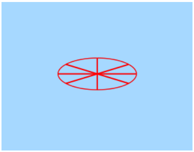


$$\Psi(t+T,x) = \Psi(t,-x)$$



- Partial reflection introduces a “crosscap” in space time:





- The spacetime is effectively *the real projective plane*, $\mathbb{R}P^2$.

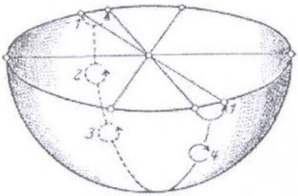


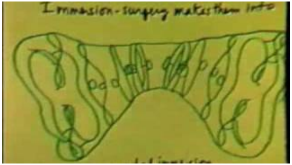
Abb. 9.

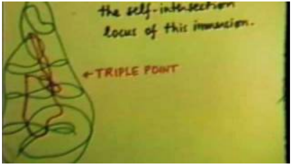
- "Interesting" mathematics... [Kapustin et al (14), Witten (15), Freed et al(14-16)]

$$8 \times \mathbb{R}P^2 \sim 0$$

- Watch a Youtube video (thanks: Dennis Sullivan):

  https://www.youtube.com/watch?v=7ZbbhBQEJmI





Topology: The connected sum of 8 copies of Boys Surface is immersion-cobordant to zero. The addition and the cobordism are illustrated here.

Higher dimensions – (3+1)d with inversion

- Consider ^{3}He-B:

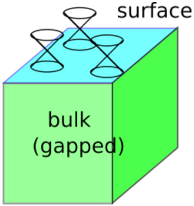
$$H = \int d^3\mathsf{k}\, \Psi^\dagger(\mathsf{k}) \mathcal{H}(\mathsf{k}) \Psi(\mathsf{k}), \quad \mathcal{H}(\mathsf{k}) = \begin{bmatrix} \frac{k^2}{2m} - \mu & \Delta\sigma\cdot\mathsf{k} \\ \Delta\sigma\cdot\mathsf{k} & -\frac{k^2}{2m} + \mu \end{bmatrix}$$

$$\Psi(\mathsf{k}) = (\psi_\uparrow(\mathsf{k}), \psi_\downarrow(\mathsf{k}), \psi_\downarrow^\dagger(-\mathsf{k}), -\psi_\uparrow^\dagger(-\mathsf{k}))^T$$

- Inversion symmetry:

$$I\psi_\sigma^\dagger(\mathsf{r})I^{-1} = i\psi_\sigma^\dagger(-\mathsf{r})$$

- Topologically protected surface Majorana cone (stable when the surface is inversion symmetric)

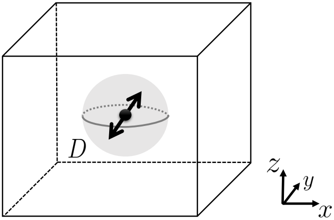


- Characterized by the integer topological invariant at non-interacting level.

Many-body topological invariant

- Previous studies indicate the non-interacting classification breaks down to $\mathbb{Z}_{16}$. Surface topological order. [Fidkowski et al (13), Metlitski et al (14), Wang-Senthil (14)]
- We consider partial inversion I_{part} on a ball D:

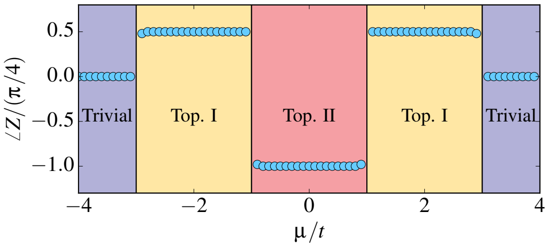
$$Z = \langle \Psi | I_{part} | \Psi \rangle$$



- The spacetime is effectively four-dimensional projective plane, $\mathbb{R}P^4$.

Calculations

- Numerics on a lattice:



- Matches with the analytical result

$$Z = \exp\left[-\frac{i\pi}{8} + \frac{1}{12}\ln(2) - \frac{21}{16}\zeta(3)\left(\frac{R}{\xi}\right)^2 + \cdots\right]$$

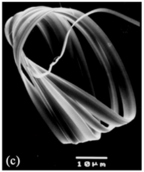
- C.f. topologically ordered surfaces: [Wang-Levin, Tachikawa-Yonekura, Barkeshli et al (16)]

Summary

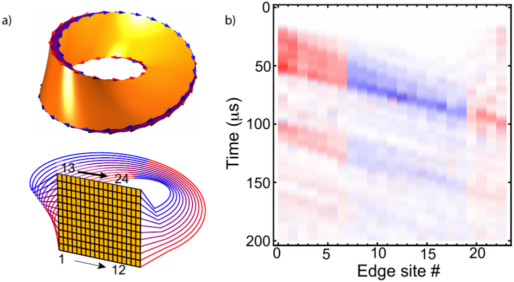
- We have succeeded in constructing many-body topological invariants for SPT phases.
- These invariants do not refer to single particle wave functions (Bloch wave functions). They are written in terms of many-body ground states $|\Psi\rangle$.
- Analogous to go from the single-particle TKNN formula to to the many-body Chern number.
- Many-body invariants in other cases (e.g., time-reversal symmetric topological insulators) can be constructed in a similar way.

Outlook

- Many future applications, in particular, in numerics.
- And ...?



NbSe$_3$ Möbius strip



[Ningyuan-Owens-Sommer-Schuster-Simon (13)]