

The Origin of Mass of the Visible and the Dark Universe

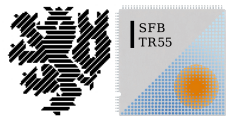
Z. Fodor

for the Budapest-Marseille-Wuppertal Collaboration:

June 9, 2017, Symposium for the 80th Birthday of Klaus Schilling

why do we have mass at all?
could the world look different?

Lattice QCD talk



Outline

- 1 Introduction
- 2 QCD (old)
- 3 Mass of the nucleon (neutron, proton)
- 4 Neutron-proton mass difference
- 5 Summary

Ordinary matter: three types of particles \Rightarrow H, He ...

ELECTRON



U-QUARK



D-QUARK



ELECTRIC



STRONG



HIGGS



All of them are massless!

ELECTRON



$$m_e=0$$

U-QUARK



$$m_u=0$$

D-QUARK



$$m_d=0$$

Why do we have mass at all?

Why do not we just fly apart with c ?

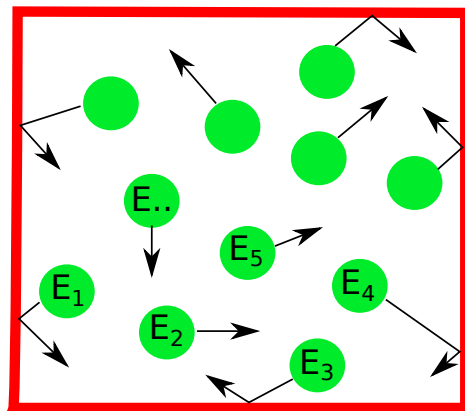
Three mechanisms

I. Strong mass

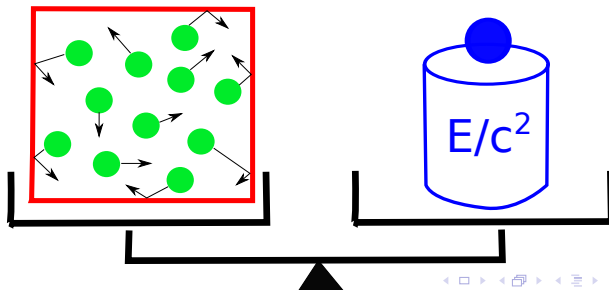
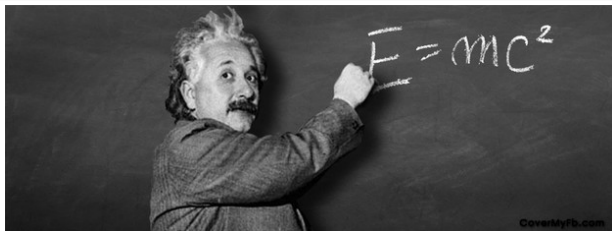
II. Electromagnetic mass

III. Mass from the Higgs-Mechanism

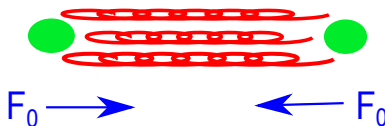
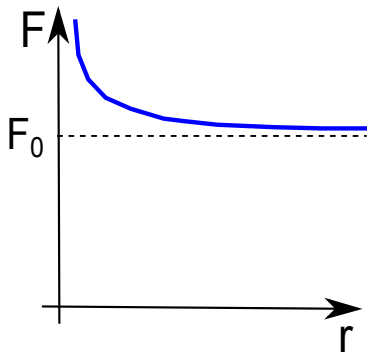
The massless box



Mass from energy



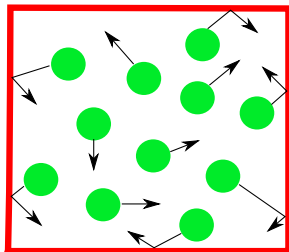
The strong force (massless box) \implies hadrons



**Quarks can not fly apart
they are confined!**

Three mechanisms

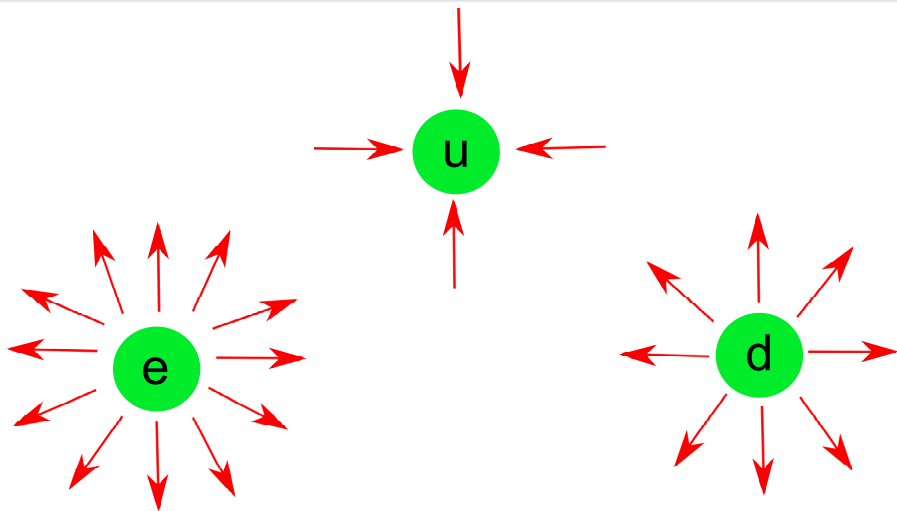
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ELECTRIC

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The electric field



Electric energy from the field energy

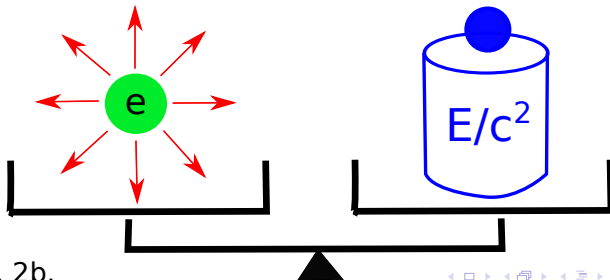
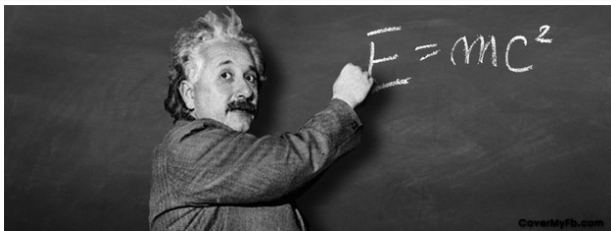
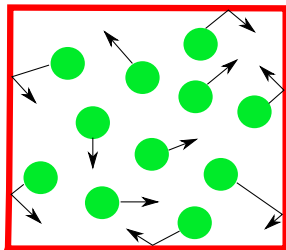


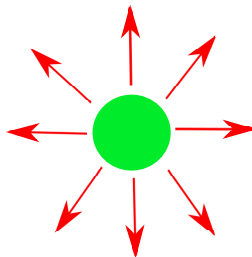
FIG. 2b.

Three mechanisms

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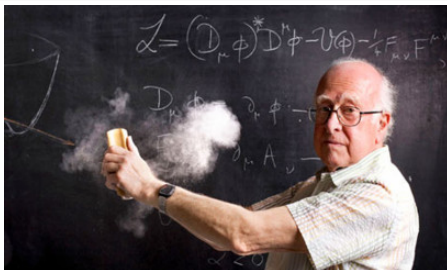
Higgs-Mechanism in 7s



“you need a better job of talking each other”

“the left hand now knows what the right hand is doing”

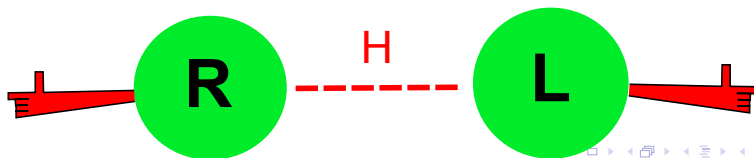
Higgs-Mechanism: Yukawa couplings



MASSLESS

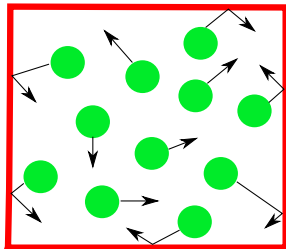


MASSIVE

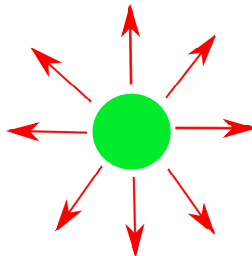


Three mechanisms

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HIGGS



Chromodynamics is a generalized electrodynamics

QCD is a generalized, extended version of QED

electrodynamics: only 1 charge, electric (positive or negative)

chromodynamics: 3 charges, call them colors (red, blue, green)
all of them can be positive or negative (not real colors)

gluons (similarly to photons) transmit the strong interaction
between quarks (which are similar to electrons)
richer structure: color of quarks can be changed by gluons (charged)

neither free gluons nor quarks were seen

fundamental degrees of freedom don't appear experimentally (later)

Lagrangian of QED and QCD

electrodynamics: electromagnetic field is given by A_μ , electron by ψ

$$-\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma_\mu(\partial^\mu + iA_\mu) + m]\psi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

chromodynamics: field A_μ is a traceless 3×3 matrix, ψ has an index

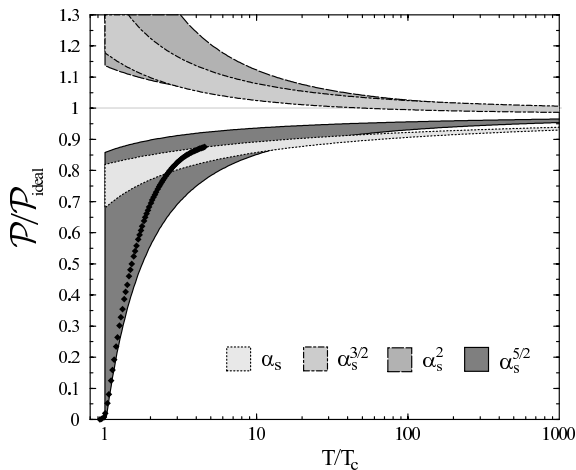
$$-\frac{1}{2g^2}\text{tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\{i\gamma_\mu(\partial^\mu + iA_\mu) + m\}\psi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu A_\nu - A_\nu A_\mu]$$

unambiguously fixed by gauge invariance: phase factor $\exp[i\alpha(x)]$

this is the classical level of the field theory, we quantize it
strongly interacting theory: difficult to solve

QCD: need for a systematic non-perturbative method

pressure at high temperatures converges at $T=10^{300}$ MeV



Lattice field theory

systematic non-perturbative approach (numerical solution):

quantum fields on the lattice

quantum theory: path integral formulation with $S = E_{kin} - E_{pot}$

quantum mechanics: for all possible paths add $\exp(iS)$

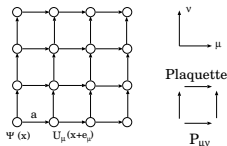
quantum fields: for all possible field configurations add $\exp(iS)$

Euclidean space-time ($t = i\tau$): $\exp(-S)$ sum of Boltzmann factors

use a space-time grid \Rightarrow formally: four-dimensional statistical system
extrapolate to the continuum limit: $a \rightarrow 0$

\Rightarrow stochastic approach, with reasonable spacing/size: solvable
theory, algorithm, CPU; easy to make it $1000\times$ more expensive

Lattice Lagrangian: gauge fields



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi}(D_\mu \gamma^\mu + m)\psi$$

anti-commuting $\psi(x)$ quark fields live on the sites
gluon fields, $A_\mu^a(x)$ are used as links and plaquettes

$$U(x, y) = \exp \left(ig_s \int_x^y dx'^\mu A_\mu^a(x') \lambda_a / 2 \right)$$

$$P_{\mu\nu}(n) = U_\mu(n) U_\nu(n + e_\mu) U_\mu^\dagger(n + e_\nu) U_\nu^\dagger(n)$$

$S = S_g + S_f$ consists of the pure gluonic and the fermionic parts

$$S_g = 6/g_s^2 \cdot \sum_{n,\mu,\nu} [1 - \text{Re}(P_{\mu\nu}(n))]$$

Lattice Lagrangian: fermionic fields

quark differencing scheme:

$$\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) \rightarrow \bar{\psi}_n\gamma^\mu(\psi_{n+e_\mu} - \psi_{n-e_\mu})$$

$$\bar{\psi}(x)\gamma^\mu D_\mu\psi(x) \rightarrow \bar{\psi}_n\gamma^\mu U_\mu(n)\psi_{n+e_\mu} + \dots$$

fermionic part as a bilinear expression: $S_f = \bar{\psi}_n M_{nm} \psi_m$

e.g. 2 degenerate light quarks (u,d) and the strange quark: $n_f = 2 + 1$

Euclidean partition function is given by the Boltzmann weights

$$Z = \int \prod_{n,\mu} [dU_\mu(x)] [d\bar{\psi}_n] [d\psi_n] e^{-S_g - S_f} = \int \prod_{n,\mu} [dU_\mu(n)] e^{-S_g} \det(M[U])$$

Importance sampling

$$Z = \int \prod_{n,\mu} [dU_\mu(n)] e^{-S_g} \det(M[U])$$

we do not take into account all possible gauge configuration

each of them is generated with a probability \propto its weight

importance sampling, Metropolis algorithm:

(all other algorithms are based on importance sampling)

$$P(U \rightarrow U') = \min [1, \exp(-\Delta S_g) \det(M[U']) / \det(M[U])]$$

gauge part: trace of 3×3 matrices (easy, **without M: quenched**)

fermionic part: determinant of $10^6 \times 10^6$ sparse matrices (hard)

more efficient ways than direct evaluation ($Mx=a$), but still hard

The mass is not the sum of the constituents' mass

usually the mass of “some ordinary thing” is just
the sum of the mass of its constituents (upto tiny corrections)

origin of the mass of the visible universe: dramatically different

proton is made up of massless gluons and almost massless quarks

quarks



3 x 5 grams

proton



1 kilogram

mass of a quark is ≈ 5 MeV, that of a proton (hadron) is ≈ 1000 MeV

Hadron spectroscopy in lattice QCD

Determine the transition amplitude between:
 having a “particle” at time 0 and the same “particle” at time t
 \Rightarrow Euclidean correlation function of a composite operator \mathcal{O} :

$$C(t) = \langle 0 | \mathcal{O}(t) \mathcal{O}^\dagger(0) | 0 \rangle$$

insert a complete set of eigenvectors $|i\rangle$

$$= \sum_i \langle 0 | e^{Ht} \mathcal{O}(0) e^{-Ht} | i \rangle \langle i | \mathcal{O}^\dagger(0) | 0 \rangle = \sum_i |\langle 0 | \mathcal{O}^\dagger(0) | i \rangle|^2 e^{-(E_i - E_0)t},$$

where $|i\rangle$: eigenvectors of the Hamiltonian with eigenvalue E_i .

and

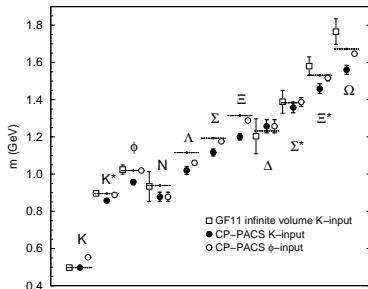
$$\mathcal{O}(t) = e^{Ht} \mathcal{O}(0) e^{-Ht}.$$

t large \Rightarrow lightest states (created by \mathcal{O}) dominate: $C(t) \propto e^{-M \cdot t}$
 \Rightarrow exponential fits or mass plateaus $M_t = \log[C(t)/C(t+1)]$

Quenched results

QCD is 40 years old \Rightarrow properties of hadrons (Rosenfeld table)
 non-perturbative lattice formulation (Wilson) immediately appeared
 "did not have 35 years"; more optimistic scientists: 10 years (fusion)
 needed 20 years even for quenched result of the spectrum (cheap)
 instead of $\det(M)$ of a $10^6 \times 10^6$ matrix trace of 3×3 matrices

always at the frontiers of computer technology:



GF11: IBM "to verify QCD"
 (10 Gflops, '92)

CP-PACS: Hitachi QCD machine
 (614 Gflops, '96)

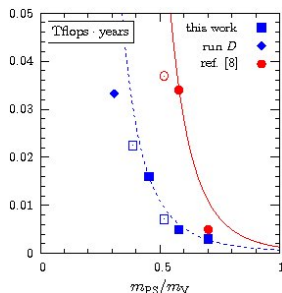
the $\approx 10\%$ discrepancy was believed
 to be a quenching effect

Difficulties of full dynamical calculations

though the quenched result can be qualitatively correct
uncontrolled systematics \Rightarrow full “dynamical” studies
by two-three orders of magnitude more expensive (balance)
present day machines offer several Pflops

no revolution but evolution in the algorithmic developments

Berlin Wall '01: it is extremely difficult to reach small quark masses:



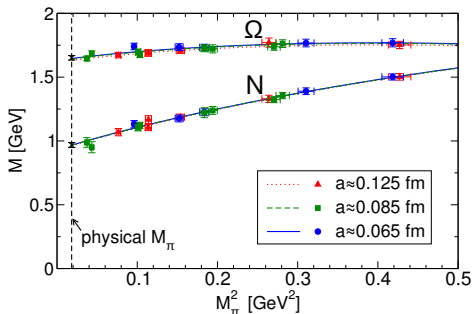
FLAG review of lattice results

Colangelo et al. Eur.Phys.J. C71 (2011) 1695

Collaboration		publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	$m_{ud,\overline{\text{MS}}}(2\text{GeV})$	$m_{s,\overline{\text{MS}}}(2\text{GeV})$
PACS-CS 10	P	★	■	■	★	<i>a</i>		2.78(27)	86.7(2.3)
MILC 10A	C	●	★	★	●	—		3.19(4)(5)(16)	—
HPQCD 10	A	●	★	★	★	—		3.39(6)*	92.2(1.3)
BMW 10AB	P	★	★	★	★	<i>b</i>		3.469(47)(48)	95.5(1.1)(1.5)
RBC/UKQCD	P	●	●	★	★	<i>c</i>		3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum et al. 10	P	●	■	●	★	—		3.44(12)(22)	97.6(2.9)(5.5)

S. Durr et al., Budapest-Marseille-Wuppertal Collaboration, Science 322 (2008) 1224

altogether 15 points for each hadrons



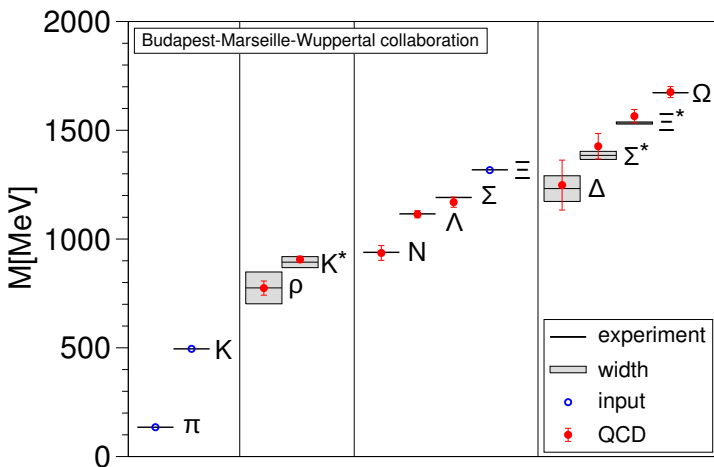
smooth extrapolation to the physical pion mass (or m_{ud})

small discretization effects (three lines barely distinguishable)

continuum extrapolation goes as $c \cdot a^n$ and it depends on the action
in principle many ways to discretize (derivative by 2,3... points)

Final result for the hadron spectrum

S. Durr et al., Science 322 1224 2008



Breakthrough of the Year

Proton's Mass 'Predicted'

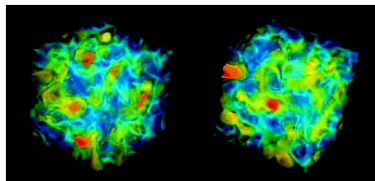
STARTING FROM A THEORETICAL DESCRIPTION OF ITS INNARDS, physicists precisely calculated the mass of the proton and other particles made of quarks and gluons. The numbers aren't new; experimenters have been able to weigh the proton for nearly a century. But the new results show that physicists can at last make accurate calculations of the ultracomplex strong force that binds quarks.

In simplest terms, the proton comprises three quarks with gluons zipping between them to convey the strong force. Thanks to the uncertainties of quantum mechanics, however, myriad gluons and quark-antiquark pairs flit into and out of existence within a

proton in a frenzy that's nearly impossible to analyze but that produces 95% of the particle's mass.

To simplify matters, theorists from France, Germany, and Hungary took an approach known as "lattice quantum chromodynamics."

They modeled continuous space and time as a four-dimensional array of points—the lattice—and confined the quarks to the points and the gluons to the links between them. Using supercomputers, they reckoned the masses of

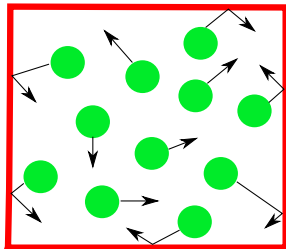


the proton and other particles to a precision of about 2%—a tenth of the uncertainties a decade ago—as they reported in November.

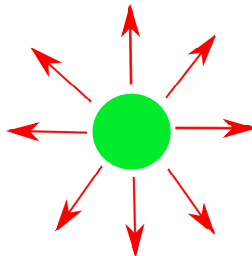
In 2003, others reported equally precise calculations of more-esoteric quantities. But by calculating the familiar proton mass, the new work signals more broadly that physicists finally have a handle on the strong force.

Three mechanisms

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Introduction to isospin symmetry

Isospin symmetry: 2+1 or 2+1+1 flavor frameworks

if 'up' and 'down' quarks had identical properties (mass, charge)

$$M_n = M_p, \quad M_{\Sigma^+} = M_{\Sigma^0} = M_{\Sigma^-}, \quad \text{etc.}$$

The symmetry is explicitly broken by

- up, down quark electric charge difference (up: $2/3 \cdot e$ down: $-1/3 \cdot e$)
 \Rightarrow proton: $uud = 2/3 + 2/3 - 1/3 = 1$ whereas neutron: $udd = 2/3 - 1/3 - 1/3 = 0$
 at this level (electric charge) the proton would be the heavier one
- up, down quark mass difference ($m_d/m_u \approx 2$): 1+1+1+1 flavor

The breaking is large on the quark's level ($m_d/m_u \approx 2$ or charges) but small (typically sub-percent) compared to hadronic scales.

These two competing effects provide the tiny $M_n - M_p$ mass difference $\approx 0.14\%$ is required to explain the universe as we observe it

Big bang nucleosynthesis and nuclei chart

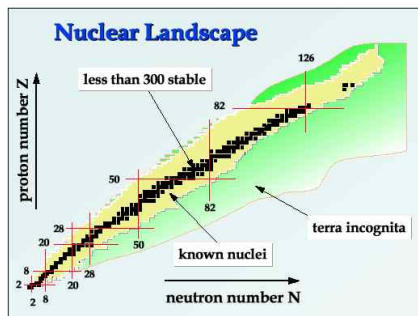
if $\Delta m_N < 0.05\%$ \rightarrow inverse β decay leaving (predominantly) neutrons
 $\Delta m_N \gtrsim 0.05\%$ would already lead to much more He and much less H
 \rightarrow stars would not have ignited as they did

if $\Delta m_N > 0.14\%$ \rightarrow much faster beta decay, less neutrons after BBN
 burning of H in stars and synthesis of heavy elements difficult

The whole nuclei chart is based
 on precise value of Δm_N

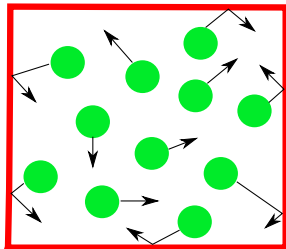
Could things have been different?

Jaffe, Jenkins, Kimchi, PRD 79 065014 (2009)

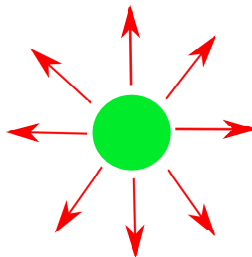


Three mechanisms

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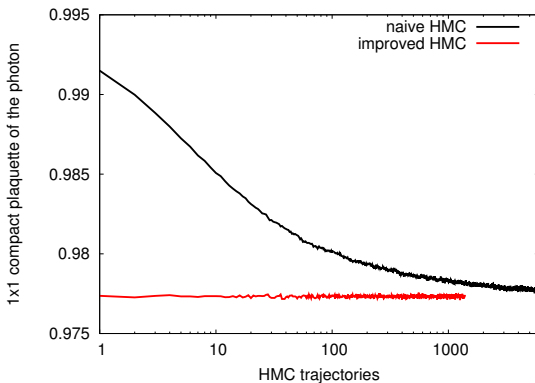
ELECTRIC



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Autocorrelation of the photon field

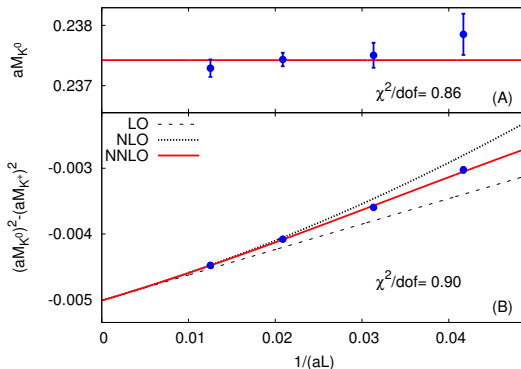


Standard HMC has $\mathcal{O}(1000)$ autocorrelation

Improved HMC has none (for the pure photon theory)

Small coupling to quarks introduces a small autocorrelation

Finite V dependence of the kaon mass



Neutral kaon shows essentially no (small $1/L^3$) volume dependence
 Volume dependence of the K splitting is perfectly described
 $1/L^3$ order is significant for kaon (baryons are not as precise)

Lattice spacings and pion masses

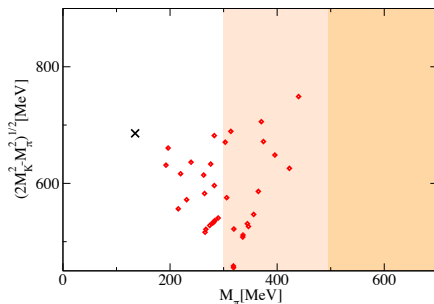
final result is quite independent of the lattice spacing & pion mass

⇒ four lattice spacings with $a=0.102, 0.089, 0.077$ and 0.064 fm

four volumes for a large volume scan: $L=2.4 \dots 8.2$ fm

five charges for large electric charge scan: $e=0 \dots 1.41$

41 ensembles with $M_\pi=195\text{--}440$ MeV (various cuts)

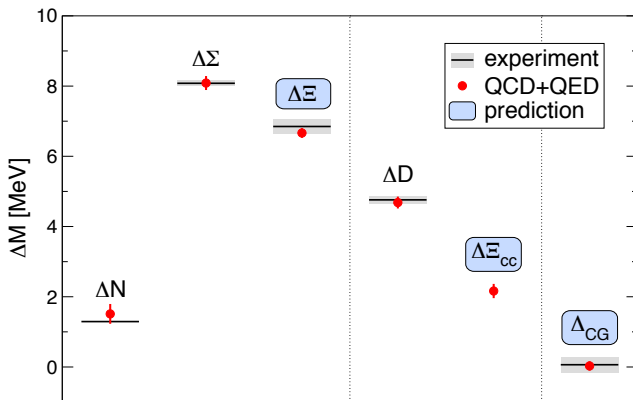


large parameter space: helps in the Kolmogorov-Smirnov analysis



Isospin splittings

splittings in channels that are stable under QCD and QED:



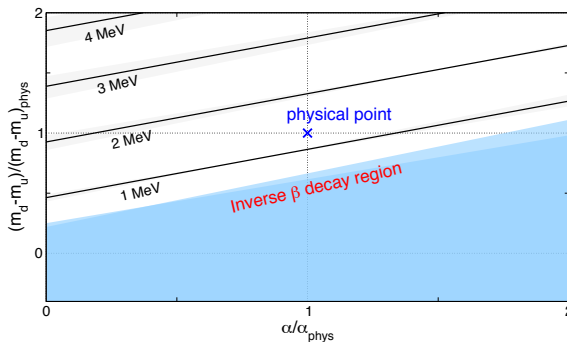
ΔM_N , ΔM_Σ and ΔM_D splittings: post-dictions

ΔM_Ξ , $\Delta M_{\Xi_{cc}}$ splittings and Δ_{CG} : predictions

Quantitative anthropics

Precise scientific version of the great question:
Could things have been different (string landscape)?

eg. big bang nucleosynthesis & today's stars need $\Delta M_N \approx 1.3 \text{ MeV}$



(lattice message: too large or small α would shift the mass)

Gauging the allure of
designer drugs p. 469

Blow-up brains for a better
inside view pp.474 & 543

Single-crystal perovskite
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Gang of three

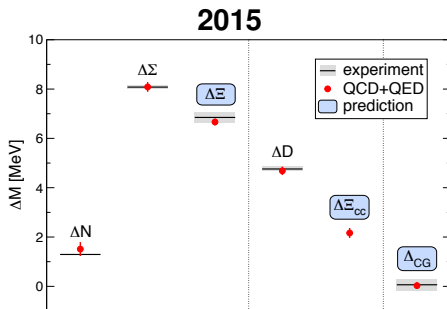
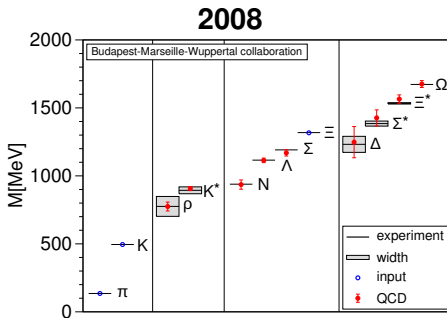
How dynactin, dynein, and
Bicaudal-D2 motor together
p. 1441



The proton



Strong + Higgs + Electro = Experiment



vision about a future, in which high precisions can be achieved for a broad spectrum of non-perturbative questions (lattice formalism)