Theories of Bad Metals

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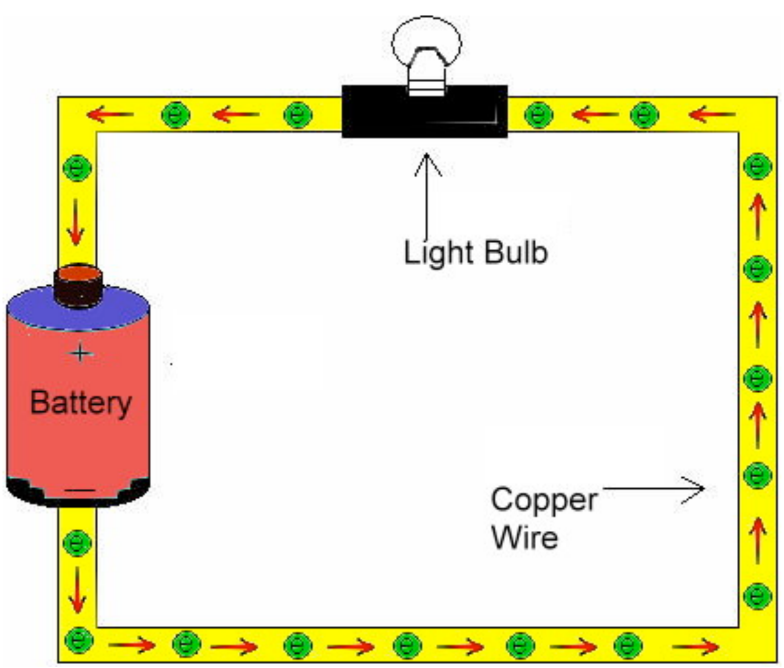
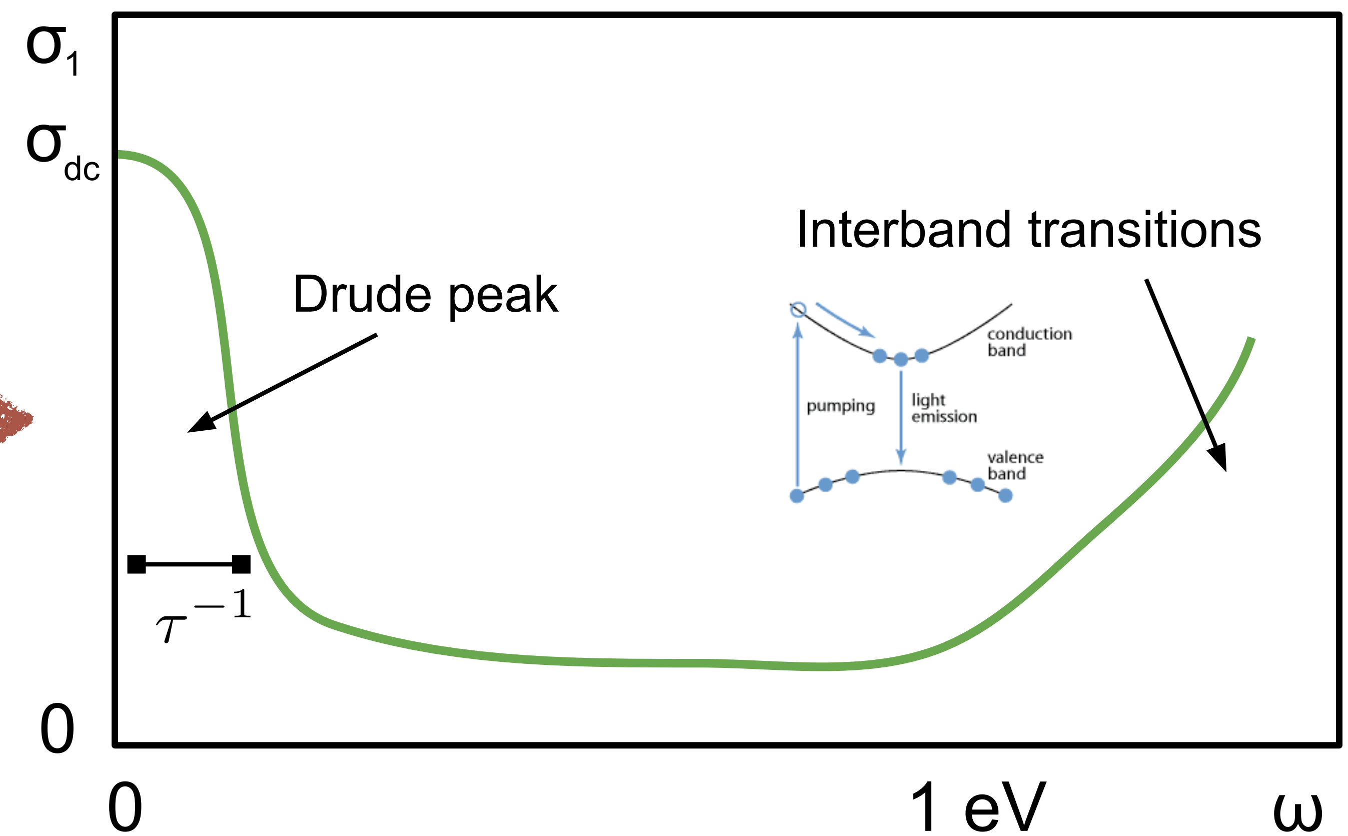
Quasiparticles and transport

- In conventional metals, the low energy excitations are weakly interacting quasiparticles.





Quasiparticles and transport

- The electrical resistivity is given by the Drude formula:

$$\rho = \frac{m}{ne^2}\frac{1}{\tau}$$

- The quasiparticle lifetime is key.

- Need $\tau \to \tau_{\mathrm{tr}}$ in general both for optical and dc conductivity. Due to:
  (a) dominance of small angle scattering that (Bloch)
  (b) 'phonon drag' which conserves momentum (Peierls).

- Usually $\tau_{\mathrm{tr}} \gg \tau$, so these effects make resistivity small.

Quasiparticles ⇒ Mott-Ioffe-Regel bound

- The Drude formula can be written as:

$$\rho = \frac{m}{ne^2}\frac{1}{\tau} \sim \frac{1}{k_F \ell}\frac{\hbar}{e^2} \qquad\qquad (d=2)$$

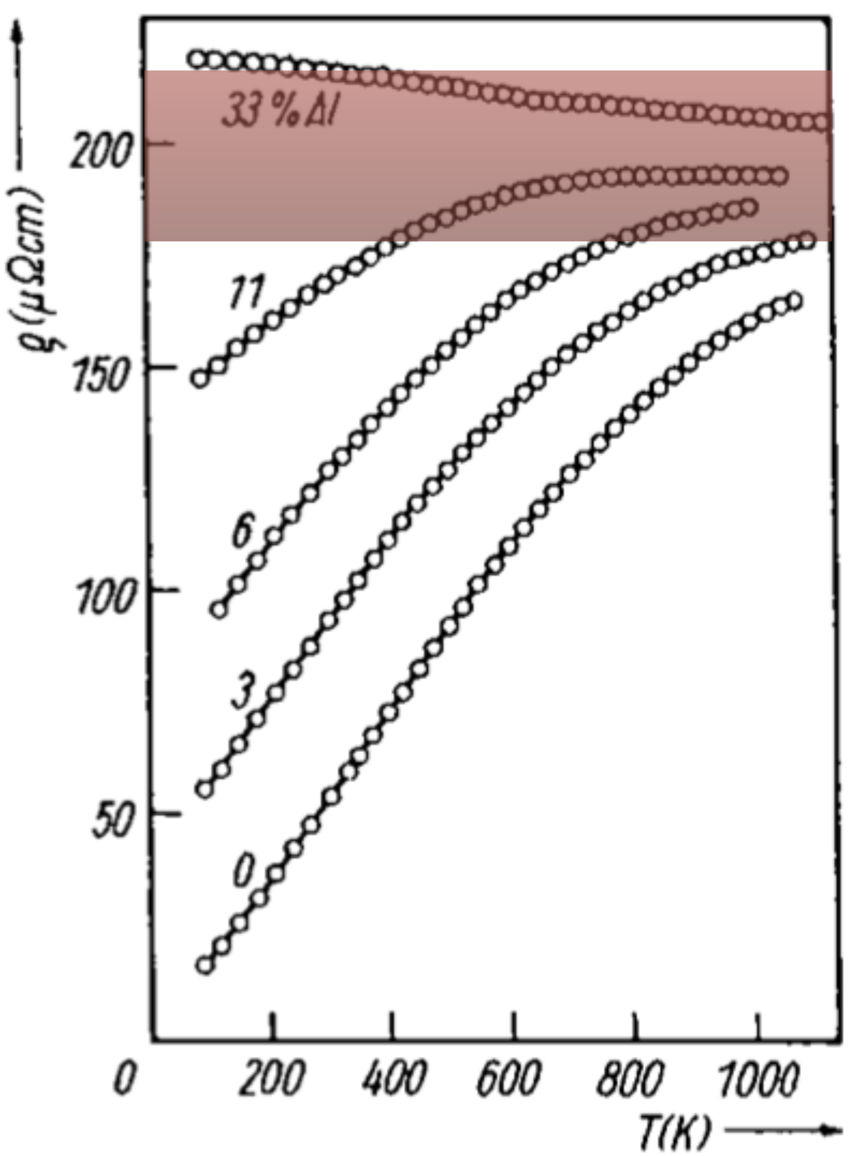
- If the quasiparticle momentum is well defined:

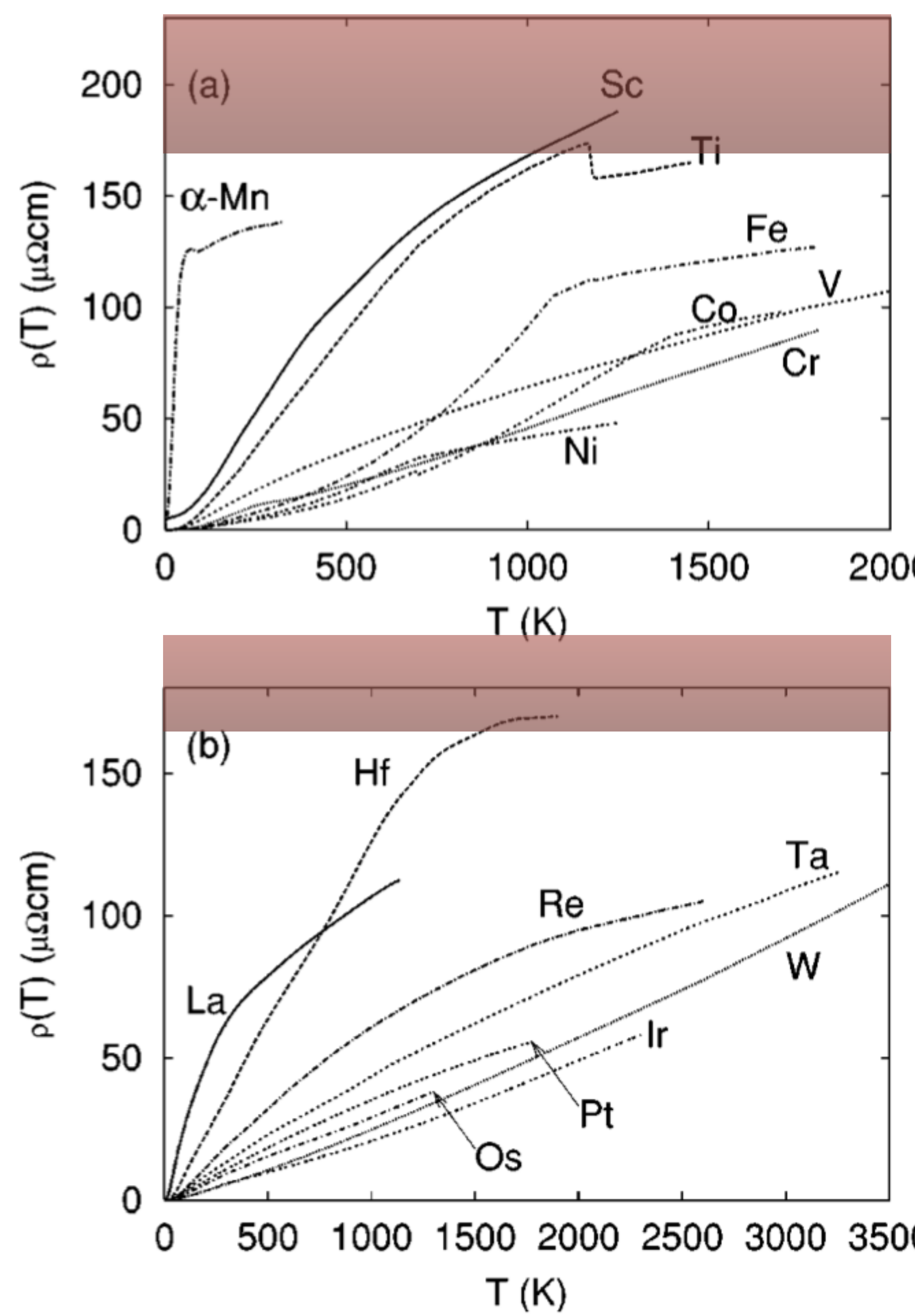
$$k_F \ell \gtrsim \Delta k_F \, \ell \gtrsim 1 \qquad \Rightarrow \qquad \rho \lesssim \frac{\hbar}{e^2}$$

- This is the Mott-Ioffe-Regel bound on the resistivity of metals. (insulators, of course, have $\rho = \infty$).

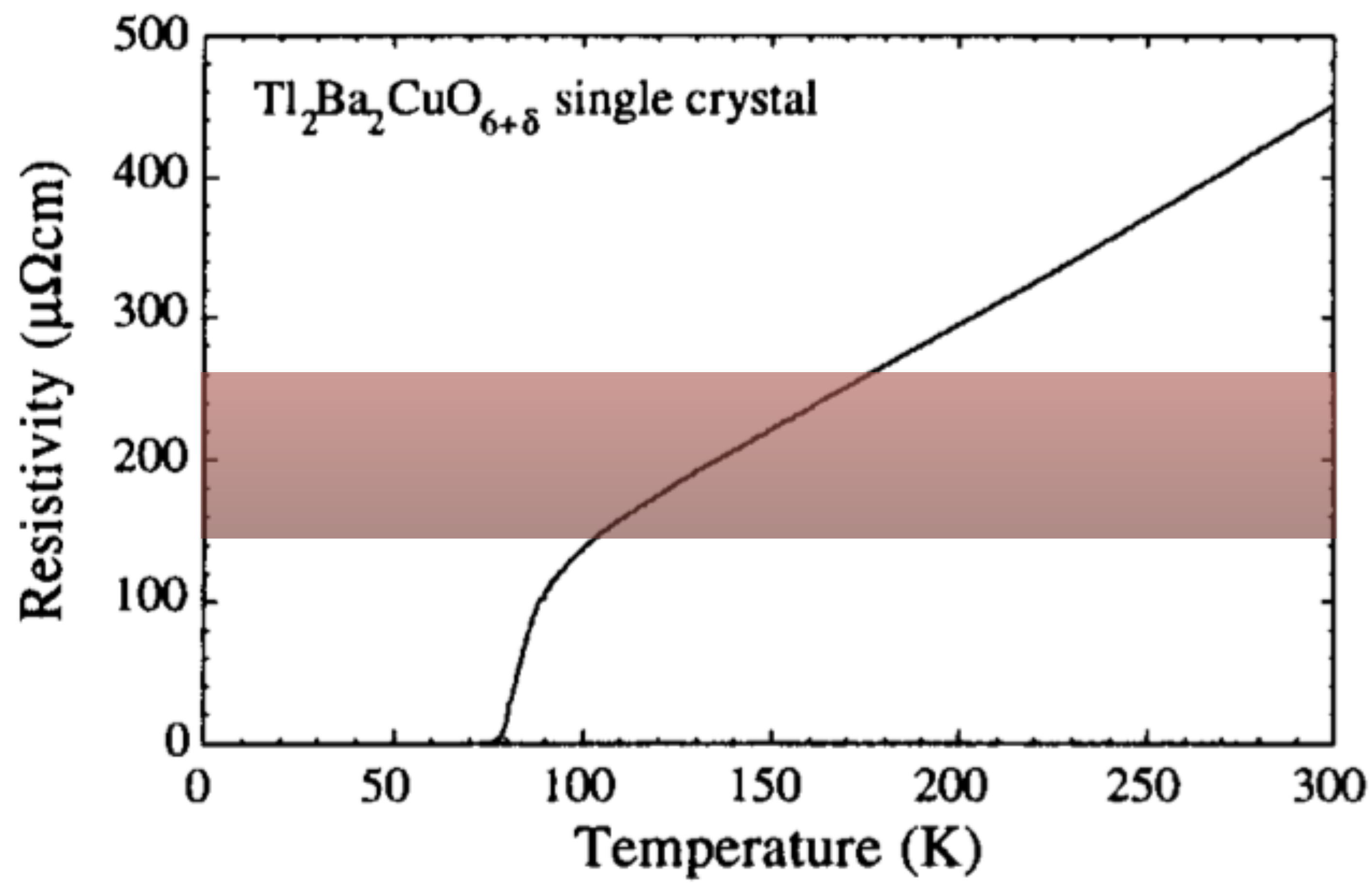
Conventional metals obey the MIR bound

- MIR bound at work [Gunnarsson et al.]





Violation of MIR bound in e.g. high-Tc



[Tyler-Mackenzie ’97]

Bad metals

- A simple interpretation is that such "bad metals" are <u>not</u> described in terms of quasiparticles, and so the MIR bound does not apply.

[e.g. Emery-Kivelson]

- What plays the role of the quasiparticle lifetime?
- Why doesn't "momentum drag" make the resistivity small?

Transport without quasiparticles?

- I will discuss two results on non-quasiparticle transport.

  (1) Transport is always bounded by the local equilibration time and a ‘Lieb-Robinson’ velocity.

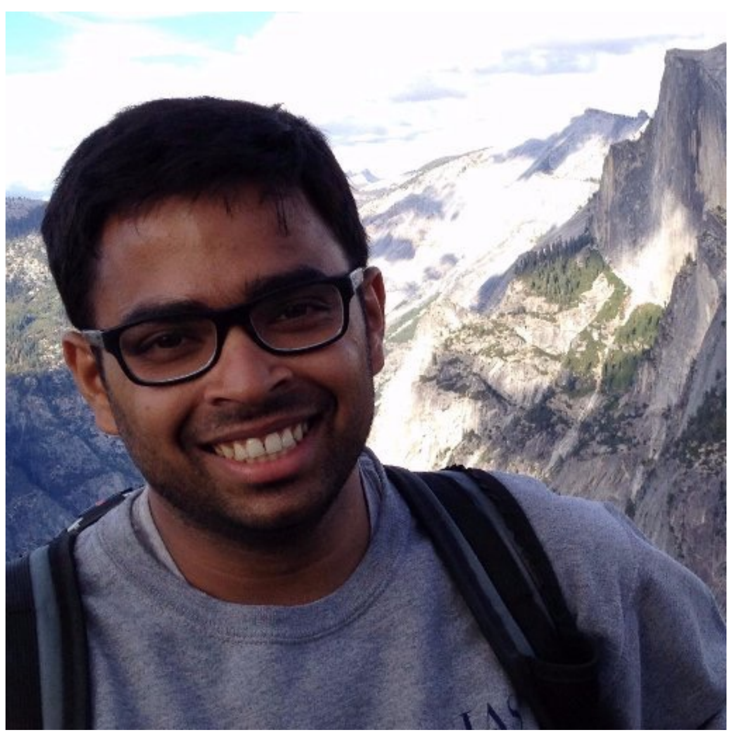
  (2) Phase fluctuating charge density waves remove the momentum zero mode and fit bad metal phenomenology.

  [If I had more time I would also discuss

  (3) ‘Imbalance modes’ due to additional conservation laws force momentum relaxation even with weak disorder.
  see 1704.07384 [cond-mat.str-el] w/ A. Lucas]

An Upper Bound on Transport

- Based on 1706.00019 [hep-th]
- With Raghu Mahajan and Thomas Hartman



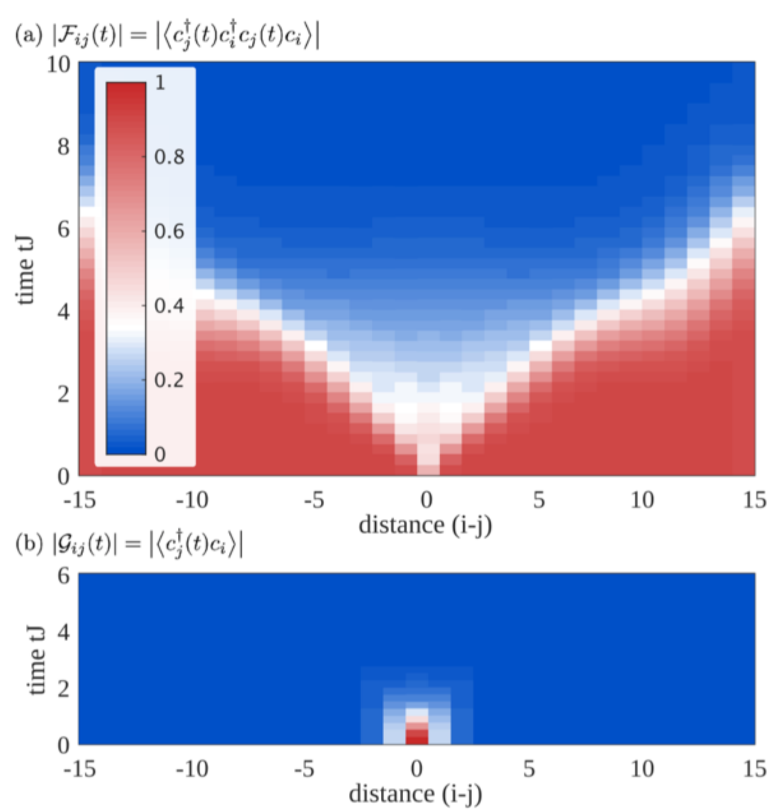
Implications of locality I

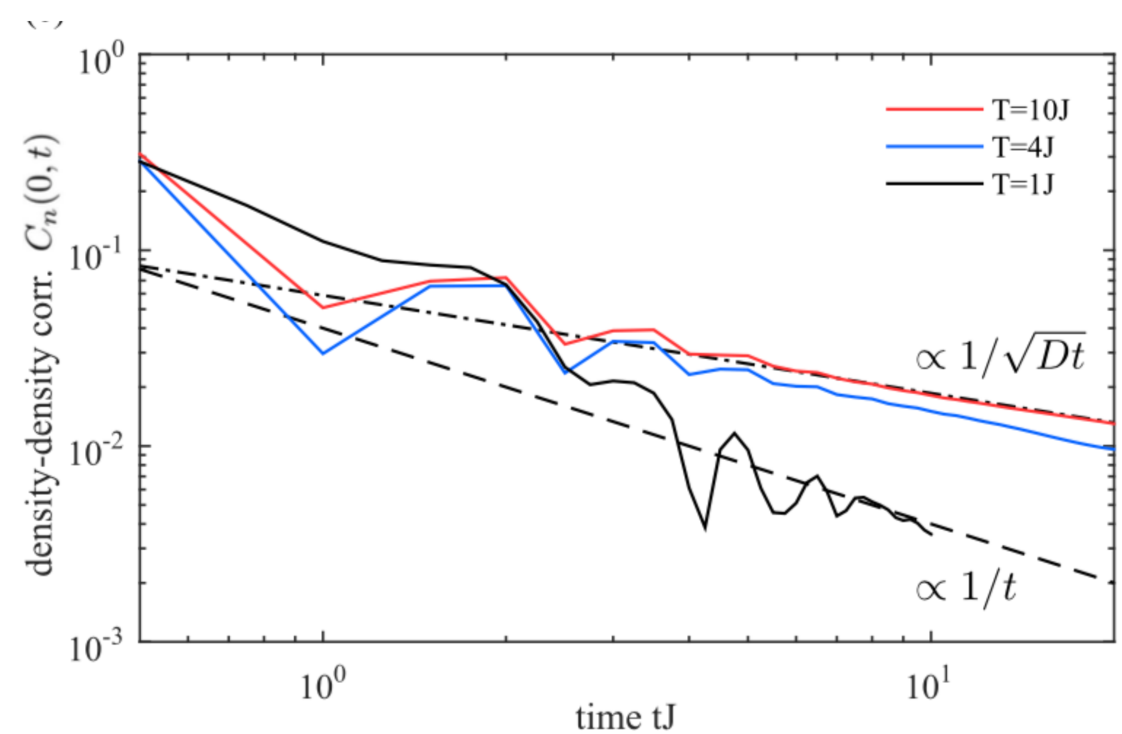
- Even non-relativistic systems have a ‘lightcone’: bounded propagation of signals from locality.

$$||[A(t,x),B(0,0)]|| \lesssim ||A||\,||B||e^{-\mu(|x|-vt)}$$

- The “Lieb-Robinson” velocity: $v \sim \dfrac{J\,a}{\hbar}$

e.g.
Bohrdt et al ’17
[Bose-Hubbard]





Implications of locality II

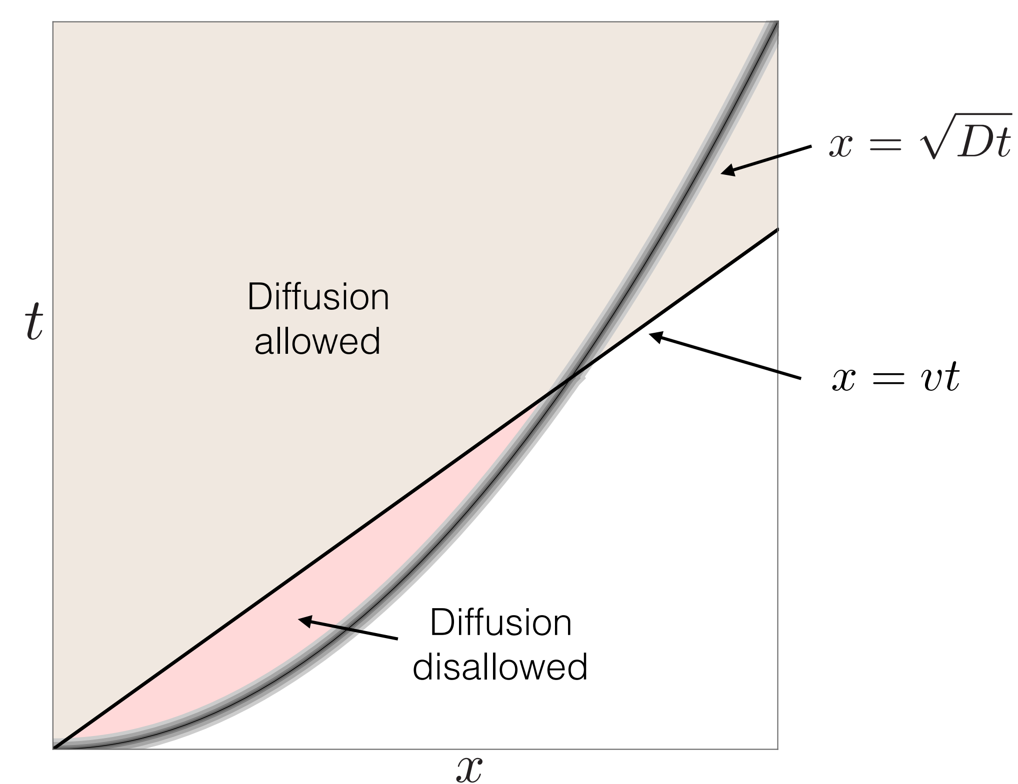
- Conserved densities diffuse (assume no sound modes):

$$\langle [n(t,x), n(0,0)] \rangle \propto \nabla^2 \frac{e^{-x^2/(4Dt)}}{t^{d/2}} \qquad (t \gtrsim \tau_{\text{eq}}, \quad |x| \gtrsim \ell_{\text{eq}}) .$$

- The diffusivity controls transport, e.g.:

$$\sigma = \chi\, D_{\text{charge}} , \qquad \kappa = c\, D_{\text{heat}} , \qquad \eta = \chi_{\pi\pi} D_{\text{momentum}} .$$

- At short times, diffusion is too fast!



Transport bound

- To avoid contradiction with the lightcone, disallowed region must not be diffusive — i.e. must occur before the local equilibration time, so that:

$$\boxed{D \lesssim v^2 \tau_{\mathrm{eq}}}$$

- In a quasiparticle system, $\tau_{\mathrm{eq}} \sim \tau \text{ or } \tau_{\mathrm{tr}}$. The inequality is essentially saturated by the Drude formula. More generally, the inequality relates transport to a relaxation timescale, without assuming the existence of quasiparticles.

Transport bound

- The microscopic velocity v is relevant for low energy dynamics in two important cases. Relativistic systems (v=c) and degenerate fermion systems ($v=v_F$).

- E.g. momentum diffusion in quark-gluon plasma (put c=1):

$$\frac{\eta}{s} \lesssim T\,\tau_{\mathrm{eq}}$$

- This bound goes the other way to the KSS bound…

- Consistent with measurements at RHIC: $0.15\ \hbar/k_B \lesssim 1.1\ \hbar/k_B$

Transport bound

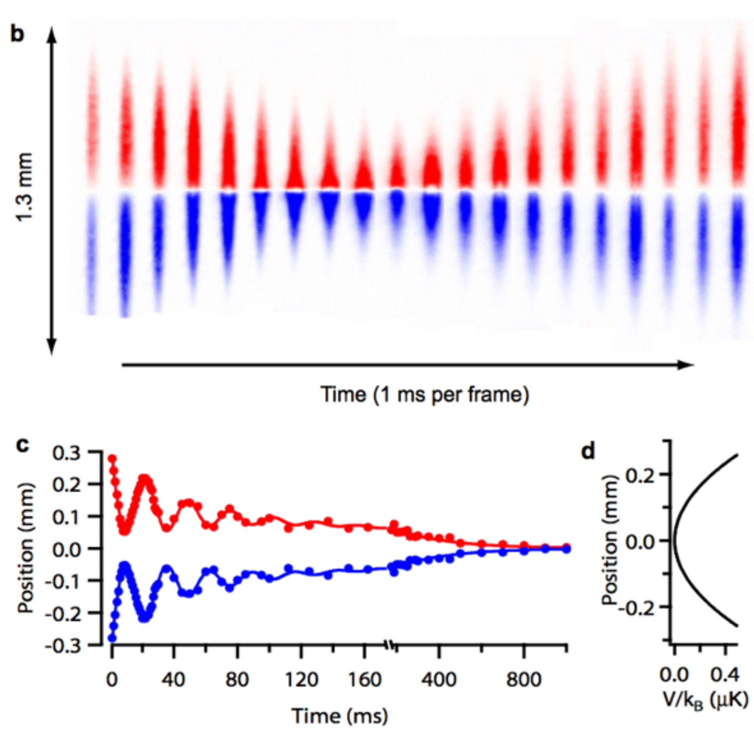
- For degenerate Fermi systems:

$$mD \lesssim E_F\,\tau_{\rm eq} \qquad \Rightarrow \qquad \frac{\eta}{n} \lesssim E_F\,\tau_{\rm eq} \qquad \text{(momentum)}$$

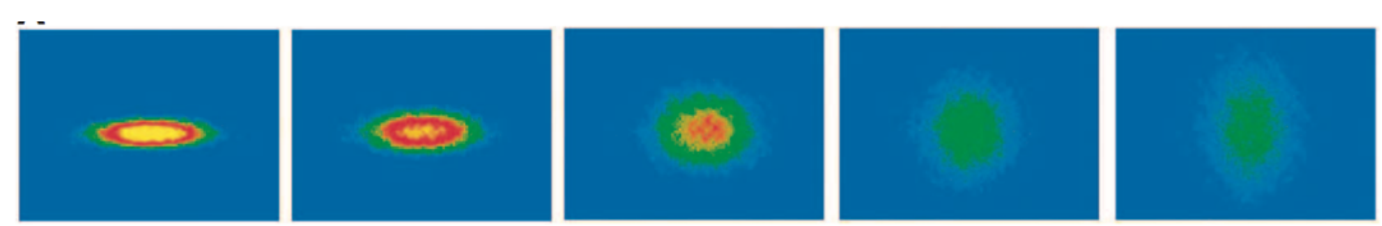
$$\rho \gtrsim \frac{m}{e^2 n}\frac{1}{\tau_{\rm eq}} \qquad \text{(charge)}$$

- Unitary cold Fermions: spin and momentum diffusion

Sommer et al. ’11

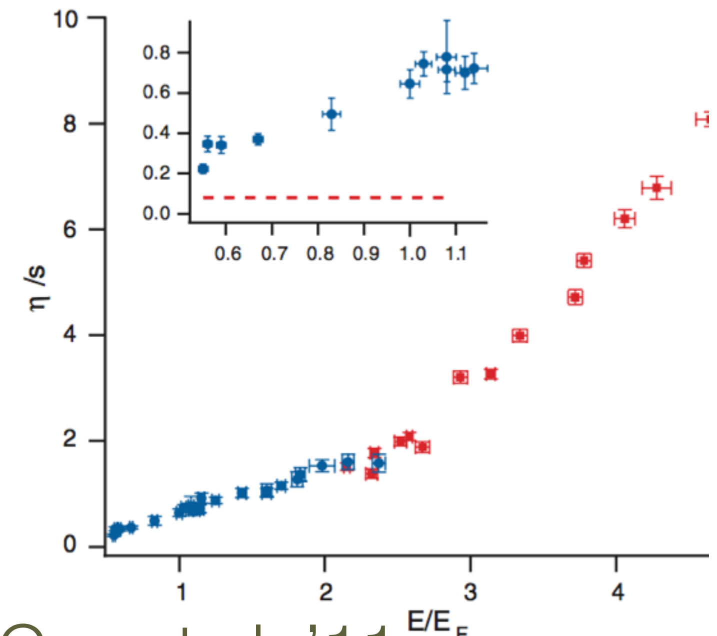


Cao et al. ’11

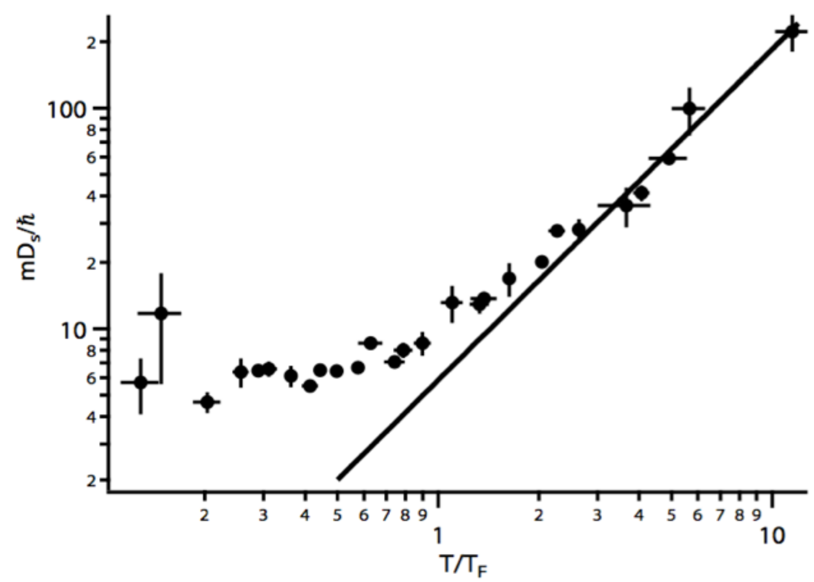


Diffusivities and relaxation rates

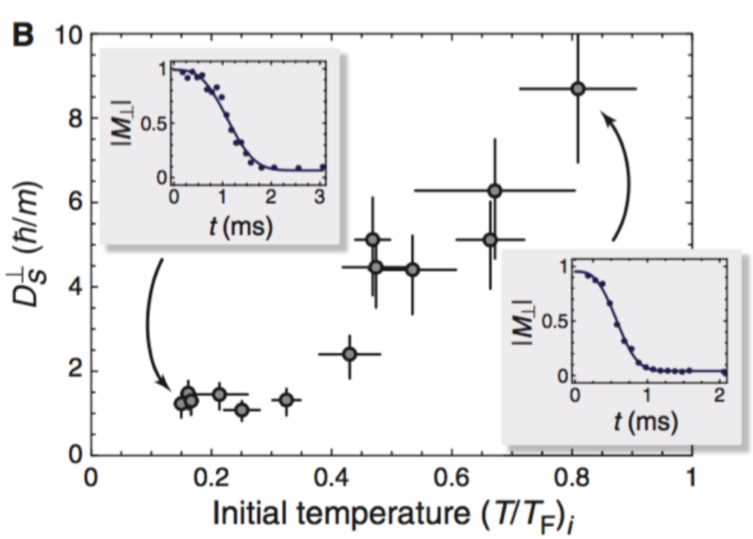
Diffusivities



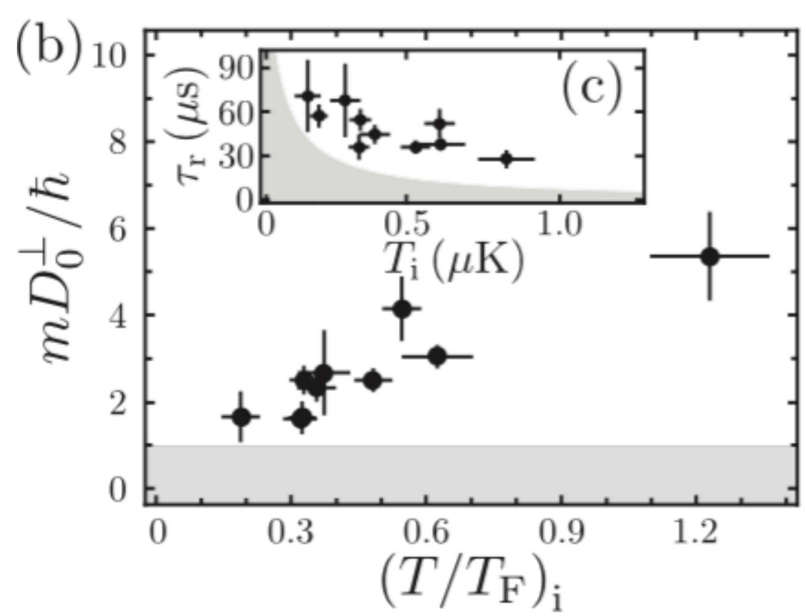
Cao et al. ’11



Sommer et al. ’11

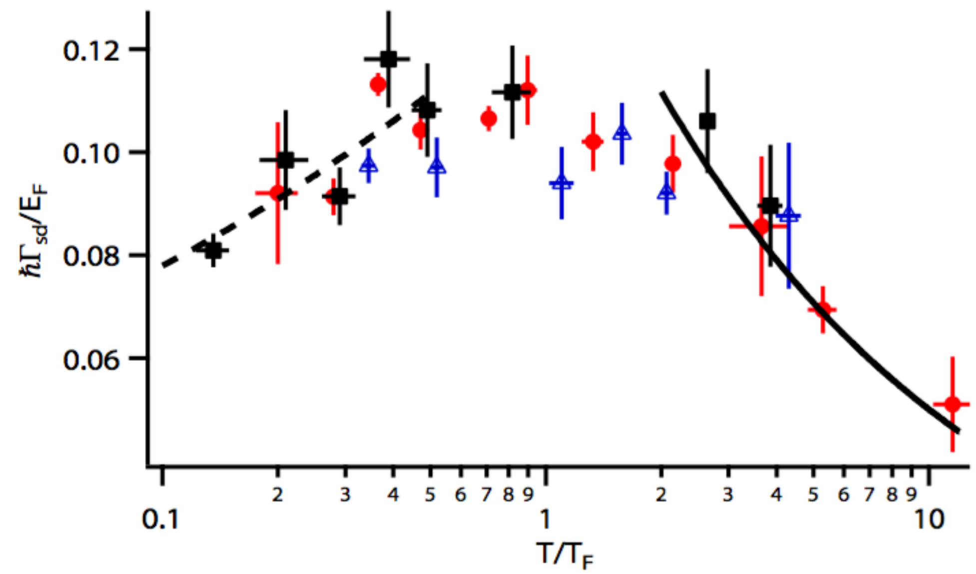


Bardon et al. ’14

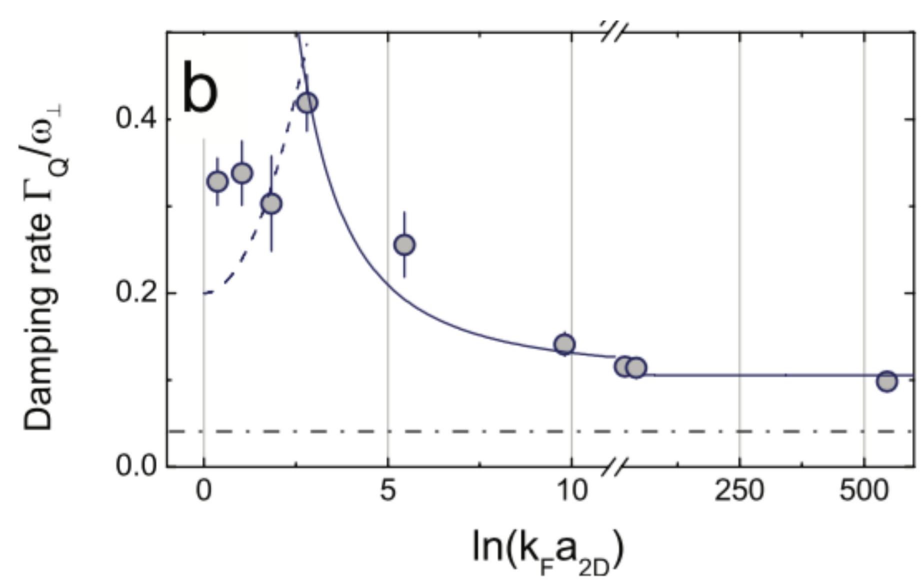


Luciuk et al. ’17

Relaxation rates (?)



Sommer et al. ’11

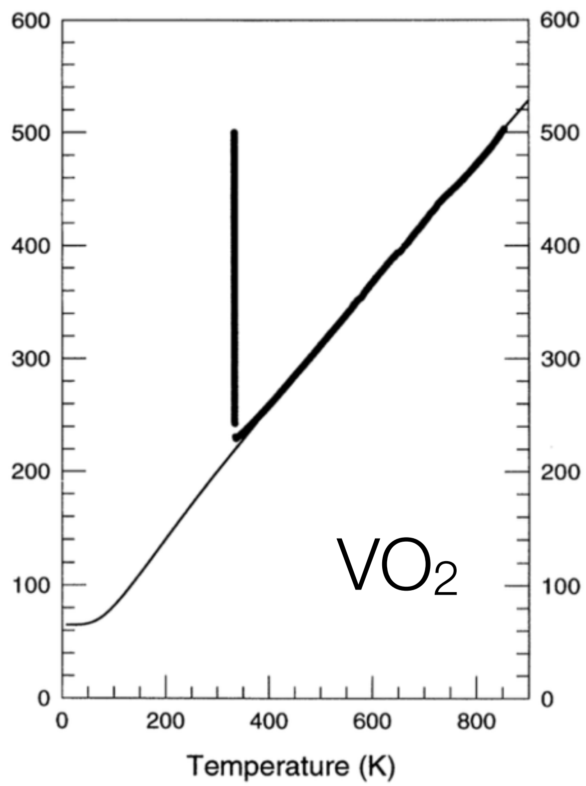


Vogt et al. ’12

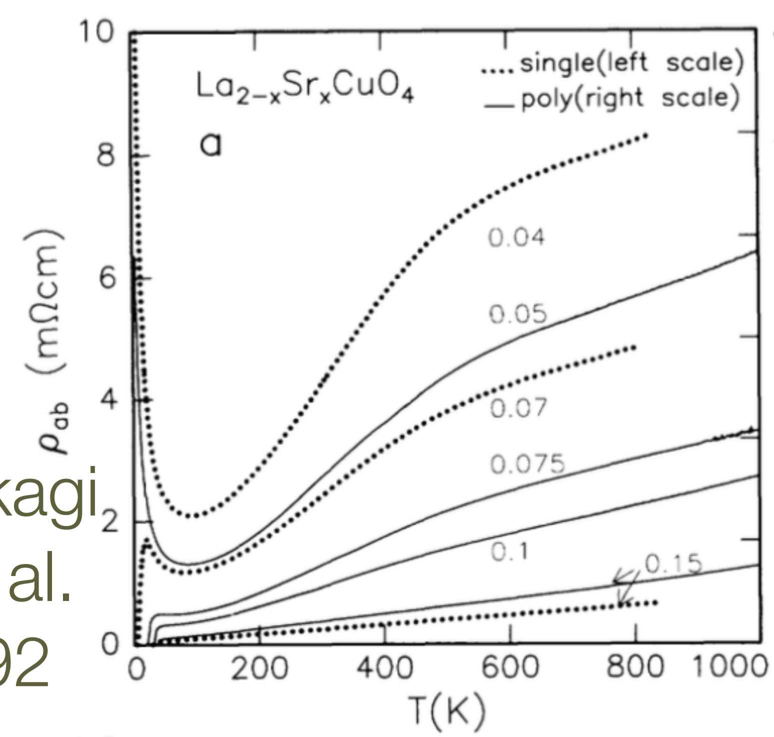
Bad metal resistivity

- Many bad metals show T-linear resistivity:

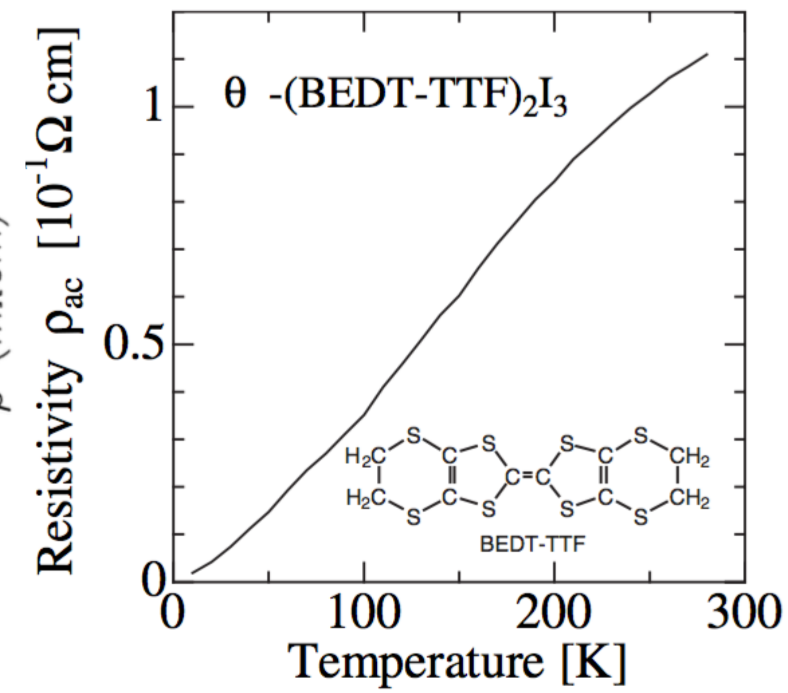
Takenaka et al. ’05

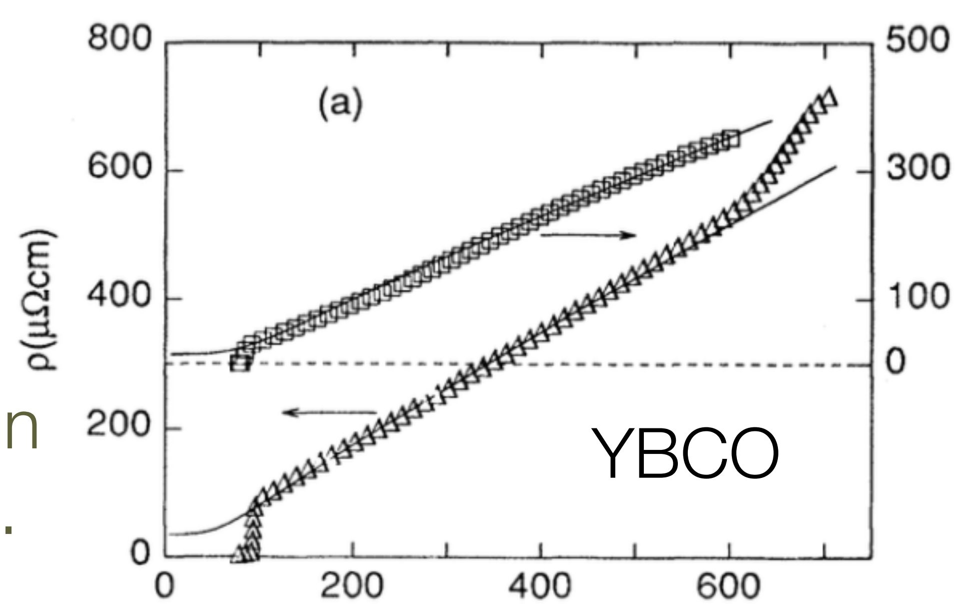


Allen et al. ’93

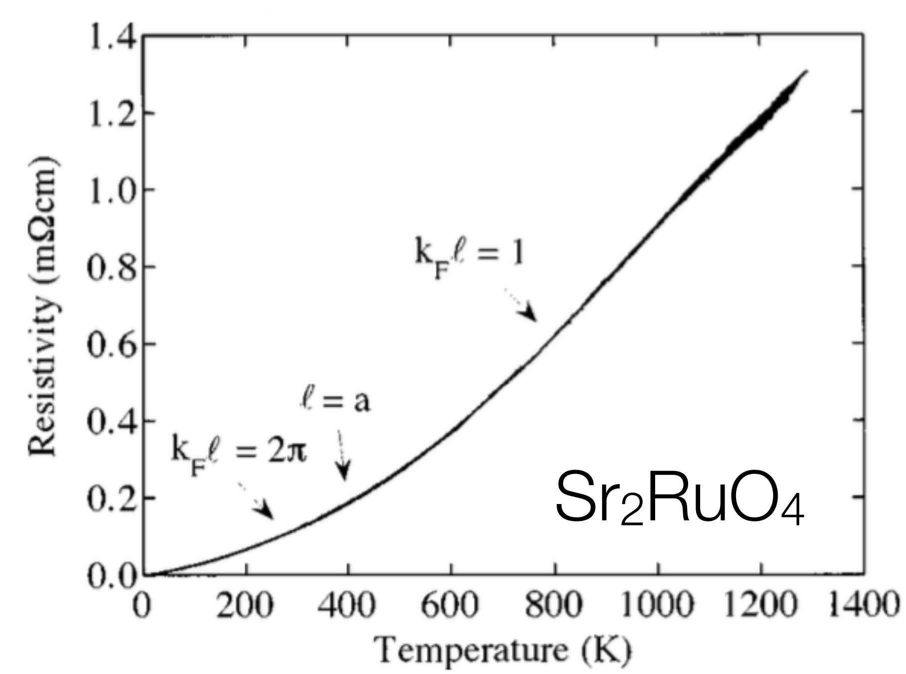


Takagi et al. ’92





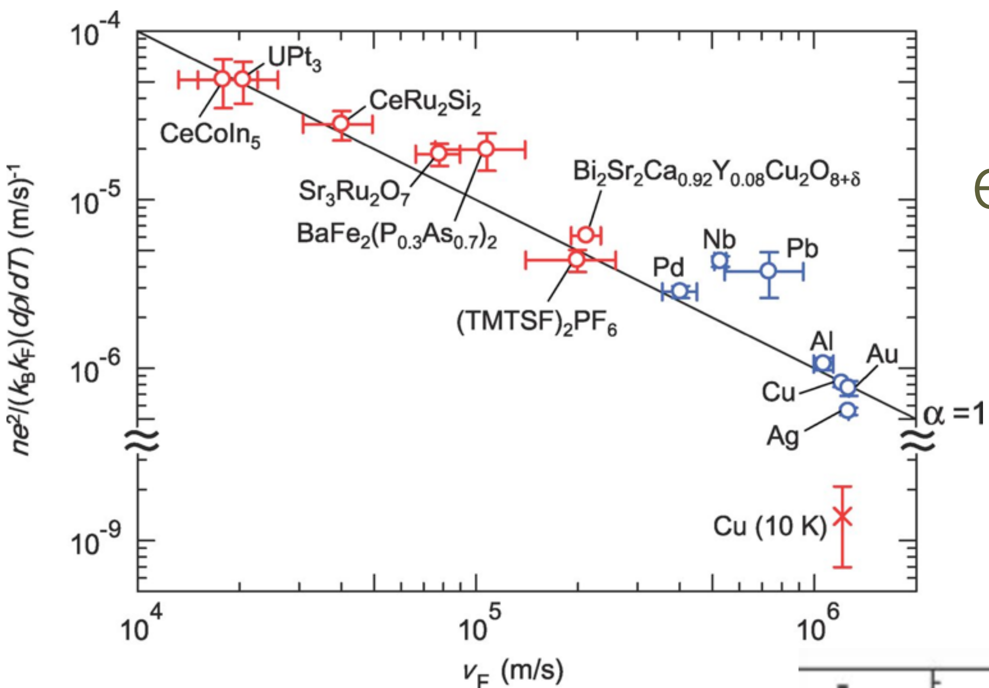
Martin et al. ’88



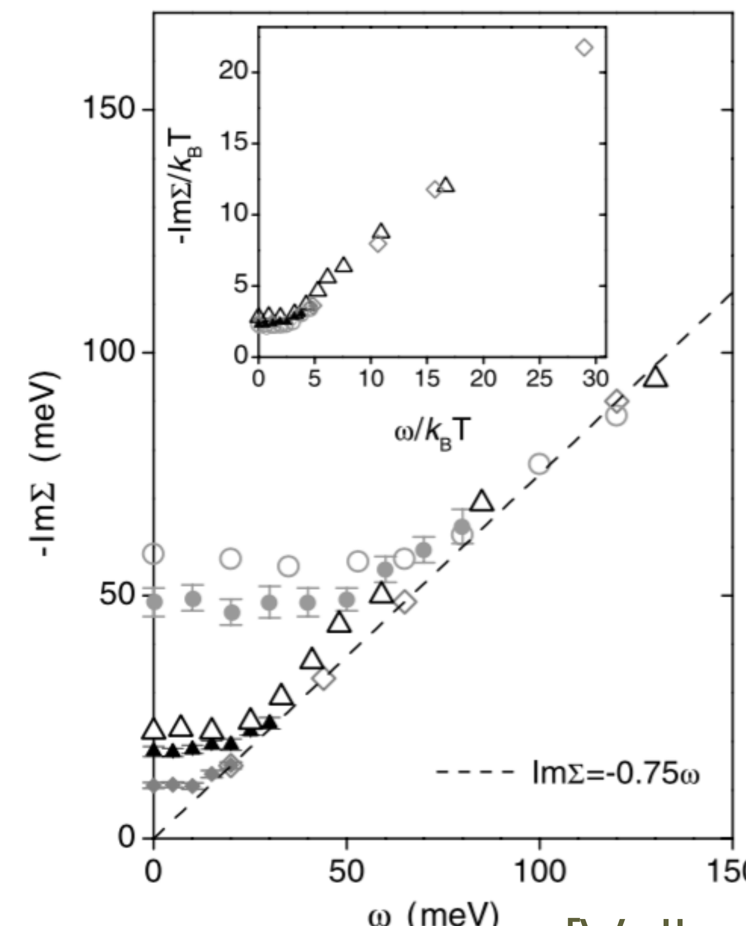
Tyler et al. ’98

Relaxation rates

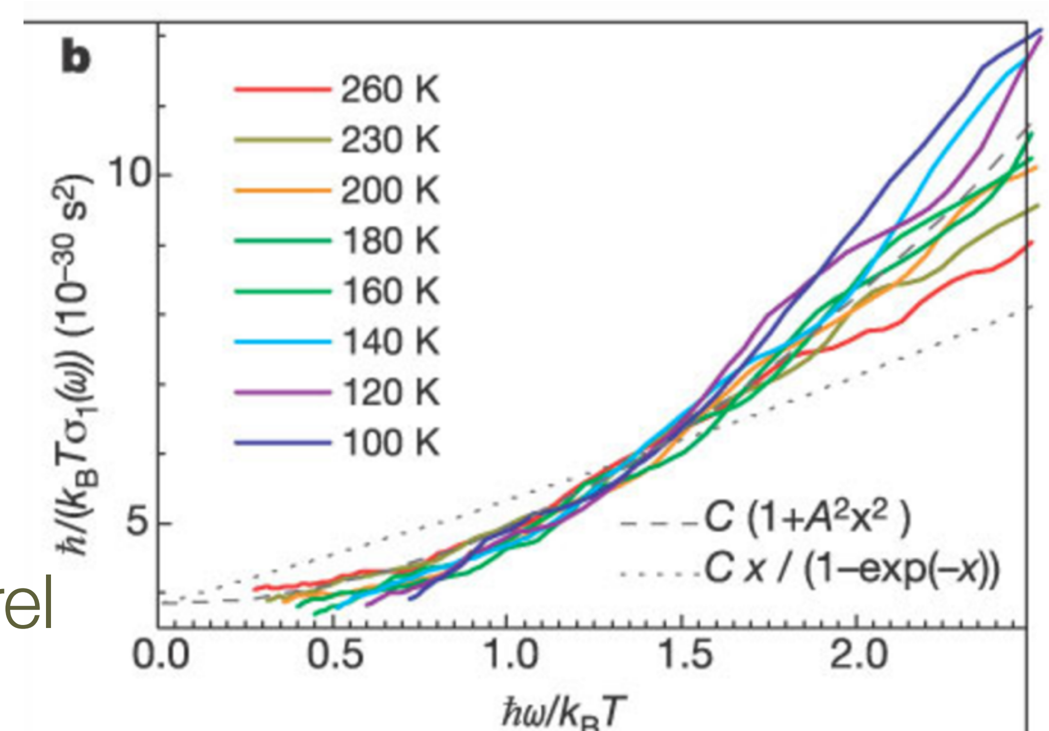
- And this T-linear resistivity is indeed due to $\tau_{\mathrm{eq}} \sim \hbar/(k_B T)$



[Bruin et al '13]



[Valla et al '99]

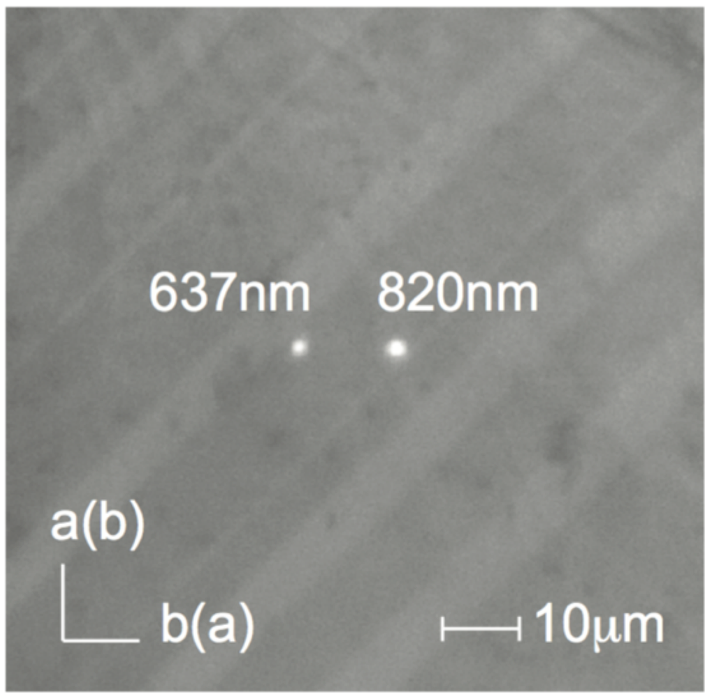


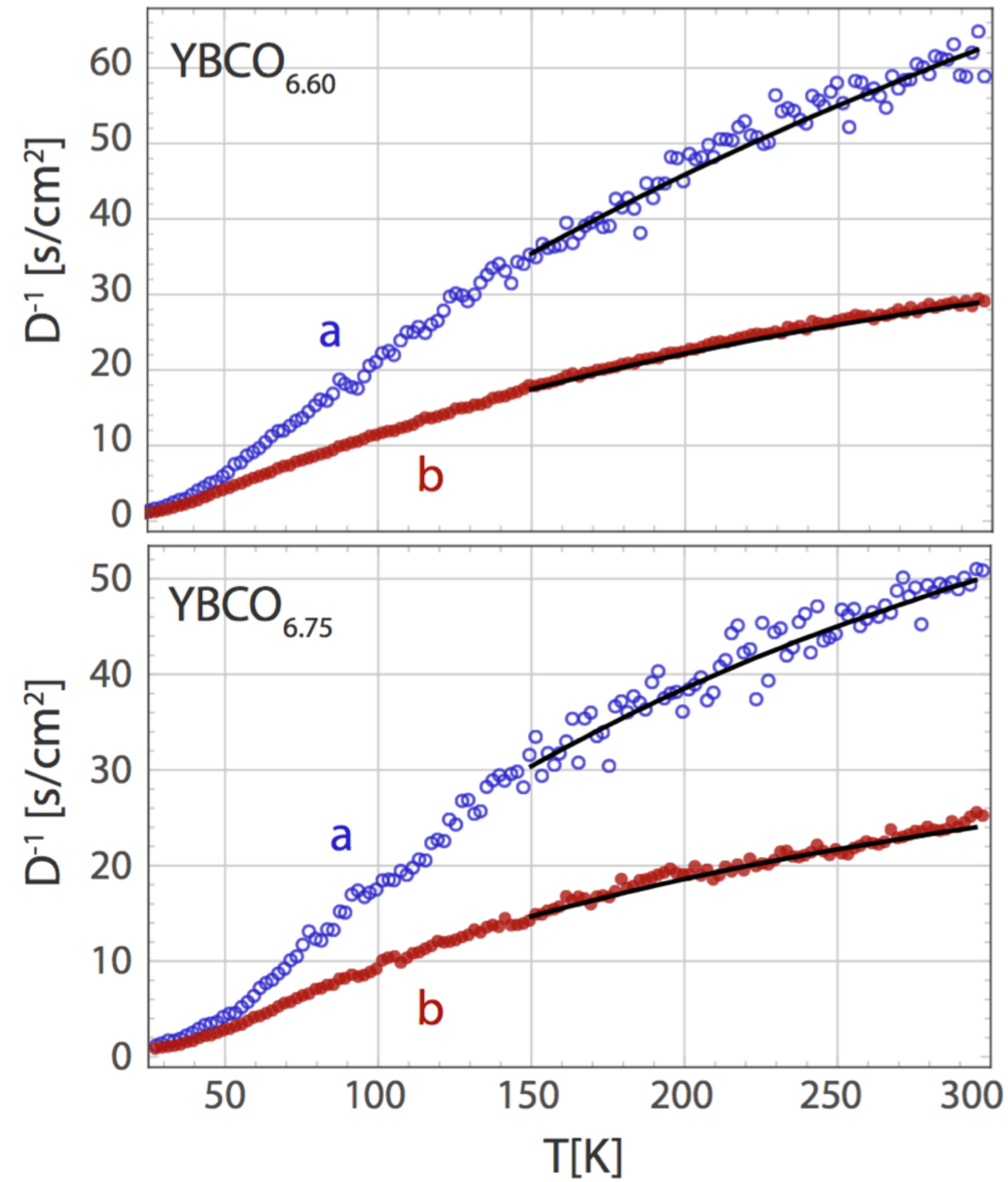
[van der Marel et al '03]

Transport bound

- The existence of the $\tau_{\text{eq}} \sim \hbar/(k_B T)$ timescale per se is not the most mysterious aspect of these materials. E.g. quantum criticality is plausibly responsible.
- What was lacking was a non-quasiparticle way to translate this into a resistivity and also explain why the resistivity is large.
- The bound achieves both of these things.
- However, some of these materials are quite clean, need to explain why a long-lived momentum does make the resistivity small with $\tau_{\text{tr}} \gg \tau$.

Thermal diffusivity in YBCO





- There are many more phonons than electrons: $c_{ph} >> c_{el}$.

- But the electrons are much faster: $v_F >> v_S$.

J.-C. Zhang, E.M. Levenson-Falk, B.J. Ramshaw, D.A. Bonn, R. Liang, W.N. Hardy, S.A. Hartnoll, A. Kapitulnik. '16

Thermal diffusivity in YBCO

- Simultaneously ‘electronic’ and ‘phononic’ character suggests a picture of an electron-phonon soup in which strong electron-phonon scattering renders both electrons and phonons ill-defined as single-particle excitations.
- Motivated by this, fit diffusivity to

$$D_{\mathrm{heat}} \sim v_B^2 \frac{\hbar}{k_B T} \,,$$

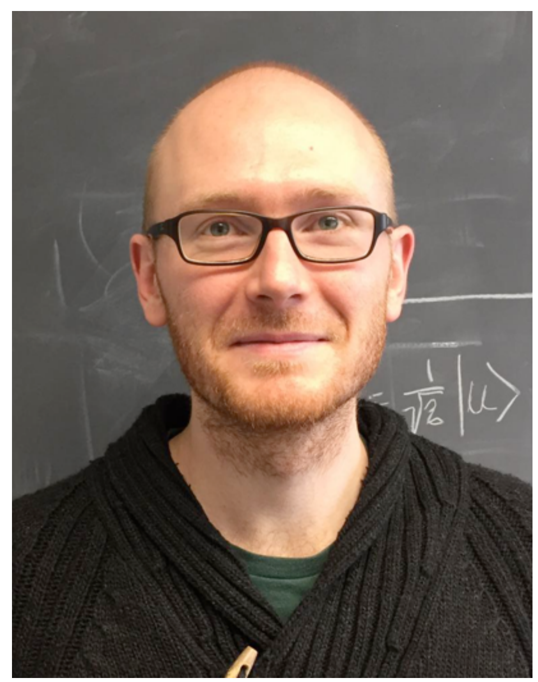
- Where:

$$v_B^2 = \alpha \, \frac{c_{\mathrm{el}}}{c} v_F^2 + \beta \, \frac{c_{\mathrm{ph}}}{c} v_s^2$$

- Excellent fit with α and β order one numbers.

Bad Metals from Fluctuating Density Waves

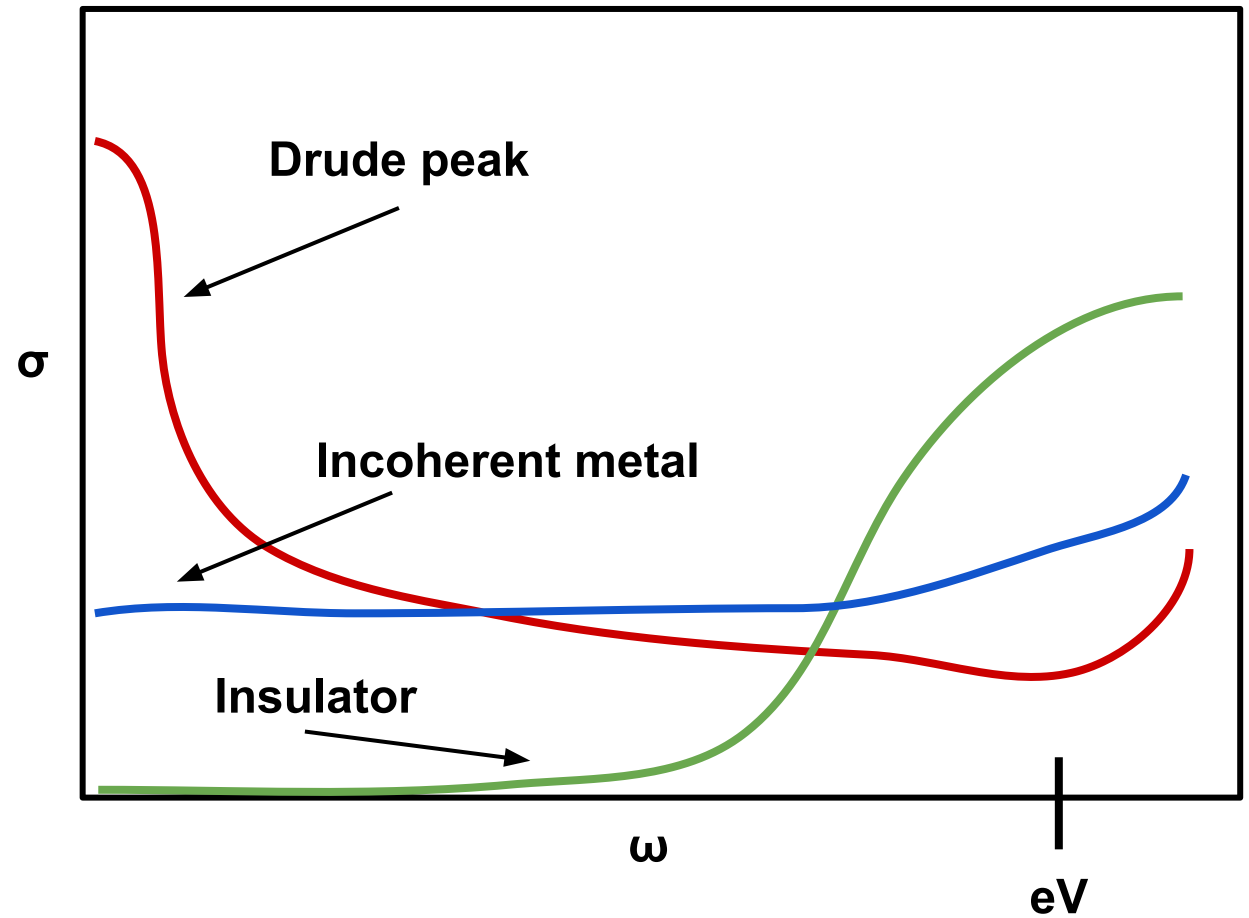
- Based on 1612.04381 [cond-mat.str-el]
- With Luca Delacrétaz, Blaise Goutéraux, Anna Karlsson



Long-lived momentum

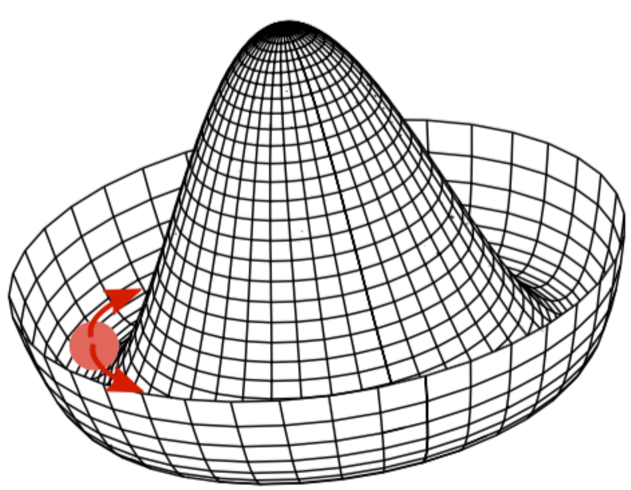
- A long-lived momentum, due to weak translation symmetry breaking, gives a sharp peak and big σ_{dc}.

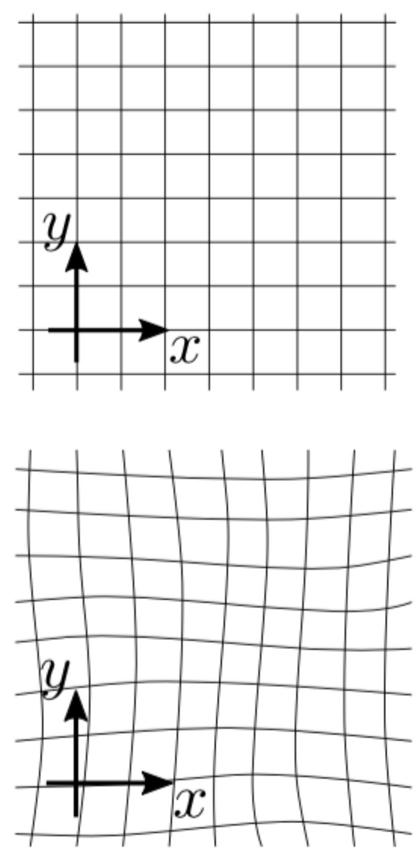
- To get a small σ_{dc}., can relax momentum strongly, but typically get insulators when you do that.

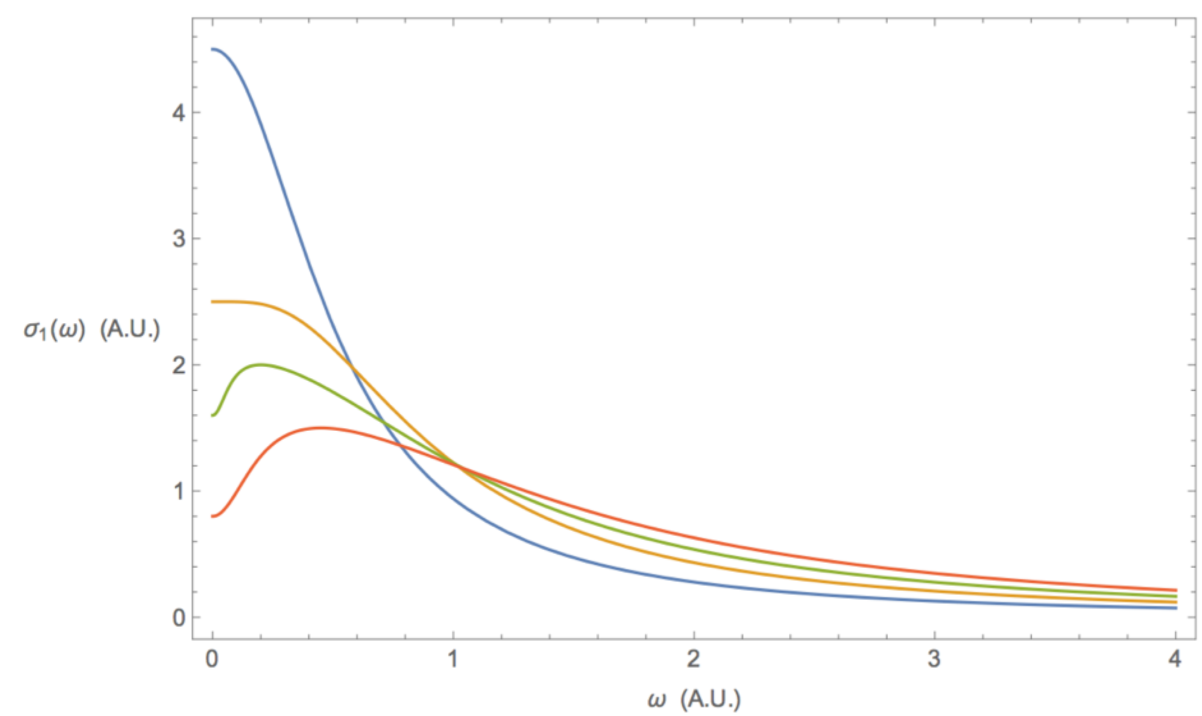


Pseudo-Goldstone bosons

- To gap the momentum mode, need to spontaneously break translations, in addition to weak explicit breaking (no other way — Pseudo-Goldstone boson!).
- If the pinning of the charge density wave is strong enough, peak moves off the axis.

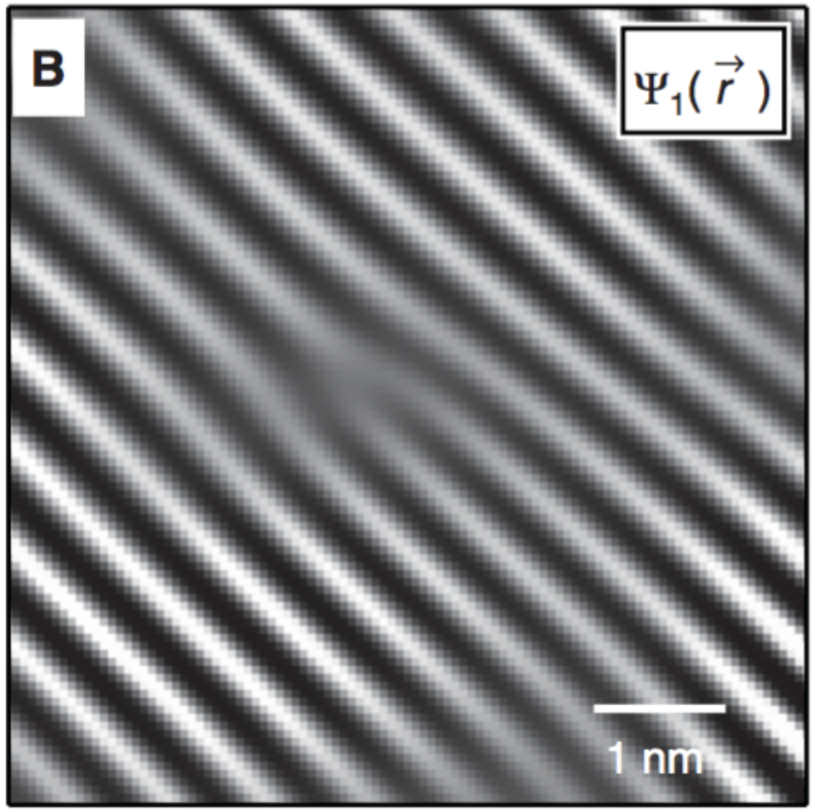






Proliferating dislocations

- Pinned charge density waves are insulators [Lee et al ’74].
- To get a bad metal crucial to allow phase-disordering due to proliferating dislocations. This can be incorporated into the long-wavelength hydrodynamic description.



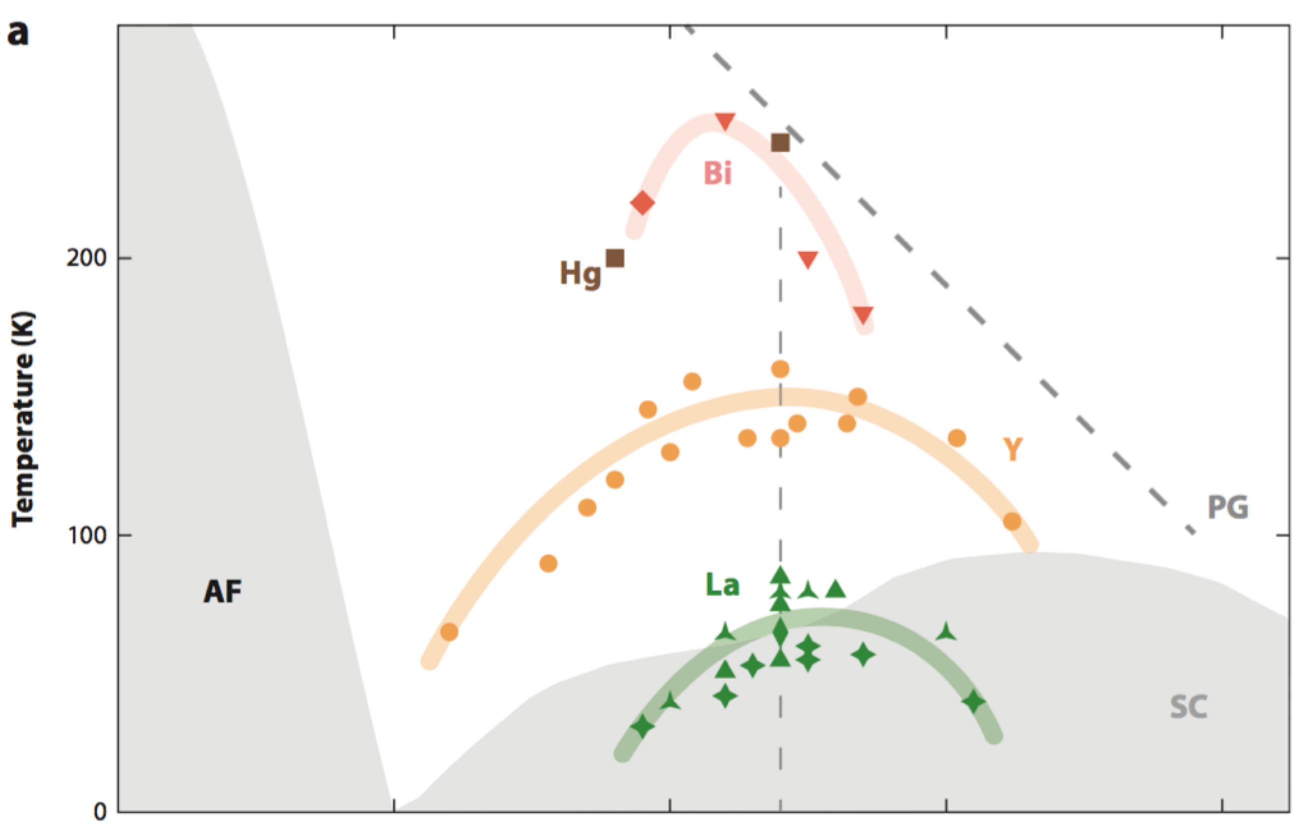
Mesaros et al. ’11

$$\sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$

$$\sigma_{\mathrm{dc}} = \sigma_o + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega}{\Omega\Gamma + \omega_o^2}$$

Charge density wave order in bad metals

- Static charge density wave order present in underdoped cuprates. Phase-disordered density waves are plausibly important across the phase diagram.



Comin et al. ’16

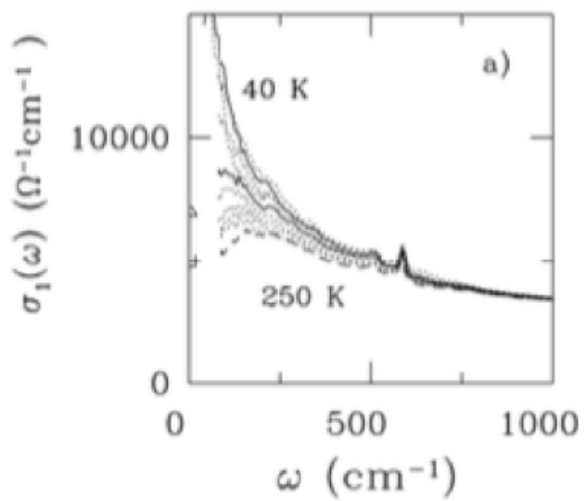
- Weight of the peak in the optical conductivity is set by the Drude weight, not the density wave stiffness.

- Ideal observable for detecting fluctuating order.

Peaks in bad metals I

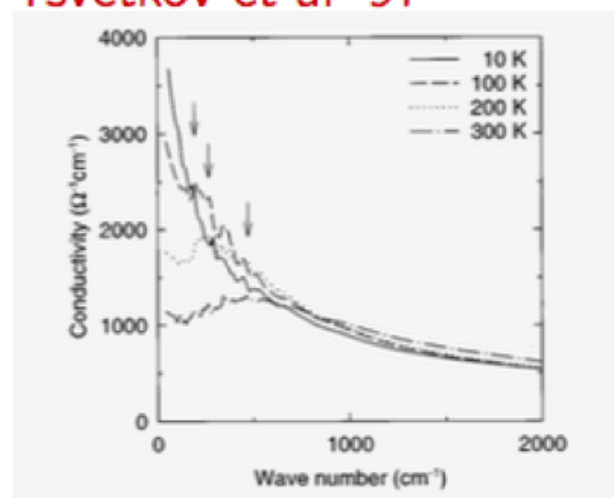
$SrRuO_3$

Kostic et al '98



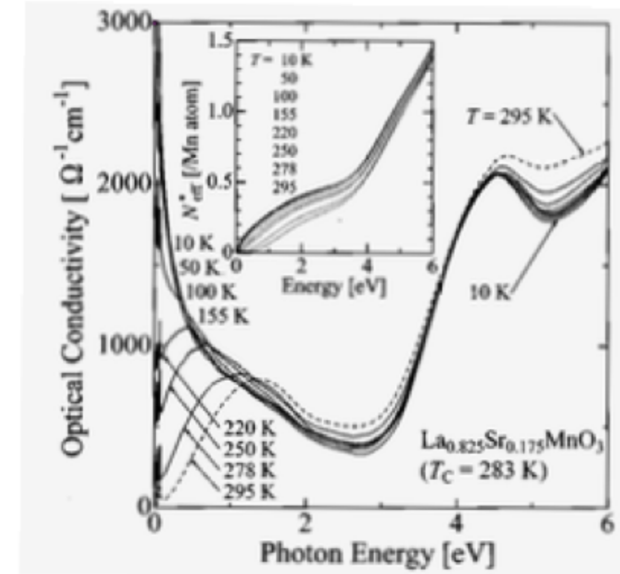
$Bi_2Sr_2CuO_6$

Tsvetkov et al '97



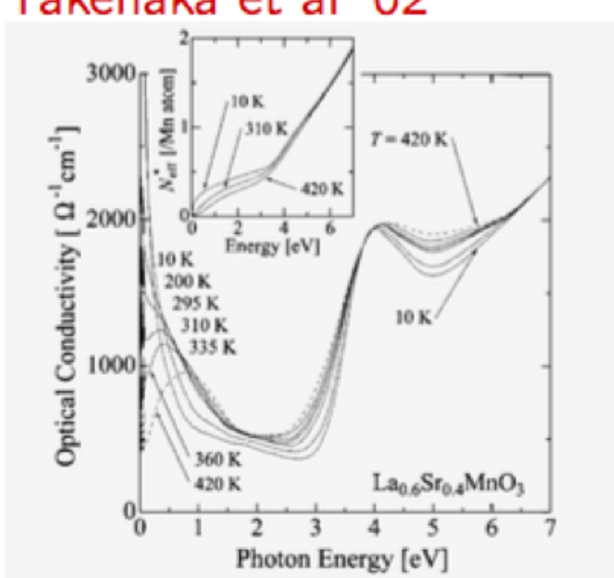
$La_{0.825}Sr_{0.175}MnO_3$

Takenaka et al '99



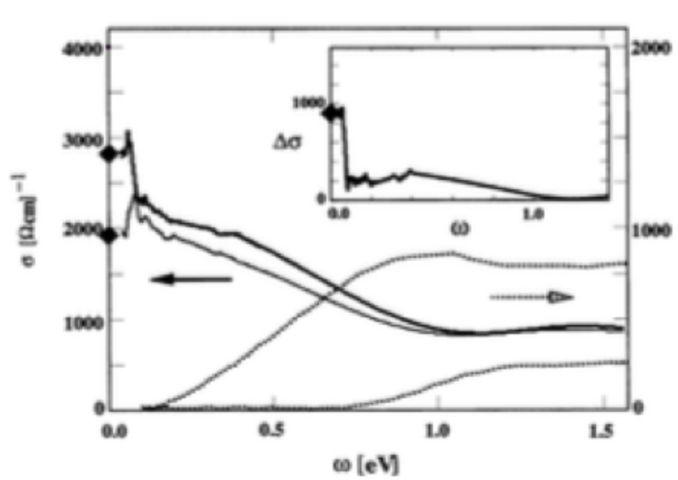
$La_{0.6}Sr_{0.4}MnO_3$

Takenaka et al '02



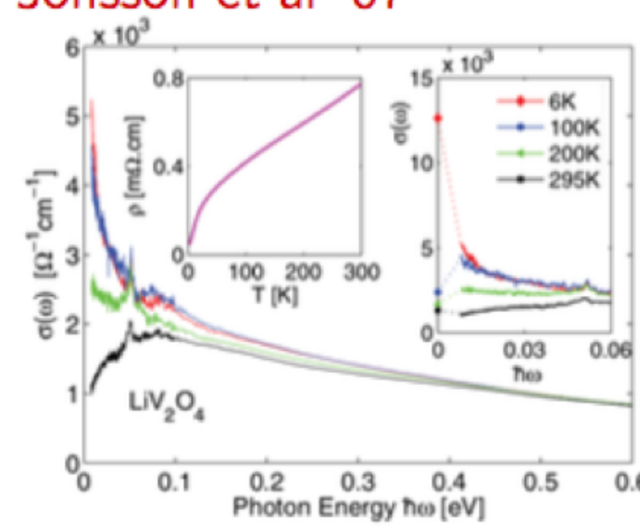
V_2O_3

Rozenberg et al '95



LiV_2O_4

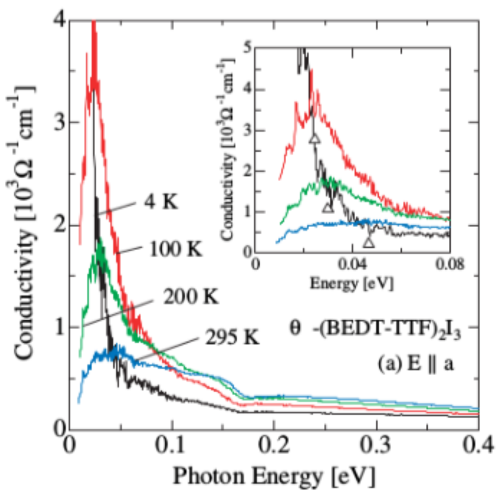
Jönsson et al '07



Peaks in bad metals II

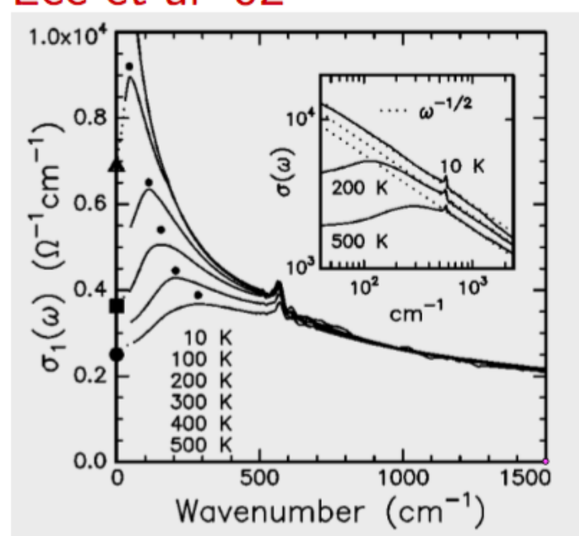
θ-(BEDT-TTF)$_2$I$_3$

Takenaka et al '05



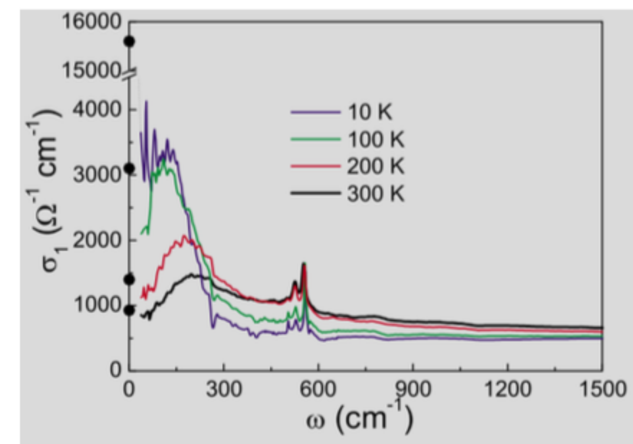
Ca_2RuO_3

Lee et al '02



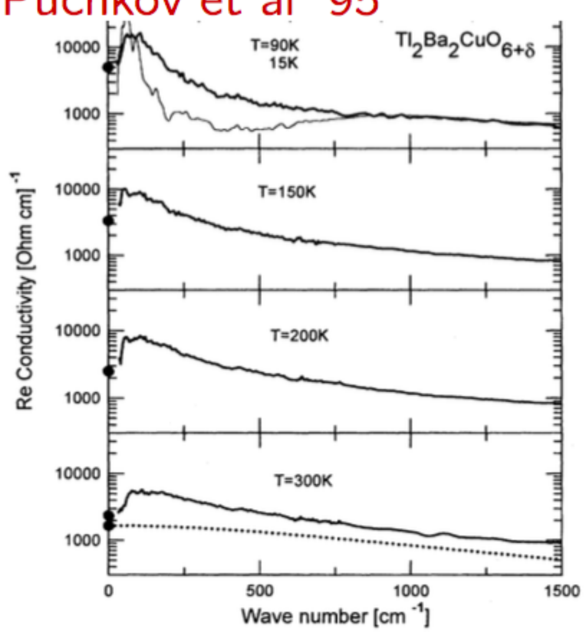
$Na_{0.7}CoO_2$

Wang et al '04



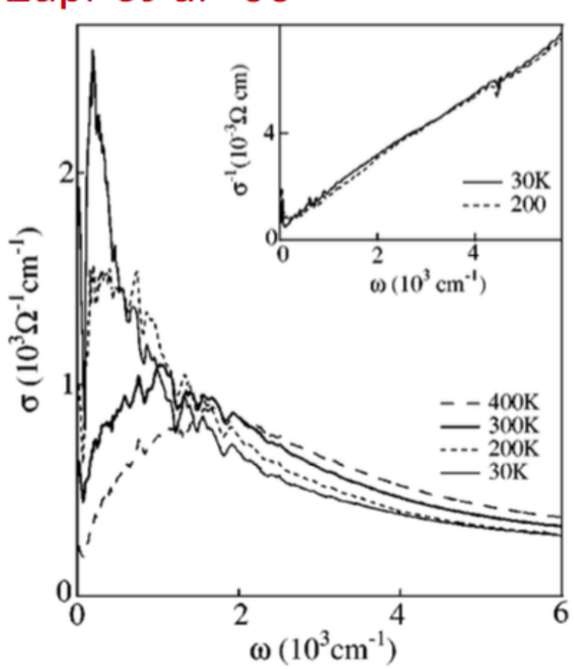
$Ti_2Ba_2CuO_{6+\delta}$

Puchkov et al '95



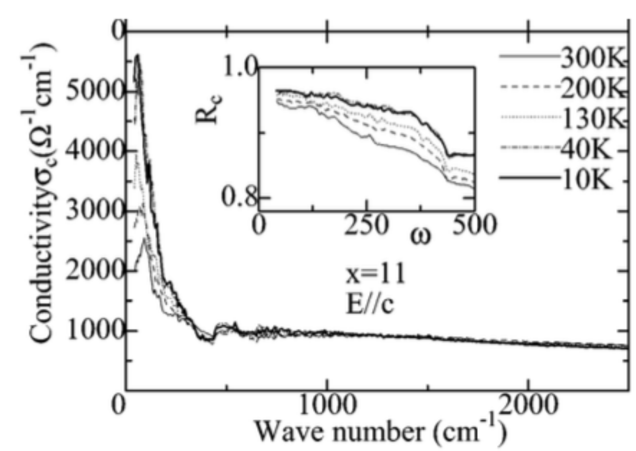
$Bi_2Sr_2CuO_6$

Lupi et al '00

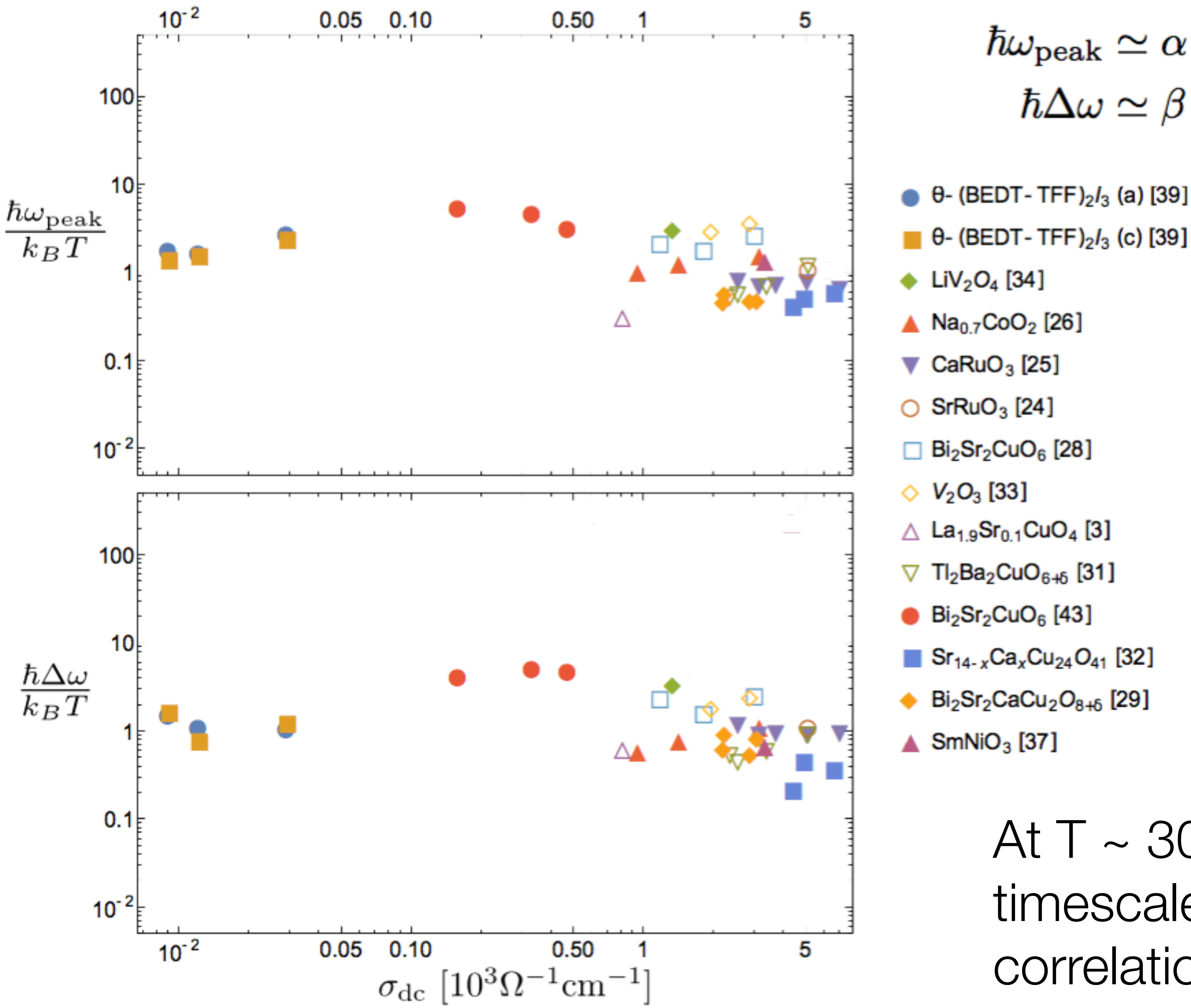


$Sr_{14-x}Ca_xCu_{24}O_{41}$

Osafune et al '99



Peaks in bad metals III



$$\hbar\omega_{\text{peak}} \simeq \alpha \cdot k_B T$$

$$\hbar\Delta\omega \simeq \beta \cdot k_B T$$

At T ~ 300K,
timescale ~ 25 fs
correlation length ~ 2.5 nm

Resistivity and diffusion

- The late time, long wavelength diffusive processes are controlled by the timescale $\Omega/(\omega_0)^2 \sim \hbar/(k_B T)$ rather than the slow momentum relaxation time $1/\Gamma$.

- In this way, the T-linear resistivity can be understood as coming from a diffusion bound, despite the presence of slow momentum relaxation.

- Upshot: bad metallic T-linear resististivity can arise due to quantum critical fluctuating density waves, even in clean systems.

Summary

- Bad metals $\Rightarrow$ need handle on non-quasiparticle transport.
- Obtained a bound on diffusion in terms of the lightcone velocity and local equilibration time.
- This bound holds in quark-gluon plasma, cold fermions at unitary and in bad metals.
- Phase-disordered charge-density waves are one mechanism for removing the effects of long-lived momentum from the dc resistivity.