The Particle Energy Losses on Polarization Radiation

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Motivation

- Beam instrumentation and beam dynamic purposes both for high-energy leptons and hadrons.
- Novel acceleration techniques using dielectric material.
- Sources of electromagnetic radiation.
- Taking into account the real experimental conditions.

Mathematical Description of PR

$$i\left[\mathbf{k}\times\mathbf{H}^{pol}(\mathbf{k},\omega)\right] = \frac{4\pi}{c}\mathbf{j}^{pol}(\mathbf{k},\omega) - i\frac{\omega}{c}\mathbf{E}^{pol}(\mathbf{k},\omega),$$

$$\left[\mathbf{k}\times\mathbf{E}^{pol}(\mathbf{k},\omega)\right] = \frac{\omega}{c}\mathbf{H}^{pol}(\mathbf{k},\omega),$$

$$i\left(\mathbf{k}\cdot\mathbf{E}^{pol}(\mathbf{k},\omega)\right) = 4\pi\rho^{pol}(\mathbf{k},\omega),$$

$$\left(\mathbf{k}\cdot\mathbf{H}^{pol}(\mathbf{k},\omega)\right) = 0.$$

Exact solution of Maxwell equations for PR field

$$\sigma(\omega) = \frac{i\omega}{4\pi}(1-\varepsilon(\omega)),$$

$$\mathbf{H}^{pol}(\mathbf{r},\omega) = \nabla\times\frac{1}{c}\int_{V_T}\sigma(\omega)\underline{\mathbf{E}^0(\mathbf{r}',\omega)}\frac{\exp\left(i\sqrt{\varepsilon(\omega)}|\mathbf{r}'-\mathbf{r}|\omega/c\right)}{|\mathbf{r}'-\mathbf{r}|}d^3r'.$$

Fourier transform of the charged particle field

$$\mathbf{E}^0(\mathbf{k},\omega) = \frac{4\pi i}{\omega}\frac{\mathbf{j}^0(\mathbf{k},\omega)\omega^2/c^2 - \mathbf{k}[\mathbf{k}\cdot\mathbf{j}^0(\mathbf{k},\omega)]}{\mathbf{k}^2-\omega^2/c^2}$$

PR energy losses

To calculate the PR energy losses, it is necessary to obtain the magnetic-field strength $\mathbf{H}^{pol}\,(\mathbf{r},\ \omega)$ and then describe this field in the form of two orthogonal components:

$$H_{\parallel}^{pol}(\mathbf{r},\omega)=\sqrt{\left(H_z^{pol}\right)^2+\left(H_x^{pol}\sin\phi+H_y^{pol}\cos\phi\right)^2},$$

$$H_{\perp}^{pol}(\mathbf{r},\omega)=H_x^{pol}\cos\phi-H_y^{pol}\sin\phi\,.$$

Fresnel coefficients

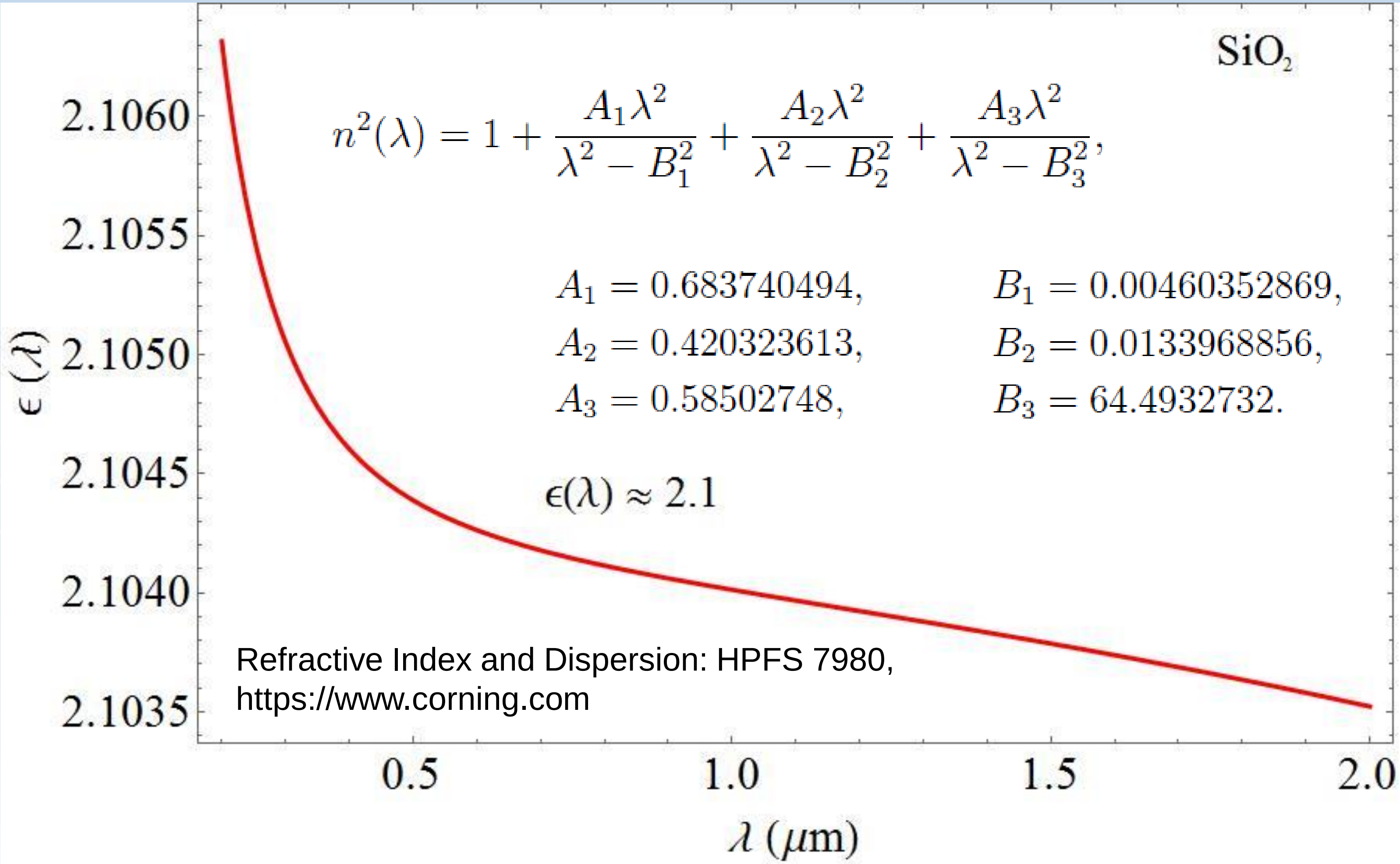
$$\frac{d^2W}{d\omega d\Omega}=\frac{cr^2}{|\varepsilon(\omega)|^2}\left(\left|\sqrt{\varepsilon(\omega)}f_E\right|^2\left|H_{\parallel}^{pol}(\mathbf{r},\omega)\right|^2+\left|f_H\right|^2\left|H_{\perp}^{pol}(\mathbf{r},\omega)\right|^2\right)$$

$$\Delta W=\int_{\Delta\Omega}d\Omega\int_{\Delta\omega}\frac{d^2W}{d\Omega d\omega}d\omega.$$

Energy losses on PR in selected angular $\Delta\Omega$ and frequency $\Delta\omega$ area.

Choice of target material

- Easy to process and production
- Suitable for vacuum
- Inexpensive
- Information about the optical properties



$$n^2(\lambda) = 1 + \frac{A_1\lambda^2}{\lambda^2 - B_1^2} + \frac{A_2\lambda^2}{\lambda^2 - B_2^2} + \frac{A_3\lambda^2}{\lambda^2 - B_3^2},$$

$A_1 = 0.683740494,$ $B_1 = 0.00460352869,$

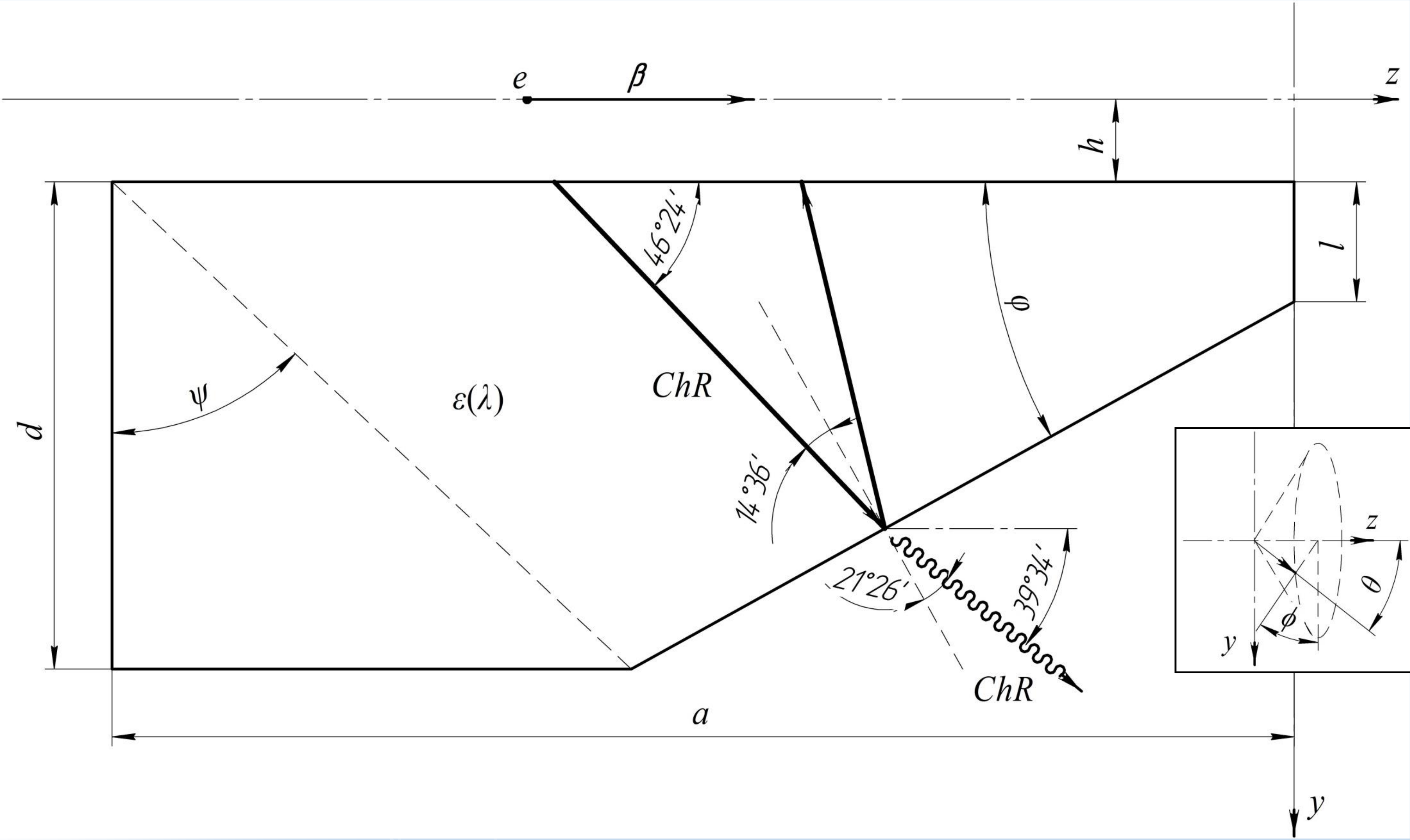
$A_2 = 0.420323613,$ $B_2 = 0.0133968856,$

$A_3 = 0.58502748,$ $B_3 = 64.4932732.$

$\epsilon(\lambda) \approx 2.1$

Refractive Index and Dispersion: HPFS 7980,
https://www.corning.com

ChR from the Complex Dielectric Target



Simulation Parameters

Parameters	Value
Electron energy	2.1 GeV ($\gamma = 4110$)
Target length a	20 mm
Target height d	10.517 mm
Vertex angle of the prism φ	29 deg
Angle of the prism ψ	46.8 deg
Wavelength λ	[0.2; 2] μm
Dielectric permittivity $\varepsilon(\lambda)$	2.1
Impact parameter h	[0.1; 1] mm

T. Lefèvre et al., Proc. IPAC'17, Copenhagen, Denmark, May 2017, paper MOPAB118, pp. 400-403.

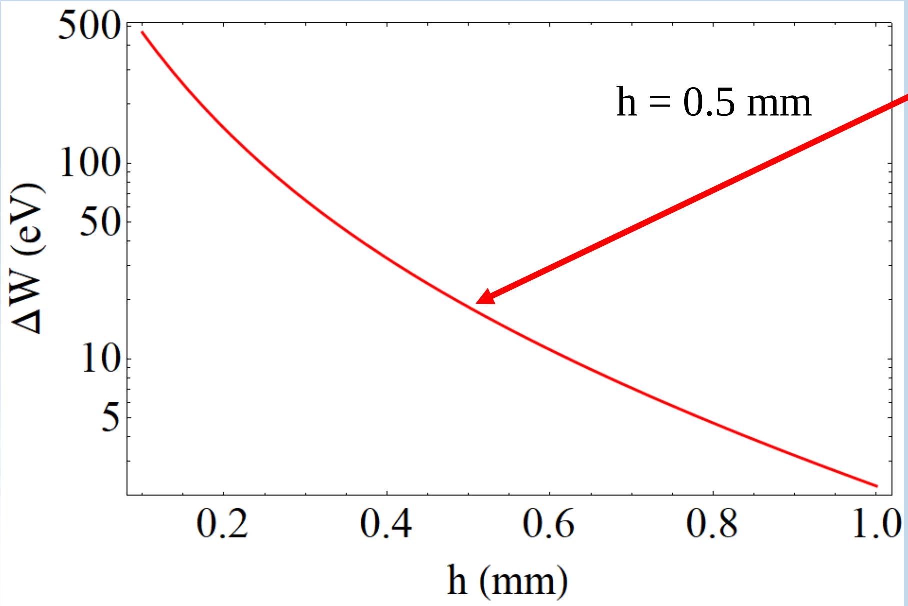
Angular distribution of ChR

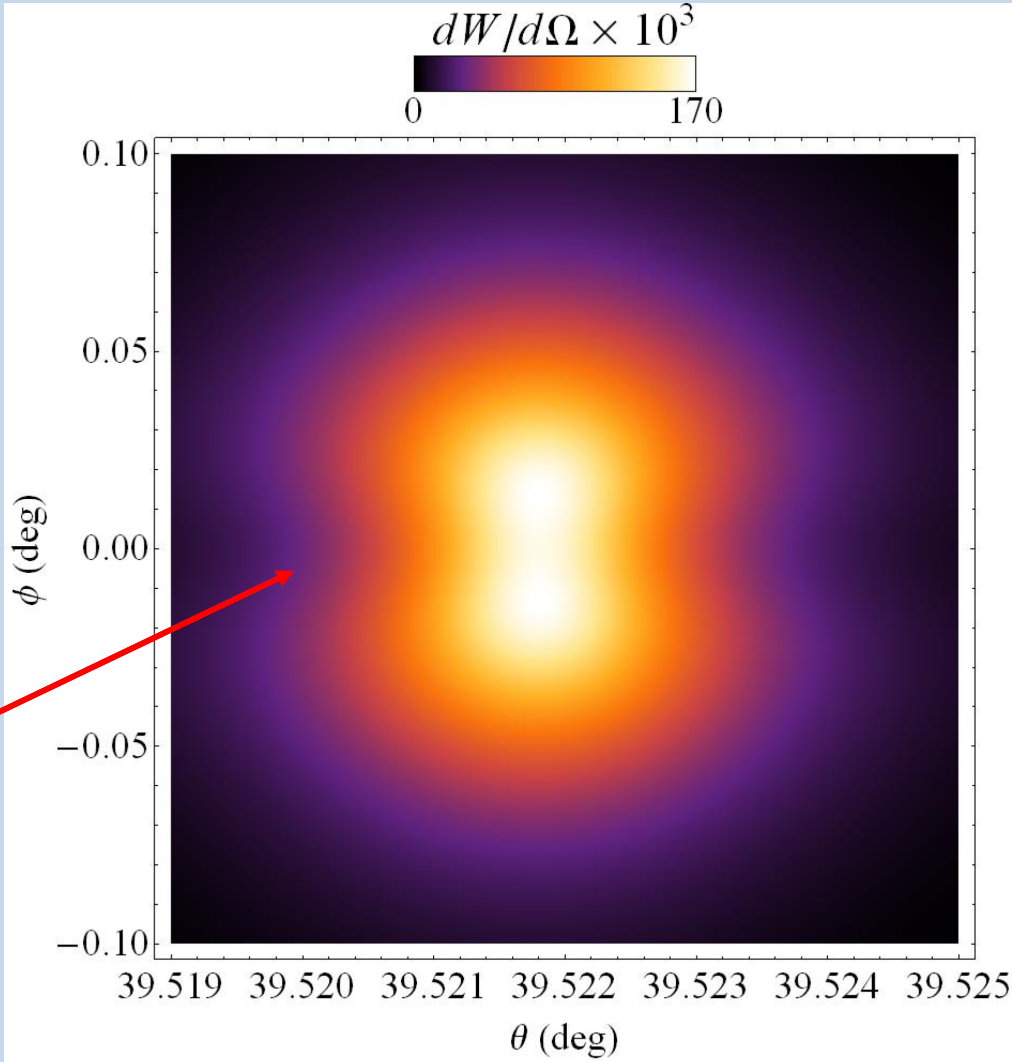
The Tamm-Frank equation:

$$\Delta W = 4\pi^2\alpha \cdot \hbar c \cdot a \cdot \left(1 - \frac{1}{\beta^2 \varepsilon(\lambda)}\right) \cdot \left(\frac{1}{\lambda_{\min}^2} - \frac{1}{\lambda_{\max}^2}\right)$$

For considered parameters ΔW = 14.9 keV.

The energy losses ΔW on Cherenkov radiation according to Polarization Currents Approach



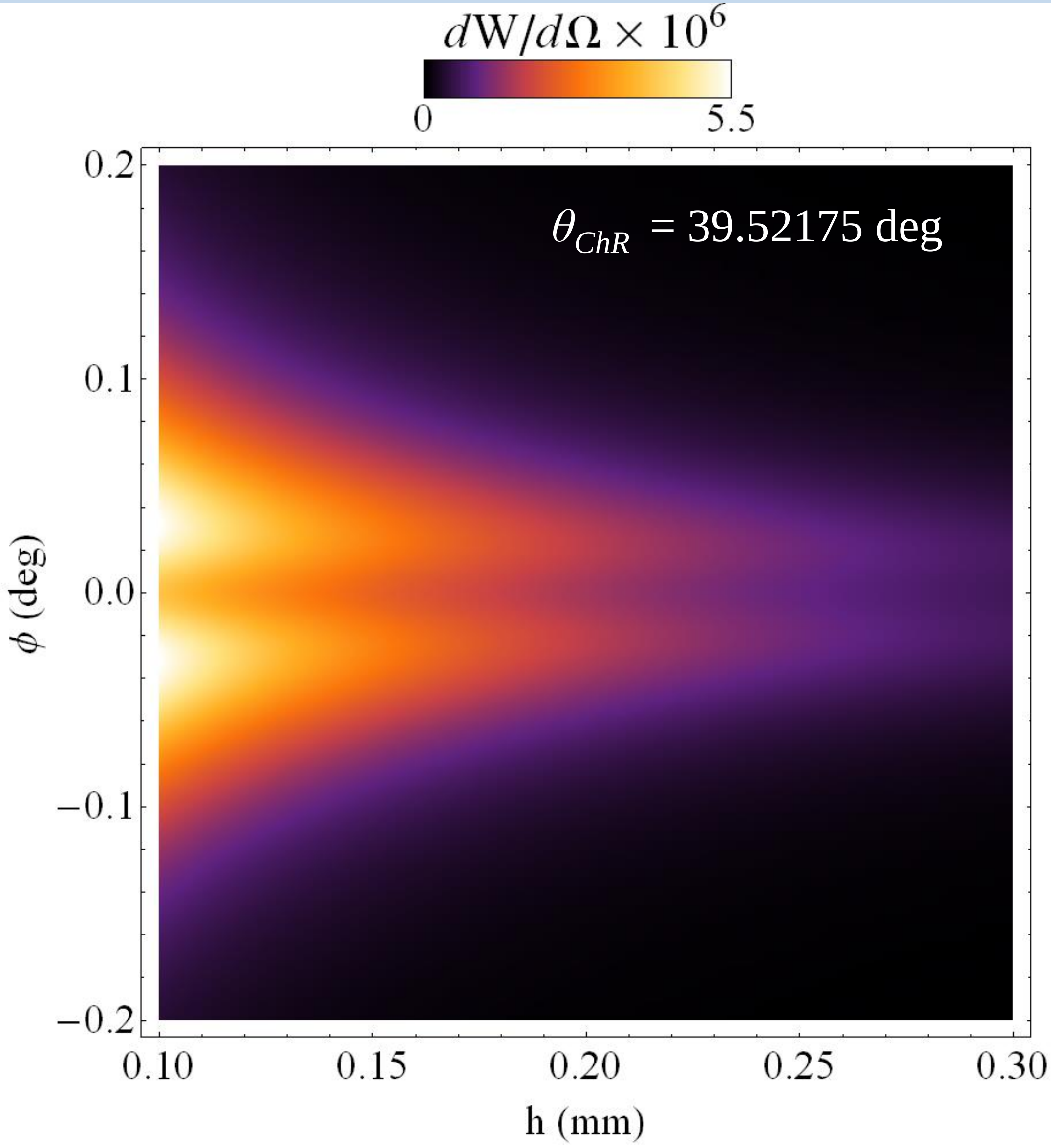


Integration domain: $\Delta\theta = \theta_{ChR} \pm 3\cdot10^{-3}$ deg

$\Delta\phi = \pm$ 0.1 deg

λ = [0.2; 2] μm

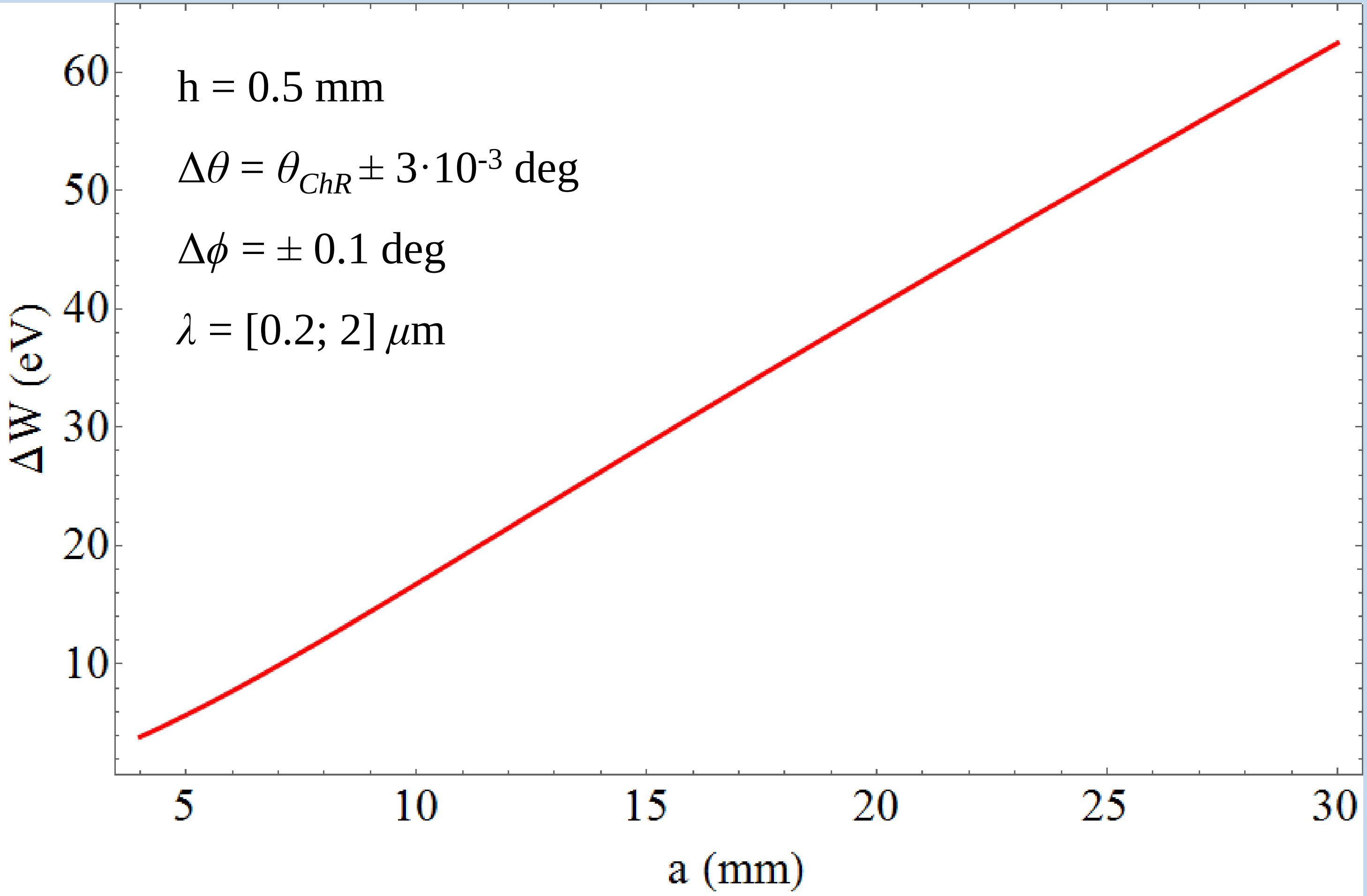
FWHM dependence on the impact parameter



The energy losses on Cherenkov radiation increase in 245 times as the impact parameter value is reduced from 1 to 0.1 mm.

Decrease of the impact parameter in considered region leads to rising of FWHM azimuthal distribution (59%).

Total energy loss versus target length



The total energy losses on Cherenkov radiation linearly depend on the target length.

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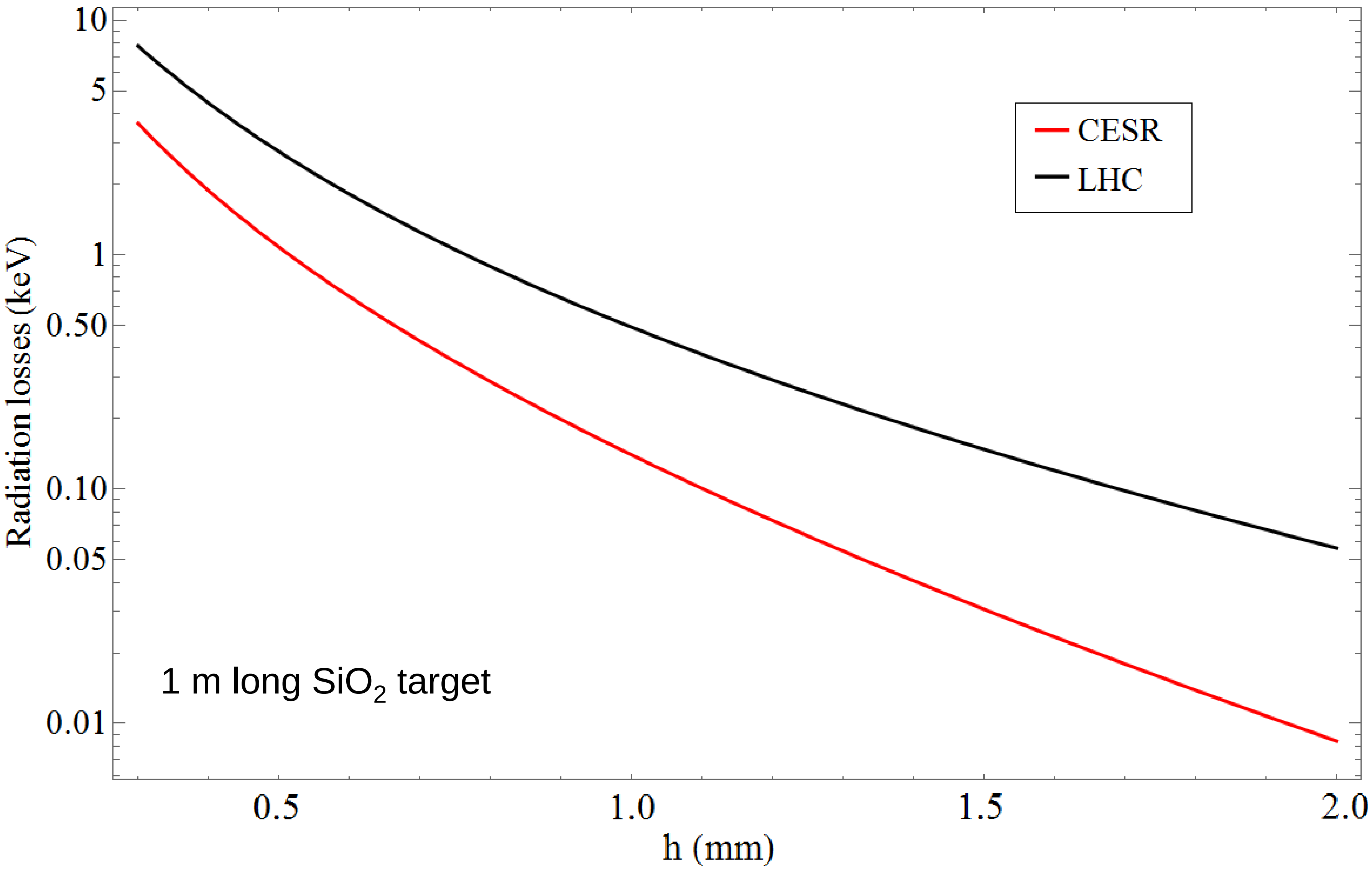
Total energy loss versus impact parameter



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Conclusion

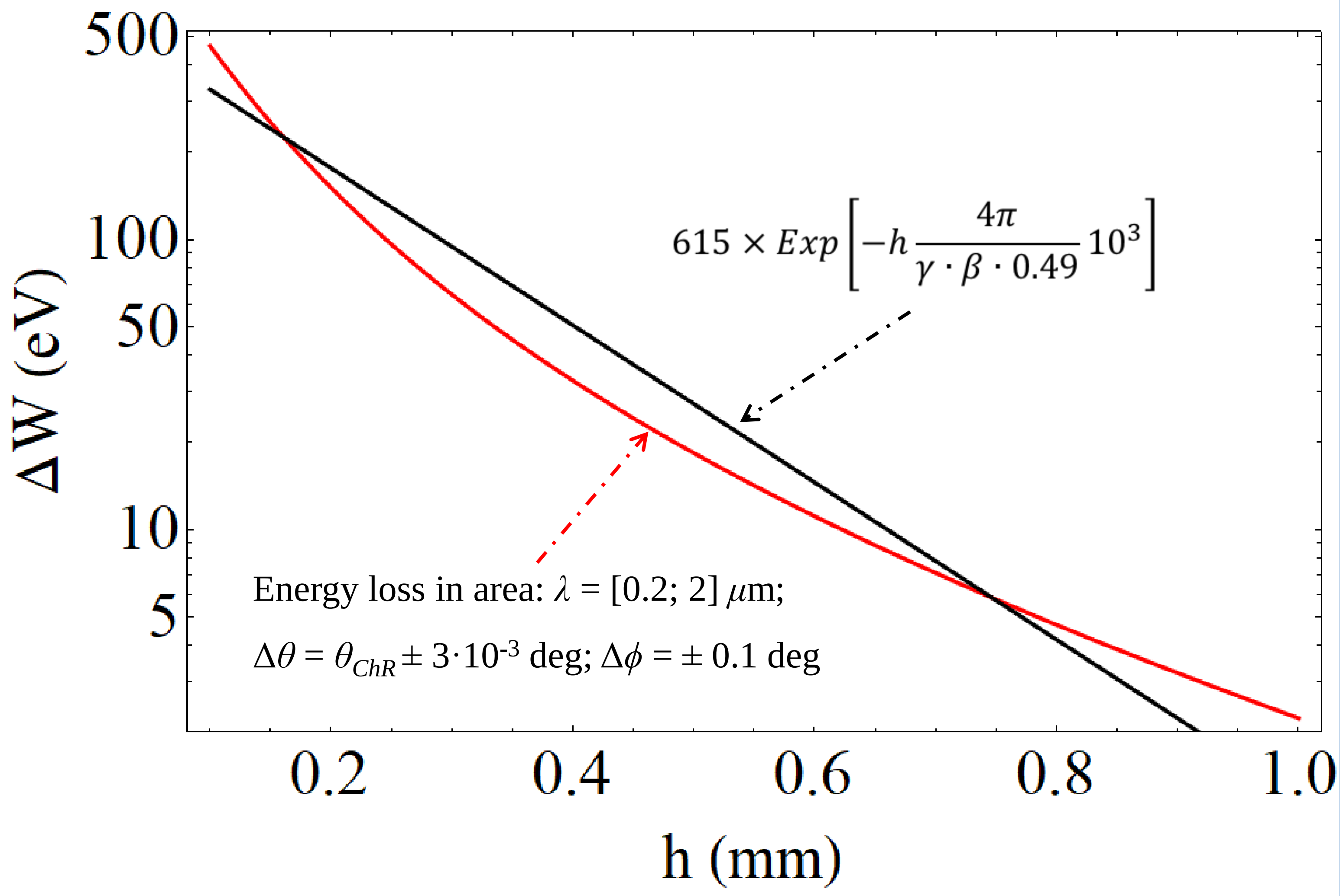
➢Decrease of the impact parameter value leads to rising of the energy losses on Cherenkov radiation in an azimuthal plane.

➢The total energy losses on Cherenkov radiation for transparent material linearly depend on the target length.

➢The exponential dependence of Cherenkov radiation energy losses on the impact parameter allows choosing optimal value of the impact parameter for selected particle energy and radiation wavelength, which minimizes the influence on the beam, but at that, provides sufficient intensity for experimental measurements.

Thank you for your attention!

Function Fit



$$615 \times Exp\left[-h\frac{4\pi}{\gamma\cdot\beta\cdot 0.49}10^{3}\right]$$

Energy loss in area: $\lambda = [0.2; 2]$ μm;

$\Delta\theta = \theta_{ChR} \pm 3\cdot10^{-3}$ deg; $\Delta\phi = \pm 0.1$ deg