Self amplified spontaneous emission from relativistic electrons in crystals: linear response analysis for PXR and channeling radiation mechanisms

Andrei Benediktovitch,
CFEL, DESY, Hamburg, Germany

Ihar Lobach, Stasis Chuchurka, Alexander Leonau
Belarusian State University, Minsk, Belarus

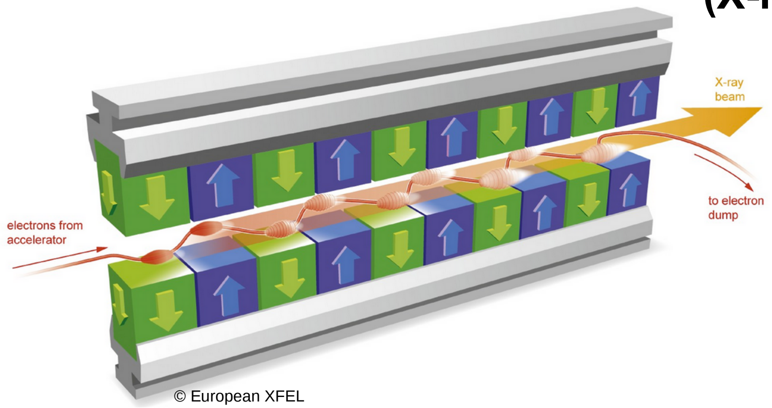
Sergei Galyamin,
St.Petersburg State University, St.Petersburg, Russian Federation

HELMHOLTZ
ASSOCIATION

DESY

Motivation

What determines size of XFEL and are there analogues for crystal abased radiation mechanisms?



(X-ray) Free Electron Laser

Radiation wavelength: $\lambda = \frac{\lambda_u}{2\gamma^2}(1 + \frac{K^2}{2})$

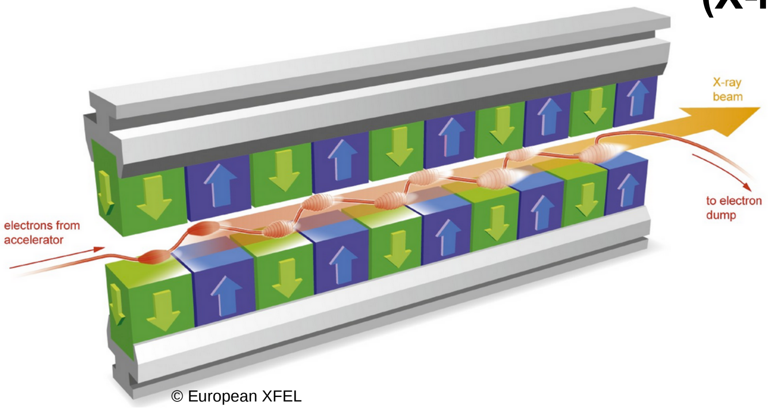
Undulator size:

$$L \sim \frac{\lambda_u}{\rho}, \quad \rho = \left(\frac{I}{\gamma^3 I_A} \frac{K^2 [JJ]^2}{32\pi} \frac{\lambda_u^2}{2\pi \sigma_z \sigma_y} \right)^{\frac{1}{3}}$$

Motivation

What determines size of XFEL and are there analogues for crystal abased radiation mechanisms?

(X-ray) Free Electron Laser

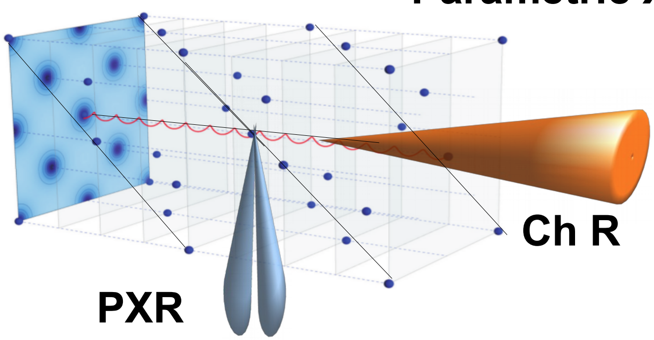


Radiation wavelength: $$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

Undulator size:

$$L \sim \frac{\lambda_u}{\rho}, \quad \rho = \left(\frac{I}{\gamma^3 I_A} \frac{K^2 [JJ]^2}{32\pi} \frac{\lambda_u^2}{2\pi\sigma_z\sigma_y}\right)^{\frac{1}{3}}$$

Parametric X-Ray Radiation (PXR) and Channeling radiation (Ch R)



Radiation wavelength: $$\lambda_{\mathrm{PXR}} = 2d\sin\Theta, \quad E > 50\mathrm{MeV}$$

$$\omega_{\mathrm{ChR}} = 2\gamma^2\Omega$$

Increment estimation:

$$\delta = -2^{-1/2}(Q\omega_\tau |g_0|)^{1/4},$$

$$\tfrac{1}{2}|\delta|\omega_\tau^{-1}[\mathrm{Im}(g_0 - \sqrt{g_\tau g_{-\tau}})]^{-1} > 1. \qquad (16)$$

One can find the following estimation from eq. (16) under the above-mentioned conditions:

$j_s \gtrsim 10^7\ \mathrm{A/cm}^2$. $=100\mathrm{pC}/(100\mathrm{um})^2/100\mathrm{fs}$

Volume 102A, number 3 PHYSICS LETTERS 7 May 1984

PARAMETRIC BEAM INSTABILITY OF RELATIVISTIC CHARGED PARTICLES IN A CRYSTAL

V.G. BARYSHEVSKY and I.D. FERANCHUK
Department of Physics, Byelorussian State University, Minsk-80, USSR

SASE for PXR – 1-st order perturbation theory

Beam susceptibility

Small perturbation of the trajectory (linear theory):

$$\boldsymbol{\beta}_j(t) = \boldsymbol{\beta} + \boldsymbol{\delta\beta}_j(t)$$
$$\boldsymbol{r}_j(t) = \boldsymbol{r}_{j0} + \boldsymbol{\beta} ct + \boldsymbol{\delta r}(t)$$
$$\dot{\boldsymbol{\delta}}\boldsymbol{r}_j(t) = c\boldsymbol{\delta\beta}_j(t)$$

Equation of motion (2-nd Newton law, relativistic):

$$\dot{\boldsymbol{\delta}}\boldsymbol{\beta}_j = \frac{-e}{mc\gamma_i}((\mathbf{1} - \boldsymbol{\beta}_j \otimes \boldsymbol{\beta}_j)\boldsymbol{E}(\boldsymbol{r}_j, t) + \boldsymbol{\beta}_j \times \boldsymbol{H}(\boldsymbol{r}_j, t))$$

Resulting current density (in k-w space)

$$\boldsymbol{j}(\boldsymbol{k}, w) = \frac{ie^2 n_b}{mc\gamma w}(\mathbf{1} + \frac{\boldsymbol{\beta} \otimes \boldsymbol{k} + \boldsymbol{k} \otimes \boldsymbol{\beta}}{w - \boldsymbol{k} \cdot \boldsymbol{\beta}} + \frac{k^2 - w^2}{(w - \boldsymbol{k} \cdot \boldsymbol{\beta})^2}\boldsymbol{\beta} \otimes \boldsymbol{\beta})\boldsymbol{E}(\boldsymbol{k}, w)$$

SASE for PXR – 1-st order perturbation theory

Dispersion equation

Direct wave (at exp[*i* ***k r***]) $$(T-\boldsymbol{k}\otimes\boldsymbol{k})\boldsymbol{E}-w^2\chi_{-g}\boldsymbol{E_g}=-Y\boldsymbol{\beta}\otimes\boldsymbol{\beta}\boldsymbol{E}$$

Diffracted wave (at exp[*i* ***(k+g) r***]) $$(T_g-\boldsymbol{k_g}\otimes\boldsymbol{k_g})\boldsymbol{E_g}-w^2\chi_g\boldsymbol{E}=0,$$

here

$$T=k^2-w^2\epsilon_0,\ T_g=k_g^2-w^2\epsilon_0$$ dispersion equations for "free" waves

$$Y=\frac{w_b^2}{\gamma}\frac{k^2-w^2}{(w-\boldsymbol{k}\cdot\boldsymbol{\beta})^2}$$ factor due to Induced current

$$w_b^2=4\pi e^2 n_b/(mc^2)$$ electron beam plasma frequency

Resulting dispersion equation for σ polarized wave:

$$(w-\boldsymbol{k}\cdot\boldsymbol{\beta})^2 D_\sigma=\frac{w_b^2}{\gamma w^2}\left(k^2-w^2\right)\left(D_\sigma-\beta_\perp^2 w^2 T_g\right)$$

here $D_\sigma=TT_g-\chi_g\chi_{-g}w^4$ - dispersion equation in case of dynamical diffraction theory

SASE for PXR – 1-st order perturbation theory

Dispersion equation

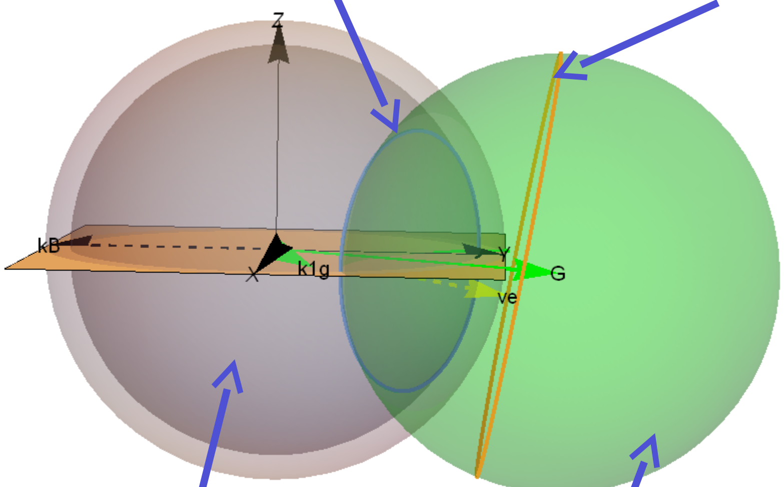
$$(w - \boldsymbol{k} \cdot \boldsymbol{\beta})^2 D_\sigma = \frac{w_b^2}{\gamma w^2} \left(k^2 - w^2\right) \left(D_\sigma - \beta_\perp^2 w^2 T_g\right)$$

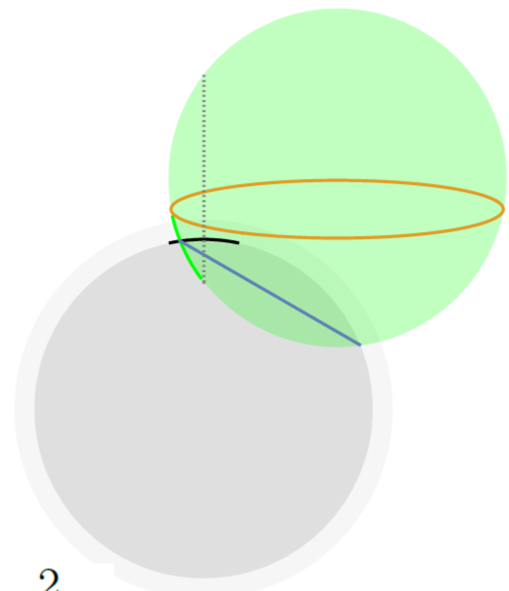
proportional to

$$\beta_z^2 (\delta_z - \delta_{\mathrm{Ch}})^2 (\delta_z - \delta_1)(\delta_z - \delta_2)(\delta_z - \delta_3)(\delta_z - \delta_4)$$

Bragg condition

Cherenkov condition

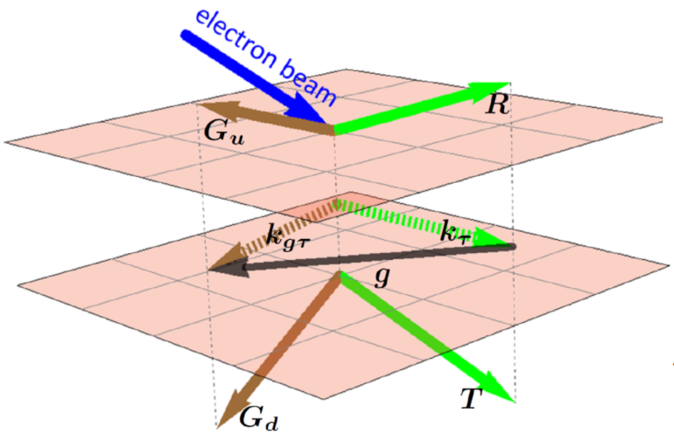




$$T = k^2 - w^2 \epsilon_0, \; T_g = k_g^2 - w^2 \epsilon_0$$

SASE for PXR – 1-st order perturbation theory

Boundary conditions



Homogeneous and inhomogeneous parts of the current:

$$(T - \boldsymbol{k} \otimes \boldsymbol{k})\boldsymbol{E} - w^2 \chi_{-g} \boldsymbol{E_g} = -Y \boldsymbol{\beta} \otimes \boldsymbol{\beta} \boldsymbol{E} + H \boldsymbol{\beta} \delta (w - \boldsymbol{k} \cdot \boldsymbol{\beta})$$

$$(T_g - \boldsymbol{k_g} \otimes \boldsymbol{k_g})\boldsymbol{E_g} - w^2 \chi_g \boldsymbol{E} = 0$$

Homogeneous and inhomogeneous parts of the field

$$\boldsymbol{E}(\boldsymbol{k}, w) = \boldsymbol{E^i}(\boldsymbol{k}, w) + \sum_m \boldsymbol{E^h_m}(w) \delta \left(k_n - k_n^{(m)}(w) \right)$$

$$\int \boldsymbol{E_c}(\boldsymbol{k_\tau} + \boldsymbol{k^c_n}, w) dk_n^c = \int \boldsymbol{E_v}(\boldsymbol{k_\tau} + \boldsymbol{k^v_n}, w) dk_n^v$$

- continuity at the interface

Field continuity

entry surface

$$\sum_m \boldsymbol{E}_m = \boldsymbol{R}$$

$$\sum_m k_z^{(m)} \boldsymbol{E}_m = -w_z \boldsymbol{R}$$

$$\sum_m \widehat{g}_m \boldsymbol{E}_m = \boldsymbol{G_u}$$

$$\sum_m k_z^{(m)} \widehat{g}_m \boldsymbol{E}_m = -w_{zg} \boldsymbol{G_u}$$

exit surface

$$\sum_m \exp(i\delta_z^{(m)} l) \boldsymbol{E}_m = \boldsymbol{T}$$

$$\sum_m \exp(i\delta_z^{(m)} l) k_z^{(m)} \boldsymbol{E}_m = w_z \boldsymbol{T}$$

$$\sum_m \exp(i\delta_z^{(m)} l) \widehat{g}_m \boldsymbol{E}_m = \boldsymbol{G_d}$$

$$\sum_m \exp(i\delta_z^{(m)} l) k_z^{(m)} \widehat{g}_m \boldsymbol{E}_m = w_{zg} \boldsymbol{G_d}$$

Current continuity

$$\widehat{\boldsymbol{J}} \boldsymbol{E_9} = \boldsymbol{J_0}$$

$$\widehat{\boldsymbol{J}} = -Y \boldsymbol{\beta} \otimes \boldsymbol{\beta}$$

$$\boldsymbol{J_0} = H \boldsymbol{\beta} / \beta_z$$

$$H = 8\pi^2 i w e / c$$

One can show that

$$\boldsymbol{E^i} \to \left(\exp\left(i\delta_z^{(9)} l\right) \frac{\delta_z^{(9)}}{\delta_z^{(9)} - \delta_z^{(10)}} + \exp\left(i\delta_z^{(10)} l\right) \frac{-\delta_z^{(10)}}{\delta_z^{(9)} - \delta_z^{(10)}} \right) \boldsymbol{E^i}$$

SASE for PXR – 1-st order perturbation theory

Deteriorative effect of emittance

Average over velocity directions: $\boldsymbol{j}(\boldsymbol{k},w) = \frac{ie^2}{mc\gamma w}\int \frac{dn_B}{\boldsymbol{d\beta}}\frac{k^2-w^2}{(w-\boldsymbol{k}\cdot\boldsymbol{\beta})^2}\boldsymbol{\beta}\otimes\boldsymbol{\beta}\boldsymbol{d\beta}\boldsymbol{E}(\boldsymbol{k},w)$

Ideal beam $\boldsymbol{j}(\boldsymbol{k},w) \sim n_e\frac{\boldsymbol{\beta}\otimes\boldsymbol{\beta}}{(w-\boldsymbol{k}\cdot\boldsymbol{\beta})^2}$

with emittance $\boldsymbol{j}(\boldsymbol{k},w) \sim n_e\frac{\langle\boldsymbol{\beta}\rangle\otimes\langle\boldsymbol{\beta}\rangle}{(w-\boldsymbol{k}\cdot\boldsymbol{\beta}-ik_y\Delta\alpha_1)(w-\boldsymbol{k}\cdot\boldsymbol{\beta}-ik_y\Delta\alpha_1-(k_z\cos\eta+k_x\sin\eta)\Delta\alpha_2)}$

SASE for PXR – 1-st order perturbation theory

Deteriorative effect of emittance

Average over velocity directions: $\boldsymbol{j}(\boldsymbol{k},w) = \frac{ie^2}{mc\gamma w}\int \frac{dn_B}{\boldsymbol{d\beta}} \frac{k^2 - w^2}{(w - \boldsymbol{k}\cdot\boldsymbol{\beta})^2} \boldsymbol{\beta}\otimes\boldsymbol{\beta} \boldsymbol{d\beta}\boldsymbol{E}(\boldsymbol{k},w)$

Ideal beam $\boldsymbol{j}(\boldsymbol{k},w) \sim n_e \frac{\boldsymbol{\beta}\otimes\boldsymbol{\beta}}{(w-\boldsymbol{k}\cdot\boldsymbol{\beta})^2}$

with emittance $\boldsymbol{j}(\boldsymbol{k},w) \sim n_e \frac{\langle\boldsymbol{\beta}\rangle\otimes\langle\boldsymbol{\beta}\rangle}{(w-\boldsymbol{k}\cdot\boldsymbol{\beta}-ik_y\Delta\alpha_1)(w-\boldsymbol{k}\cdot\boldsymbol{\beta}-ik_y\Delta\alpha_1-(k_z\cos\eta+k_x\sin\eta)\Delta\alpha_2)}$

Numerical example:

Si, (040)

E=1 GeV

I=0.4 kA

s=0.1 μm x 0.1 μm

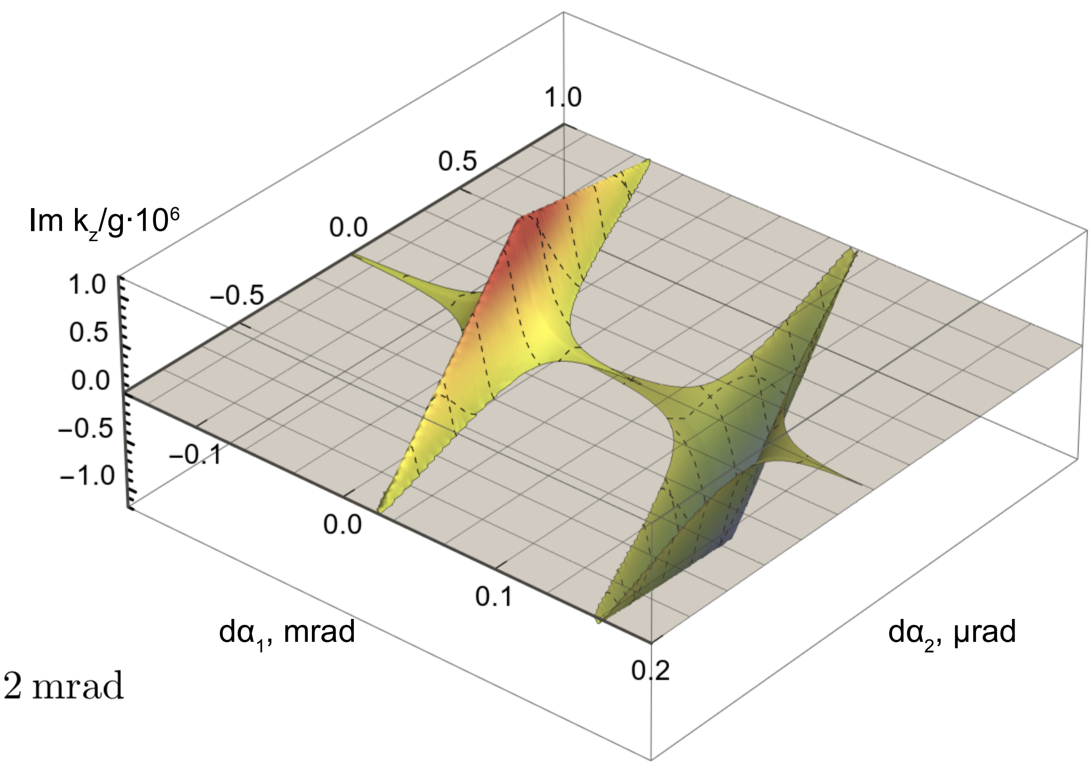
Δα=0.25 mrad x 0.25 mrad

ψ=1 deg;

Resulting peak gain

L_g=20 μm, Lg/ψ=1 mm

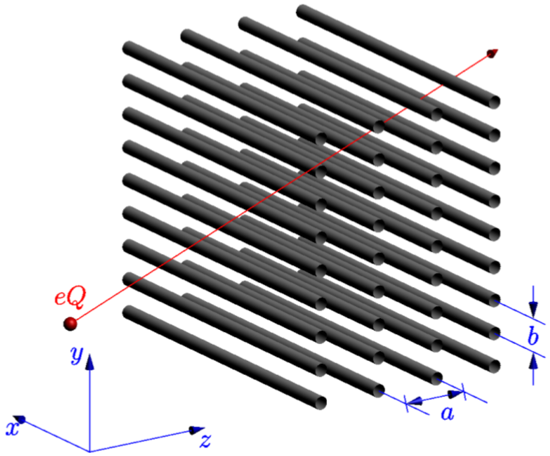
Multiple scattering $\langle\theta_s\rangle \sim \frac{E_s}{E}\sqrt{\frac{L}{L_R}} \sim 2\,\mathrm{mrad}$



SASE for PXR – 1-st order perturbation theory

Analogs in THz range

Artificial crystal in ThZ range



One can have

$$\chi_g \sim 10^{-2}..10^{-3}$$

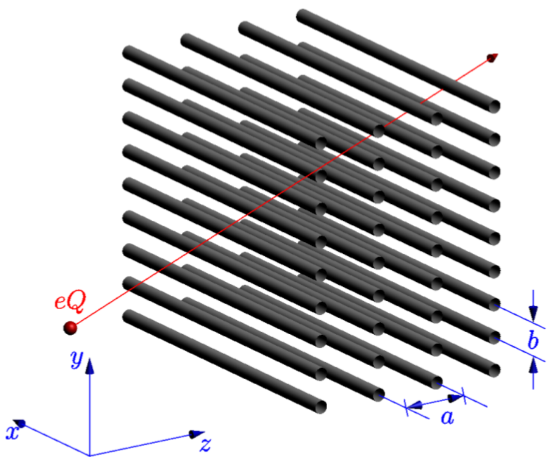
And apply the same formalism.

V.G.Baryshevsky, E.A.Gurnevich, *Cherenkov and parametric (quasi-Cherenkov) radiation produced by a relativistic charged particle moving through a crystal built from metallic wires*, NIM B, 402, 30-34 (2017)

SASE for PXR – 1-st order perturbation theory

Analogs in THz range

Artificial crystal in ThZ range



One can have

$$\chi_g \sim 10^{-2}..10^{-3}$$

And apply the same formalism.

Numerical example:

a=b=0.1 mm

$\chi_g=2.3 \cdot 10^{-3}$

E=50 MeV

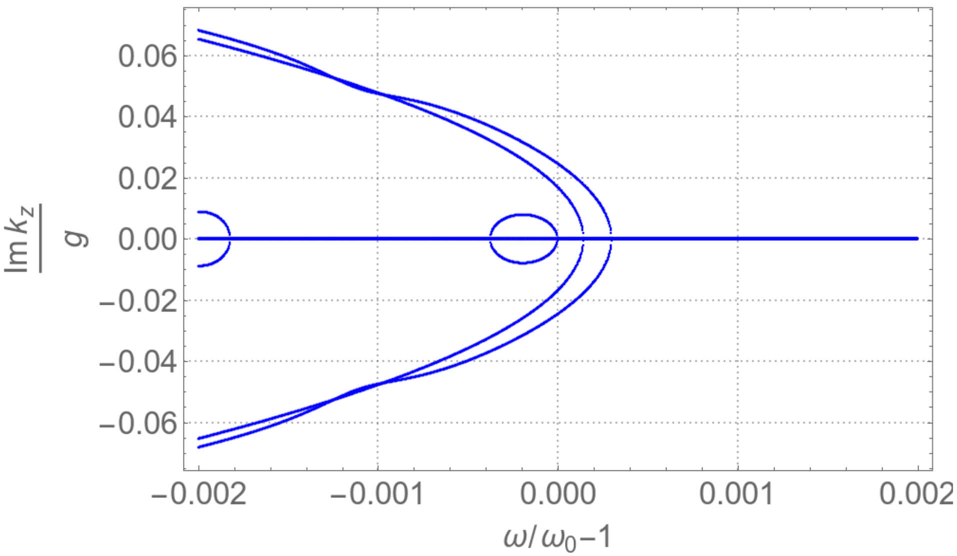
Q=20 nC

σ_x=1 mm

σ_y=1 mm

σ_z=1 mm

ψ=0.6 deg;



Resulting peak gain

L_g=1.6 mm, Lg/ψ=15 cm

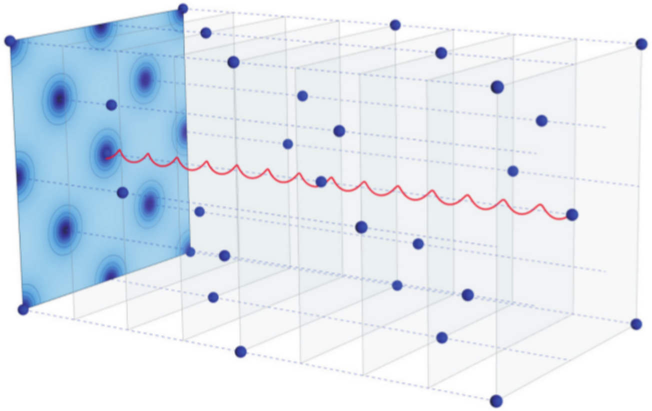
V.G.Baryshevsky, E.A.Gurnevich, *Cherenkov and parametric (quasi-Cherenkov) radiation produced by a relativistic charged particle moving through a crystal built from metallic wires*, NIM B, 402, 30-34 (2017)

SASE for channeling radiation – 1-st order perturbation theory

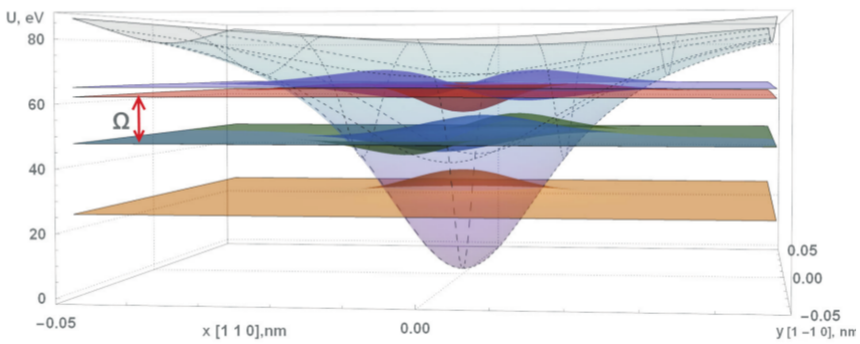
Axial channeling and channeling radiation

Transverse motion of relativistic electron can be described by Schrodinger equation with effective mass γ m

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2\gamma m}\Delta_{x,y} + U(x,y)\right)\psi$$



Sketch of axial channeling along [001] axes in Si crystal



Axes potential and energy levels for 25 MeV electrons

The transition between the transverse energy levels results in spontaneous channeling radiation. The radiation properties are close to that of undulator radiation:

radiation frequency: $\omega = \dfrac{2\gamma^2\Omega}{1+\gamma^2\theta^2}$

peak photon density: $\dfrac{dN}{d^2\vec{n}d\omega/\omega} = 4\alpha K_u^2 N_u^2 \gamma^2 P_e$

effective undulator parameter: $K_u = \gamma\dfrac{\Omega d_{eg}}{c} \sim 0.02$

effective number of undulator periods: $N_u = \dfrac{1}{2\pi}\dfrac{\omega L}{2\gamma^2}$

SASE for channeling radiation – 1-st order perturbation theory

Equations from first principles

- starting from Hamiltonian for the field and electrons in axial potential,
- quantizing by imposing equal-time commutation relations,
- from Heisenberg equation one obtains ***operator*** equations:

SASE for channeling radiation – 1-st order perturbation theory

Equations from first principles

- starting from Hamiltonian for the field and electrons in axial potential,
- quantizing by imposing equal-time commutation relations,
- from Heisenberg equation one obtains ***operator*** equations:

Field:

$$\Delta\hat{\vec{A}}(\vec{r},t) - \frac{\epsilon(\vec{r})}{c^2}\frac{\partial^2\hat{\vec{A}}(\vec{r},t)}{\partial t^2} - \vec{\nabla}(\vec{\nabla}\cdot\hat{\vec{A}}(\vec{r},t)) =$$

$$\frac{4\pi i}{c}\sum_i e\Omega\hat{\sigma}_{eg}^{(i)}(t)[\vec{d}_{eg} + \frac{i}{\Omega}\vec{u}(\vec{d}_{eg}\cdot\vec{\nabla})]\delta(\vec{r}-\vec{r}_0^{(i)}-\vec{u}t) + h.c.$$

SASE for channeling radiation – 1-st order perturbation theory

Equations from first principles

- starting from Hamiltonian for the field and electrons in axial potential,
- quantizing by imposing equal-time commutation relations,
- from Heisenberg equation one obtains ***operator*** equations:

Field:

$$\Delta\hat{\vec{A}}(\vec{r},t) - \frac{\epsilon(\vec{r})}{c^2}\frac{\partial^2\hat{\vec{A}}(\vec{r},t)}{\partial t^2} - \vec{\nabla}(\vec{\nabla}\cdot\hat{\vec{A}}(\vec{r},t)) =$$

$$\frac{4\pi i}{c}\sum_i e\Omega\hat{\sigma}_{eg}^{(i)}(t)[\vec{d}_{eg} + \frac{i}{\Omega}\vec{u}(\vec{d}_{eg}\cdot\vec{\nabla})]\delta(\vec{r} - \vec{r}_0^{(i)} - \vec{u}t) + h.c.$$

Channeled electrons (two level systems) :

$$\frac{\partial\hat{\sigma}_{eg}^{(i)}(t)}{\partial t} = i\Omega\hat{\sigma}_{eg}^{(i)}(t) + \hat{\sigma}_{zz}^{(i)}(t)\frac{e\Omega}{\hbar c}[\vec{d}_{eg} + \frac{i}{\Omega}\vec{u}(\vec{d}_{eg}\cdot\vec{\nabla})]\hat{\vec{A}}(\vec{r},t)|_{\vec{r}=\vec{r}_0^{(i)}+\vec{u}t} + h.c.$$

$$\frac{\partial\hat{\sigma}_{zz}^{(i)}(t)}{\partial t} = \hat{\sigma}_{eg}^{(i)}(t)\frac{2e\Omega}{\hbar c}[\vec{d}_{eg} + \frac{i}{\Omega}\vec{u}(\vec{d}_{eg}\cdot\vec{\nabla})]\hat{\vec{A}}(\vec{r},t)|_{\vec{r}=\vec{r}_0^{(i)}+\vec{u}t} + h.c.$$

The operator describing electron state are:

coherences $\hat{\sigma}_{eg} = |e\rangle\langle g|$, occupation inversion $\hat{\sigma}_{zz} = |e\rangle\langle e| - |g\rangle\langle g|$

SASE for channeling radiation – 1-st order perturbation theory

0-th order approximation: spontaneous radiation

When radiation backaction on channeling states is negligible the dynamic of electron variable is trivial: $\hat{\sigma}_{eg}(t) = e^{i\Omega t}\hat{\sigma}_{eg}(-\infty),\ \hat{\sigma}_{zz}(t) = \hat{\sigma}_{zz}(-\infty)$ .The quantum-mechanical averages are $\langle\hat{\sigma}_{eg}(-\infty)\rangle = 0, \langle\hat{\sigma}_{eg}(-\infty)\hat{\sigma}_{ge}(-\infty)\rangle = P_e$. The radiation field is due to $j_{sp} \sim \hat{\sigma}_{eg}(-\infty)$

With help of Green function for wave equation one can obtain

$$\frac{dN}{d^2\vec{n}d\omega} \sim \langle\vec{A}\vec{A}^*\rangle \sim \langle\hat{\sigma}_{eg}(-\infty)\hat{\sigma}_{ge}(-\infty)\rangle \sim P_e$$

SASE for channeling radiation – 1-st order perturbation theory

0-th order approximation: spontaneous radiation

When radiation backaction on channeling states is negligible the dynamic of electron variable is trivial: $\hat{\sigma}_{eg}(t) = e^{i\Omega t}\hat{\sigma}_{eg}(-\infty),\ \hat{\sigma}_{zz}(t) = \hat{\sigma}_{zz}(-\infty)$ .The quantum-mechanical averages are $\langle\hat{\sigma}_{eg}(-\infty)\rangle = 0, \langle\hat{\sigma}_{eg}(-\infty)\hat{\sigma}_{ge}(-\infty)\rangle = P_e$. The radiation field is due to $j_{sp} \sim \hat{\sigma}_{eg}(-\infty)$

With help of Green function for wave equation one can obtain

$$\frac{dN}{d^2\vec{n}d\omega} \sim \langle\vec{A}\vec{A}^*\rangle \sim \langle\hat{\sigma}_{eg}(-\infty)\hat{\sigma}_{ge}(-\infty)\rangle \sim P_e$$

1-st order approximations: beam susceptibility

Assume $\hat{\sigma}_{zz}(t) = \hat{\sigma}_{zz}(-\infty)$, performing integration of operator equation one obtains

$$\hat{\sigma}^{(i)}_{eg}(t) = \hat{\sigma}^{(i)}_{eg}(-\infty)e^{i\Omega t} + \int_{-\infty}^{t} dt'\hat{\sigma}^{(i)}_{zz}(-\infty)\frac{e\Omega}{\hbar c}[\vec{d}_{eg} + \frac{i}{\Omega}\vec{u}(\vec{d}_{eg}\cdot\vec{\nabla})]\hat{\vec{A}}(\vec{r},t')|_{\vec{r}=\vec{r}^{(i)}_0+\vec{u}t'}e^{i\Omega(t-t')}$$

Substituted in equations for field it results in linear susceptibility. In *k,ω* space susceptibility is

$$\chi^{(b)}(\vec{k},\omega) = \chi_b\frac{\omega}{\omega - \vec{k}\vec{v} - \Omega}\vec{a}\otimes\vec{a}, \quad \chi_b = \frac{4\pi ec}{\hbar\omega^3}j_b\frac{\Omega^2 d^2_{eg}}{c^2}(P_e - P_g),\ \vec{a} = \frac{\vec{d}_{eg}}{d_{eg}} + \vec{v}\frac{\vec{k}\vec{d}_{eg}}{\Omega d_{eg}},\ \vec{d}_{eg} = \langle e|\vec{r}|g\rangle$$

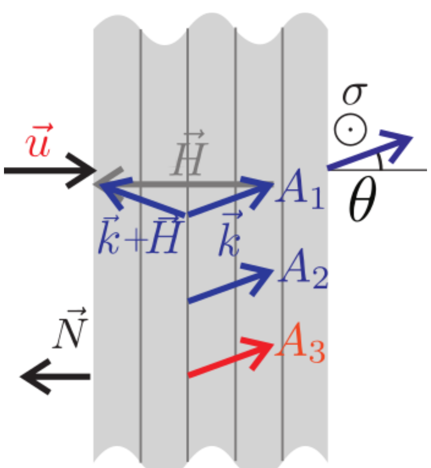
SASE for channeling radiation – 1-st order perturbation theory

Wavefields and dispersion equation

Consider periodic permittivity $\epsilon(\vec{r}) = 1 + \chi_0 + \chi_H e^{i\vec{H}\vec{r}} + \chi_{-H} e^{-i\vec{H}\vec{r}}$ due to crystallographic order and assume that the emitted radiation is close to Bragg conditions for reciprocal lattice vector H (a two-wave approximation):

$$[X_k I - \vec{k} \otimes \vec{k} - \frac{\omega^2}{c^2}\chi^{(b)}(\vec{k},\omega)] \cdot \vec{A}_k(\vec{k},\omega) - \frac{\omega^2}{c^2}\chi_{-H}\vec{A}_H(\vec{k},\omega) = \frac{4\pi}{c}\vec{j}_{(sp)}(\vec{k},\omega)$$

$$[X_H I - (\vec{k}+\vec{H}) \otimes (\vec{k}+\vec{H})] \cdot \vec{A}_H(\vec{k},\omega) - \frac{\omega^2}{c^2}\chi_H \vec{A}_k(\vec{k},\omega) = 0,$$



Here the dispersion equations for incident and diffracted wavefields are

$$X_k = k^2 - \frac{\omega^2}{c^2}(1+\chi_0),\ X_H = (\vec{k}+\vec{H})^2 - \frac{\omega^2}{c^2}(1+\chi_0)$$

The noise current (resulting in spontaneous radiation) is

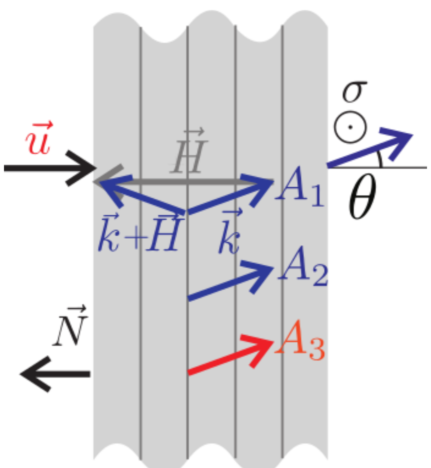
$$\vec{j}_{(sp)}(\vec{k},\omega) = 2\pi i e \Omega d_{eg}\vec{a}\delta(\omega - \vec{k}\vec{u} - \Omega)\sum_i e^{-i\vec{k}\vec{r}_0^{(i)}}\hat{\sigma}_{eg}^{(i)}(-\infty).$$

SASE for channeling radiation – 1-st order perturbation theory

Dispersion equation and boundary conditions

The dispersion equation can be factorized for σ and π polarizations, for σ one obtains

$$[X_k X_H - \frac{\omega^4}{c^4}\chi_H\chi_{-H}](\omega - \vec{k}\vec{u} - \Omega) = \chi_b X_H \frac{\omega^3}{c^2}.$$



Solution of the wave field inside the crystal takes the following form:

$$\vec{A}(\vec{k},\omega) = \sum_s A_s(\vec{k},\omega)\vec{e}_s(\vec{k},\omega)\delta(k_z - k_z^{(s)}(\vec{k}_{||},\omega))$$

To find A_s one needs boundary conditions for the field and in addition for the current density, since the number of dispersion equation roots is larger by one compared to dynamical diffraction case:

$$8\pi^2 ie\Omega d_{eg}\sum_i e^{-i\vec{k}\vec{r}_0^{(i)}}\hat{\sigma}_{eg}^{(i)}(-\infty) = \sum_s A_s \frac{[X_k X_H - \frac{\omega^4}{c^4}\chi_H\chi_{-H}]}{X_H\omega^2/c^2}\Big|_{k_z=k_z^{(s)}}.$$

SASE for channeling radiation – 1-st order perturbation theory

Dispersion equation: numerical example and general view

Here following parameters were used:

E=25 MeV

axial channeling in Si along [001]

Ω=14.3 eV

P_e-P_g= 6.5%

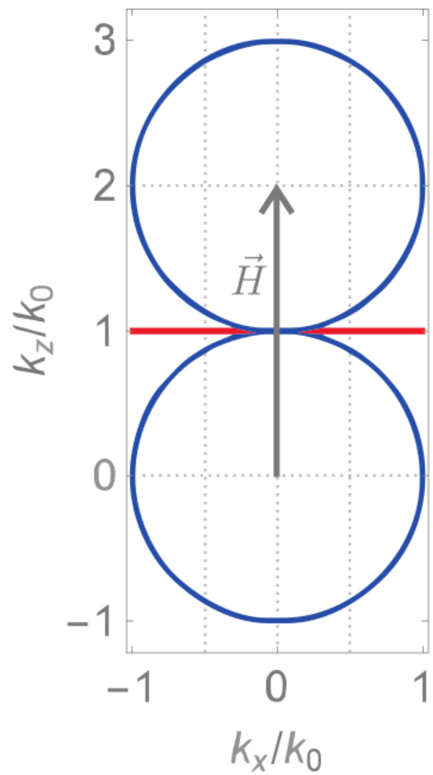
B=$1.7\cdot10^{19}$ A m^{-2} rad^{-2} (*)

resulting beam susceptibility

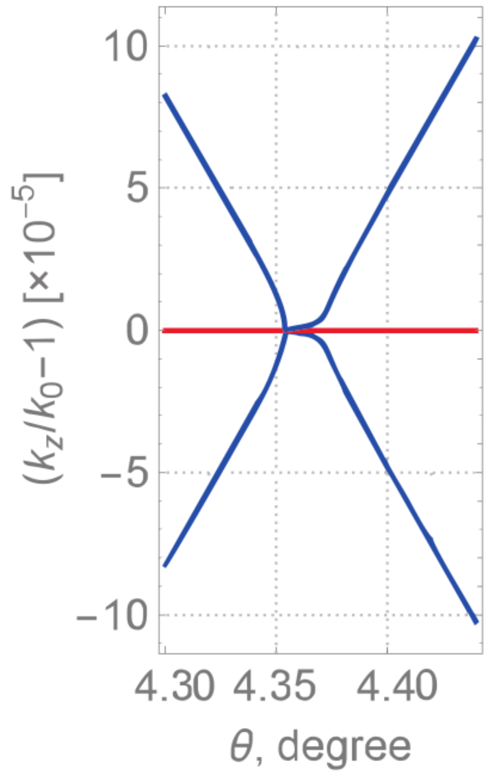
$\chi_b=4.6\cdot10^{-13}$

Bragg condition for (004) at $\Theta_B=\pi/2$

x-ray photons ω=4.5 keV



General view of dispersion surface



dispersion surface close to Bragg peak

blue - solutions corresponding to dynamical diffraction,

red - to channeling radiation condition

(*)F.Li, J.F.Hua, et al.,*Generating high-brightness electron beams via ionization injection by transverse colliding lasers in a plasma-wakefield accelerator*, Phys. Rev. Lett., 111, 015003 (2013)

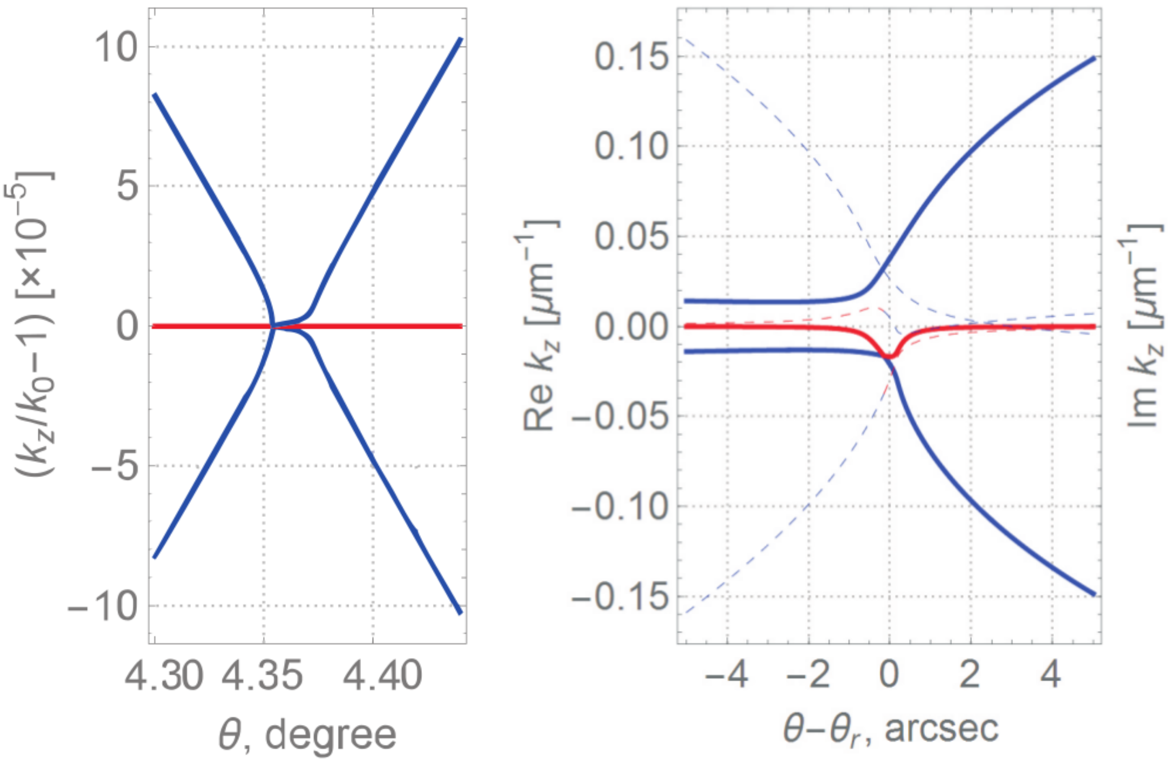
SASE for channeling radiation – 1-st order perturbation theory

Dispersion equation: numerical example and detailed view

The largest instability increment is at roots intersection that takes place at the edge of Darwin table. For considered case:

$$\mathrm{Im}k_z^{(r)} = \frac{\omega}{c}\frac{\chi_b}{2\mathrm{Im}\chi_0(1-e^{-W})}$$

Bragg diffraction leads to increase of instability increment due to decrease of effective absorption – an analogue of Borrmann effect.



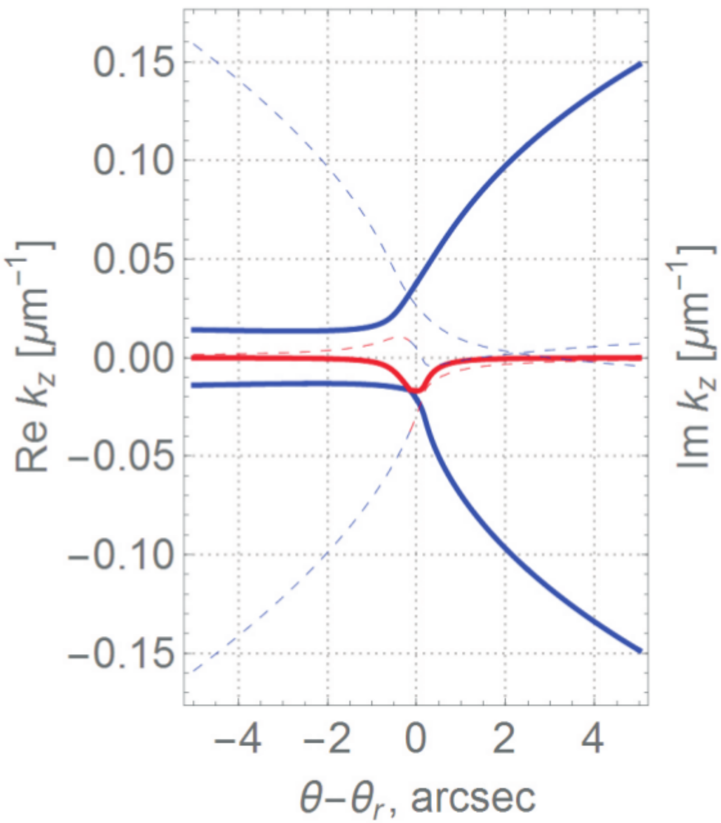
dispersion surface close to Bragg peak

detailed view near the intersection, dip of red line results in instability increment

blue - solutions corresponding to dynamical diffraction,
red - to channeling radiation condition

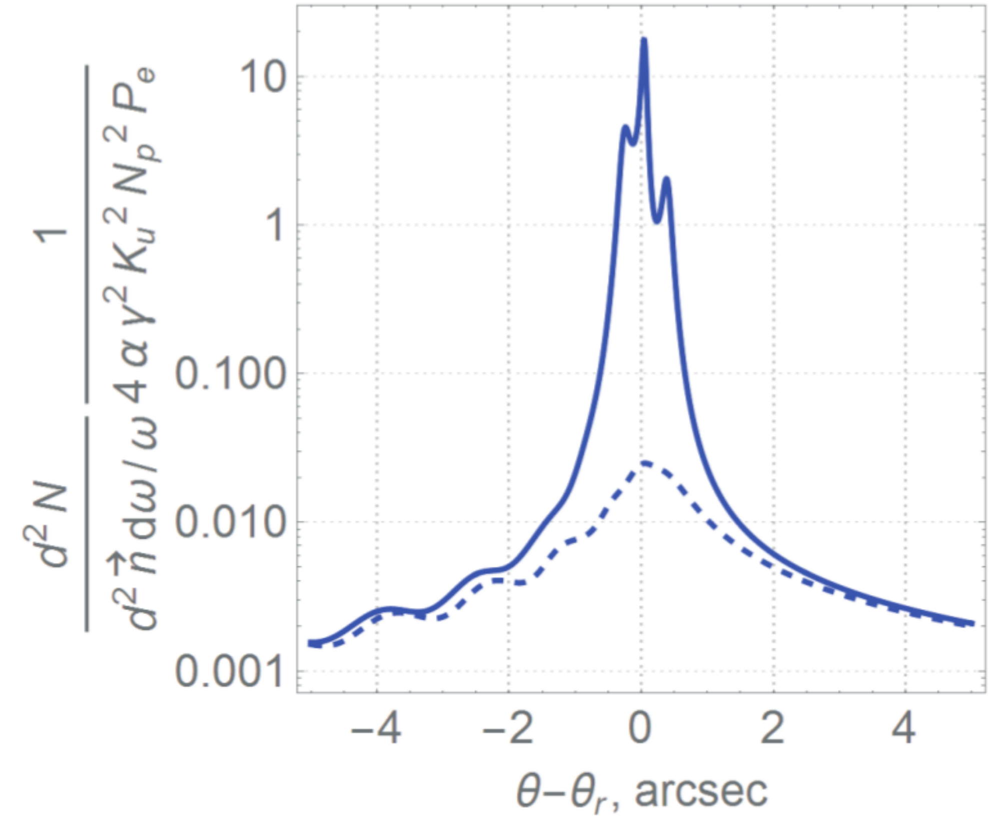
SASE for channeling radiation – 1-st order perturbation theory

Numerical example of SASE within 1-st order perturbation theory



detailed view near the intersection, dip of red line results in instability increment

Gain length L_g~60 μm



Radiation intensity profile from 200μm crystal slab, solid line represents SASE effect, dashed line corresponds to spontaneous radiation

Conclusions

SASE for PXR – 1-st order perturbation theory

-beam divergence deteriorates the effect crucially

-in THz range the effect could survive

SASE for Channeling radiation – 1-st order perturbation theory

-QM operator equations should be solved, methods developed for X-ray ASE could be applied

-within 1-st order perturbation theory one can described by means of effective medium -> dispersion equations -> boundary conditions

-Bragg diffraction helps to decrease the absorption length by analogue of Borrmann effect

...

Thank you for your attention!