Universal RG Flows Across Dimensions

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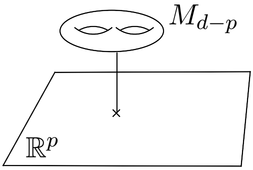
Based on
1) 1511.09462 with N. Bobev and F. Benini
2) 1704.xxxxx with N. Bobev
3) Work in progress with N. Bobev & V. Min, and F. Azzurli & A. Zaffaroni.

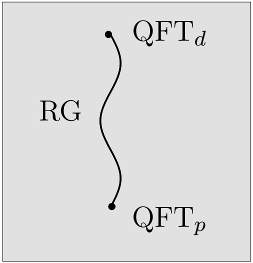
DESY

April 20, 2017

Basic Setup

Consider a QFT on $M_d = \mathbb{R}^p \times M_{d-p}$ and flow to IR:





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3) What is the holographic description?

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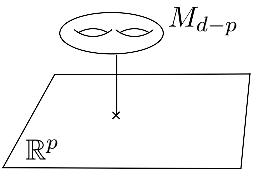

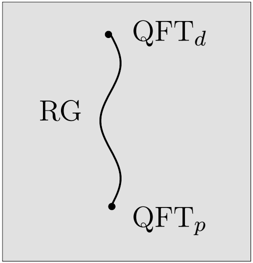

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- Recently, great interest in QFTs on M_d
- Exact results by localization in various dimensions **[Pestun, Kapustin et al., Benini et al., Doroud et al.]** Check of dualities and AdS/CFT
- Taking $M_d = M_{d-p} \times \tilde{M}_p$ leads to new dualities $T_{d-p} \leftrightarrow \tilde{T}_p$ **[Alday-Gaiotto-Tachikawa, Gadde-Pomini-Rastelli-Razamat, Dimofte-Gaiotto-Gukov, $\cdots$]**
- Compactification leads to large classes of SCFTs from higher dimensions **[Maldacena-Núñez, Bershadsky-Johansen-Sadov-Vafa, Gaiotto, Bah et al., Crichigno-Benini-Bobev,$\cdots$]**
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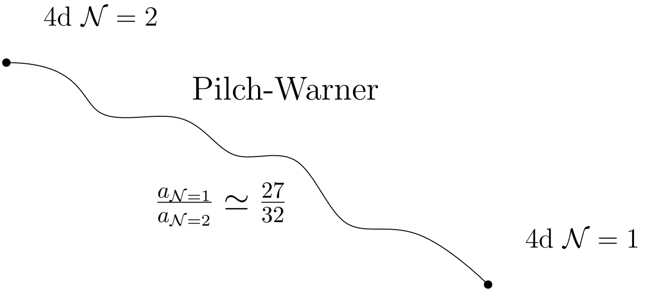
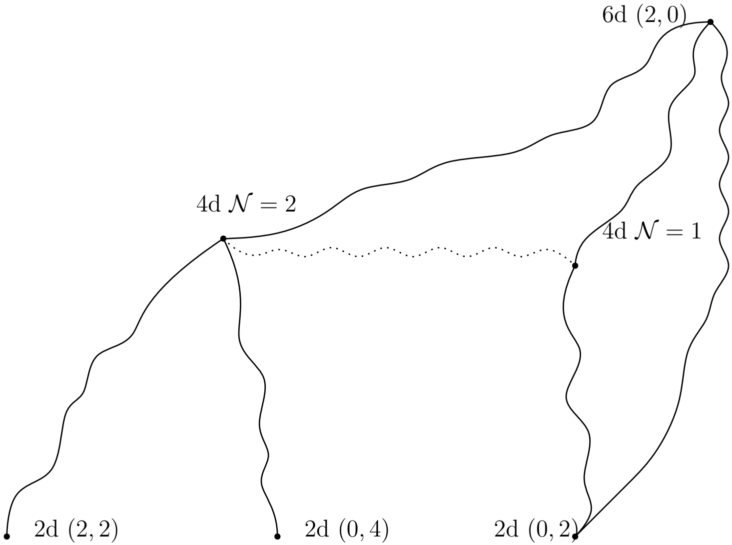
What do we mean by “universal”?

- Prototypical example of a (4d→4d) universal flow:



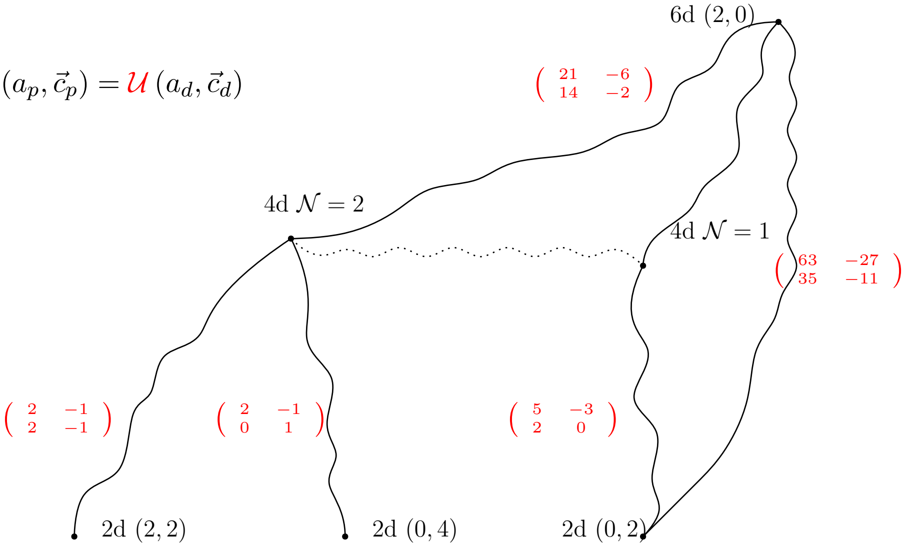
- First found for $\mathcal{N} = 4 \to \mathcal{N} = 1$ flow **[Anselmi-Freedman et al. 1997]**
- Later proven in more generality for $\mathcal{N} = 2 \to \mathcal{N} = 1$ flows **[Tachikawa-Wecht 2009]**

Universal flows across (even) dimensions



Universal flows across (even) dimensions

$$(a_p, \vec{c}_p) = \mathcal{U}\,(a_d, \vec{c}_d)$$



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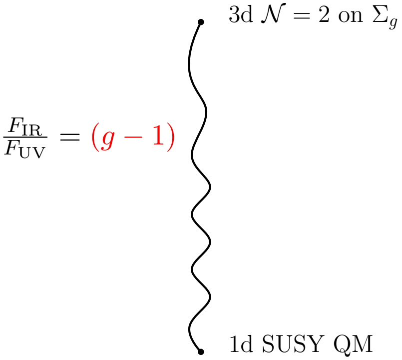

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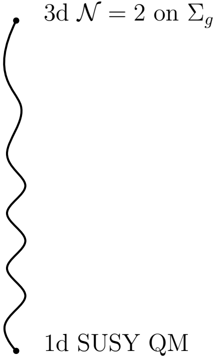

$$\frac{F_{\text{IR}}}{F_{\text{UV}}} = (g - 1)$$

- Many more universal relations predicted by holography

Review tools

Tool 1: Topological twist [Witten 1988]

To preserve SUSY on M_d:

$$(\partial_\mu + \omega_\mu)\epsilon = 0$$

Generically, no solutions. But, if global R-symmetry, turn background on:

$$(\partial_\mu + \omega_\mu + A_\mu^{back})\epsilon = 0 \quad \xrightarrow{A_\mu^{back} = -\omega_\mu} \quad \partial_\mu \epsilon = 0$$

Comments:

- SUSY only partially preserved
- Spin of fields *shifted*: $(\partial_\mu + (s - q)\,\omega_\mu)\phi$
- If $M_d = \mathbb{R}^p \times \mathcal{M}_{d-p}$ twist only along $\mathcal{M}_{d-p}$

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- Global symmetry $S \times G_R$ (Lorentz×R-symmetry)
- Twist amounts to choosing embedding

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Tool 2: Anomalies

Anomalies are robust nonperturbative observables. Interested in two kinds of anomalies:

- 't Hooft R-symmetry anomalies $\langle \partial_\mu j_R^\mu \rangle \neq 0$. In 4d and 2d:

$$k_{RRR}, k_{RR}, \cdots$$

- Weyl Anomalies

$$\langle T^\mu_\mu \rangle_{M_d} \sim aE + c_i W_i$$

- If SUSY $\Rightarrow \{T_{\mu\nu}, j_R^\mu\} \Rightarrow$

(a, c_i) related to $k's$

- Downside: Only available in *even* dimensions

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Field Theory

$$\text{4d } \mathcal{N} = 1 \text{ on } \Sigma_{g>1}$$

$$\rightsquigarrow$$

$$\text{2d } \mathcal{N} = (0,2)$$

Universal relation [Benini-Bobev-MC 2015]

To establish relation among central charges, 3 simple steps:

1) R-symmetry is $U(1)_R$. Assume *no Abelian flavor symmetry*:

$$I_6 = \frac{k_{RRR}}{6} c_1(R)^3 - \frac{k_R}{24} c_1(R)\, p_1(T_4)$$

2) Twist on $\Sigma_\mathfrak{g}$: $U(1)_\Sigma \subset U(1)_R \Rightarrow$

$$c_1(R) \to c_1(R) + \frac{1}{2} \mathrm{dVol}(\Sigma_g)$$

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This leads to (*anomaly matching*)

$$k_{RR} = (g-1)k_{RRR}\,, \qquad k = (g-1)k_R$$

Assuming fixed point in UV and IR: (*Ward identities*)

$$\begin{aligned} \text{4d}: &\qquad a_{4d} = \frac{9}{32}k_{RRR} - \frac{3}{32}k_R\,, \qquad c_{4d} = \frac{9}{32}k_{RRR} - \frac{5}{32}k_R \\ \text{2d}: &\qquad c_r = 3k_{RR}\,, \qquad c_r - c_l = k \end{aligned}$$

gives:

$$\begin{pmatrix} c_r \\ c_l \end{pmatrix} = \frac{16(g-1)}{3}\begin{pmatrix} 5 & -3 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} a_{4d} \\ c_{4d} \end{pmatrix}$$

- True also for compactification of $\mathcal{N} = 2, 4$.
- In the large–N limit:

$$c_r \simeq c_l \simeq \frac{32}{3}(g-1)a_{4d}$$

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$$\text{2d } \mathcal{N} = (2, 2)$$

R-symmetry is now $SU(2) \times U(1)_r$. Thus, more twists are possible:

$$\text{Pick} \qquad U(1)_\Sigma \subset \underbrace{U(1)_{R_3}}_{\alpha} \times \underbrace{U(1)_r}_{\beta}$$ [Kapustin 2006]

Assuming IR fixed point one shows: [Bobev-MC 2017]

- For α–twist:

$$\text{2d } \mathcal{N} = (2,2): \qquad \begin{pmatrix} c_r \\ c_l \end{pmatrix} = 12\,(g-1) \begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} a_{4d} \\ c_{4d} \end{pmatrix}$$

  Note $c_r = c_l = 3(g-1)d_G$ with $d_G \equiv 4(2a_{4d} - c_{4d})$ dim. of Coulomb branch of 4d theory [Shapere-Tachikawa 2008]

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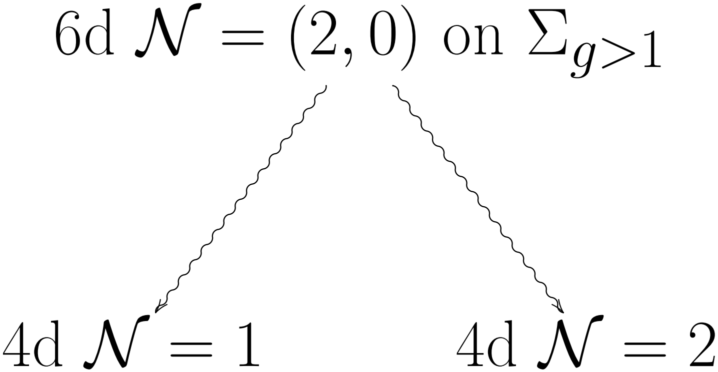
$$\text{2d } \mathcal{N} = (0,4): \qquad \begin{pmatrix} c_r \\ c_l \end{pmatrix} = 24\,(g-1) \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_{4d} \\ c_{4d} \end{pmatrix}$$

- $\frac{1}{3}\alpha + \frac{4}{3}\beta$–twist equals $\mathcal{N} = 1$ twist


6d $\mathcal{N} = (2,0)$ on $\Sigma_{g>1}$
4d $\mathcal{N} = 1$
4d $\mathcal{N} = 2$

6d→4d and 6d→2d

Pick $SO(2)_\Sigma \subset SO(2) \times SO(2) \subset SO(5)_R$

4d $\mathcal{N}=2$: $$\begin{pmatrix} a_{4d} \\ c_{4d} \end{pmatrix} = \frac{(g-1)}{72} \begin{pmatrix} 21 & -6 \\ 14 & -2 \end{pmatrix} \begin{pmatrix} a_{6d} \\ c_{6d} \end{pmatrix}$$

4d $\mathcal{N}=1$: $$\begin{pmatrix} a_{4d} \\ c_{4d} \end{pmatrix} = \frac{(g-1)}{384} \begin{pmatrix} 105 & -33 \\ 49 & -1 \end{pmatrix} \begin{pmatrix} a_{6d} \\ c_{6d} \end{pmatrix}$$

Similarly, compactifying on Kähler M_4:

2d $\mathcal{N}=(0,2)$: $$\begin{pmatrix} c_r \\ c_l \end{pmatrix} = \frac{(P_1+2\chi)}{96} \begin{pmatrix} 63 & -27 \\ 35 & -11 \end{pmatrix} \begin{pmatrix} a_{6d} \\ c_{6d} \end{pmatrix}$$

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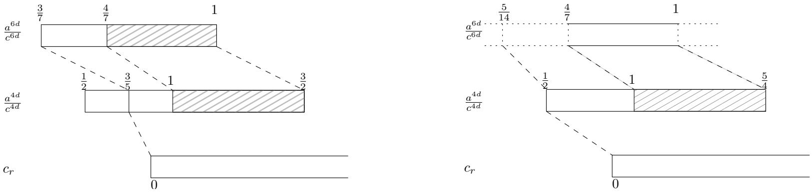
Comment: Hofman-Maldacena Bounds

These are 4d bounds on $\frac{a_{4d}}{c_{4d}}$ from energy positivity **[Hofman-Maldacena '08, '16]**



Universal flows map 4d values to (interesting?) values in 6d and 2d

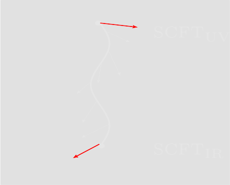
Which flows are *not* universal?

Consider theories with flavor symmetries G_F with generators F_i

- Infinite family of twists:

$$T_{back} = T_R^{\text{conf.}} + b_i F_i$$

- Complication: *Mixing* of flavor and R-symmetry along flow:



$$R_{\text{IR}} = R_{UV} + \epsilon_i(b) F_i \,, \qquad \epsilon_i = ?$$

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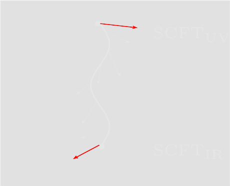

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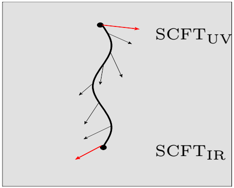

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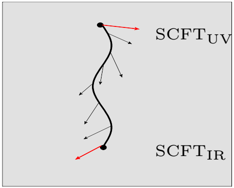

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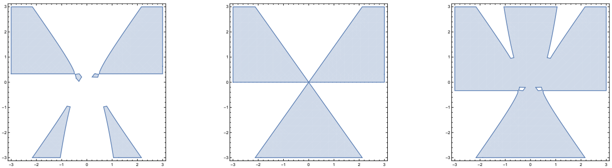
Theories unitary ($c_{l,r} > 0$) only in regions of parameter space:



Regions in $b_{1,2}$ plane ($b = 0$) where $c_R > 0$ (for $\kappa = \{1, 0, -1\}$)

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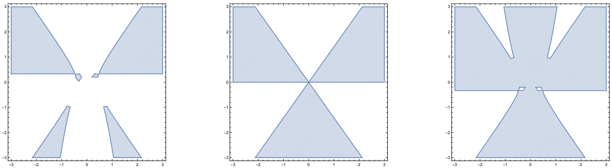

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Holography

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1) Establish *existence* of RG flows (at large N)

2) When anomalies not available, holography predicts

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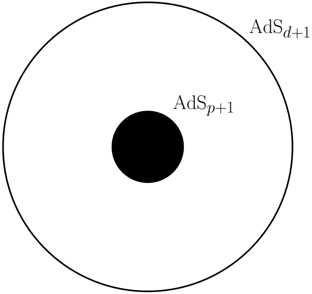

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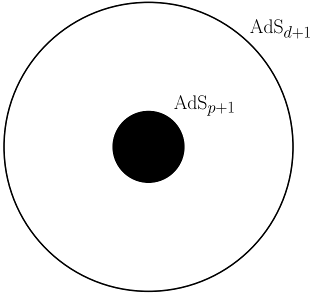

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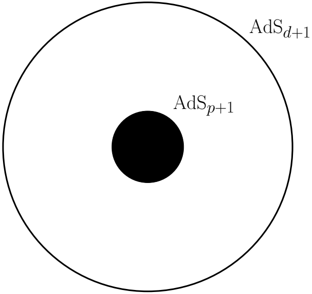

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5d $\mathcal{N} = 2$ *minimal* gauged SUGRA $(g_{\mu\nu}\,, A_\mu)$. Ansatz:

$$ds_5^2 = e^{2f(r)}(-dt^2 + dz^2 + dr^2) + e^{2g(r)} ds^2_{\Sigma_{g>1}}$$

$$F = dA = -\frac{1}{3}\mathrm{dVol}_{\Sigma_{g>1}}$$

AdS_3 fixed point: $e^{2f} = \frac{1}{r^2}e^{2f_0}\,, e^{2g} = e^{2g_0}$. Central charge:

[Brown-Henneaux]

$$c_R \simeq c_L \simeq \frac{3L_{\mathrm{AdS}_3}}{2G_N^{(3)}} \simeq \frac{32}{3}(g-1)a_{4d}\,!$$

(used holographic relations $a_{4d} \simeq c_{4d} \simeq \frac{\pi}{8G_N^{(5)}}$)

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Black holes in AdS_4 describe flows from 3d $\mathcal{N}=2$ to 1d SUSY QM

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In *massive IIA*, new solution:

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$$e^{2\lambda} \equiv (\cos(2\alpha)+3)^{1/2}(\cos(2\alpha)+5)^{1/8}\,,$$

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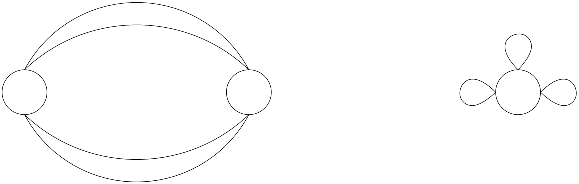
Uplifting to either M-theory or massive IIA, dual CFT_3's very different:



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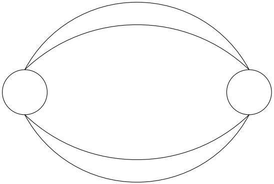

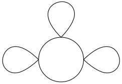

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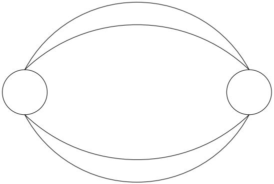

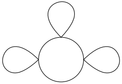

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Matrix Models

Can we test this in field theory?

- Localization on S^3: [Kapustin-Willett-Yaakov 2009]

$$Z_{S^3} = \int du e^{-ik\pi \mathrm{Tr} a^2} \prod_{\alpha} 2\sinh(\pi\alpha(a)) \prod_{\rho\in\mathcal{R}} \frac{1}{\cosh(\pi\rho(a))}$$

where $a = u + i(\Delta - 1/2)$.

- Localization on $S^1 \times \Sigma_{\mathfrak{g}}$ [Benini-Zaffaroni 2016]

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Can we test this in field theory?

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$$Z_{S^3} = \int du e^{-ik\pi \mathrm{Tr} a^2} \prod_{\alpha} 2\sinh(\pi\alpha(a)) \prod_{\rho\in\mathcal{R}} \frac{1}{\cosh(\pi\rho(a))}$$

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At large N: [Morteza-Zaffaroni 2016]

$$F_{S^1\times\Sigma_{\mathfrak{g}}}(\Delta_I,\mathfrak{n}_I) = (\mathfrak{g}-1)F_{S^3}(\Delta_I/\pi) + \sum_I \left(\frac{\mathfrak{n}_I}{1-\mathfrak{g}} - \frac{\Delta_I}{\pi}\right)\frac{\pi}{2}\frac{\partial}{\partial\Delta_I}F_{S^3}(\Delta_I/\pi)$$

Universal twist amounts to: $\mathfrak{n}_I = (1-\mathfrak{g})\Delta_I/\pi \Rightarrow$

$$F_{S^1\times\Sigma_{\mathfrak{g}}} = (\mathfrak{g}-1)F_{S^3}$$

Perfect match! [Azzurli-Bobev-MC-Min–Zaffaroni 2017]

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Summary & Outlook

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- Existence of $\mathcal{U}$niversal RG flows across (even) dimensions (exact, finite N)
- Strong evidence across odd dimensions (large N for now)
- Holography predicts nontrivial matrix model relations
- Large number of new AdS_2 M-theory/massive IIA backgrounds with CFT duals
- 5d $\rightarrow$ 3d, 1d? New relations among matrix models? $F_{S^5}/F_{\Sigma_g \times S^3}$?
- Flows between even-odd dimensions? $F_{IR} \propto a_{UV}$?

THANK YOU!