Six-dimensional CFTs and M5-branes

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Introduction

- M5-branes: open problem in string theory

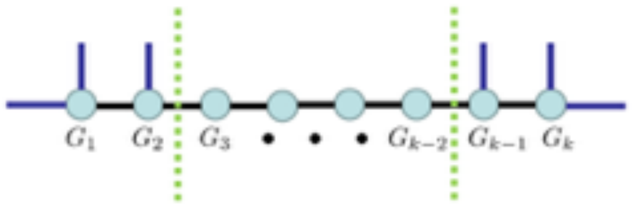
$\mathcal{N} = (2,0)$ SCFT$_6$

Recently: progress in $\mathcal{N} = (1,0)$ SCFT$_6$

- Classification results

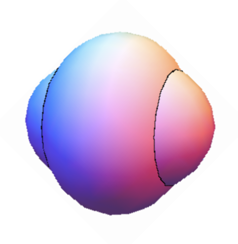
in F-theory [Heckman, Morrison, Rudelius, Vafa '15]

perturbative [Bhardwaj '15]

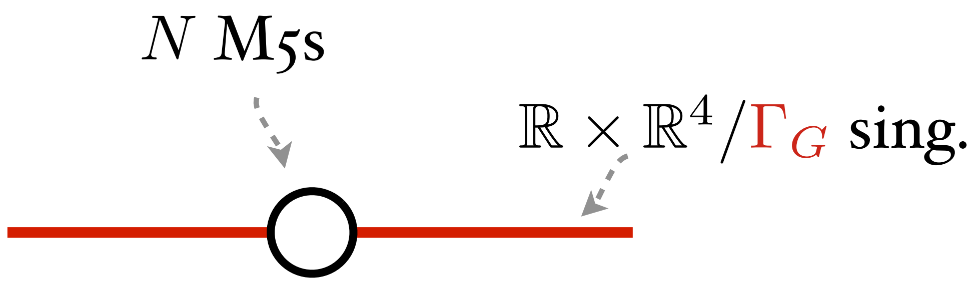


- AdS7 duals

[Apruzzi,Fazzi,Rosa,AT'13; Gaiotto, AT '14; Apruzzi,Fazzi,Passias,Rota'15;Cremonesi,AT'15]



- Applications: M5-branes at singularities



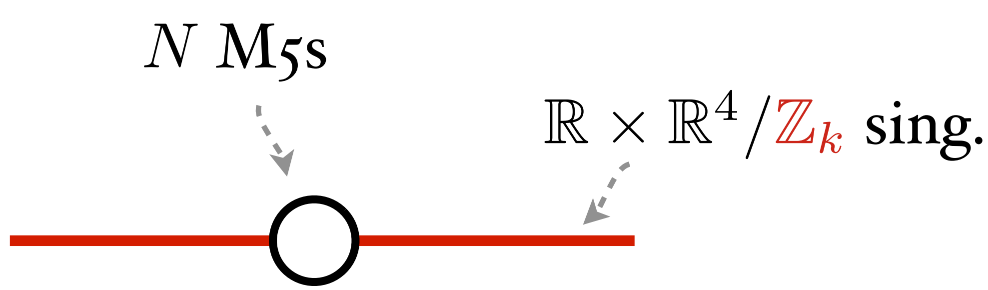
Part I: M5 'Fractionation'

Part II: More general 'nilpotent' phenomena

$$T^{G,N}_{\rho_{\mathrm{L}},\rho_{\mathrm{R}}}$$

nilpotent $\in G$

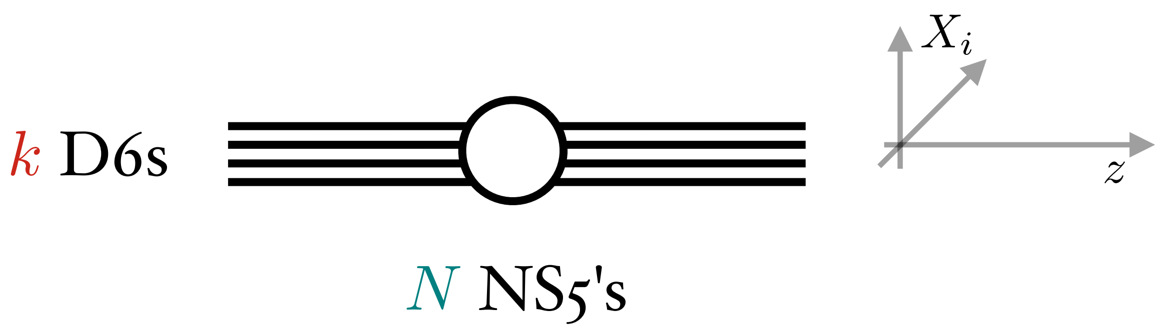
I. M5s on singularities



- superconformal
- $\mathcal{N} = (1, 0)$ supersymmetry
- number of dof: $a \sim k^2 N^3$

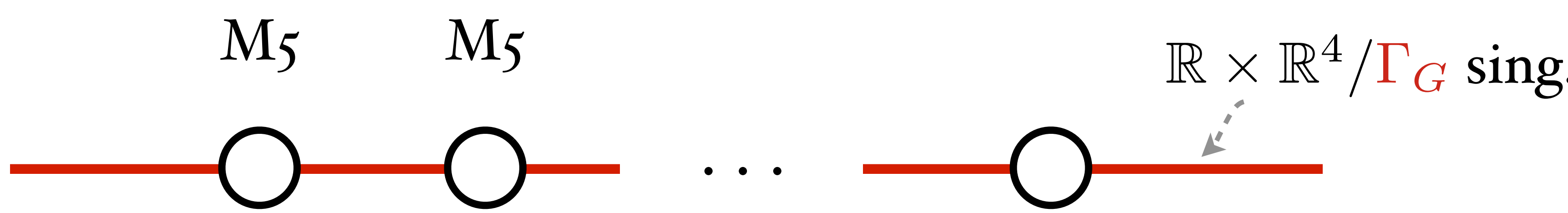
- If we reduce to IIA:

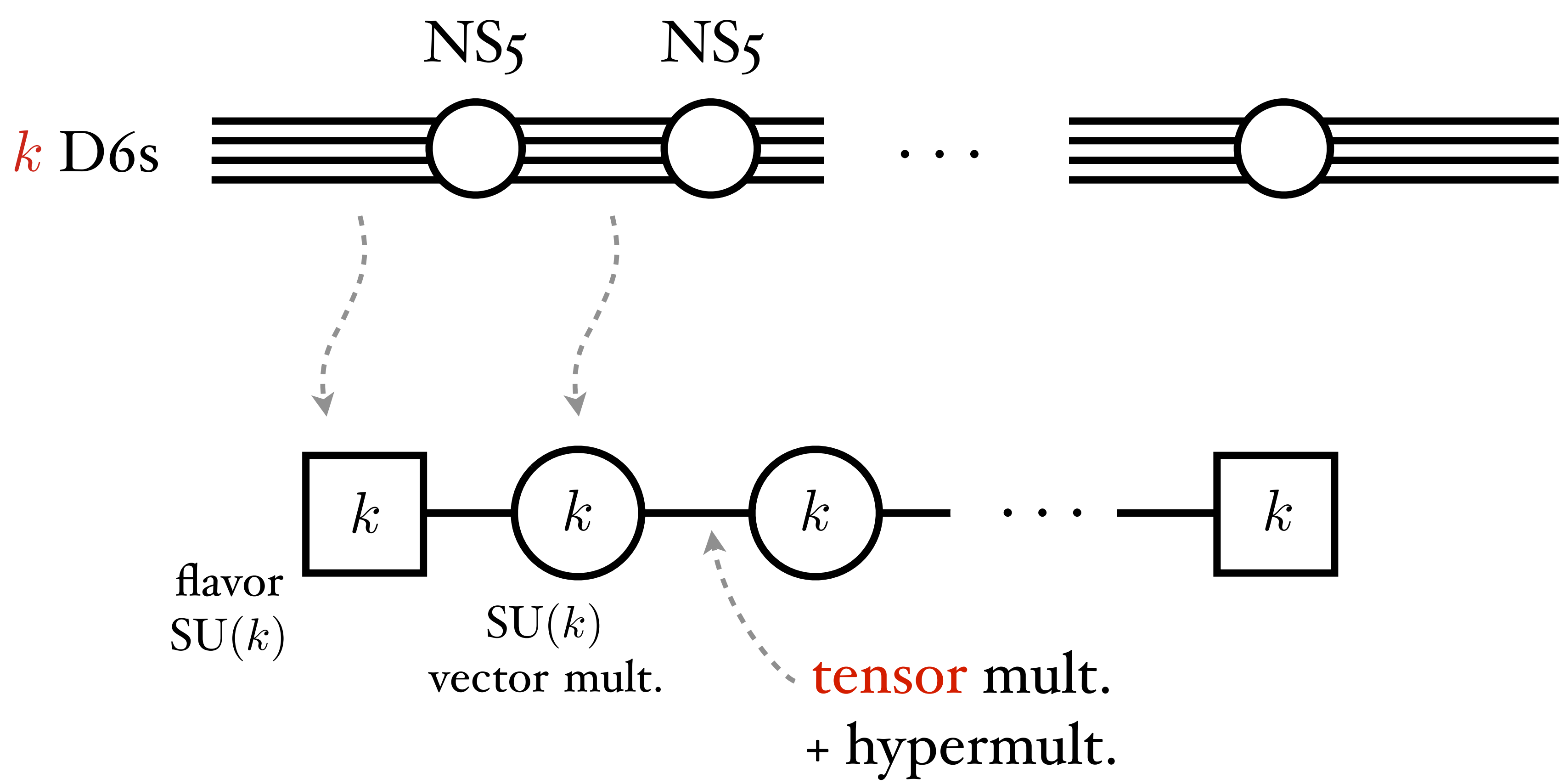
BPS equations on D6:
Nahm equations



$$\partial_z X_i = \epsilon_{ijk}[X_j, X_k]$$

- ‘Effective description’ with gauge groups by separating M5s or NS5s





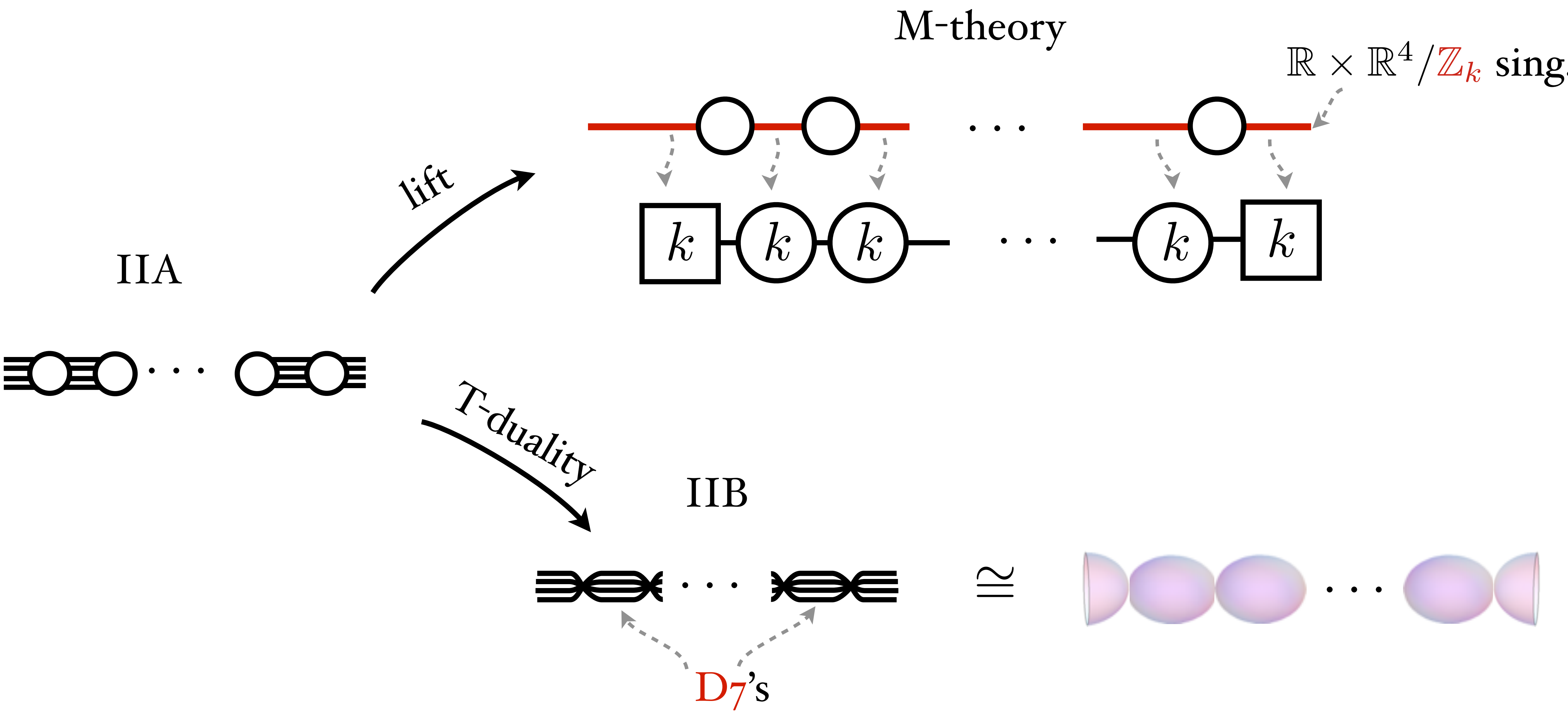
vector mult. (λ_+, A_μ)

hypermult. (q, ψ_-)

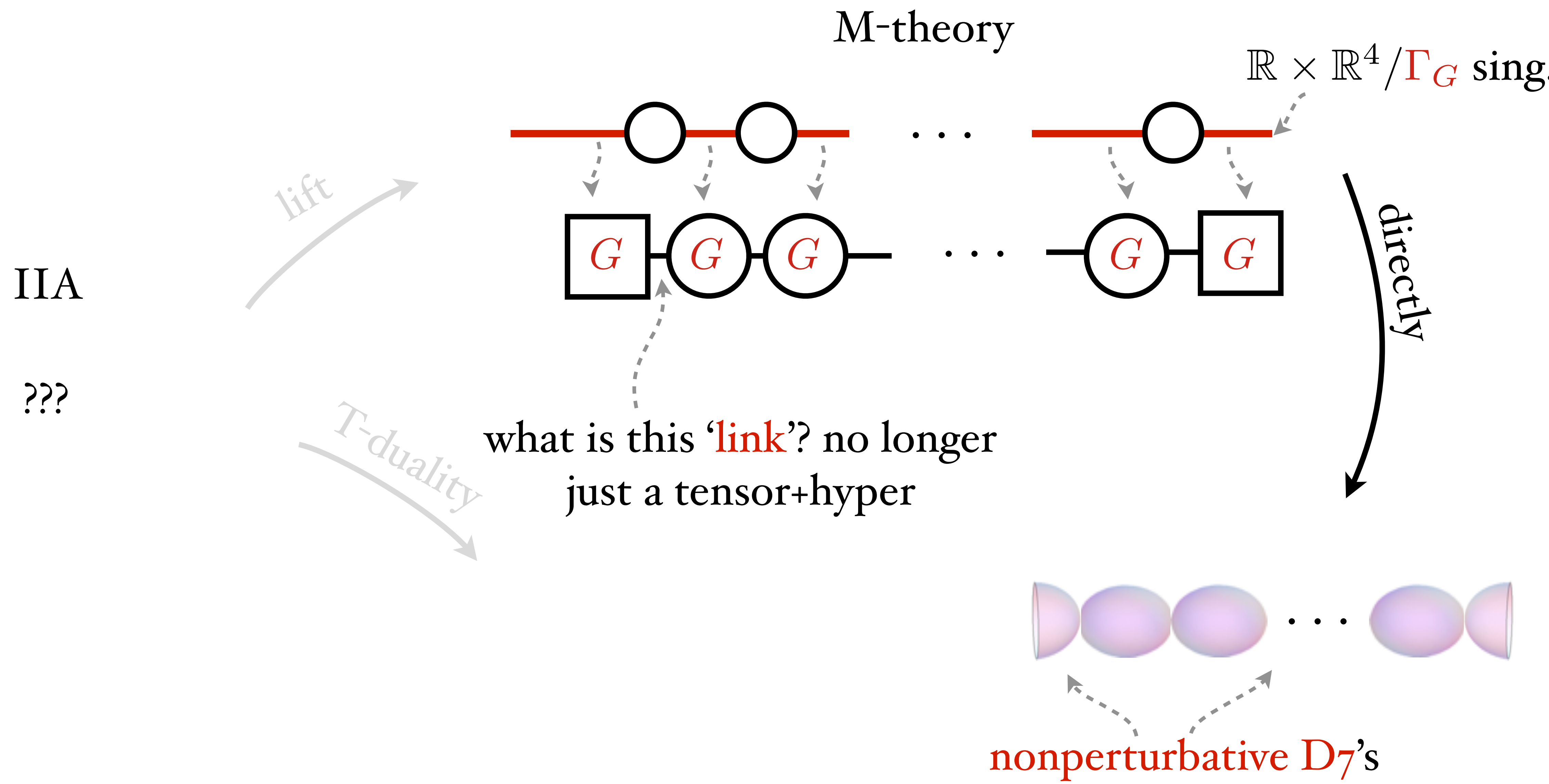
tensor mult. $(\phi, \chi_-, b^+_{\mu\nu})$

$\mathcal{L} \supset (\phi_{i+1} - \phi_i)\mathrm{Tr}F_i^2$ $\phi_i = x^6$ positions of NS5's

coincident NS5s =
strong coupling point: CFT

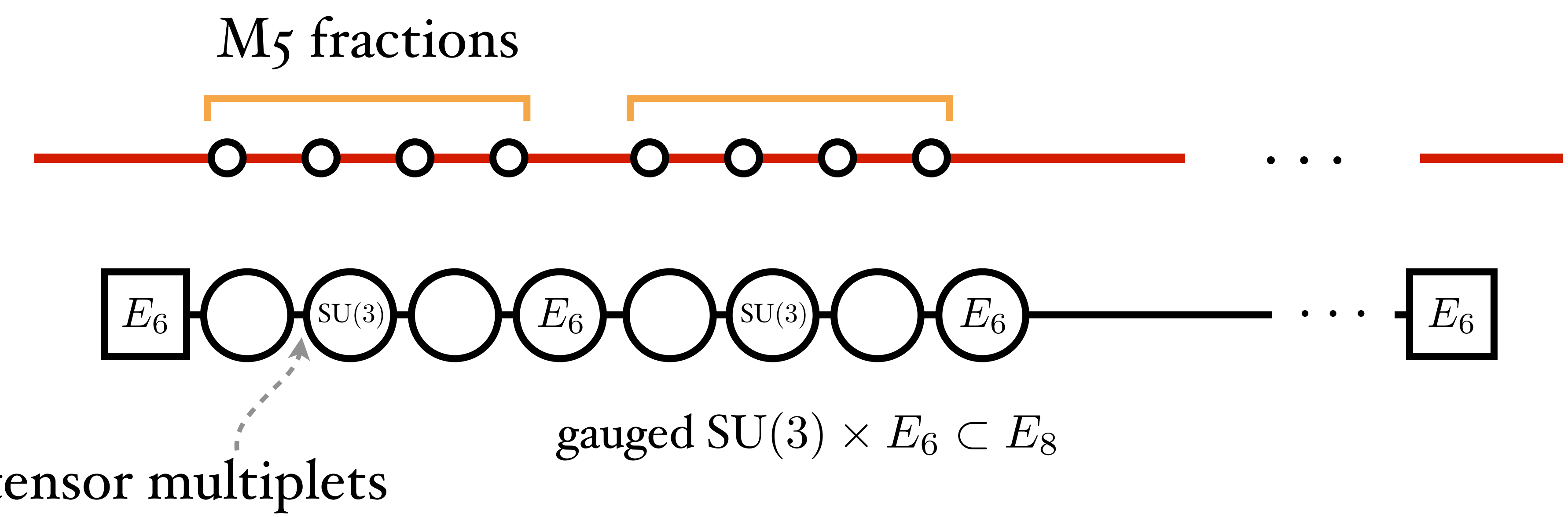

M-theory
$\mathbb{R} \times \mathbb{R}^4/\mathbb{Z}_k$ sing.
lift
k
IIA
T-duality
IIB
$\cong$
D7's

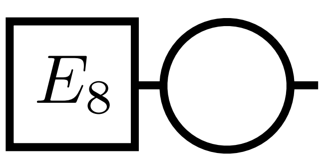
- F-theory allows to include more general gauge groups



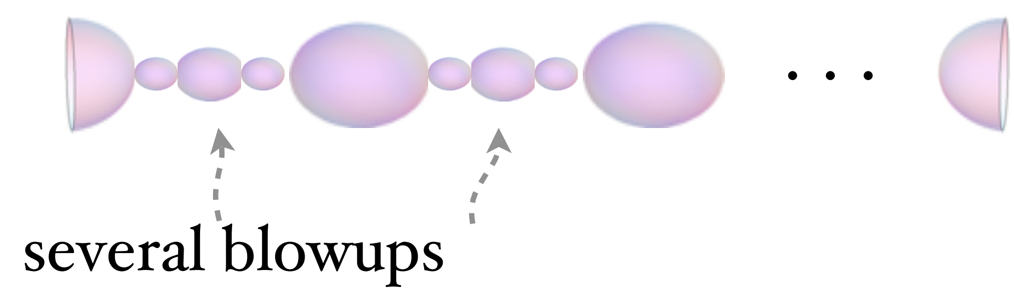
F-theory predicts new phenomenon: M5 fractionation

[del Zotto, Heckman, AT, Vafa '14]



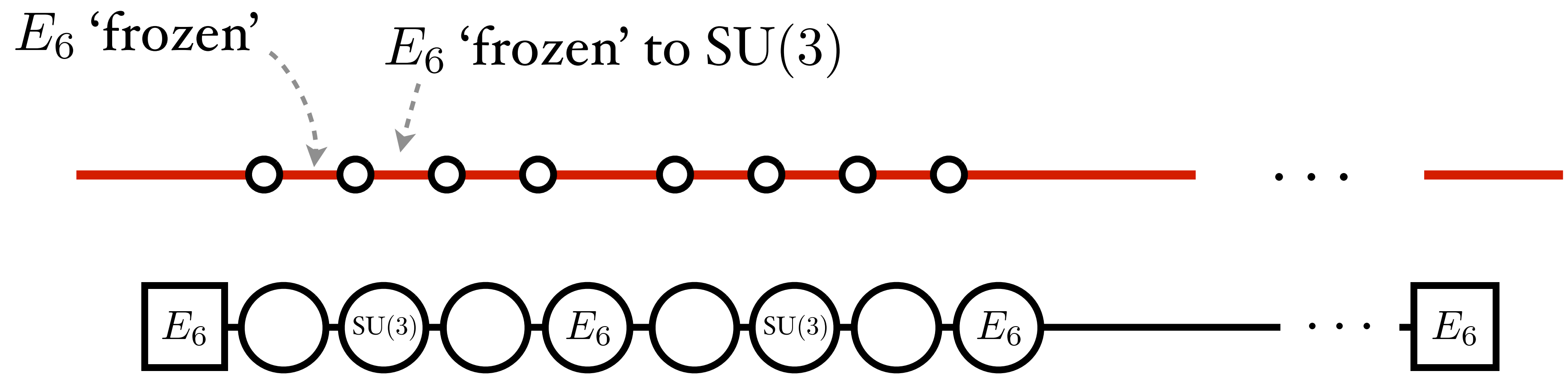


= "E-string"
(no gauge group)
E_8 flavor symmetry



It also appears for M5's near M9

a ‘discrete flux’ is created whenever
a fractional M5 is crossed



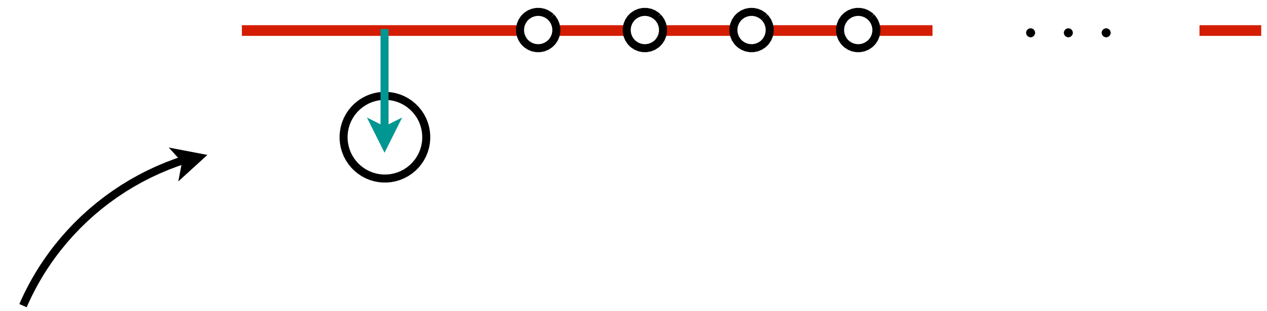
dual to ‘fractional M2’:
domain walls for gauge triples
[Ohmori, Shimizu, Tachikawa, Yonekura ’15]

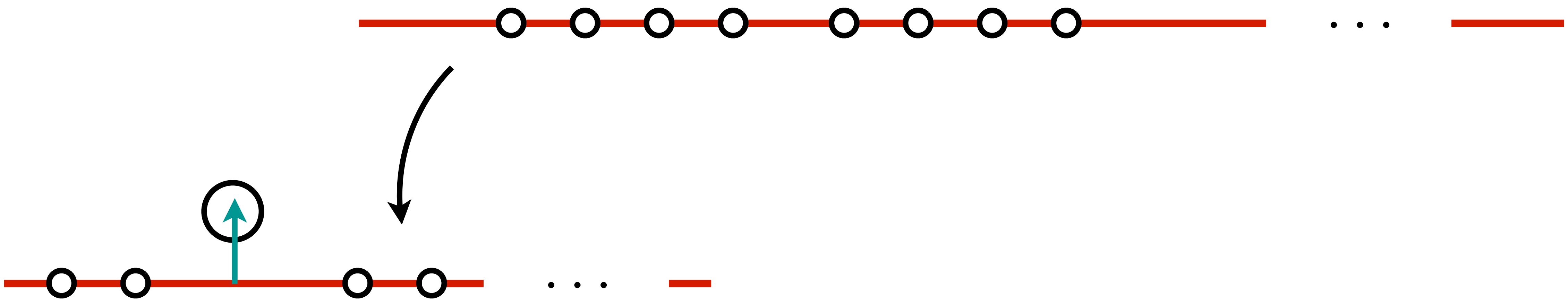
	#fractions
$\mathrm{SO}(2k)$	2
E_6	4
E_7	6
E_8	12

fractions can recombine in various ways

anomaly-matching arguments

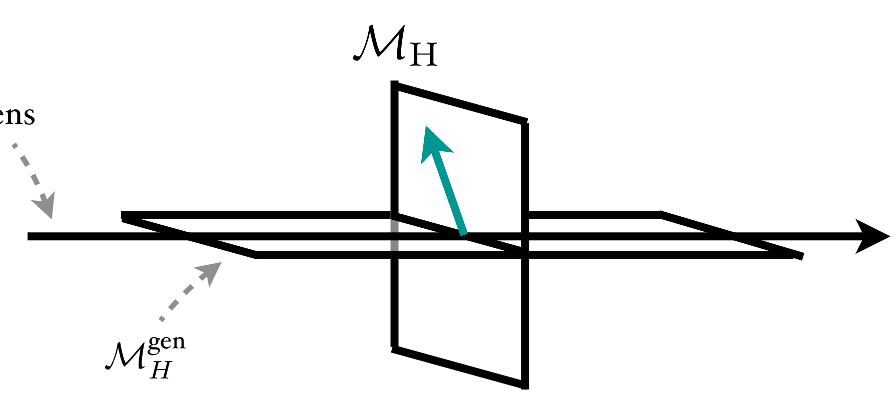
[Mekareeya, Ohmori, Shimizu, AT wip.]





field theory interpretation:

positions of M5 fractions: $\mathcal{M}_{\text{tens}}$

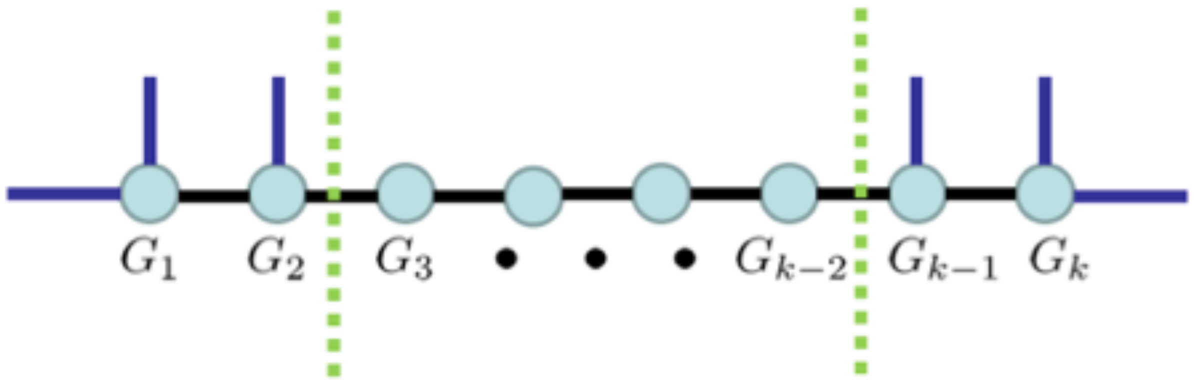


M5s

$$\dim(\mathcal{M}_H) = N + \dim(G)$$

?

- General classification:



[Heckman, Morrison, Rudelius, Vafa '15]

nodes = D or E gauge groups

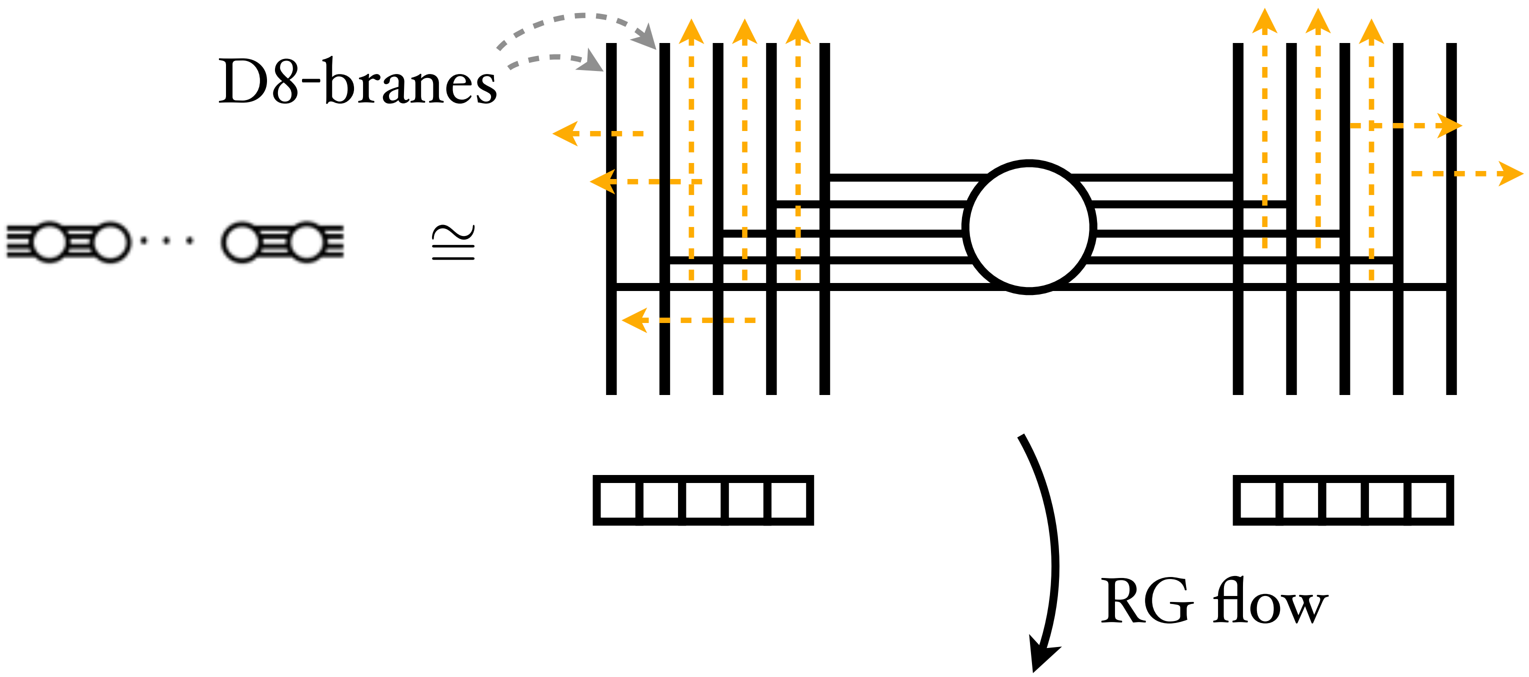
links = chains of SU groups, or 'conformal matter'

- Gauge groups go 'up and then down':

$$G_1 \subseteq G_2 \subseteq \ldots \subseteq G_m \supseteq \ldots \supseteq G_{k-1} \supseteq G_k.$$

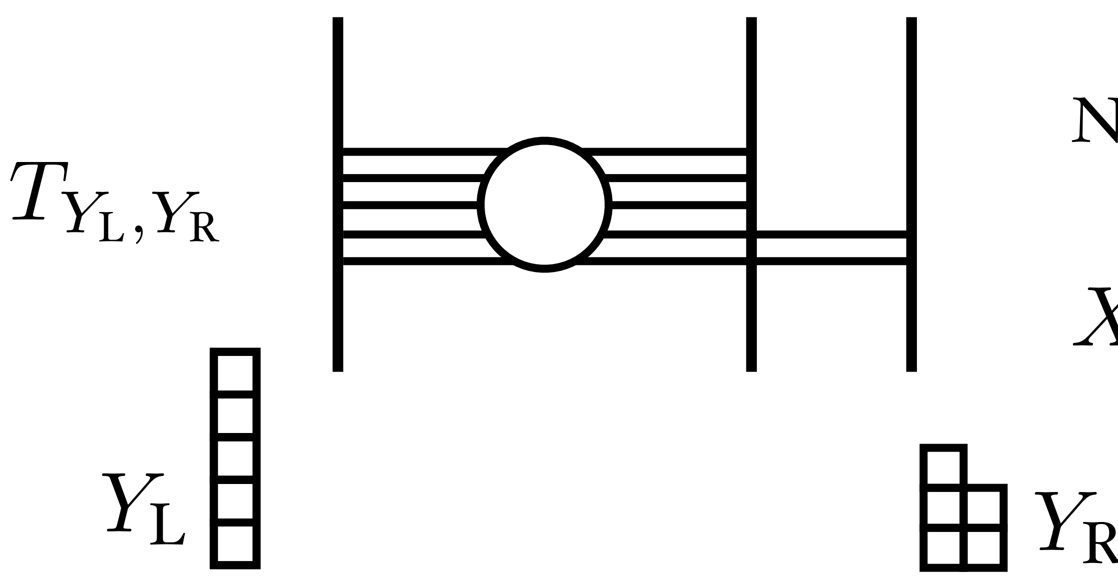
II. T-branes

- alternative realization: each D6 ends on a single D8 [Gaiotto, Witten '08; Gaiotto, AT '14]



D8-branes
||
Dirichlet boundary cond. for X_i

move away some D6 segments:
a new conformal theory!



Nahm pole boundary conditions:

$$X^i \sim \frac{t_i}{z}$$

su(2) subalgebra of su(k)

≀||

partition

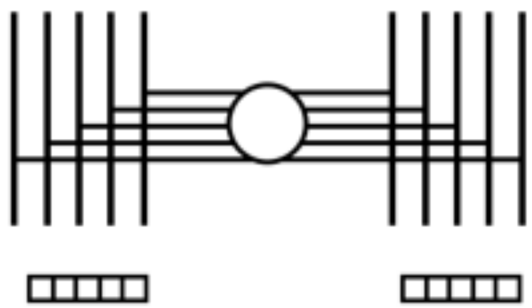
for ex.

$$t^1 + i t^2 = \left(\begin{array}{ccc|cc} & 1 & & & \\ & & 1 & & \\ & & & & \\ \hline & & & & 1 \\ & & & & \end{array}\right)$$

[Gaiotto, Witten '08; Gaiotto, AT '14]

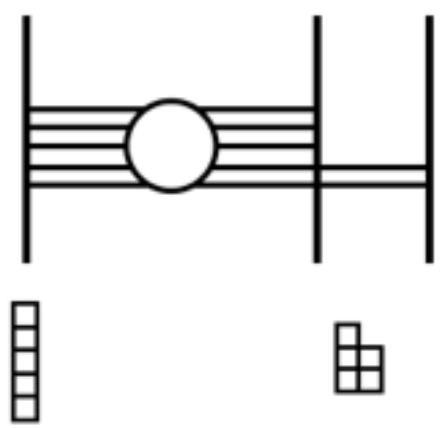
[(Blum,)Intriligator '97, Hanany, Zaffaroni '97, Brunner, Karch '97...]

these flows are triggered by vevs in the ‘missing’ directions



$$\dim(\mathcal{M}_H) = N + \dim(G)$$

RG flow

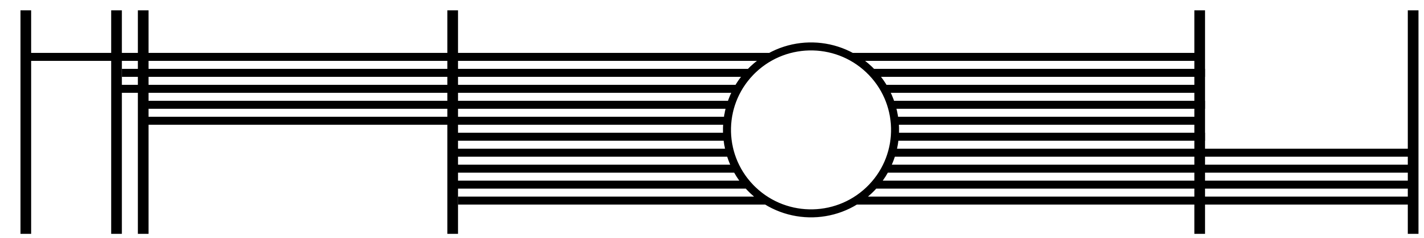


$$\dim(\mathcal{M}_H) = N + \dim(G) - d_{Y_\mathrm{L}} - d_{Y_\mathrm{R}}$$

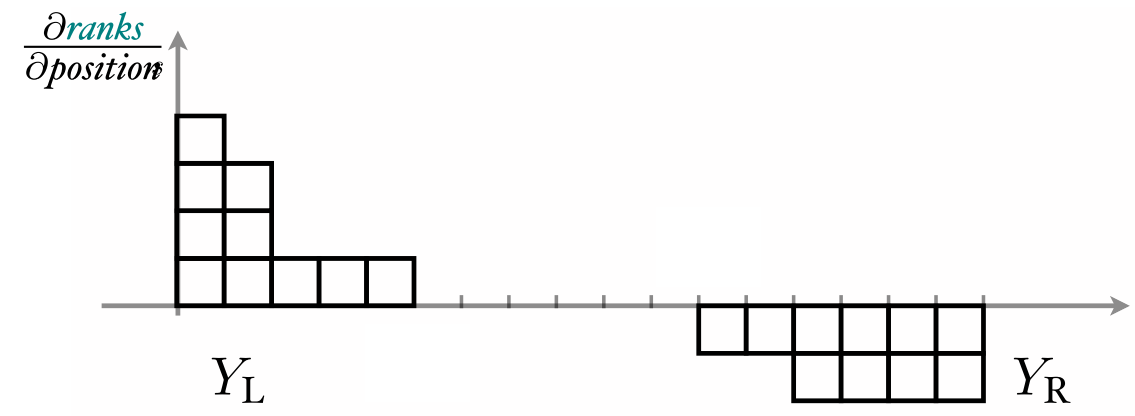
dimensions of nilpotent orbits

AdS7 duals

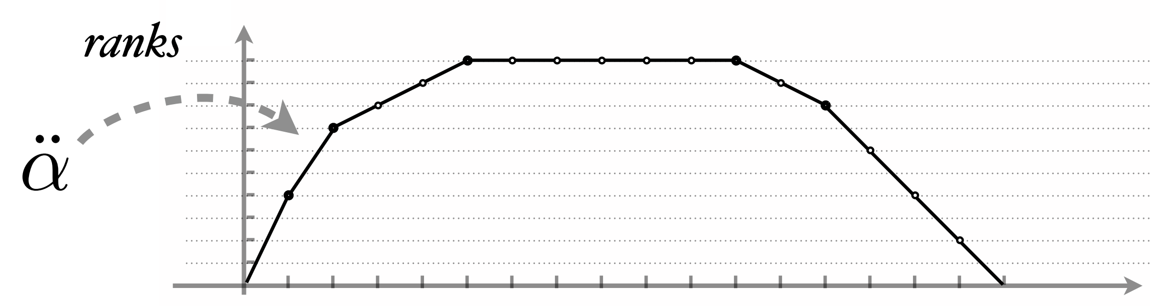
[Apruzzi,Fazzi,Rosa,AT'13; Gaiotto, AT '14; Apruzzi,Fazzi,Passias,Rota'15;Cremonesi,AT'15]



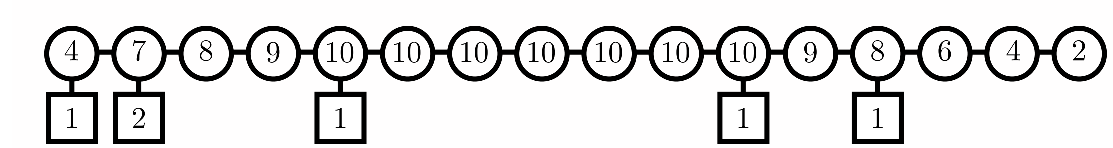
- Start with partitions:



- 'integrate': gauge groups.

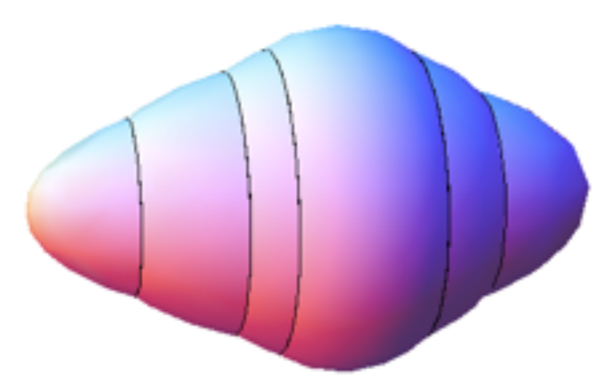


$$G_1 \subseteq G_2 \subseteq \ldots \subseteq G_m \supseteq \ldots \supseteq G_{k-1} \supseteq G_k.$$



$$ds^2 = 8\sqrt{-\frac{\ddot{\alpha}}{\alpha}}ds^2_{\mathrm{AdS}_7} + \underbrace{\sqrt{-\frac{\alpha}{\ddot{\alpha}}}dz^2 + \frac{\alpha^{3/2}(-\ddot{\alpha})^{1/2}}{\sqrt{2\alpha\ddot{\alpha}-\dot{\alpha}^2}}ds^2_{S^2}}$$

'3d generalized Sasaki–Einsten'



A check: anomalies

[Cremonesi, AT '15]

- Cancel gauge anomalies [Green,Schwarz,West'86, Sagnotti '92]

[Intriligator '14, Ohmori, Shimizu, Tachikawa, Yonekura '14]

- Compute global $\underbrace{\mathrm{SU}(2)_{\mathrm{R}}}$ and $\underbrace{\text{gravitational anomaly}}$

conformal anomaly a

[Cordova, Dumitrescu, Intriligator '15]

$\langle T^\mu_\mu \rangle \sim a$ Euler+ Weyl comb.

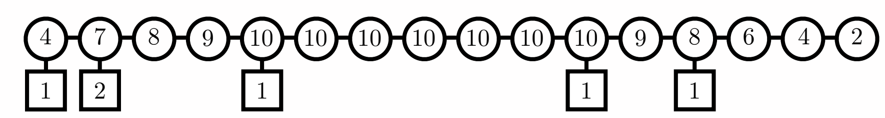
Cartan of $\mathrm{SU}(N)$

ranks of gauge groups

- $a = \frac{192}{7} \sum_{i,j} C^{-1}_{ij} r_i r_j +$ subleading

$\wr$

(# gauge groups)^3



reproduces the famous cubic scaling.

- We proved it always reproduces the holographic result. Heuristically:

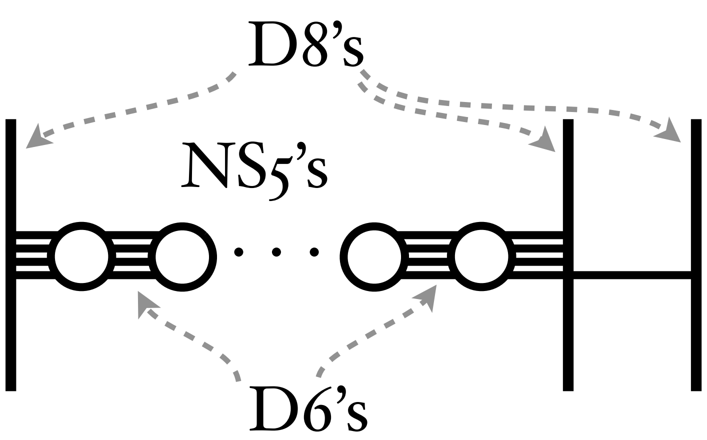
$C_{ij} = 2\delta_{ij} - \delta_{i-1,j} - \delta_{i+1,j}$ 'discrete double derivative'

$$\sum \underbrace{r_i}_{\ddot{\alpha}} \underbrace{C^{-1}_{ij} r_j}_{\alpha} \longrightarrow \int \ddot{\alpha}\alpha \;\propto \int_{M_3} e^{5A-2\phi} \quad \checkmark$$

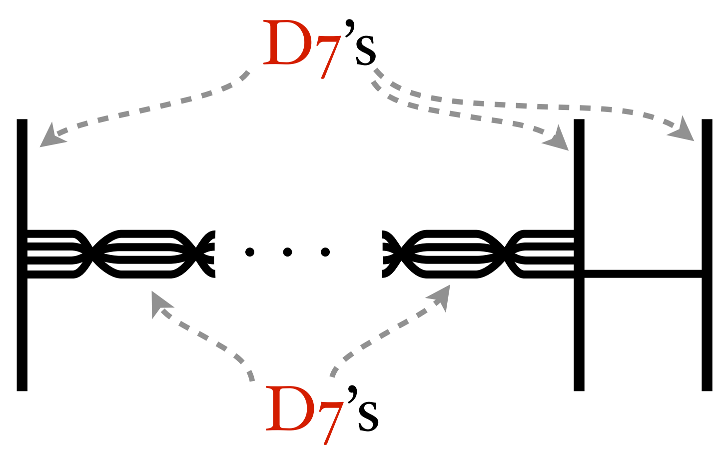
IIB dual:

[del Zotto, Heckman, AT, Vafa '14]

T-duality

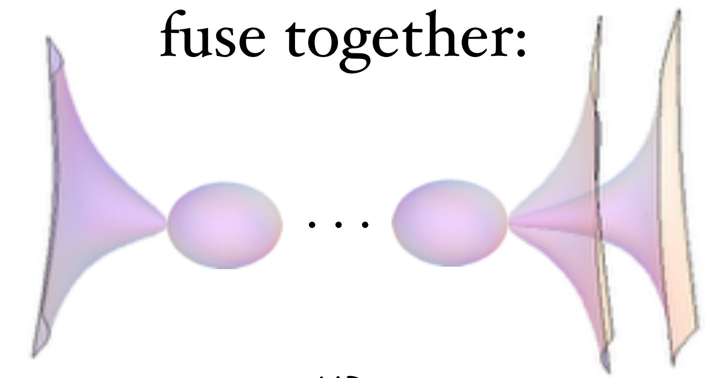


$\rightarrow$

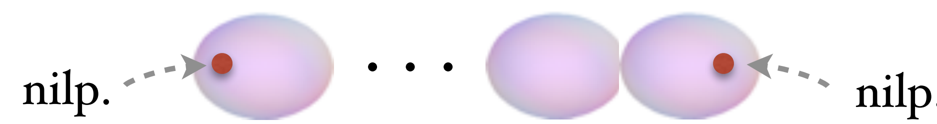


but actually the D7's fuse together:

$\cong$



$\cong$



Nahm's equations $\rightarrow$ Hitchin's equations

Chain of curves,
decorated by Hitchin poles

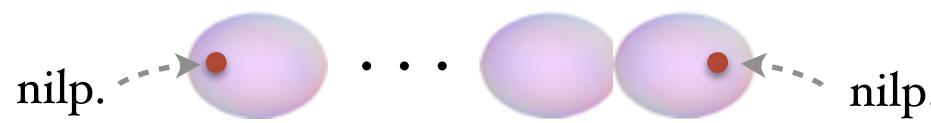
$$\phi = X^1 + iX^2 \sim \frac{t}{z - z_0}$$

residue is nilpotent

$|||$

"T-brane"

F-theory predicts
[del Zotto, Heckman, AT, Vafa '14]



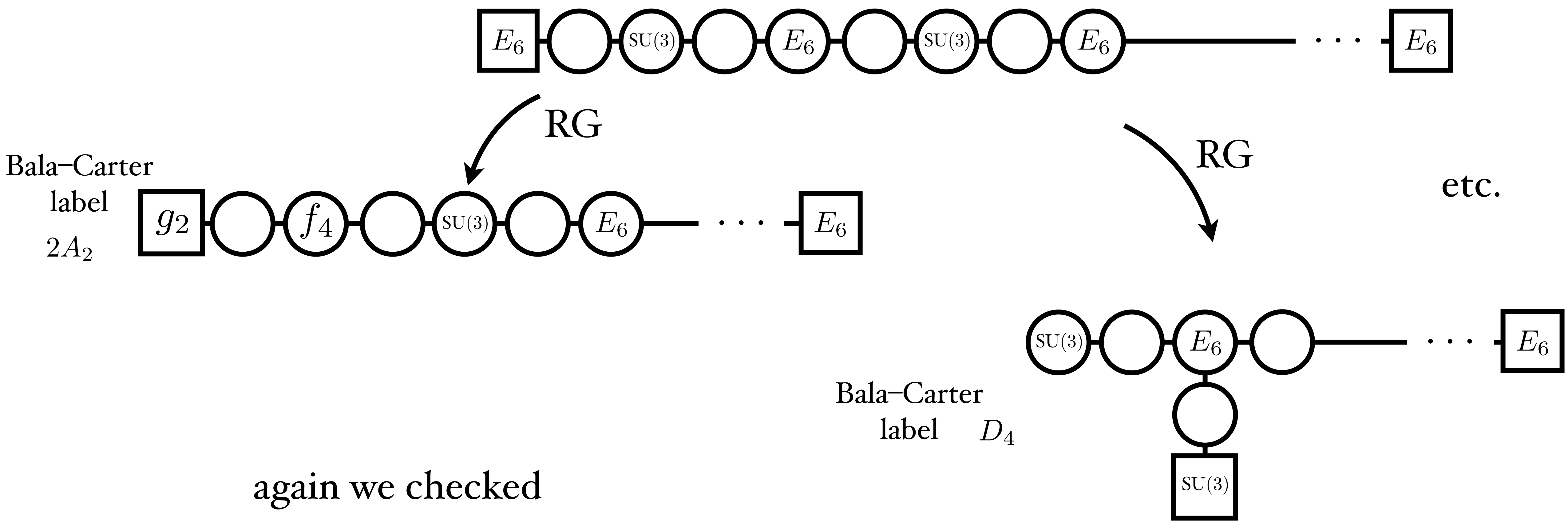
$\exists$ SCFT $T^{G}_{\underline{Y_\mathrm{L}},\underline{Y_\mathrm{R}}}$ any ADE

nilpotent elem. $\in G$

[classified by mathematicians: "Bala–Carter labels"]

for example, E_6:

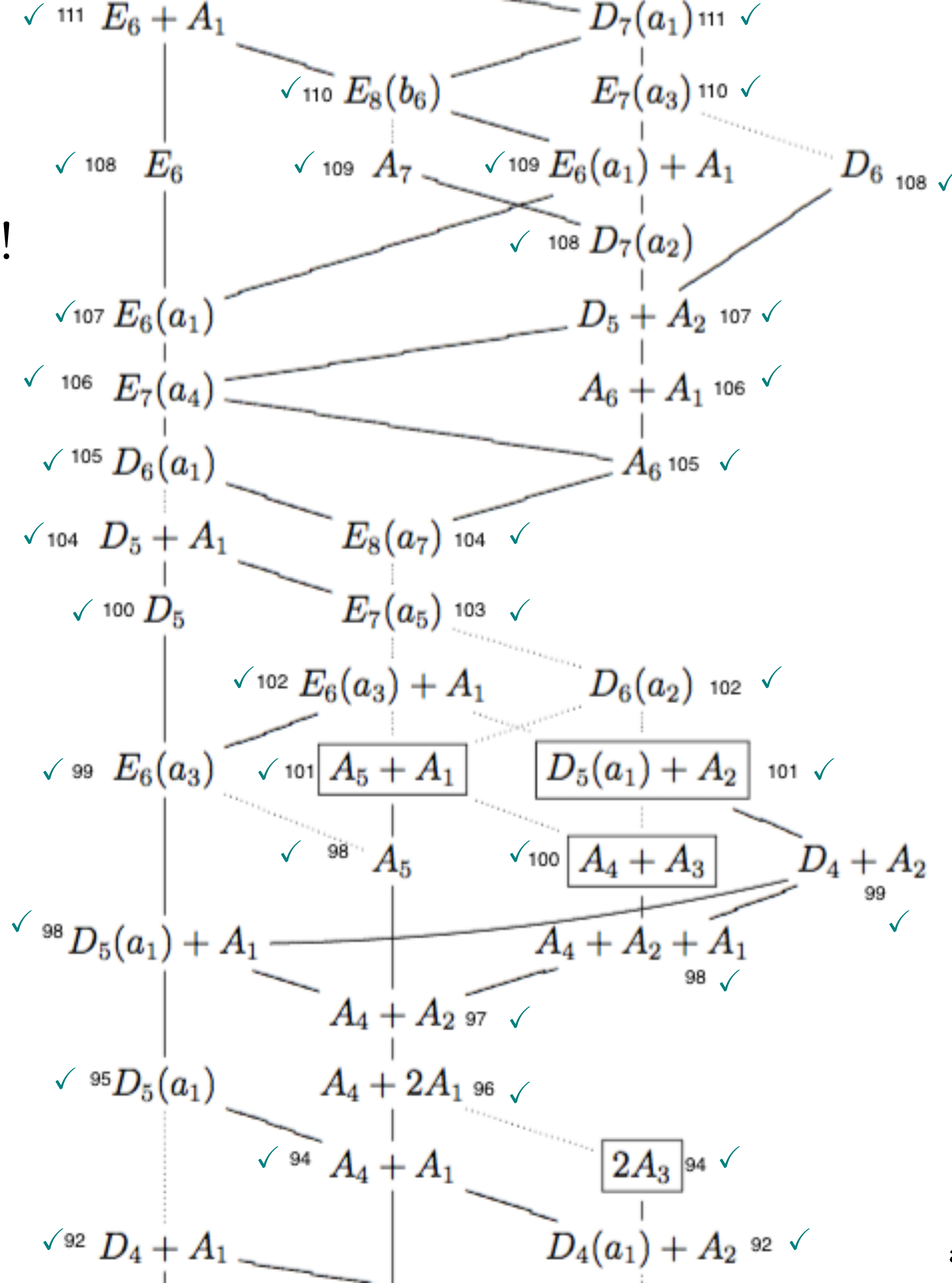
[Heckman, Rudelius, AT '16]



again we checked

$$\dim(\mathcal{M}_H) = N + \dim(G) - d_{Y_\mathrm{L}} - d_{Y_\mathrm{R}}$$

we really checked!

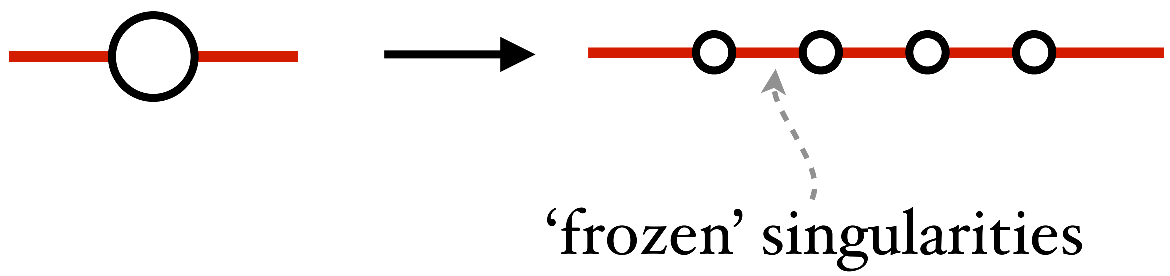


[all dimensions are quaternionic]

Conclusions

- Recent progress in 6d SCFTs ⇨ interesting phenomena for M5 on singularities

- M5 fractionation

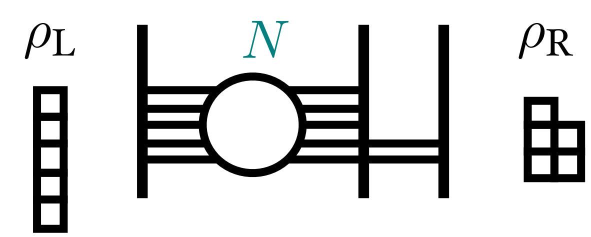


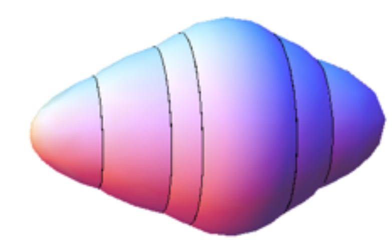
- More general Higgs phenomena

$T^{G,N}_{\rho_{\mathrm{L}},\rho_{\mathrm{R}}}$

nilpotent ADE elements

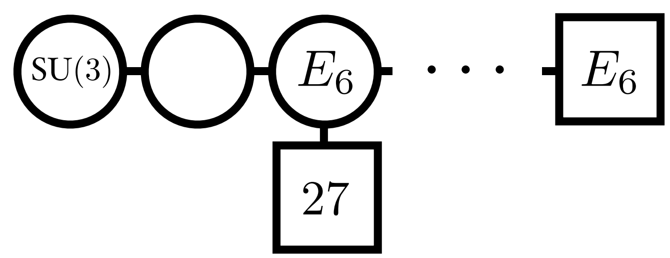
- In the A_k case:
  - nilpotent = pattern of D6s ending on D8s
  - AdS7 duals systematically known





- In the D_k, E_k cases:

  more exotic ingredients



- What implications for M5-dynamics?