

N=1 superconformal theories from six dimensions

Simone Giacomelli

ICTP

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Based on: M. Fazzi and S.G. arXiv:1609.08156 [hep-th],
S.G. arXiv:1409.3077 [hep-th],
G. Bonelli, S.G., K. Maruyoshi and A. Tanzini
arXiv:1307.7703 [hep-th].

Field theories in 4d from six dimensions

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We can build a large class of SCFT's in four dimensions by compactifying the $\mathcal{N} = (2, 0)$ theory on a Riemann surface with punctures (i.e. codimension two defects).

D. Gaiotto '09.

This approach simplifies the study of dualities and sheds light on the theories studied holographically by Maldacena and Nunez.

D. Gaiotto and J. Maldacena; F. Benini, Y. Tachikawa and B. Wecht '09.

In 2012 Bah, Beem, Bobev and Wecht (BBBW) found a large class of $\mathcal{N} = 1$ theories generalizing MN solutions, which they studied holographically and (in part) field theoretically.

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Topological twist: Consider $\mathcal{N} = (2, 0)$ of type A_{N-1}

$$SO(3, 1) \times SO(2) \times SO(2) \times SO(3) \subset SO(5, 1) \times SO_R(5)$$

Identify $SO(2)_{\text{diag}}$ with the holonomy of a Riemann surface $\mathcal{C}$
 $\rightarrow$ 4d $\mathcal{N} = 2$ SCFT with $SO(2) \times SO(3)$ global symmetry!
Equivalently, consider M-theory on $\mathbb{R}^7 \times T^*(\mathcal{C})$ and wrap N M5
branes on $\mathbb{R}^4 \times \mathcal{C}$.

Pants decomposition of $\mathcal{C} \rightarrow$ Three-punctured spheres (i.e.
matter sectors) connected by tubes (i.e. $SU(N)$ gauging).

Punctures carry global symmetry (subgroup of $SU(N)$) and are
classified by partitions of N .

Class $\mathcal{S}$ theories

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T_N is the three-punctured sphere with $SU(N)^3$ symmetry. All other trinions can be obtained from T_N via higgsing.

$$\tilde{q}_1 q_1 \leftrightarrow \mu_A; \quad \tilde{q}_{N-1} q_{N-1} \leftrightarrow \mu_B; \quad q_1 \dots q_{N-1} \leftrightarrow Q_{ij1}$$

From F-terms on the lagrangian side we get

Chiral ring relations for T_N theory

$$\begin{aligned} \text{Tr } \mu_A^k &= \text{Tr } \mu_B^k = \text{Tr } \mu_C^k, \\ (\mu_A)_i^{i'} Q_{i'jk} &= (\mu_B)_j^{j'} Q_{ij'k} = (\mu_C)_k^{k'} Q_{ijk'}, \end{aligned}$$

T_N theory

$\mathcal{N}=1$ superconformal theories from six dimensions

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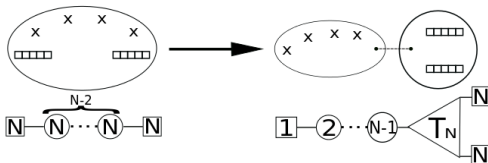
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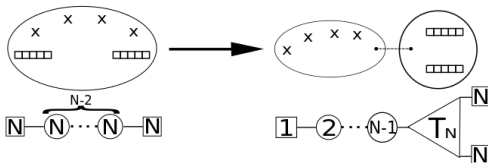
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We consider M-theory on $\mathbb{R}^5 \times CY_3$, where

$$CY_3 = L_1(\mathcal{C}) \oplus L_2(\mathcal{C}); \quad \deg(L_1) + \deg(L_2) = 2g - 2$$

and wrap N M5 branes on $\mathcal{C}$. We obtain an $\mathcal{N} = 1$ SCFT!

- We compactify two directions: $\mathbb{R}^5 \rightarrow \mathbb{R}^3 \times S^1 \times S^1$,
- go to Type IIA shrinking the S^1 wrapped by the M5s,
- T-duality on the other circle $\rightarrow$ D5 branes on $\mathbb{R}^3 \times S^1 \times \mathcal{C}$.

$$S = \int d^2z \epsilon^{\bar{i}j} \text{Tr}(\Phi_i D_{\bar{z}} \Phi_j); \quad \Phi_i \in \Gamma(L_i \otimes \text{Ad}(SU(N))).$$

Moduli space of vacua = Moduli space of generalized Hitchin system: $(D_{\bar{z}}\Phi_i = 0; \quad [\Phi_1, \Phi_2] = 0)$

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Spectral curve

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- $D_{\bar{z}}\Phi_1 = 0 \rightarrow \det(x_1 - \Phi_1) = 0$
- $D_{\bar{z}}\Phi_2 = 0 \rightarrow \det(x_2 - \Phi_2) = 0$

As in $\mathcal{N} = 2$ case the coefficients of spectral equations are vev of chiral operators.

- $[\Phi_1, \Phi_2] = 0 \rightarrow$ chiral ring relations for the $\mathcal{N} = 1$ theory.
Geometrically, the spectral curve is N-sheeted cover of $\mathcal{C}$.

The superpotential of the 4d theory is given by the integral of holomorphic three-form $\Omega = dx_1 dx_2 dz$

E. Witten '97.

$$\mathcal{W} = \int_B \Omega; \quad \partial B = \Sigma \cup \Sigma_0.$$

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All these theories have $U(1)_1 \times U(1)_2$ global symmetry (rotating the fibers of L_1 and L_2).

- We assume the R-symmetry is a combination of them.
- The superpotential has R-charge $2 \rightarrow R(\Omega) = 2$.

As a consequence $R(x_1) = 1 + \epsilon$ and $R(x_2) = 1 - \epsilon$.

For $\mathcal{N} = 1$ SCFTs a and c can be extracted from 't Hooft anomalies:

Anselmi, Freedman, Grisaru, Johansen '97.

$$a = \frac{3}{32}(3 \operatorname{Tr} R^3 - \operatorname{Tr} R); \quad c = \frac{1}{32}(9 \operatorname{Tr} R^3 - 5 \operatorname{Tr} R).$$

a-maximization: We compute the trial central charge $a(\epsilon)$ and then maximize w.r.t. ϵ .

K. Intriligator, B. Wecht '03.

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New three-punctured spheres

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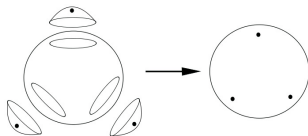
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We get a three-punctured sphere by gluing three punctured disks to a sphere with three holes.

Depending on how we glue the line bundles we get either $\mathcal{N} = 2$ or $\mathcal{N} = 1$ three-punctured spheres $\implies$ We assign a $\pm$ sign to the sphere and to each puncture!

K. Yonekura '13.

$$p \equiv \deg(L_1) = s_1 - p_1; \quad q \equiv \deg(L_2) = s_2 - p_2.$$

There are two types of gluing:

- Connecting spheres of the same kind $\rightarrow \mathcal{N} = 2$ gauging.
- Connecting spheres of opposite kind $\rightarrow \mathcal{N} = 1$ gauging with superpotential $\text{Tr } \mu_1 \mu_2$.

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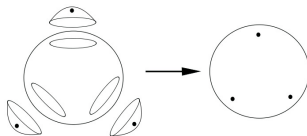
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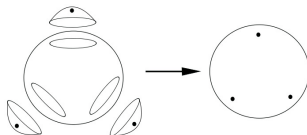
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$\mathcal{N} = 1$ three-punctured spheres are defined as follows:

- Given a sphere of one kind, we replace all punctures of the other kind with maximal punctures ($\mathcal{N} = 2$ theory).
- For every “replaced” puncture we add a multiplet M in the adjoint of $SU(N)$ with superpotential $\text{Tr } \mu M$.
- We turn on a suitable nilpotent vev for each M .

SCFT's are labelled by $(s_1, s_2, p_1, p_2)!$

“Swap duality” of Gadde, Maruyoshi, Tachikawa, Yan (2013).

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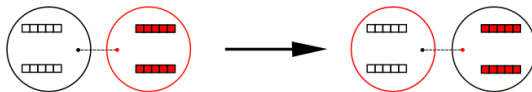
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Accessible VS Inaccessible theories

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We can construct SCFT's labelled by surfaces (without punctures) of any genus and $p > 0$.

Bah, Beem, Bobev, Wecht '12.

Gluing spheres of type $\pm$ we get surfaces without punctures with $g > 1$ and $p(=s_+), q(=s_-) > 0$

We call these theories accessible!

How can we construct inaccessible theories?

Accessible VS Inaccessible theories

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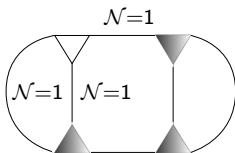
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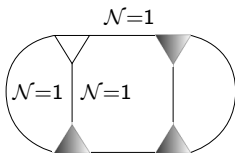
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Start from T_N with a “flipping field” M ($p = q = -1$) and close the puncture giving a nilpotent to M , getting a theory with $SU(N)^2$ symmetry we call D_N .

- R-symmetry mixes with a cartan of the broken $SU(N)$.
- Some components of μ combine with $SU(N)$ current.
- All components of M except $M_2, \dots, M_N$ decouple (**Goldstone multiplets**).

$$\langle M \rangle + M = \begin{pmatrix} 0 & 1 & 0 & \dots \\ M_2 & 0 & 1 & \dots \\ \vdots & \ddots & \ddots & 1 \\ M_N & \dots & M_2 & 0 \end{pmatrix}$$

Central charges = T_N contribution (with new R-symmetry) + contribution from multiplets M_i .

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$$\langle M \rangle + M = \begin{pmatrix} 0 & 1 & 0 & \dots \\ M_2 & 0 & 1 & \dots \\ \vdots & \ddots & \ddots & 1 \\ M_N & \dots & M_2 & 0 \end{pmatrix}$$

Central charges = T_N contribution (with new R-symmetry)
+ contribution from multiplets M_i .

Inaccessible theories revisited

$\mathcal{N}=1$ superconformal theories from six dimensions

Simone Giacomelli

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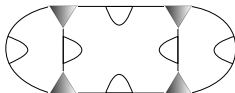
$\mathcal{N} = 2$ theories

$\mathcal{N} = 1$ theories

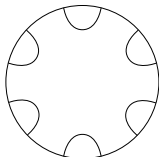
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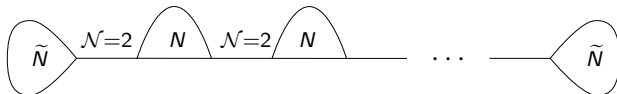
Theory with $g = 3$, $p = 10$ and $q = -6$



Theory with $g = 1$ and $p = -q = 6$



Theory with $g = 0$, $p = 2 + n$ and $q = -(4 + n)$



Central charges and unitarity bound violations

N=1 superconformal theories from six dimensions

Simone Giacomelli

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The a and c central charges of our models match those of inaccessible theories computed by BBBW integrating the anomaly polynomial of the 6d theory on $\mathcal{C}$!

PUZZLE: The theory with $g = 0$, $N = 2$ and $p = 1$ violates the Maldacena-Hofman bound ($0.5 \leq a/c \leq 1.5$)!

This theory is $\widetilde{D}_2$, which describes a set of free chiral multiplets, so there are emergent $U(1)$ symmetries!

We find unitarity bound violations in several other cases:

- Theories on the sphere with $p = 1$ ($\widetilde{D}_N$) $\forall N$.
- Theories on the sphere with $p = 2$ and $N > 2$.
- Theory on the torus with $p = 1$ and $N = 2$.

The latter is $SU(2)$ SYM with 2 adjoints and a singlet plus a free chiral multiplet ($\mathcal{W} = \text{Tr}([A, B]^2) + x \text{Tr} B^2$).

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Inaccessible theories and M2-branes

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Our SCFT's always include a “heavy operator” associated with an M2-brane wrapping $\mathcal{C}$. Its dimension can be computed (at large N) holographically!

For accessible theories it is the product of trifundamentals Q_{ijk} of T_N .

For inaccessible theories we would like to include Q_{ijk} operator of D_N theory, however...

PUZZLE

The trifundamental of T_N decomposes as N bifundamentals, all with different R-charge! Which one should we use?

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T_N theory with “flipping fields”

$\mathcal{N}=1$ superconformal theories from six dimensions

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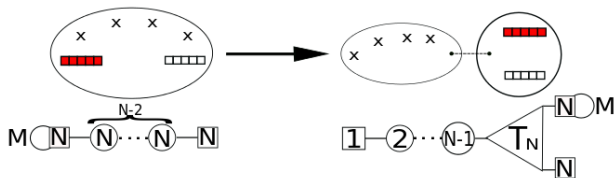
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F-terms equations on lagrangian side imply

$$Mq_1 \dots q_{N-1} = 0; \quad q_1 \dots q_{N-1}(\tilde{q}_{N-1}q_{N-1}) = 0$$

Chiral ring relations for $T_N + M$

$$\text{Tr } \mu_B^k = \text{Tr } \mu_C^k = 0,$$

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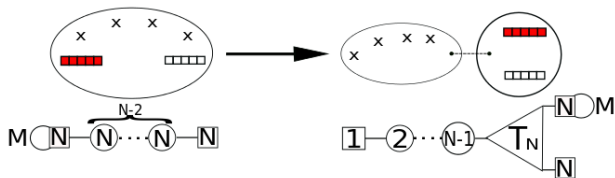
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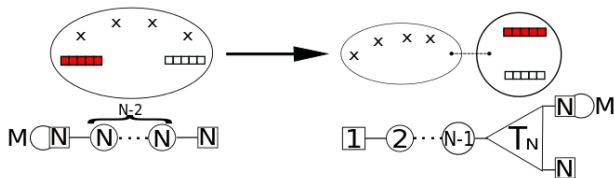
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$$M_i^{i'} Q_{i'jk} = 0$$

Q_{ijk} ($\forall i > 1$) can be written in terms of M and Q_{1jk} only!

R-charge of heavy operators: $(N-1)(g-1 + \frac{q-p}{2}\epsilon)$
In agreement with holographic computation!

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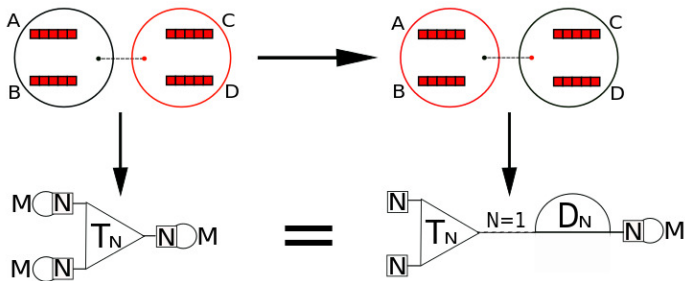
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T_N with three “flipping fields”

We start from a variant of the “swap duality” and close a puncture

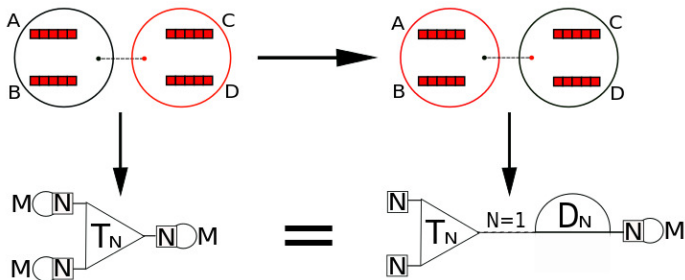


Exploiting T_N 's chiral ring relations: $(\mu_A)_i^{i'} Q_{i'jk} = (\mu_B)_j^{j'} Q_{ij'k}$ we find

$$(M_A)_i^{i'} Q_{i'jk} = (M_B)_j^{j'} Q_{ij'k} = (M_C)_k^{k'} Q_{ijk'}.$$

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We found a large class of $\mathcal{N} = 1$ SCFT's by wrapping M5-branes on a Riemann surface. By giving vev to fields charged under the global symmetries we can change the Chern class of the $U(1)$ bundles. This allows to find a QFT interpretation of all BBBW theories!

Possible future directions:

- More general geometries (e.g. non decomposable rank two bundles).
- What about other $\mathcal{N} = (2, 0)$ 6d theories?
- Most $\mathcal{N} = (1, 0)$ 6d theories have global symmetries and this construction should be possible in all these cases.

Thank You!

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