Gravitational Wave Oscillations in Bigravity

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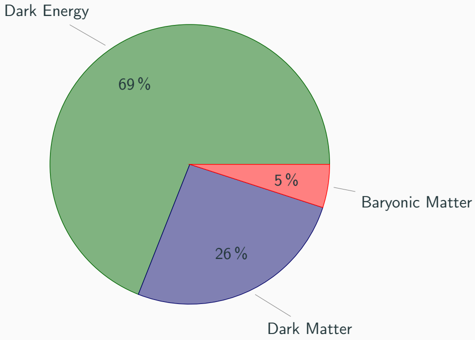
PRL **119** (2017), 111101

Max-Planck-Institut
für Kernphysik

The observed Universe[Planck, 2015]



- 95% of the Universe is ‘dark’, i.e. we do not understand it
- all evidence is based on *gravitational* interactions
- no new particles seen at colliders, direct detection, etc.

$\Rightarrow$ Better understanding of modifications of GR

Cosmology is a great playground for new ideas!

Bigravity in a nutshell - How to give the graviton a mass?

1.) linearized:[Fierz & Pauli, 1939] $m_g^2(h_{\mu\nu}h^{\mu\nu} - h_\mu^{\mu 2})$

2.) restore invariance under $h_{\mu\nu} \mapsto h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$: $h_{\mu\nu} \to h_{\mu\nu} - 2\partial_{(\mu}\chi_{\nu)}$

"Stückelberg trick": $\chi_\nu \mapsto \chi_\nu + \frac{1}{2}\xi_\nu$

Non-linear extension:

3.) define a reference metric $f_{\mu\nu} = \partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab}$

4.) couple it to the physical metric: $\left(\mathbb{1} - g^{-1}f\right)^\mu{}_\nu$

(cf. $h_{\mu\nu} \to h_{\mu\nu} - 2\partial_{(\mu}\chi_{\nu)}$)

5.) find consistent combinations of $\mathbb{X} = \sqrt{g^{-1}f}$:[de Rham et al., 2010]

$$\int \mathrm{d}^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\mathbb{X}\right)$$

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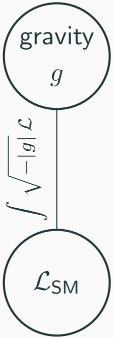
$$\int \mathrm{d}^4x \sqrt{-\det g} \sum_{n=0}^{4} \beta_n e_n(\mathbb{X}) + \int \mathrm{d}^4x \sqrt{-\det f} R(f)$$ [Hassan & Rosen, 2011]

Bigravity (see A. Schmidt-May's talk)



$$S = \int \mathrm{d}^4x \sqrt{-\det g}\,\left(M_g^2 R(g) + \mathcal{L}_{\mathsf{SM}}\right)$$

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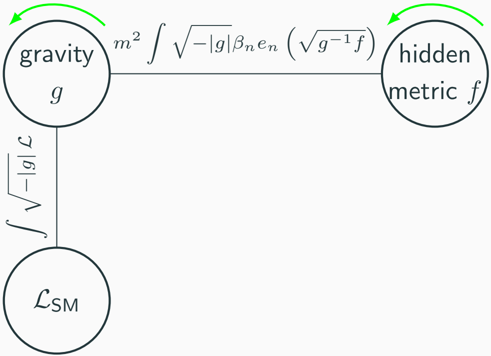

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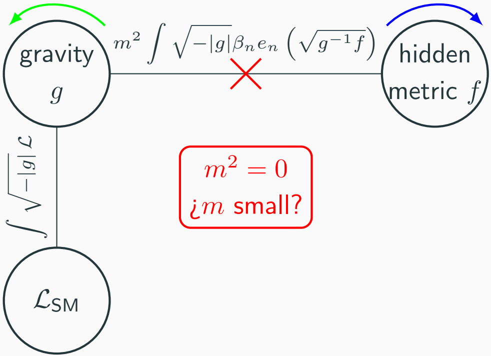

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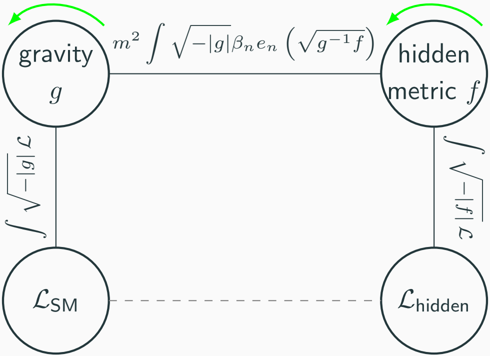

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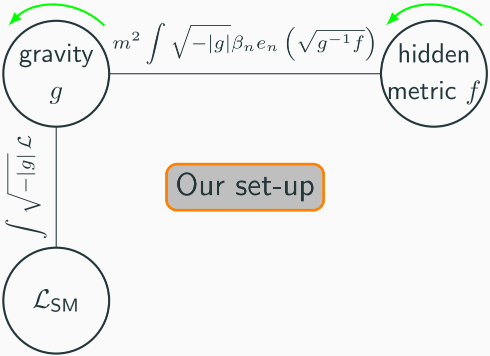

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Quick facts

Bigravity . . .

- . . . propagates 7 d.o.f. (2 for massless, 5 for a massive spin-2 field)
- . . . is ghost-free, i.e. consistent, to all orders[Hassan & Rosen, 2010/'11]
- . . . implements the "Vainshtein" mechanism:[Vainshtein, 1972]
  for $r < r_V$, GR is restored by strong coupling/non-linear effects!
- . . . contains an 'ordinary' DM candidate[Babichev et al., 2016]
- . . . degravitates the CC[Dvali, 2007; MP & Smirnov, 2016]
- . . . is a very young framework that leads to new effects, such as GW oscillations[Berezhiani et al., 2007; Hassan et al., 2013, Max et al., 2017]

Background Cosmology

Cosmology in Bigravity [von Strauss et al., 2011]

double FRW-like ansatz: $\mathrm{d}\vec{x} = \frac{\mathrm{d}r^2}{1-\kappa r^2} + r^2 \mathrm{d}\Omega^2$

$$\mathrm{d}s^2 \equiv g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a(t)^2 \mathrm{d}\vec{x}^2$$

$$\mathrm{d}\tilde{s}^2 \equiv f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -c(t)^2\,\mathrm{d}t^2 + b(t)^2 \mathrm{d}\vec{x}^2 \; \cancel{-2D(t)\,\mathrm{d}t\mathrm{d}r}$$

Gives three independent equations for $H \equiv \frac{\dot{a}}{a}$, $J \equiv \frac{\dot{b}}{b}$, $y \equiv \frac{b}{a}$:

$$-3\left(H^2 + \frac{\kappa}{a^2}\right) + \overbrace{m^2 \sin^2\theta\left[\beta_0 + 3\beta_1 y + 3\beta_2 y^2 + \beta_3 y^3\right]}^{\equiv \Lambda(y)} = -\frac{\rho}{M_g^2}$$

$$-3\left(\frac{J^2}{c^2} + \frac{\kappa}{b^2}\right) + \underbrace{m^2 \cos^2\theta\left[\beta_1 y^{-3} + 3\beta_2 y^{-2} + 3\beta_3 y^{-1} + \beta_4\right]}_{\equiv \tilde{\rho}(y)/M_f^2} = 0$$

$$\text{Bianchi :} \quad \underbrace{(c\,H - J)}_{\stackrel{!}{=}0}\underbrace{\left[\beta_1 + 2\beta_2 y + \beta_3 y^2\right]}_{\equiv \Gamma(y)} = 0$$

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Cosmology continued

Combining Einstein(g) and Einstein(f), using $J = c\,H$, yields an *algebraic* equation for y, which can be solved, e.g. iteratively for small $\frac{\rho}{m^2 M_g^2}$:

$$y \simeq y_* - \frac{\rho(t)}{3m^2 M_g^2} \frac{y_*^3}{\Gamma_*(\cos^2\theta + y_*^2 \sin^2\theta) - 2\frac{\tilde{\rho}_* y_*^4}{3m^2 M_{\tilde{g}}^2}}$$

and

$$c \simeq 1 - (1+\omega)\frac{\rho(t)}{m^2 \Gamma_* M_{\mathsf{Pl}}^2} \frac{y_*^2}{\frac{2\tilde{\rho}_* y_*^4}{3m^2 M_f^2 \Gamma_*} - \cos^2\theta}$$

At late times and for 'typical values', $y = y_*$ and $c = 1$, which allows an analytic treatment!

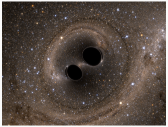
Gravitational Wave Oscillations

Existing bounds on the graviton mass

Solar system tests $\lambda_g > 2.8 \cdot 10^{12}\,\text{km},\ m_g < 7.2 \cdot 10^{-23}\,\text{eV}$

Weak lensing $\lambda_g > 2 \cdot 10^{21}\,\text{km},\ m_g < 6 \cdot 10^{-32}\,\text{eV}$

rely on a Yukawa potential $\propto e^{-m_g\, r}$



GW150914 $\lambda_g > 4.2 \cdot 10^{11}\,\text{km},\ m_g < 1.2 \cdot 10^{-22}\,\text{eV}$

due to a modified dispersion relation $v_g = \sqrt{1 - \frac{m_g^2}{E^2}}$

But do these bounds apply here?

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$r_V > r_{\text{solarsystem}} \Rightarrow \lambda_g > 10^{13}\,\text{km}$

Weak lensing $\lambda_g > 2 \cdot 10^{21}\,\text{km},\ m_g < 6 \cdot 10^{-32}\,\text{eV}$

work in progress!

rely on a Yukawa potential $\propto e^{-m_g r}$

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GW oscillations!

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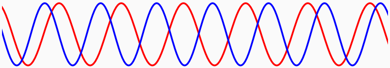
Gravitational wave vs. neutrino oscillations

$\nu_e = c_{e1}\nu_1 + c_{e2}\nu_2$

$\nu_e \to \nu_\mu \to \dots$



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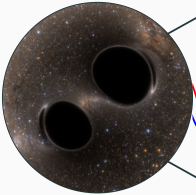
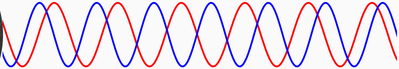
$\delta g = c_{g1}h_1 + c_{g2}h_2$



$\delta g \to \delta f \to \delta g \dots$



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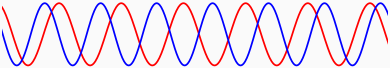

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Production matter basis | Propagation mass basis | Detection matter basis

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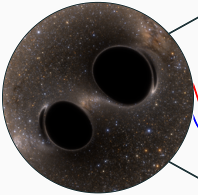

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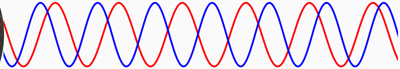


$\delta g = c_{g1}h_1 + c_{g2}h_2$

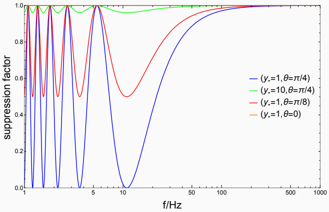
GW oscillations in detail

$$\delta g'' + k^2 \delta g + \sin^2\theta\, m^2\, \Gamma_* a^2 (\delta g - \delta f) = 0$$
$$\delta f'' + k^2 \delta f + \cos^2\theta\, \frac{m^2\,\Gamma_*}{y_*^2} a^2 (\delta f - \delta g) = 0$$

$$h_1'' + k^2 h_1 = 0$$
$$h_2'' + k^2 h_2 + m_g^2\, h_2 = m_g^2 \eta h_1$$

$$\delta g(t, \vec{k}) = \frac{\cos^2\theta \cos(k\,t) + \sin^2\theta y_*^2 \cos\left(\sqrt{k^2 + m_g^2}\, t\right)}{\cos^2\theta + y_*^2 \sin^2\theta}$$
$$\delta f(t, \vec{k}) = \frac{\cos^2\theta \cos(k\,t) - \cos^2\theta \cos\left(\sqrt{k^2 + m_g^2}\, t\right)}{\cos^2\theta + y_*^2 \sin^2\theta}$$

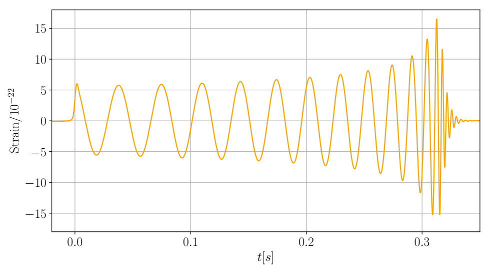
Interference yields a frequency dependent modulation of the amplitude.



$$\cos\theta \equiv \frac{M_{\text{eff}}^2}{M_g^2},\ \sin\theta \equiv \frac{M_{\text{eff}}^2}{M_f^2},\ \sqrt{k^2 + m_g^2} \simeq k + \frac{m_g^2}{2k}$$

GW oscillations – amplitude modulation

GW150914 num. GR by Einstein Toolkit



$$\left\langle \delta g^2(t,k) \right\rangle_{T_0 \ll T \ll T_*} = \frac{\cos^4\theta}{(\cos^2\theta + y_*^2 \sin^2\theta)^2} \left[1 + 2\, y_*^2 \tan^2\theta\, \cos\left(\delta\omega\, T\right) + y_*^4 \tan^4\theta\right],$$

where $\delta\omega \equiv \frac{m_g^2}{2k} \ll k$, $T_0 \equiv \frac{2\pi}{k}$ and $T_* \equiv \frac{2\pi}{\delta\omega}$

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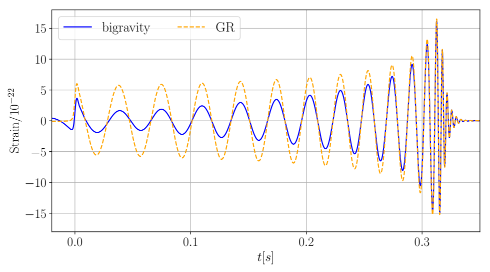

$$m_g = 10^{-22}\,\text{eV},\ y_* = 1,\ \theta = \pi/4$$

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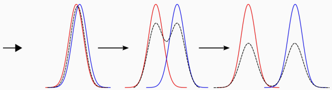
GW oscillations – decoherence

When the GWs travel over large distances:



large redshift or m_g $\longrightarrow$ frequency dependence is lost in favor of constant suppression

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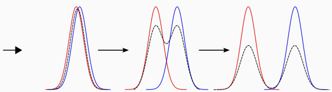

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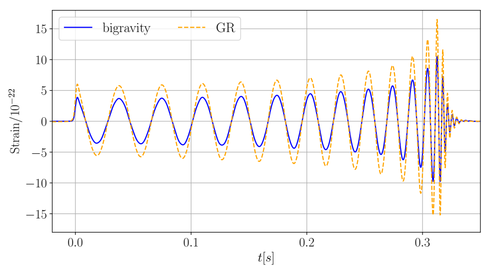


Caution:

indistinguishable from larger redshift z!

$\Rightarrow$ Study distribution of events **or** coincidence with optical signal

$$m_g = 10^{-19}\,\text{eV},\ y_* = 1,\ \theta = \pi/4$$

GW oscillations – summary so far

- The length scale of this modulation is $L_{\text{osc}} \sim \frac{2f}{m_g^2}$ (cf. ν oscillations)

- For $m_g \ll f$ the dispersion relation is nearly unaltered wrt GR

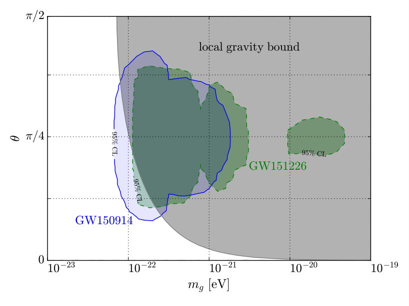
- Decoherence of oscillations ($E/\hbar \sim 100\,\text{Hz}$):

$$L_{\text{coh}} \sim \left(\frac{10^{-21}\,\text{eV}}{m_g}\right)^2 \text{Gpc}$$

- Use wavefrom of GW signals to constrain the (m_g, θ)-space
  - → large masses, decoherence, suppressed amplitude ($\sim$ larger z)
  - → small masses, freq. dependent modulation of amplitude, compare with GR

Constraining the Bigravity parameter space

- Compare waveform to 2015 LIGO events
- High-mass regime
  $\to$ decoherence:
  need more events
- Low-mass regime
  $\to$ freq. dependence:
  comparable bounds to LIGO
  ($m_g < 1.2 \cdot 10^{-22}$ eV)



(local gravity bound: solar system bound $\times \sin\theta$)

With more events & precision, all mass ranges can be probed!

Thank you!

Back-up slides

Einstein equations & Bianchi constraint

$$G(g)_{\mu\nu} + m^2 \sin^2(\theta) \sum_{n=0}^{3} \beta_n {V_g^{(n)}}_{\mu\nu} = M_g^{-2} T_{\mu\nu}$$

$$G(f)_{\mu\nu} + m^2 \cos^2(\theta) \sum_{n=1}^{4} \sqrt{|g|/|f|} \beta_n {V_f^{(n)}}_{\mu\nu} = 0,$$

where $\sin^2(\theta) = \frac{M_{\text{eff}}^2}{M_g^2}$, $\cos^2(\theta) = \frac{M_{\text{eff}}^2}{M_f^2}$, and the interaction or mass terms

$V_{g/f}$ follow from the variation of the e_n in $\mathcal{S}$.

Also, if the energy-momentum-tensor is to be conserved,

$$\nabla_g^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \nabla_g^\mu {V_g^{(n)}}_{\mu\nu} = 0$$

$$\nabla_f^\mu G(f)_{\mu\nu} = 0 \quad \Rightarrow \quad \nabla_f^\mu {V_f^{(n)}}_{\mu\nu} = 0$$

Back-up – Einstein Eqs. continued

Recall that $\mathbb{X}^{\mu}{}_{\nu} = \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu}$:

$$
\begin{aligned}
V^{(0)}(g)^{\mu}{}_{\nu} =& \delta^{\mu}{}_{\nu}, \\
V^{(1)}(g)^{\mu}{}_{\nu} =& \mathrm{tr}\left(\mathbb{X}\right)\delta^{\mu}{}_{\nu} - \mathbb{X}^{\mu}{}_{\nu}, \\
V^{(2)}(g)^{\mu}{}_{\nu} =& \left(\mathbb{X}^2\right)^{\mu}{}_{\nu} - \mathrm{tr}\left(\mathbb{X}\right)\mathbb{X}^{\mu}{}_{\nu} + \frac{\delta^{\mu}{}_{\nu}}{2}\left[\mathrm{tr}\left(\mathbb{X}\right)^2 - \mathrm{tr}\left(\mathbb{X}^2\right)\right], \\
V^{(3)}(g)^{\mu}{}_{\nu} =& -\left(\mathbb{X}^3\right)^{\mu}{}_{\nu} + \mathrm{tr}\left(\mathbb{X}\right)\left(\mathbb{X}^2\right)^{\mu}{}_{\nu} - \frac{1}{2}\left[\mathrm{tr}\left(\mathbb{X}\right)^2 - \mathrm{tr}\left(\mathbb{X}^2\right)\right]\mathbb{X}^{\mu}{}_{\nu} + \\
& + \frac{\delta^{\mu}{}_{\nu}}{6}\left[\mathrm{tr}\left(\mathbb{X}\right)^3 - 3\,\mathrm{tr}\left(\mathbb{X}\right)\mathrm{tr}\left(\mathbb{X}^2\right) + 2\,\mathrm{tr}\left(\mathbb{X}^3\right)\right]
\end{aligned}
$$

Back-up – Einstein Eqs. continued

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V^{(1)}(f)^{\mu}{}_{\nu} =& \mathbb{X}^{\mu}{}_{\nu}, \\
V^{(2)}(f)^{\mu}{}_{\nu} =& -\left(\mathbb{X}^2\right)^{\mu}{}_{\nu} + \mathrm{tr}\left(\mathbb{X}\right)\mathbb{X}^{\mu}{}_{\nu}, \\
V^{(3)}(f)^{\mu}{}_{\nu} =& \left(\mathbb{X}^3\right)^{\mu}{}_{\nu} + \mathrm{tr}\left(\mathbb{X}\right)\left(\mathbb{X}^2\right)^{\mu}{}_{\nu} + \frac{1}{2}\left[\mathrm{tr}\left(\mathbb{X}\right)^2 + \mathrm{tr}\left(\mathbb{X}^2\right)\right]\mathbb{X}^{\mu}{}_{\nu}, \\
V^{(4)}(f)^{\mu}{}_{\nu} =& \delta^{\mu}{}_{\nu}
\end{aligned}
$$

Einstein equations – linearised

For a specific choice of $\vec{\beta} = (3, -1, 0, 0, +1)$, one recovers the FP mass term by expanding around the Minkowski background η

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{\delta g_{\mu\nu}}{M_g}, \;\; f_{\mu\nu} = \eta_{\mu\nu} + \frac{\delta f_{\mu\nu}}{M_f}$$

$$\begin{aligned}\mathcal{S}_{\mathsf{mass}} =& m^2 M_{\mathsf{eff}}^2 \int \mathrm{d}^4x \sqrt{-|g|} \sum_{n=0}^{4} \beta_n e_n(\mathbb{X}) \\ \simeq& - m^2 M_{\mathsf{eff}}^2 \int \mathrm{d}^4x \left[\left(\frac{\delta g}{M_g} - \frac{\delta f}{M_f} \right)^{\mu\nu} \left(\frac{\delta g}{M_g} - \frac{\delta f}{M_f} \right)_{\mu\nu} - \right. \\ & \left. - \left(\frac{\delta g^\mu{}_\mu}{M_g} - \frac{\delta f^\mu{}_\mu}{M_f} \right)^2 \right]\end{aligned}$$

Einstein equations – linearised 2

Therefore, the mass eigenstates are

$$\begin{pmatrix} u \\ v \end{pmatrix} \equiv \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{pmatrix} \begin{pmatrix} \delta g \\ \delta f \end{pmatrix}$$

$$\Rightarrow \mathcal{S}_{\text{mass}} = -\frac{m^2}{8} \int \mathrm{d}^4x \left[u^{\mu\nu} u_{\mu\nu} - (u^{\mu}{}_{\mu})^2 \right],$$

while v remains <u>massless</u>.

two interesting limits

$\theta \to \frac{\pi}{2} \;\Leftrightarrow\; M_g \to \infty$: massive gravity limit

$\theta \to 0 \;\Leftrightarrow\; M_f \to \infty$: GR limit, but *NO* discontinuity!

Cosmology – Bi-Friedmann Equations

At late times, everything combines to (almost) standard cosmology

$$\left(H^2 + \frac{\kappa}{a^2}\right) = \frac{1}{3}\Lambda_{\mathsf{eff}} + \frac{\rho}{3M_{\mathsf{Pl}}^2}$$

$$\Lambda_{\mathsf{eff}} \equiv m^2 \sin^2\theta \left(\beta_0 + 3\beta_1 y_* + 3\beta_2 y_*^2 + \beta_3 y_*^3\right)$$

$$M_{\mathsf{Pl}}^2 = M_g^2 \frac{\cos^2\theta + y_*^2 \sin^2\theta - \frac{2\rho_f^* y_*^4}{3m^2 M_f^2 \Gamma_*}}{\cos^2\theta - \frac{2\rho_f^* y_*^4}{3m^2 M_f^2 \Gamma_*}}$$

For very early times, $\rho \to \infty$ and the physical solution yields $y \to 0$, i.e.

$$\left(H^2 + \frac{\kappa}{a^2}\right) = \frac{1}{3} m^2 \sin^2\theta \beta_0 + \frac{\rho}{3M_g^2}$$