Loop effects in QED in space with a black hole

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Overview

Schwarzschild black hole with quantum fields

- ▷ Schwarzschild-black-hole evaporation
- ▷ Fermion- and vector-field propagator

Radiative corrections in QED

- ▷ Photon self-energy
- ▷ Electron/positron self-energy

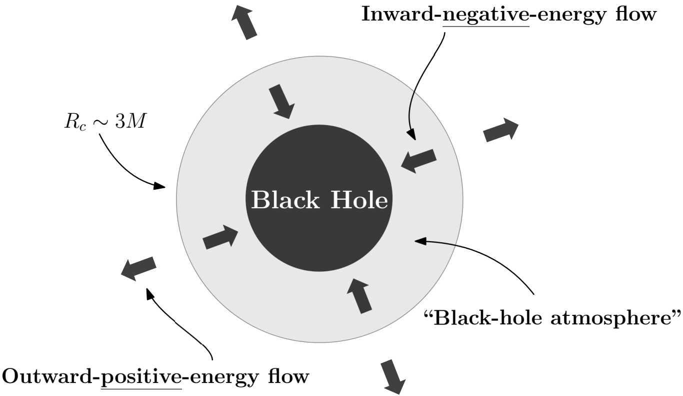
One-loop effects induced by small black holes

- ▷ How small?
- ▷ Reminder: Hot neutral $e^- e^+$ plasma
- ▷ Debye-like screening
- ▷ Modified photon dispersion relation
- ▷ Modified electron dispersion relation

Concluding remarks

Schwarzschild black hole with quantum fields

▷ **Sketch of Schwarzschild-black-hole evaporation**



W.G. Unruh, Phys. Rev. D**15**, 365 (1977)
P. Candelas, Phys. Rev. D**21**, 2185 (1980)
S.B. Giddings, Phys. Lett. B**754**, 39 (2016)

Schwarzschild black hole with quantum fields

▷ **Fermion- and vector-field propagator**

Black hole evaporate through quantum fields

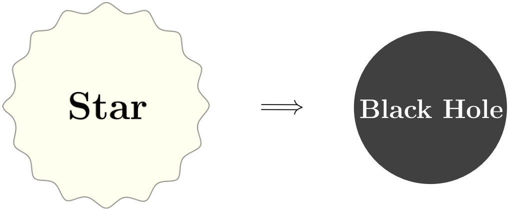
$$\Updownarrow$$

Field propagators get a correction describing the energy flux

Schwarzschild black hole with quantum fields

▷ **Fermion- and vector-field propagator: Electron/positron field**

Under the gravitational collapse



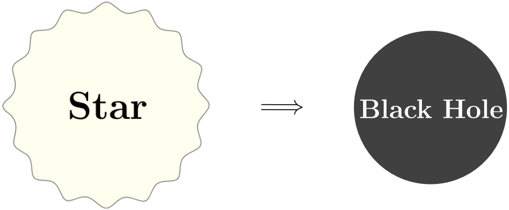
fermion propagator in near- & far-horizon region becomes

$$\underbrace{\frac{i(\not{p}+m)}{p^2-m^2+i\varepsilon}}_{\text{empty space}} \Longrightarrow \underbrace{\frac{i(\not{p}+m)}{p^2-m^2+i\varepsilon} - 4\pi^2 g_R \frac{|\mathbf{p}|(\not{p}+m)}{e^{\beta|p_0|}+1}\,\delta\big(\mathbf{p}-(p_0^2-m^2)^{\frac{1}{2}}\mathbf{n}\big)}_{\text{space with a black hole}}$$

Schwarzschild black hole with quantum fields

- **Fermion- and vector-field propagator: Photon field**

Under the gravitational collapse



photon propagator in near- & far-horizon region becomes

$$\underbrace{\frac{-i\eta_{\mu\nu}}{k^2+i\varepsilon}}_{\text{empty space}} \Longrightarrow \underbrace{\frac{-i\eta_{\mu\nu}}{k^2+i\varepsilon} - 4\pi^2 g_R \frac{|\mathbf{k}|\eta_{\mu\nu}}{e^{\beta|k_0|}-1}\,\delta(\mathbf{k}-k_0\mathbf{n})}_{\text{space with a black hole}}$$

Schwarzschild black hole with quantum fields

▷ **Fermion- and vector-field propagator: Parameters g_R and β**

The parameters g_R and β can be shown to be given by

$$g_R \approx \frac{27r_H^2}{16}\begin{cases} +1/R^2\,, & R \gg r_H\,, \\ -4/r_H^2\,, & R \sim r_H\,, \end{cases}$$

and

$$\beta = \frac{1}{T_H}\begin{cases} 1\,, & R \gg r_H\,, \\ 2\,, & R \sim r_H\,, \end{cases}$$

where T_H is the Hawking temperature parameter.

V.A. Emelyanov, Annalen der Physik, 1700078 (2017)

Radiative corrections in QED

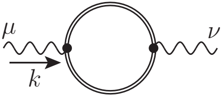
▷ **Vacuum polarization tensor**

The classical Maxwell equation changes due to radiative corrections. In momentum space, it reads

$$\left[k^2\eta^{\mu\nu} - k^\mu k^\nu + \Pi^{\mu\nu}(k)\right]A_\nu(k) \quad = \quad 0\,,$$

where vacuum polarization tensor is pictorially given by

$i\Pi^{\mu\nu}(k) \quad =$



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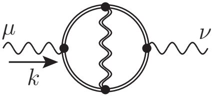

$+\,2$

$+\,O\left(\alpha^3\right)$

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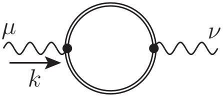

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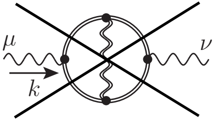

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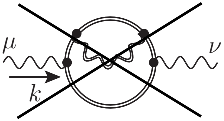


$+\,O(\alpha^2)$

Radiative corrections in QED

▷ **Vacuum polarization tensor**

The vacuum polarization tensor can be represented as follows:

$$\Pi^{\mu\nu}(k) = \pi_T(k_0, \mathbf{k})P^{\mu\nu} + \pi_L(k_0, \mathbf{k})Q^{\mu\nu} ,$$

where $P^{\mu\nu}$ & $Q^{\mu\nu}$ are projections being orthogonal to each other. The photon propagator then aquires the following form:

$$G_{\mu\nu}(k_0, \mathbf{k}) = \frac{-iP_{\mu\nu}}{k^2 - \pi_T(k_0, \mathbf{k}) + i\varepsilon} + \frac{-iQ_{\mu\nu}}{k^2 - \pi_L(k_0, \mathbf{k}) + i\varepsilon}$$

H.A. Weldon, Phys. Rev. D**26**, 1394 (1982)

Radiative corrections in QED

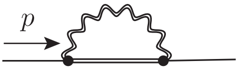
▷ **Electron/positron self-energy**

Loop diagrams in case of the fermion propagator leads to

$$\frac{i}{\not{p}-m+i\varepsilon} \implies \frac{i}{\not{p}-m-\Sigma(\not{p})+i\varepsilon},$$

where, at the leading order of the approximation, it holds that

$$-i\Sigma(\not{p}) \quad = \quad$$



$$+\,O\left(\alpha^2\right)$$

M.E. Peskin & D.V. Schroeder, *Introduction to QFT* (1995)

Small black holes: One-loop effects

▷ **How small?**

A black hole is said to be small if

$$T_H \gg m_e$$

In this case, the electron/positron field can be considered as being effectively massless (hard-thermal-loop approximation).

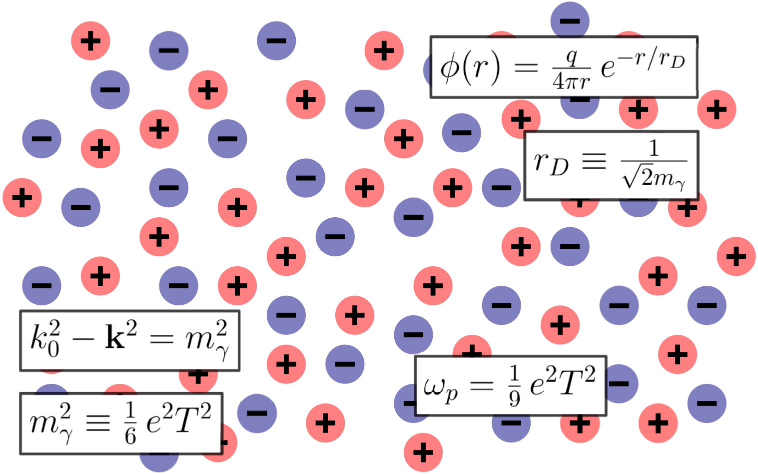
In other words, small black holes have a mass in the following range:

$$10^{10}\,\mathrm{g} \lesssim M \ll 10^{16}\,\mathrm{g}\,,$$

where the lower bound is to have a quasi-equilibrium approximation.

Small black holes: One-loop effects

▷ **Reminder: Isotropic neutral electron-positron plasma at** $T \gg m_e$



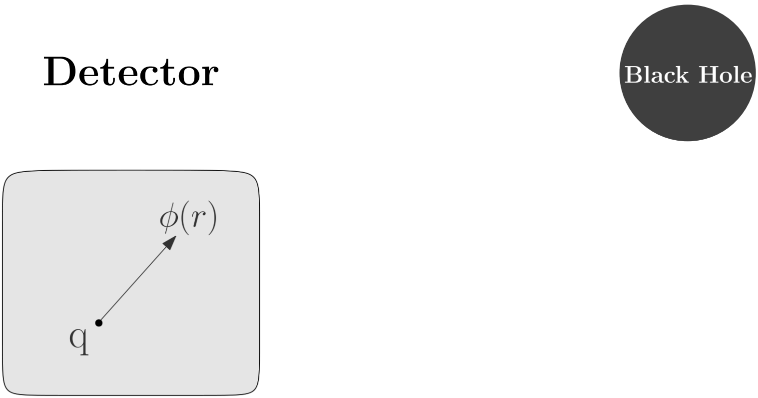
M. Le Bellac, *Thermal Field Theory* (1996)

Small black holes: One-loop effects

- ▷ **Debye-like screening: Spacetime with an evaporating black hole**

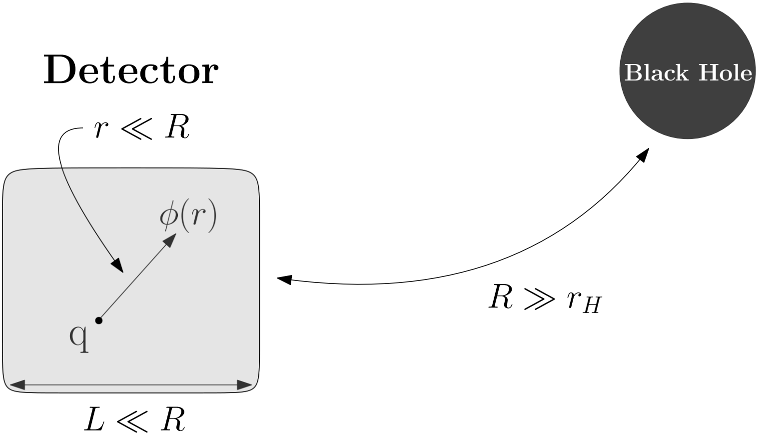
Small black holes: One-loop effects

- **Debye-like screening: Spacetime with an evaporating black hole**



Small black holes: One-loop effects

▷ **Debye-like screening: Spacetime with an evaporating black hole**



Small black holes: One-loop effects

▷ **Debye-like screening: Spacetime with an evaporating black hole**

In the far-from-horizon region, a point-like charge q at rest has the following (modified due to $\pi_L(0,\mathbf{k}) \neq 0$) electrostatic potential:

$$\phi(r) \approx \frac{q}{4\pi r}\, e^{-r/r_D} \quad \text{with} \quad r_D \equiv \frac{2}{\pi m_\gamma}\,,$$

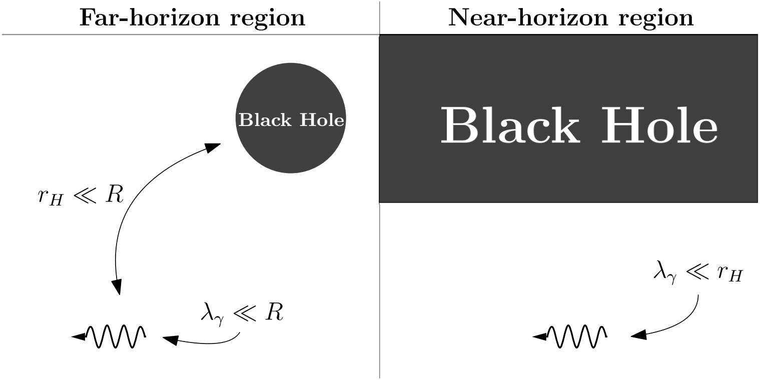
where

$$m_\gamma^2 \equiv \frac{1}{6}\, e^2 T_L^2 \sim \alpha T_H^2 \left(\frac{r_H}{R}\right)^2 \sim \frac{\alpha}{R^2}\,.$$

V.A. Emelyanov, Nucl. Phys. B**919**, 110 (2017)
Nucl. Phys. B**921**, 796 (2017)

Small black holes: One-loop effects

▷ **Modified photon dispersion relation**



Small black holes: One-loop effects

▷ **Modified photon dispersion relation**

Due to $\pi_T(k_0, \mathbf{k}) \neq 0$ in the limit $|\mathbf{k}| \to k_0$, the photon propagator gets its pole shifted, namely

$$k_0^2 - \mathbf{k}^2 \;=\; m_\gamma^2 \;\approx\; \frac{9\alpha}{128\pi} \times \begin{cases} +1/R^2\,, & R \gg r_H\,, \\ -1/r_H^2\,, & R \sim r_H\,, \end{cases}$$

where R is a distance to the black-hole centre.

V.A. Emelyanov, Nucl. Phys. B**919**, 110 (2017)
Nucl. Phys. B**921**, 796 (2017)
arXiv:hep-th/1703.05078

Small black holes: One-loop effects

▷ **Modified electron dispersion relation**

The pole structure of the electron propagator is also modified:

$$p_0^2 - \mathbf{p}^2 \quad \approx \quad m_e^2 + \frac{27\alpha}{256\pi} \times \begin{cases} +1/R^2\,, & R \quad \gg \quad r_H\,, \\ -1/r_H^2\,, & R \quad \sim \quad r_H\,, \end{cases}$$

where R is a distance to the black-hole centre.

Concluding remarks

Is the Debye-like screening effect testable?

There are two problems:

• It is far from clear whether small black holes exist in nature.

• If existent, it is unclear how long one needs to wait to have at least one of these in the neighbourhood of earth at the distance not larger than roughly 10^3 km.

Concluding remarks

Is the Einstein causality violated in the near-horizon region?

In general, a negative mass-squared term implies either instability or causality violation.

The instability is caused by modes with $|\mathbf{k}| \leq m_\gamma \ll 1/r_H$, but our approximation holds only for modes with $|\mathbf{k}| \gg 1/r_H$.

It means that there is a natural IR cutoff being of order $1/r_H$. *A posteriori* we know that modes with momentum higher or equal than local space-time curvature is immaterial for particle physics.

Thus, it seems that locality is violated near the event horizon of evaporating black holes.