Dark matter direct detection at one loop and connection to neutrino masses

Juan Herrero García

University of Adelaide, CoEPP

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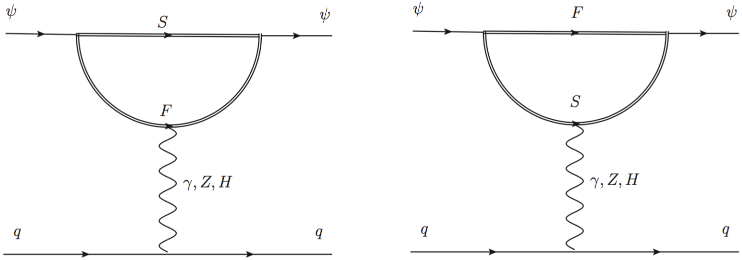
To appear soon, with C. Hagedorn, E. Molinaro and M. Schmidt.

Motivation for loopy direct detection

- The are **no DM signals** in direct/indirect detection or at the LHC.
- One explanation is that direct detection occurs at **one loop**.
- Next generation (liquid noble gas) DD detectors could probe it.
- A **fermionic singlet** DM ψ (like a bino) is a simple example.



Simplified models for a fermion singlet with DD at one loop

Dark sector	Field	SU(3)$_{\rm C}$	SU(2)$_{\rm L}$	U(1)$_{\rm Y}$	$G_{\rm dm}$
Dark matter	ψ	1	1	0	$I^{\psi}_{\rm dm}$
Dark scalar	S	1	$I^S_{\rm L}$	Y^S	$I^S_{\rm dm}$
Dark fermion	F	1	$I^S_{\rm L}$	Y^S	$I^{\psi}_{\rm dm} \otimes I^S_{\rm dm}$

- $\mathcal{L}$ reads ($H \equiv$ Higgs doublet)

$$\mathcal{L}_\psi = i\,\overline{\psi}\,\partial\!\!\!/\,\psi - m_\psi\,\overline{\psi}\,\psi + i\,\overline{F}\,D\!\!\!\!/\,F - m_F\,\overline{F}\,F + (D_\mu S)^\dagger\,D^\mu S$$
$$- \left(y_1\,\overline{F_{\rm R}}\,S\,\psi_{\rm L} + y_2\,\overline{F_{\rm L}}\,S\,\psi_{\rm R} + \text{H.c.}\right) - \mathcal{V}(S,H)\,,$$

where

$$\mathcal{V}(S,H) \supset \lambda_{HS}\,(H^\dagger H)(S^\dagger S) \to \lambda_{HS}\,h\,v(S^\dagger S)\,.$$

- We assume that only ψ is stable, with F and S being unstable.

Particular models with SM fields

1. $S \to H$: Mixing ψ-F_0, so tree-level H/Z.
2. $F \to e_R$ or L_L. ψ (or S) have $L = 1$: flavored DM [Agrawal, Kopp...]
   - **LFV, EDM/AMMs.** Option: LF conserved ψ_α-ℓ_α (or just τ)
   - **LNV.** $G_{\rm dm} = Z_2$, $L = 2$ by Majorana ψ and $\lambda_5(S^\dagger H)^2$: m_ν, ScM [Ma].
3. $F \to \nu_R$. $\nu_{\rm R}$, and ψ (or S), have $L = 1$. New mixings ν_L-$\nu_{\rm R}$:

$$\mathcal{L}_{\nu_{\rm R}} \; = \; -\overline{L_L}\, Y_\nu\, \nu_{\rm R}\, \tilde{H} \; - \frac{1}{2}\, \overline{\nu}^c_{\rm R} M_{\rm R} \nu_{\rm R} \; + {\rm H.c.}\,.$$

   - DD from H/Z penguins. DM-ν_R portal [Batell, Macias, Escudero...].

Specially interesting models of type 2, where F is a SM lepton:

- No problems with charged stable particles.
- m_ν may also be generated.
- Study of example with $F \to L_L$: the Generalized Scotogenic model.

Effective operators for direct detection

- Short-range:

$$\mathcal{O}_{\rm SI}^{S} = (\overline{\psi}\psi)(\overline{q}q), \qquad \mathcal{O}_{\rm SI}^{V} = (\overline{\psi}\gamma^\mu\psi)(\overline{q}\gamma_\mu q)\,,$$
$$\mathcal{O}_{\rm SD}^{AV} = (\overline{\psi}\gamma^\mu\gamma_5\psi)(\overline{q}\gamma_\mu\gamma_5 q), \qquad \mathcal{O}_{\rm SD}^{\rm T} = (\overline{\psi}\sigma^{\mu\nu}\psi)(\overline{q}\sigma_{\mu\nu}q)\,.$$

- $\mathcal{A}_\mathcal{D} \equiv (\overline{\psi}\gamma^\mu\psi)(\partial^\nu F_{\mu\nu}) \equiv \mathcal{O}_{\rm SI}^{V}$ by EOM.
- Long-range, dipoles:

$$\mathcal{O}_{\rm mag} = \frac{e}{8\pi^2}(\overline{\psi}\sigma^{\mu\nu}\psi)F_{\mu\nu}, \qquad \mathcal{O}_{\rm edm} = \frac{e}{8\pi^2}(\overline{\psi}\sigma^{\mu\nu}i\gamma_5\psi)F_{\mu\nu}\,,$$

with coefficients μ_ψ (magnetic) and d_ψ (electric).

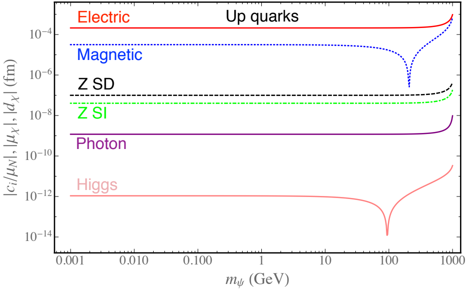
- **Majorana** ψ: $\mathcal{O}_{\rm SI}^{\rm S}$ (H), $\mathcal{O}_{\rm SD}^{\rm AV}$ (Z), anapole $\mathcal{A}_\mathcal{M} \equiv (\partial^\nu F_{\mu\nu})\,(\overline{\psi}\gamma^\mu\gamma_5\psi)$.

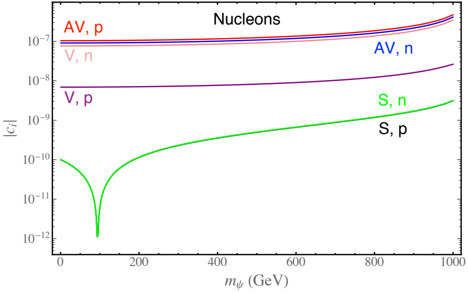
Analytical expressions valid for general models provided in the paper:

- All contributions have to be considered simultaneously.
- Comparison to literature [Berlin, Chang, Agrawal, Kumar, Schmidt, Kopp, Ibarra...]

Wilson coefficients with up quarks (nucleons) left (right)

$m_{F\,(S)} = 400\,(650)$ GeV, $y_V = 1.1 + i$, $y_A = 1.4 - i$, $\lambda_{HS} = 2$.

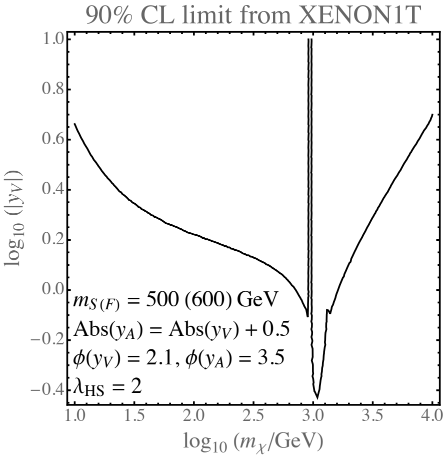




- d_ψ dominates if Yukawas have imaginary parts, $\gtrsim 10^{-4}$.
- Higgs (Z) $\sim 10^{-12}$ ($\gtrsim 10^{-8}$).
- S (Higgs) $\gtrsim 10^{-10}$, AV (Z) $\gtrsim 10^{-7}$.
- $V, p \propto V, n/10$ due to cancellation of γ and Z in V, p.

Preliminary limits on $|y_V|$ versus m_ψ from DD experiments

We use the public code by Cirelli et al [arXiv:1307.5955], which uses the total # of events, rescaling exposures.



The Generalized Scotogenic Model (GScM)

- Simple example of loopy DD with **radiative ν masses**.
- *From the trees to the forest: a review of radiative neutrino mass models*, by Cai, JHG, Schmidt, Vicente, Volkas; arXiv:1706.08524.
- DM ψ is Dirac, $F \equiv L_L$, $S = \Phi, \Phi'$. **Dark global** U(1)$_{\rm DM}$:

Field	SU(3)$_{\rm C}$	SU(2)$_{\rm L}$	U(1)$_{\rm Y}$	U(1)$_{\rm DM}$
Φ	1	2	1/2	q
Φ'	1	2	-1/2	q
ψ	1	1	0	q

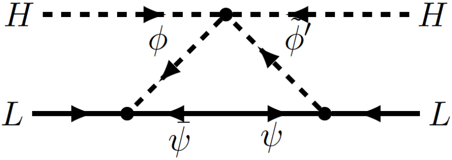
- Just **one** ψ needed. $y_{\Phi^{(\prime)}}$ are 3-component vectors. $\mathcal{L}$ reads:

$$\mathcal{L}_\psi \supset i\,\overline{\psi}\,\partial\!\!\!/\,\psi - m_\psi\,\overline{\psi}\,\psi - \left(y_\Phi^\alpha\,\overline{\psi}\,\tilde{\Phi}^\dagger\,L_L^\alpha + (y_{\Phi'}^\alpha)^*\,\overline{\psi}\,\tilde{\Phi}'^\dagger \tilde{L}_L^\alpha + \text{H.c.}\right).$$

$$V \supset \lambda_{H\Phi\Phi'}\left[(H^\dagger\tilde{\Phi}')(H^\dagger\Phi) + \text{H.c.}\right] \quad\longrightarrow\quad \sin 2\theta \propto \lambda_{H\Phi\Phi'}\,.$$

- Two neutral scalars $\eta_0^{(\prime)}$, and two charged $\eta^{(\prime)\pm}$ that do not mix.

Majorana ν masses due to loop of neutral scalars and DM



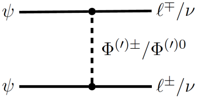
- $y_\Phi\, y'_\Phi\, \lambda_{H\Phi\Phi'}\, m_\Psi$ violate L by 2 units:

$$\mathcal{M}_\nu^{\alpha\beta} = \frac{\sin 2\theta\, m_\psi}{32\,\pi^2}\left(y_\Phi^\alpha\, y_{\Phi'}^\beta + y_{\Phi'}^\alpha y_\Phi^\beta\right)\left[\frac{m_{\eta_0}^2}{m_{\eta_0}^2 - m_\psi^2}\log\frac{m_{\eta_0}^2}{m_\psi^2} - (\eta_0 \leftrightarrow \eta'_0)\right]$$

- $\mathcal{M}_\nu$ is rank 2, so one massless ν and two massive

$$m_\nu^\pm \quad\propto\quad \left(|\mathbf{y_\Phi}|\ |\mathbf{y_{\Phi'}}| \pm |\mathbf{y_\Phi}\cdot\mathbf{y}_{\Phi'}^\dagger|\right).$$

DM s-wave annihilations into leptons and LFV



$$\simeq \frac{1}{64\pi m_\psi^2}\left| y^i_\Phi y^{j*}_\Phi \frac{m_\psi^2}{m_{\eta^\pm}^2 + m_\psi^2} - y^{i*}_{\Phi'} y^j_{\Phi'} \frac{m_\psi^2}{m_{\eta'^\pm}^2 + m_\psi^2}\right|^2$$

- $\eta^{(\prime)\pm}$ also generate $\ell_\alpha \to \ell_\beta\,\gamma$ at one loop $\propto |y^{\beta*}_\Phi\, y^\alpha_\Phi / m^2_{\eta^\pm}|^2$.
- Using the weakest LFV limit, $\tau \to \mu\,\gamma$:

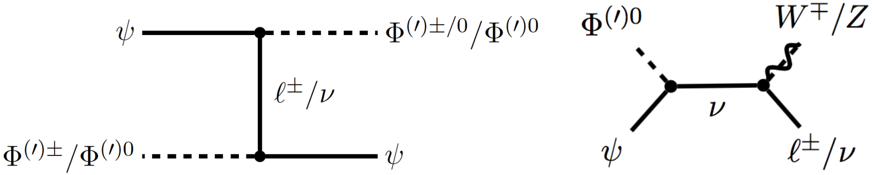
$$\langle v\sigma_{\psi\bar\psi\to\tau^-\mu^+}\rangle \lesssim 7\cdot 10^{-2}\left(\frac{B(\tau\to\mu\,\gamma)}{4.4\cdot 10^{-8}}\right)\left(\frac{3\cdot 10^{-26}\,\mathrm{cm}^3/\mathrm{s}}{\langle v\sigma\rangle_{\mathrm{th}}}\right)\left(\frac{m_\psi}{100\,\mathrm{GeV}}\right)^2$$

Annihilations into leptons are not large enough:

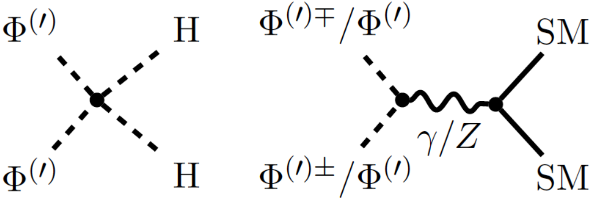
DM overproduced $\to$ need another mechanism for the relic abundance.

Coannihilations with scalars

- If $(m_\psi - m_{\Phi^{(\prime)}})/m_{\Phi^{(\prime)}} \ll 1$, $\Phi^{(\prime)}/\psi$ in equilibrium, coannihilations:



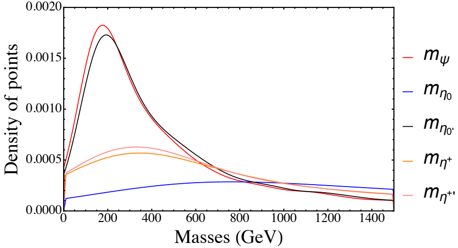
- For smaller $y_{\Phi^{(\prime)}}$, annihilations of the scalar partners dominate:

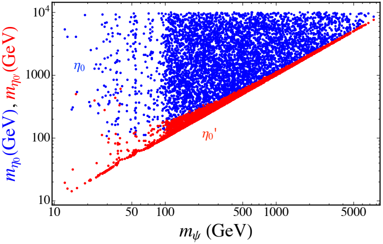


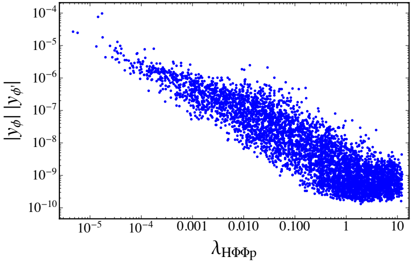
$$\sigma_{eff} = r_\psi^2\, \sigma_{\psi\psi} + r_\psi r_\Phi\, \sigma_{\psi\Phi} + r_\Phi^2\, \sigma_{\Phi\Phi}$$ [Griest]

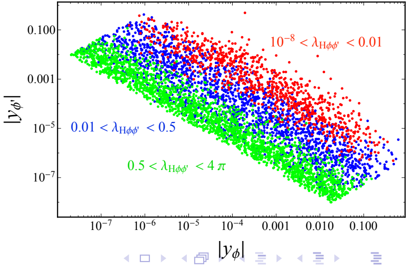
Masses for coannihilations

Using MicroOMEGAs. All points obey V stability, LFV, 3σ of ν masses and mixings, $\Omega_{\rm DM}$ and EWPT. Similar results for IO and NO.

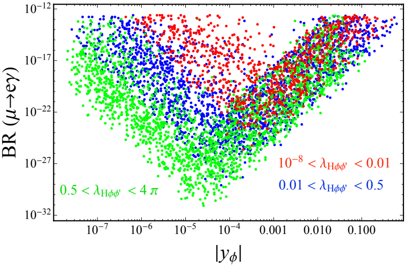


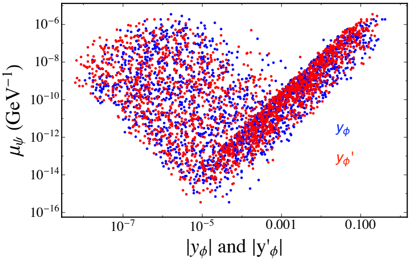


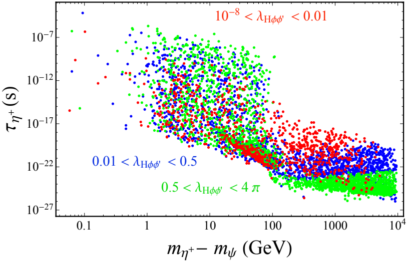




LFV, μ_ψ and lifetime of the charged scalars







$\tau_{\eta^{(\prime)+}}$ can be large: displaced vertices, ionising tracks [see A. Plascencia's talk].

Conclusions

- No DD signal: maybe DM is a fermion singlet, with **DD at 1 loop?**
- For Dirac DM, magnetic and electric dipoles are currently bounded.
- Several possible simplified models, but only a few are consistent with no charged stable particles.
- Most interesting ones have SM leptons in the loop, so there may be a **connection to neutrino masses**, like the ScM or the GScM.
- Due to **LFV**, relic abundance set by **coannihilations** with the scalars.
- DD is very suppressed, but charged scalars at LHC can be long-lived.

THANKS!

Back-up slides

The potential

The most general scalar potential invariant under $\mathrm{U}(1)_{\mathrm{DM}}$ is

$$\begin{aligned}\mathcal{V} = &- m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + m_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + m_{\Phi'}^2 \Phi'^\dagger \Phi' \\ &+ \lambda_{H\Phi} (H^\dagger H)(\Phi^\dagger \Phi) + \lambda_{H\Phi'} (H^\dagger H)(\Phi'^\dagger \Phi') + \lambda_{\Phi\Phi'} (\Phi^\dagger \Phi)(\Phi'^\dagger \Phi') \\ &+ \lambda_{H\Phi,2} (H^\dagger \Phi)(\Phi^\dagger H) + \lambda_{H\Phi',2} (H^\dagger \tilde{\Phi}')(\tilde{\Phi}'^\dagger H) + \lambda_{\Phi\Phi',2} (\Phi^\dagger \tilde{\Phi}')(\tilde{\Phi}'^\dagger \Phi) \\ &+ \lambda_{\Phi'} (\Phi'^\dagger \Phi')^2 + \lambda_{H\Phi\Phi'} \left[(H^\dagger \tilde{\Phi}')(H^\dagger \Phi) + \mathrm{H.c.} \right] ,\end{aligned}$$

with $H \equiv (h^+,\ (h+v)\sqrt{2})^T$ being the SM Higgs doublet after EWSB.

The physical scalars

- The SM Higgs boson h.
- Two (complex) neutral scalars η_0 and η_0', combinations of ϕ_0 and ϕ_0':

$$\eta_0 = \sin\theta\,\phi_0 \;+\; \cos\theta\,\phi_0'\,, \qquad \eta_0' = -\;\cos\theta\,\phi_0 \;+\; \sin\theta\,\phi_0'\,,$$

with $\tan 2\theta = 2\,c/(b\,-\,a)$, where $c = -\,1/2\,\lambda_{H\Phi\Phi'}\,v^2$ and

$$a = m_\Phi^2 + \frac{1}{2}\,v^2\,(\lambda_{H\Phi} \;+\; \lambda_{H\Phi,2})\;, \quad b = {m_\Phi'}^2 + \frac{1}{2}\,v^2\left(\lambda_{H\Phi'} \;+\; \lambda_{H\Phi',2}\right)\;,$$

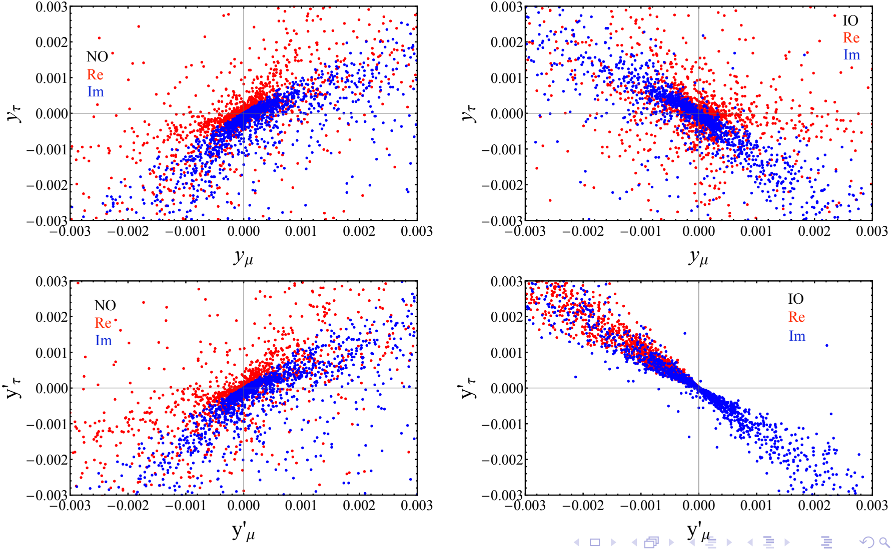
with masses

$$m^2_{\eta_0\,(\eta_0')} = \frac{1}{2}\left(a \;+\; b \;+ (-)\,\sqrt{(a\;-\;b)^2\;+\;4\,c^2}\right)\,.$$

- Two charged scalars $\eta^\pm \equiv \phi^\pm$ and $\eta'^\pm \equiv \phi'^\pm$, for $\lambda_{H\Phi\Phi'} \ll \lambda_i$:

$$m^2_{\eta_0} \simeq m^2_{\eta^\pm} \;+\; \frac{1}{2}\lambda_{H\Phi,2}\,v^2, \qquad m^2_{\eta_0'} \simeq m^2_{\eta'^\pm} \;+\; \frac{1}{2}\lambda_{H\Phi',2}\,v^2\,.$$

Left) NO: pattern $y_\tau^{(\prime)} \approx y_\mu^{(\prime)}$. **Right)** IO: pattern $y_\tau^{(\prime)} \approx -y_\mu^{(\prime)}$.



Can the relic abundance be set by annihilations?

- Need to break the proportionality $\ell_\alpha \to \ell_\beta \, \gamma \propto \langle v\sigma \rangle_{\rm th}$.
- $\langle v\sigma \rangle_{\nu_i \bar{\nu}_j} \gg \langle v\sigma \rangle_{\ell_i \bar{\ell}_j}$, with small LFV [Boehm, Hambye, Farzan, Arhrib].

$$\left\langle v\sigma_{\nu_i\bar{\nu}_j} \right\rangle \simeq \frac{1}{128\pi m_\psi^2 (1+\delta_{ij})} \left| y_\Phi^i y_\Phi^{j*} \left(\frac{m_\psi^2 s_\theta^2}{m_{\eta^0}^2 + m_\psi^2} + \frac{m_\psi^2 c_\theta^2}{m_{\eta'^0}^2 + m_\psi^2} \right) \right.$$
$$\left. - y_{\Phi'}^{i*} y_{\Phi'}^{j} \left(\frac{m_\psi^2 c_\theta^2}{m_{\eta^0}^2 + m_\psi^2} + \frac{m_\psi^2 s_\theta^2}{m_{\eta'^0}^2 + m_\psi^2} \right) \right|^2$$

- Need large hierarchy in masses:

$$\mathcal{O}(1)\,\text{MeV} \simeq m_\psi \lesssim m_{\eta'^0} \ll m_{\eta^0},\, m_{\eta'^\pm},\, m_{\eta^\pm} \sim \mathcal{O}(0.1-10)\,\text{TeV}.$$

Can the relic abundance be set by annihilations?

- To suppress $Z \to \eta_0' \eta_0'^*$ need $\cos(2\theta) \approx 0$. But then need $y \ll y'$ to suppress m_ν, which makes LFV too large...
- Also too large T parameter, which grows with the scalar splittings:

$$T = \frac{1}{16\pi^2 \alpha v^2} \Big\{ 2\, s_\varphi^2 \mathcal{F}(m_{\eta^+}^2, m_{\eta^0}^2) + 2\, c_\varphi^2 \mathcal{F}(m_{\eta^+}^2, m_{\eta'^0}^2) +$$

$$+ 2\, c_\varphi^2 \mathcal{F}(m_{\eta'^+}^2, m_{\eta^0}^2) + 2\, s_\varphi^2 \mathcal{F}(m_{\eta'^+}^2, m_{\eta'^0}^2) \Big\} .$$

Up to now we did not find any allowed region:

MeV DM into ν is not possible, or extremely fine-tuned.

Dominant interactions: electric/magnetic dipole moments

- For Dirac ψ, in the limit $m_\psi \ll m_F < m_S$, dipoles:

$$\mu_\psi \approx -\frac{Q_F}{4\,m_S}\left(|y_V|^2 - |y_A|^2\right) x_F\,\frac{1 - x_F^2 + 2\,\ln x_F}{(1 - x_F^2)^2}\,,$$
$$d_\psi \approx -\frac{Q_F}{2\,m_S}\,\mathrm{Im}[y_V^*\,y_A]\,x_F\,\frac{1 - x_F^2 + 2\,\ln x_F}{(1 - x_F^2)^2}\,.$$

where

$$x_F \equiv \frac{m_F}{m_S} \quad \text{and} \quad y_{V\,(A)} = \frac{y_2 + (-)y_1}{2}\,.$$

- All general expressions provided in the paper.
- Similar results in literature [Chang, Agrawal, Schmidt, Kopp, Ibarra...]

Stability of the dark sector

- For global $G_{\rm dm}$, DM may decay via EFT $\bar{\psi}\tilde{H}^{\dagger}(\not{D}L)$ [Mambrini], depending on m_{ψ} and UV completion. We assume DM is stable.
- In principle two of the three new states can be stable: ψ, and S or F.
- In the case of F being lighter (e.g. $m_S \geq m_F + m_{\psi}$):
  1. If F is neutral under the SM group, it also contributes to the DM.
  2. If F is charged (and uncharged) under the SM (dark) group, they must decay, mixing SM leptons [Dissauer] or via EFT.
  3. If F is a SM lepton, these are stable (e, ν_1), into which S decays.
  4. Also if F is a RH neutrino, it will mix with SM neutrinos and decay.
- Similar discussion for S being lighter (can also be the SM Higgs).

Can the $U(1)_{\mathrm{DM}}$ symmetry be gauged?

Z' options:

1. Massless: radiation and large self-interactions. Strong limits.
2. Mass from a Stueckelberg mechanism.
3. Spontaneously broken by scalar σ, which mixes with H: strong bounds from DD and invisible decays. Remnant Z_2.

Even if kinetic mixing induced $\epsilon(\Lambda_{\mathrm{UV}}) = 0$, it is induced at one loop:

$$|\epsilon| \gtrsim \frac{\sqrt{\alpha_Y \alpha_D}}{4\pi} \left| \ln\left(\frac{m_\Phi}{m_{\Phi'}}\right) \right| \sim 10^{-4}\,.$$

Strong limits on the kinetic mixing.
$\longrightarrow$ **We will focus on the global symmetry case.**

Boltzmann equations for coannihilations

$$\frac{dn_\psi}{dt} = -3Hn_\psi - \langle \sigma_{eff} v \rangle \left(n_\psi^2 - n_{\psi,eq}^2\right) \quad \text{with:}$$

$$\sigma_{eff} = r_\psi^2 \, \sigma_{\psi\psi} + r_\psi r_\Phi \, \sigma_{\psi\Phi} + r_\Phi^2 \, \sigma_{\Phi\Phi} \ ,$$

$$r_\psi = \frac{4}{g_{eff}}, \ \ r_\Phi = \frac{g_{eff}^\Phi}{g_{eff}}, \ \ g_{eff} = 4 + g_{eff}^\Phi, \ \ g_{eff}^\Phi = g_\Phi \left(\frac{m_\Phi}{m_\psi}\right)^{3/2} e^{-\frac{m_\Phi - m_\psi}{T}} \ .$$

Interactions in the dark sector for a discrete symmetry

- For dark discrete Z_2 symmetry ($\psi \to -\psi$), ψ can also have a Majorana mass (also F if it is an SU(2) singlet with $Y = 0$).
- We can identify $\psi_L \to \psi_R^c$ and $S \to \tilde{S} \equiv i\sigma_2 S^*$.
- In this case, $y_1 = y_2 \equiv y$:

$$\mathcal{L}_\psi = i\,\overline{\psi}\,\partial\!\!\!/\,\psi - \frac{1}{2}\,m_\psi\,\overline{\psi^c}\,\psi - \frac{1}{2}\,m_F\,\overline{F^c}\,F - \left(y\,\overline{F}\,S\,\psi + \text{H.c.}\right).$$

Comparison to the Scotogenic Model: the Z_2 case

- Instead of a $\mathrm{U}(1)_{\mathrm{DM}}$, it has a Z_2. We can identify $\psi \leftrightarrow \psi^c$ and $\Phi' \leftrightarrow \tilde{\Phi}$. It works with one Majorana ψ and one scalar doublet Φ:

$$\mathcal{L}_\psi = i\,\overline{\psi}\,\partial\!\!\!/\,\psi - m_\psi\,\overline{\psi^c}\,\psi - \left(y^\alpha\,\overline{\psi}\,\tilde{\Phi}^\dagger\,L_\alpha + \text{H.c.}\right).$$

- The scalar potential becomes

$$\mathcal{V} = -m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + m_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \\ + \lambda_{H\Phi}(H^\dagger H)(\Phi^\dagger \Phi) + \lambda_{H\Phi,2}(H^\dagger \Phi)(\Phi^\dagger H) + \frac{\lambda_{H\Phi,3}}{2}\left[(H^\dagger \Phi)^2 + \text{H.c.}\right]$$

- Many studies: [Ma, Restrepo, Ibarra, Molinaro, Schwetz, Toma...].
- For scalar DM [Re(Φ)/ Im(Φ)], DD by the Z is inelastic [Hambye...].
- For fermionic DM ψ, strong constraints from LFV. Some options are coannihilation [Schmidt, Toma...], freeze-in [Molinaro...]... DD at one loop.

The Scotogenic Model: neutrino masses

- The mass matrix is given by:

$$(\mathcal{M}_\nu)_{\alpha\beta} \;=\; \sum_k \frac{y_{\alpha k}\, y_{\beta k}\, m_{\psi_k}}{16\,\pi^2}\; F_{\rm ScM}(m_{\phi_0^R}, m_{\phi_0^I}, m_{\psi_k})\,,$$

where

$$F_{\rm ScM}(m_{\phi_0^R}, m_{\phi_0^I}, m_{\psi_k}) \;\equiv\; \left(\frac{m_{\phi_0^R}}{m_{\phi_0^R} - m_{\psi_k}^2} \log\frac{m_{\phi_0^R}^2}{m_{\psi_k}^2} - (\phi_0^R \leftrightarrow \phi_0^I) \right)$$

- In the limit $m_{\phi_0^R}^2 - m_{\phi_0^I}^2 = \lambda_{H\Phi,3} v^2 \ll m_0^2 = (m_{\phi_0^R}^2 + m_{\phi_0^I}^2)/2$, we get:

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{\lambda_{H\Phi,3} v^2}{16\,\pi^2} \sum_k \frac{y_{\alpha k}\, y_{\beta k}\, m_{\psi_k}}{m_0^2 - m_{\psi_k}^2} \left[1 - \frac{m_{\psi_k}^2}{m_0^2 - m_{\psi_k}^2} \log\frac{m_0^2}{m_{\psi_k}^2} \right].$$

- It needs two $\psi_{1,2}$ (and one Φ) in order to generate at least two ν masses, while the Generalized version needs only one ψ (but two Φ).

Direct detection

Dominated by the photon long-range magnetic operator ($d_\psi = 0$):

$$\mu_\psi \overset{m_\ell \ll m_\eta}{\simeq} \frac{m_\psi}{16} \left\{ -\frac{|y_\Phi^\alpha|^2}{m_{\eta^\pm}^2} \left[1 + 2\frac{m_{\ell_\alpha}^2}{m_{\eta^\pm}^2} \ln\left(\frac{m_{\ell_\alpha}^2}{m_{\eta^\pm}^2} \right) \right] + \frac{|y_{\Phi'}^\alpha|^2}{m_{\eta^{\pm\prime}}^2} \left[1 + 2\frac{m_{\ell_\alpha}^2}{m_{\eta^{\pm\prime}}^2} \ln\left(\frac{m_{\ell_\alpha}^2}{m_{\eta^{\pm\prime}}^2} \right) \right] \right\} .$$