Integrable strings beyond symmetric spaces and AdS/CFT

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Plan

- Yang-Baxter deformed (string) sigma models
- .. in string theory
- .. in AdS/CFT

A string on $\mathrm{AdS}_5 \times \mathrm{S}^5$

- Generic string sigma model (2d QFT on worldsheet)

$$S = \int d^2\sigma(\sqrt{h}h^{\alpha\beta}g_{\mu\nu} - \epsilon^{\alpha\beta}B_{\mu\nu})\partial_\alpha x^\mu \partial_\beta x^\nu + \text{fermions}$$

  target-space coordinates x^μ, world-sheet coordinates σ^α

- $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring: symmetric space G/H sigma model

- Target space Lie group G: "principal chiral model"

$$g(x^\mu) \in G, \quad A_\alpha = g^{-1}\partial_\alpha g \in \mathfrak{g}$$

$$S = \int d^2\sigma \sqrt{h}h^{\alpha\beta}\mathrm{Tr}(A_\alpha A_\beta), \quad \text{i.e.} \quad \mathrm{Tr}(A_\alpha A_\beta) = g_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu$$

  manifests left global G symmetry

Symmetric sigma models and integrability

- G, G/H models are integrable:

$$\text{e.o.m.} \Leftrightarrow \partial_\alpha L_\beta - \partial_\beta L_\alpha - [L_\alpha, L_\beta] = 0$$

  with $L_\alpha = L_\alpha(x^\mu, z) \in \mathfrak{g}$, z *extra* parameter

- For G model, e.g.

$$L_\alpha(z) = -\frac{z^2}{1-z^2} A_\alpha + \frac{z}{1-z^2} \frac{h_{\alpha\beta}}{\sqrt{h}} \epsilon^{\beta\gamma} A_\gamma$$

- From here can try quantum integrability: exact S matrices, spectra, correlation functions, ...

Deformation – classical r matrices

G, G/H models can be deformed while preserving integrability

Input is solution of the classical Yang-Baxter equation (CYBE)

$$[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0$$

- $r \in \mathfrak{g} \otimes \mathfrak{g}$, $r = r^{ij} t_i \otimes t_j$, t_i generators of $\mathfrak{g}$
- CYBE over $\mathfrak{g} \otimes \mathfrak{g} \otimes \mathfrak{g}$
- $r_{12} = r \otimes 1$, etc.

(Inhomogeneous case exists as well)

Yang-Baxter sigma models

- Translate r matrix to operator $R : \mathfrak{g} \to \mathfrak{g}$

$$R(x) = \mathrm{Tr}_2[r(1 \otimes x)]$$

- Yang-Baxter sigma model action:

Klimcik
Delduc, Magro, Vicedo
Matsumoto, Yoshida

$$S = \int d^2\sigma(\sqrt{h}h^{\alpha\beta} - \epsilon^{\alpha\beta})\mathrm{Tr}[A_\alpha O^{-1}(A_\beta)]$$

where

$$O = 1 - \mathrm{Ad}_g^{-1} \circ R \circ \mathrm{Ad}_g$$

- Still integrable: simply replace A by $O^{-1}(A)$ in Lax pair
- Breaks left G symmetry to group generated by $\{t\}$

$$R([t, x]) = [t, R(x)]$$

Deformed target space

From G, G/H model with metric g we get

$$g^{\rm YB} + B^{\rm YB} = g(1 - rg)^{-1}$$

where r is in Killing vector rep

- $t_a \longleftrightarrow K_a = t_a^\mu \partial_\mu$
- $r = r^{ij} t_i \otimes t_j \longrightarrow r^{\mu\nu} \partial_\mu \otimes \partial_\nu = r^{ij} t_i^\mu t_j^\nu \partial_\mu \otimes \partial_\nu$

Reminiscent of TsT transformation ($\in O(d,d)$)

YB deformations in string theory

- TsT transformations correspond to abelian YB deformations ($r = r^{ij} t_i \otimes t_j$ with $[t_i, t_j] = 0$) Osten, ST
- General homogeneous YB deformations $\leftrightarrow$ non-abelian T duality
  Hoare, Tseytlin; Borsato, Wulff
- Not all models are Weyl invariant!

$$r^{ij}[t_i, t_j] = 0 \leftrightarrow \text{Weyl} \qquad \text{(unimodularity)}$$

Borsato, Wulff

Homogeneous YB deformations in AdS/CFT

- TsT can yield new AdS/CFT pairs
  e.g. Lunin-Maldacena background $\leftrightarrow$ β-def SYM

- Introduces nontrivial product in SYM
  e.g. $\Phi_i \star \Phi_j = e^{i\beta(Q_i^{(1)} Q_j^{(2)} - Q_i^{(2)} Q_j^{(1)})} \Phi_i \Phi_j$ Lunin, Maldacena

- $\exists$ nontrivial $\star$-product for each homogeneous r matrix:
  YB deformations of $\mathrm{AdS}_5 \times \mathrm{S}^5 \leftrightarrow$ SYM with NC $\star$-products ST

Summary

- YB sigma models as integrable deformations of $G, G/H$ models
- In string theory (TsT, nonabelian T duality, unimodularity)
- Homogeneous YB sigma models $\leftrightarrow$ NC SYM

Outlook

- Brane construction for all YB deformations of $\mathrm{AdS}_5 \times \mathrm{S}^5$
- Classify all unimodular deformations for $\mathfrak{psu}(2,2|4)$
  - Are there inhomogeneous unimodular models (strings)?
- Twists at quantum string level? e.g. spectra?
- Integrability on field theory side?

Examples

- Inhomogeneous $r = \sigma_1 \wedge \sigma_2 = \sigma_1 \otimes \sigma_2 - \sigma_2 \otimes \sigma_1$, deforming S^2

$$ds^2 = \frac{1}{1+\eta^2 r^2}\left(\frac{dr^2}{1-r^2} + (1-r^2)d\phi^2\right)$$

- Homogeneous $r = h_1 \wedge h_2$, h_i Cartans of $\mathfrak{so}(4)$, deforming S^3

$$ds^2 = \frac{1}{1+\eta^2 \sin^2(2\theta)}\left(\cos^2\theta d\phi_1^2 + \sin^2\theta d\phi_2^2\right) + d\theta^2$$

$$B = -\frac{\eta \sin^2(2\theta)}{1+\eta^2 \sin^2(2\theta)} d\phi_1 \wedge d\phi_2$$

YB target space

Killing vector rep of $\mathfrak{g}$

- $t_a \longleftrightarrow K_a = t_a^\mu \partial_\mu$
- $A_\alpha = A_\mu \partial_\alpha x^\mu$
- $r = r^{ij} t_i \otimes t_j \longrightarrow r^{\mu\nu} \partial_\mu \otimes \partial_\nu = r^{ij} t_i^\mu t_j^\nu \partial_\mu \otimes \partial_\nu$

Question

$$g_{\mu\nu}^{\mathrm{YB}} + B_{\mu\nu}^{\mathrm{YB}} = \mathrm{Tr}[A_\mu \frac{1}{1 - \mathrm{Ad}_g^{-1} \circ R \circ \mathrm{Ad}_g}(A_\nu)] = ??$$

Killing vector property $K_a g = t_a g \Rightarrow t_a^\mu A_\mu = \mathrm{Ad}_g^{-1}(t_a)$

$$\mathrm{Ad}_g^{-1} \circ R \circ \mathrm{Ad}_g(A_\mu) = r^{\rho\nu} A_\rho \mathrm{Tr}(A_\nu A_\mu) = r^{\rho\nu} A_\rho g_{\nu\mu} = (Arg)_\mu$$

YB target space:

$$g^{\mathrm{YB}} + B^{\mathrm{YB}} = g(1 - rg)^{-1}$$