Quantum Transitions Through Cosmological Singularities

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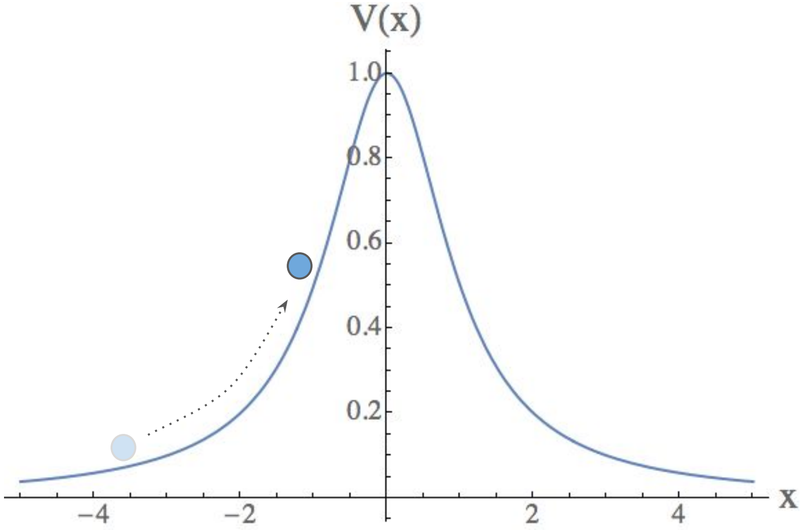
[1605.02751], [1701.05399]

MAX-PLANCK-GESELLSCHAFT

Studienstiftung des deutschen Volkes

Tunneling in 1D QM

Classically: (E < V)



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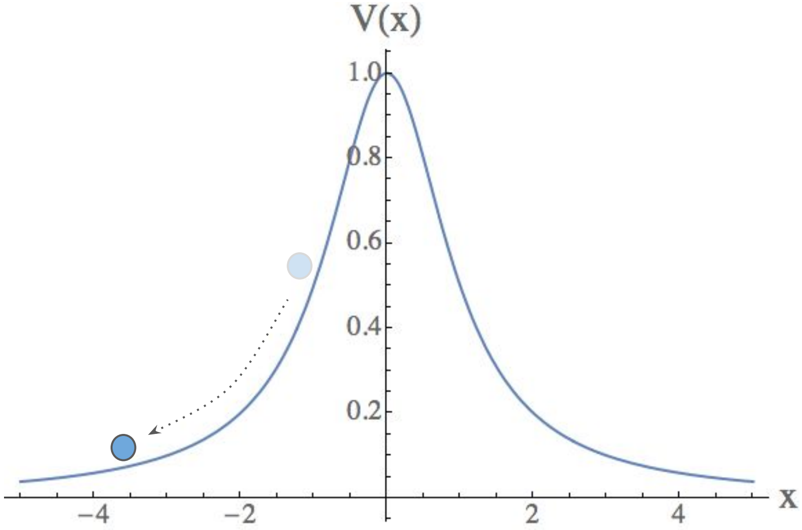

Tunneling in 1D QM

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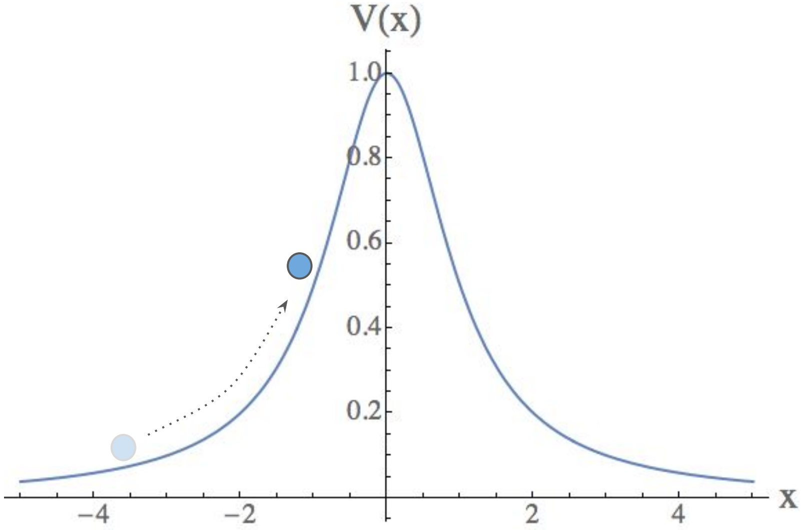

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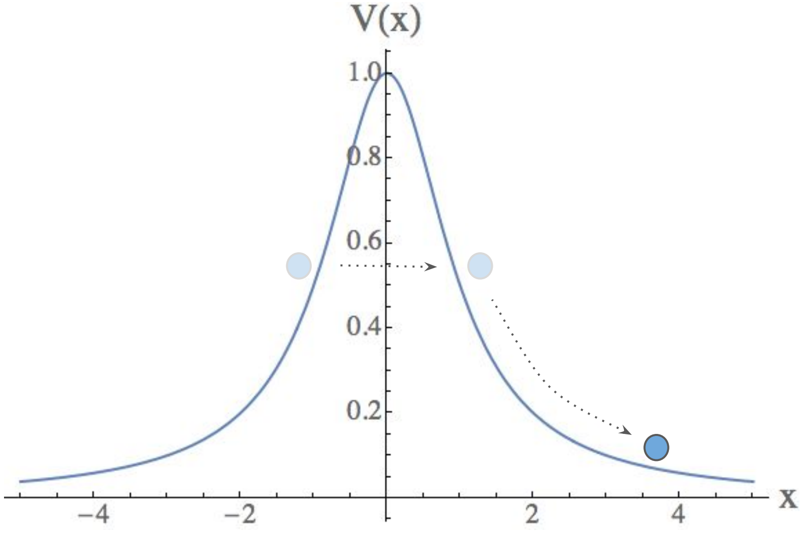

The wavefunction has non-zero support on the right-hand side

Path-Integral Framework

$$\langle x_f, t_f \mid x_i, t_i \rangle = \mathcal{N} \int_{x_i,t_i}^{x_f,t_f} D[x(t)]\, e^{iS}$$

“Transition amplitude = Sum over all paths (even crazy ones) weighted by the action”

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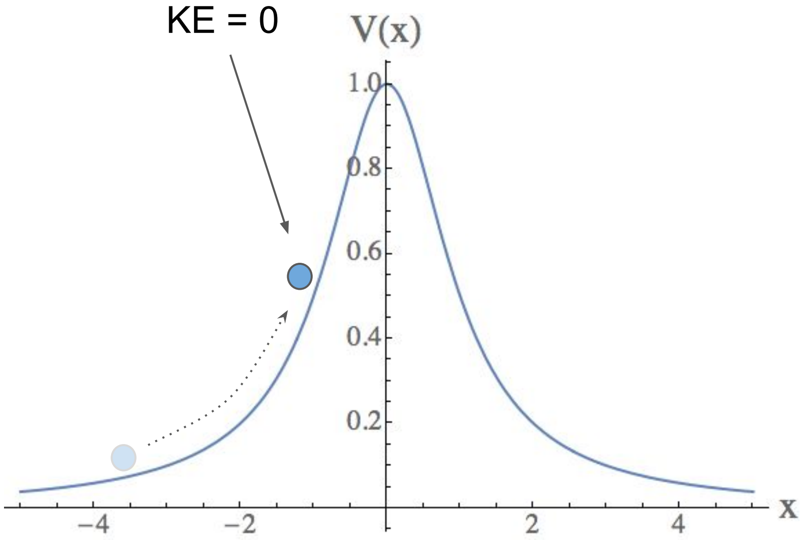
$$S = \int \mathrm{d}t \left(\frac{1}{2}\dot{x}^2 - V(x) \right)$$

Coleman's "Instanton" Method



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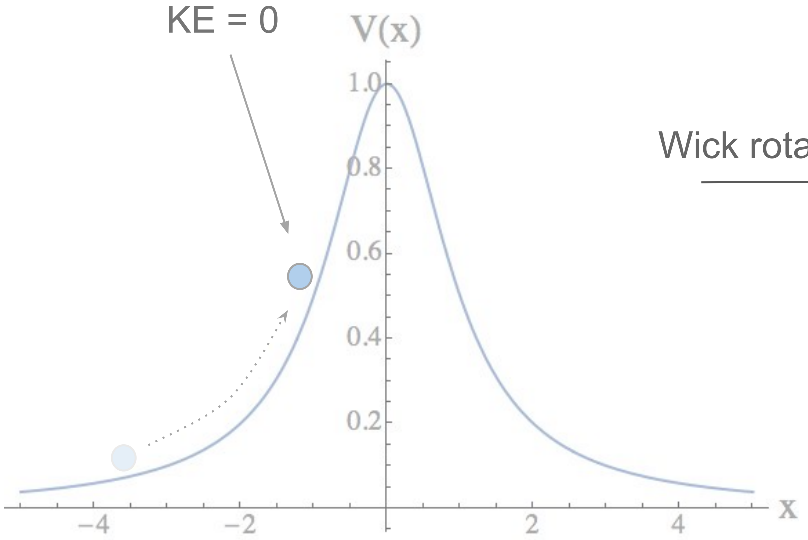
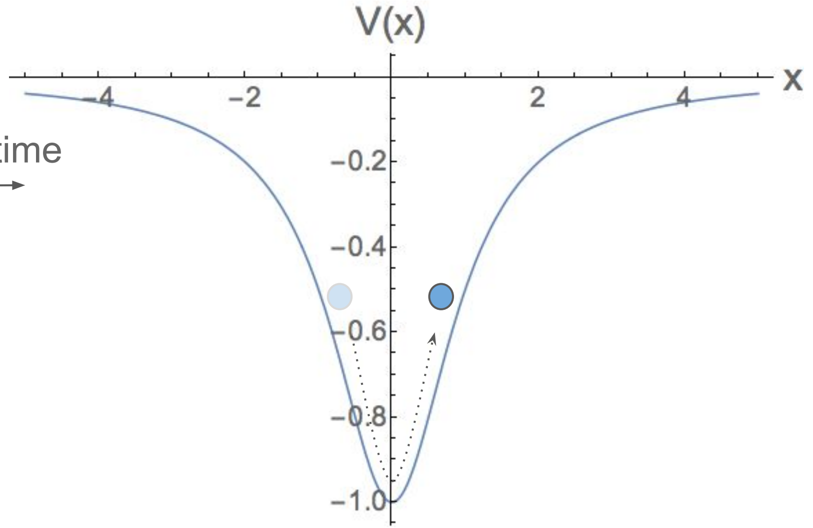


Wick rotation to Eucl. time



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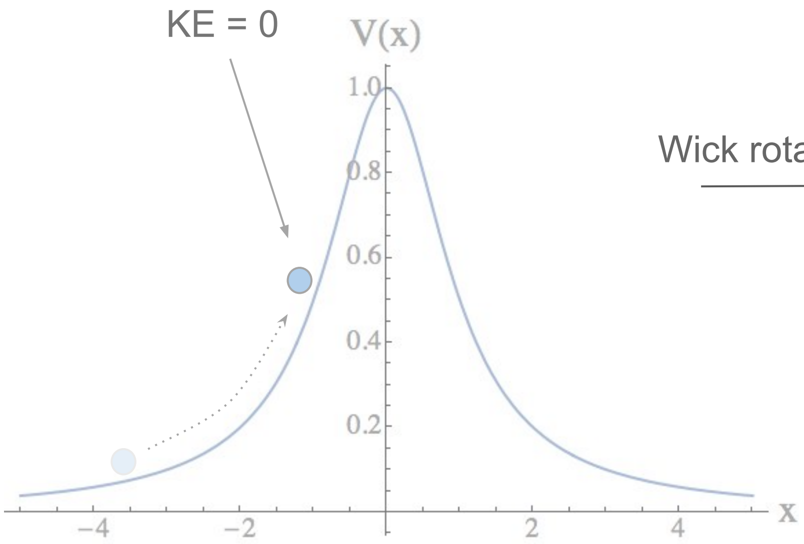

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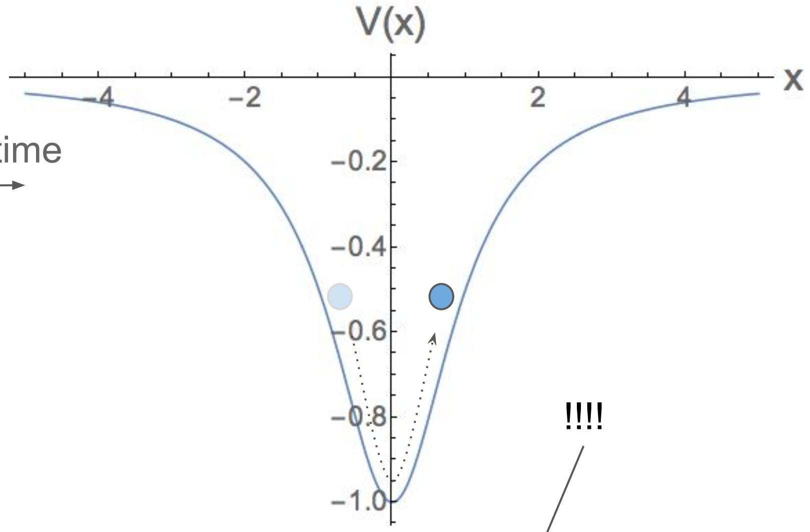

$$S = \int \mathrm{d}t \left(\frac{1}{2}\dot{x}^2 - V(x) \right)$$

$$S_E \sim \int d\tau \left(\frac{1}{2}\dot{x}^2 + V(x) \right)$$

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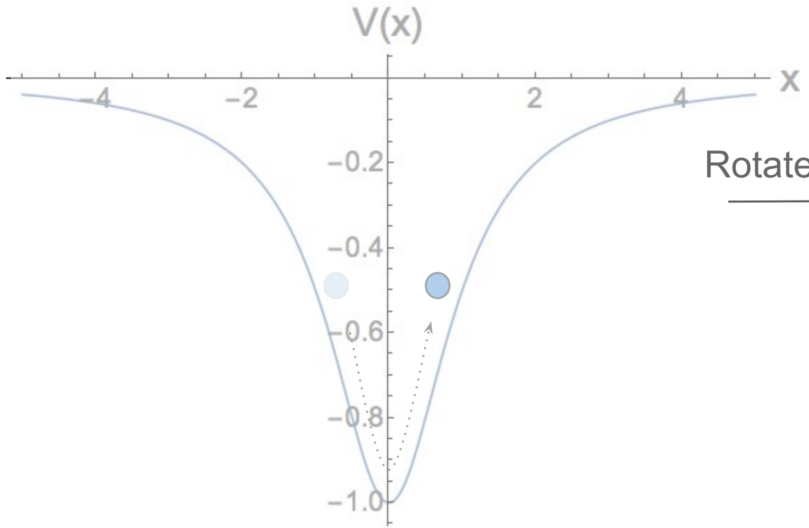
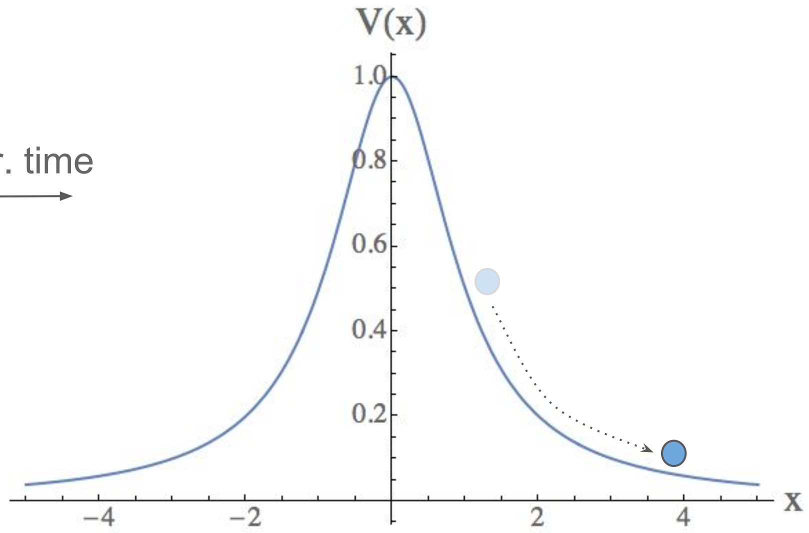


Rotate back to Lor. time



Insert Eucl. solution at an instant in time. Otherwise Lorentzian solution.

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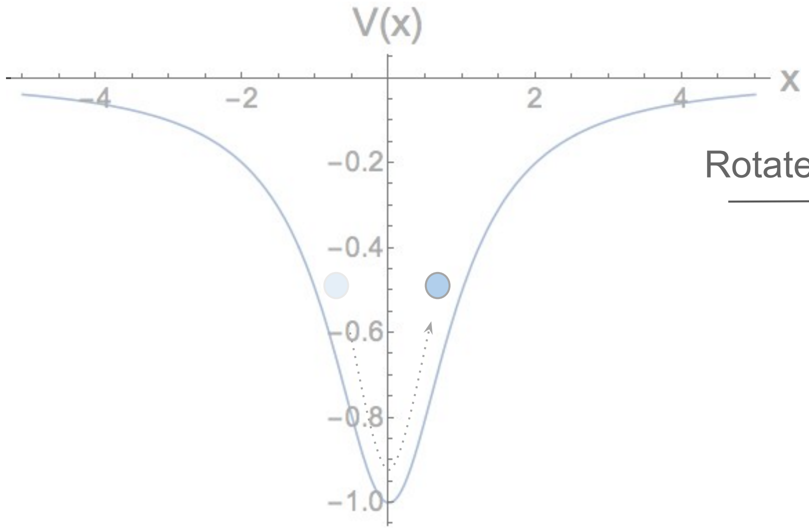

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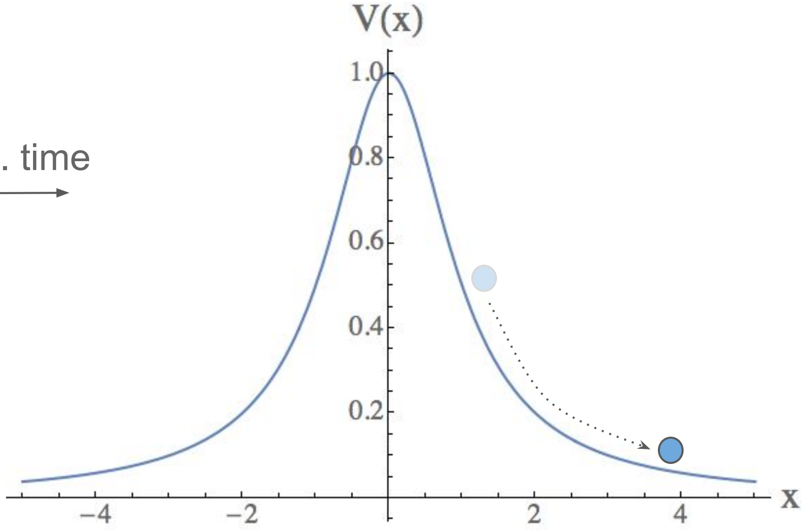


Insert Eucl. solution at an instant in time. Otherwise Lorentzian solution.

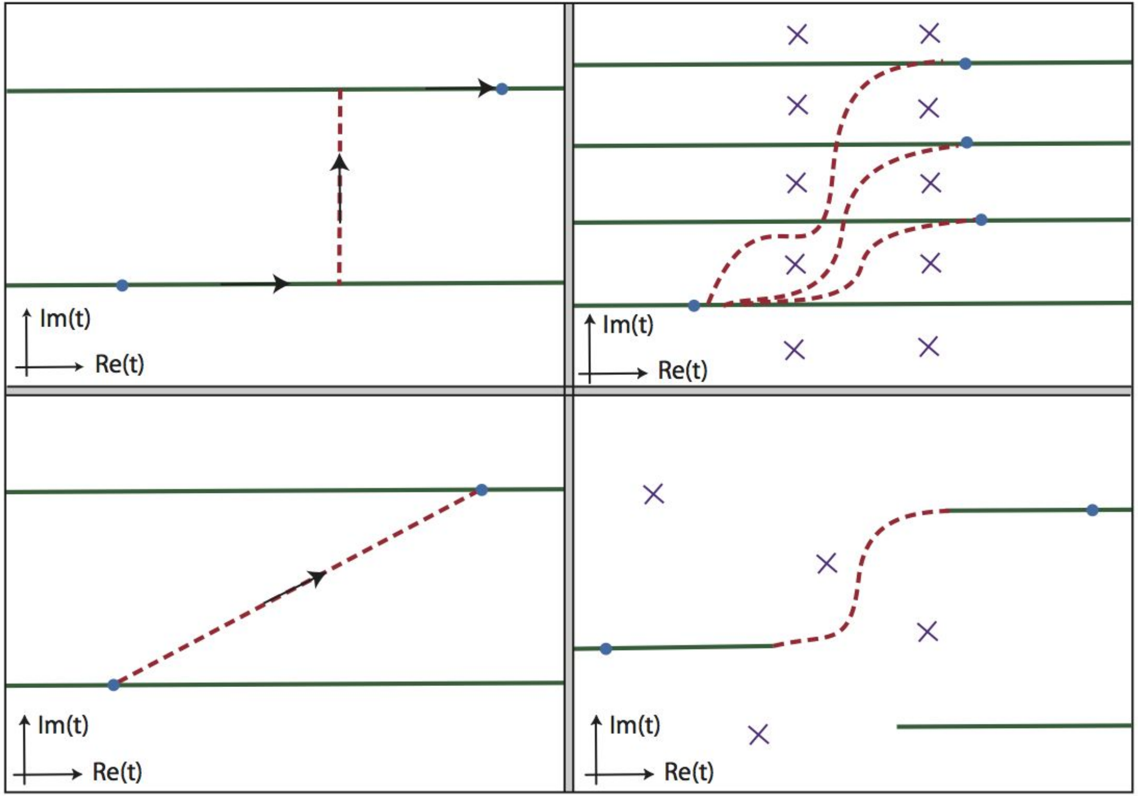
Semi-classically - probability given by instanton action

Generalizable?

- Good:
  - Works also in QFT and with gravity
- Bad:
  - Seems unmotivated
  - Not continuous
  - What about singularities

⟹ Use general complexified time: $d\tau = N dt$

Overview



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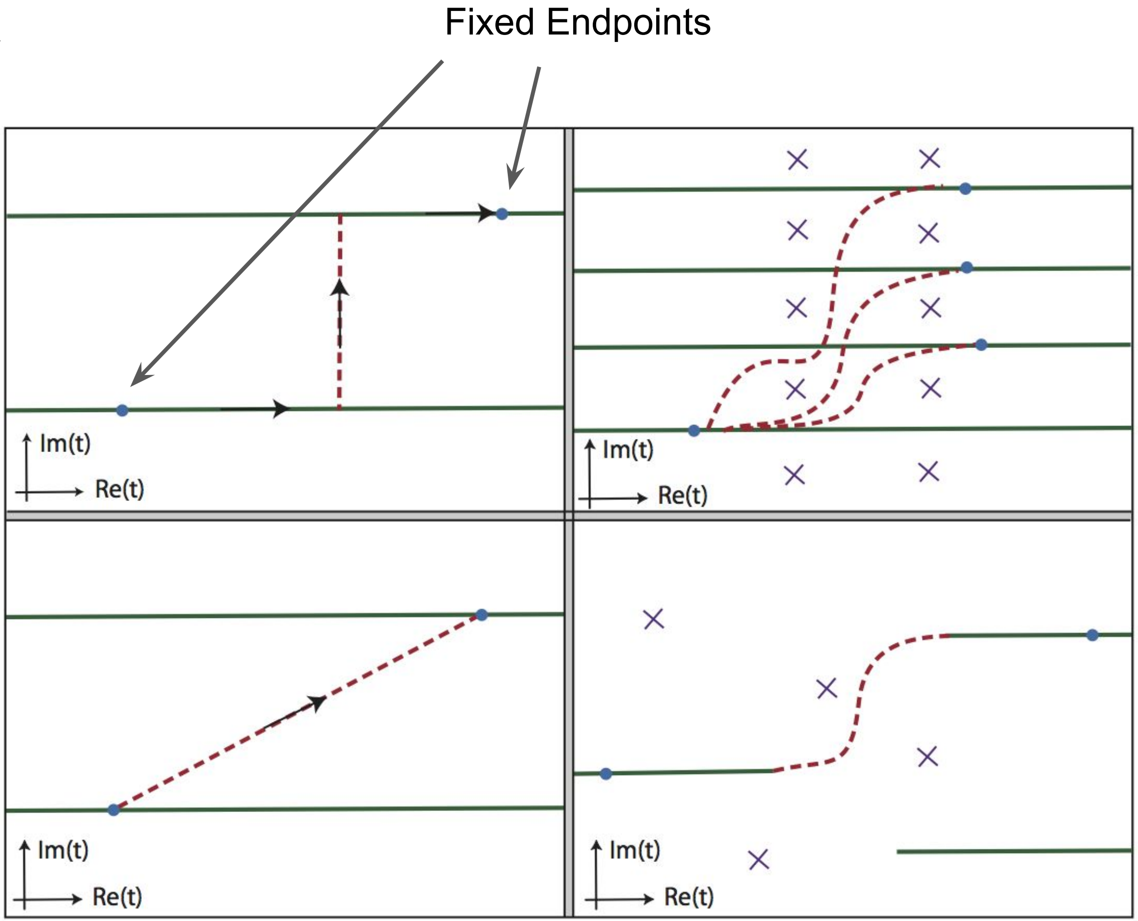

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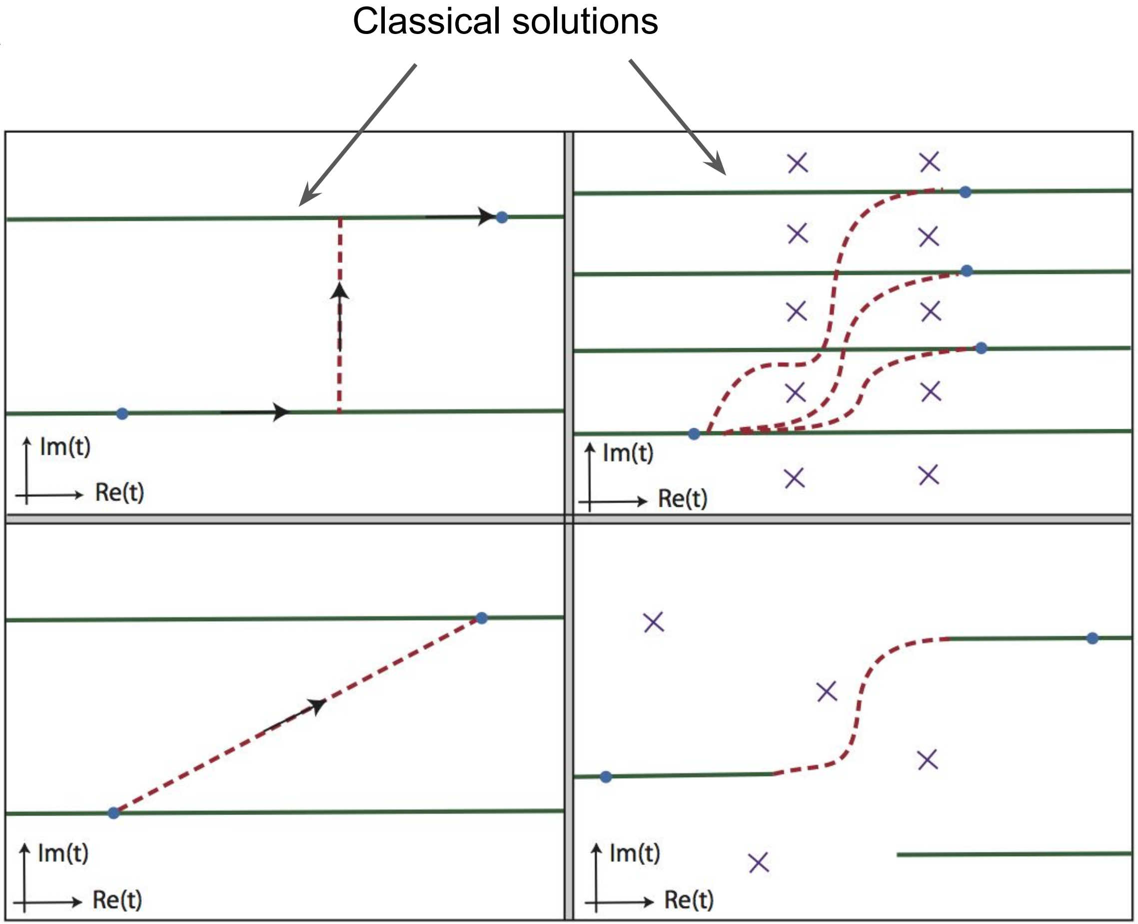

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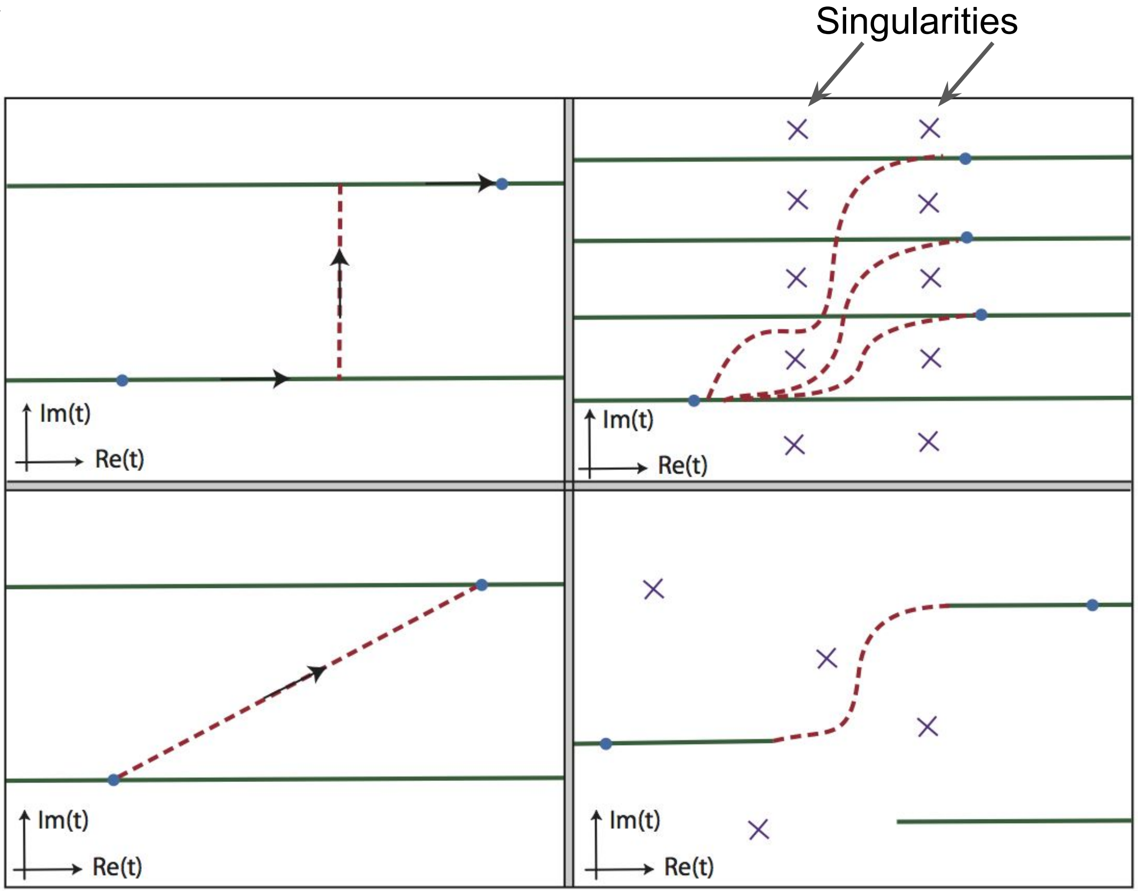
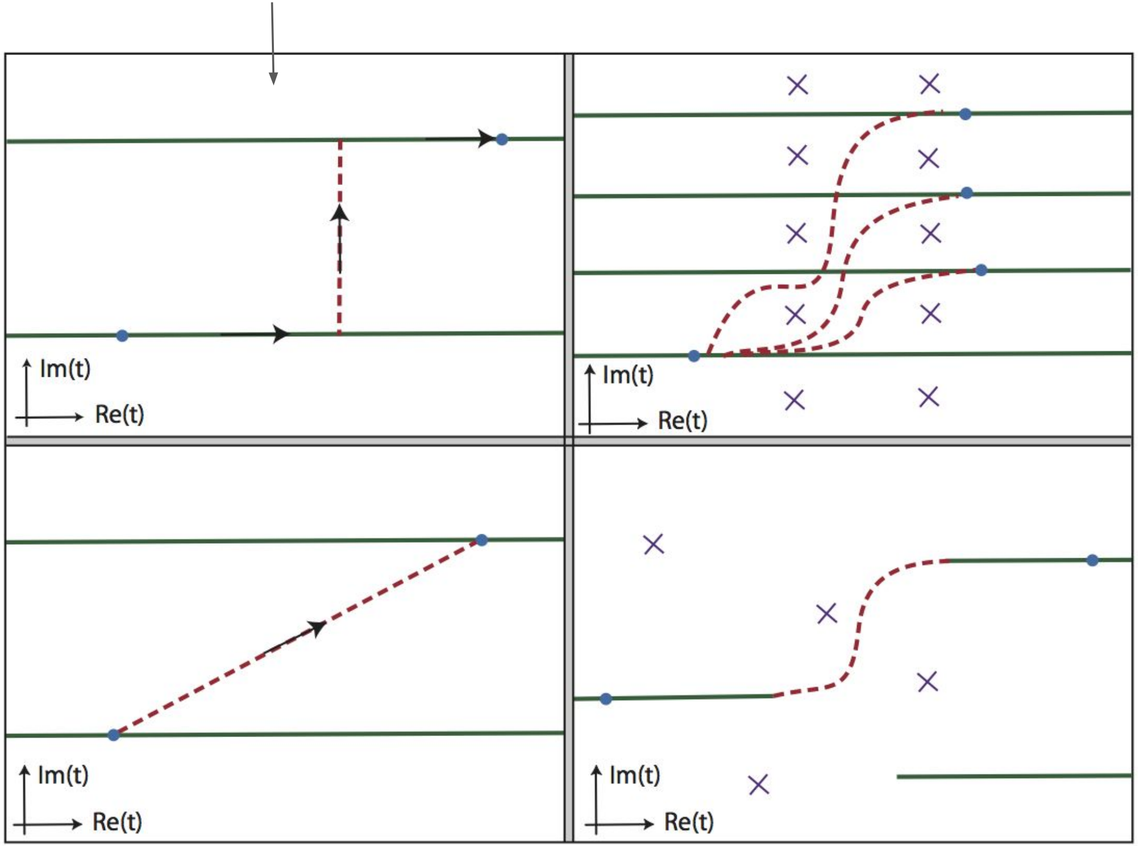


Overview

Coleman



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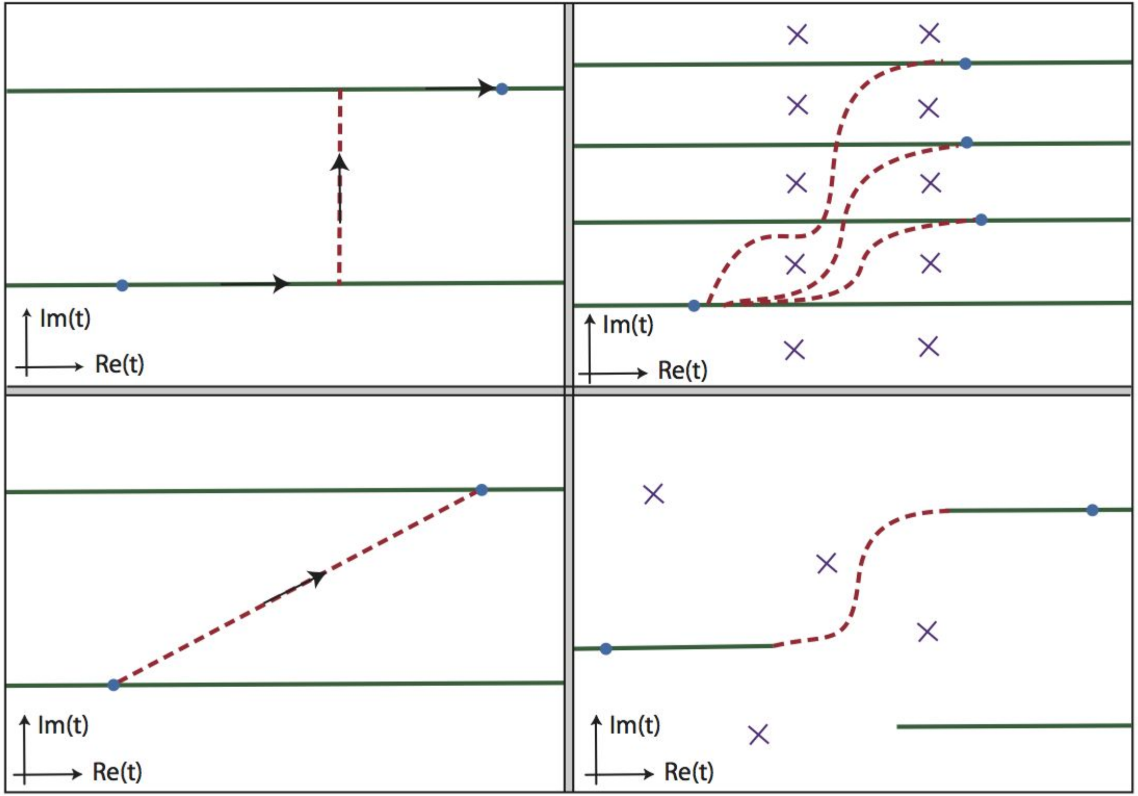

Do all paths matter?

Cauchy’s Theorem - deformed paths are equivalent when there are no singularities present.

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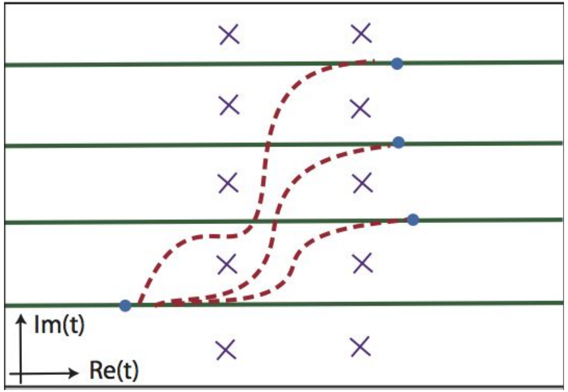
Should we sum over all inequivalent paths?



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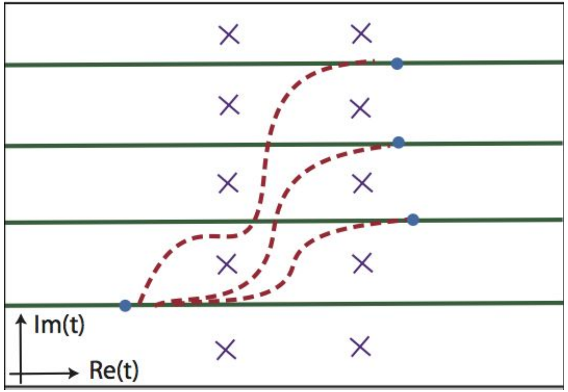

A: No but have a prescription for which paths contribute to tunneling

Which path matters?

Evaluate perturbations!

Briefly return to Coleman's description...

Which path matters?

Evaluate perturbations!

Expand action to second order around the saddle point

$$S_E[x, x_{,\tau}] = S_E[x_{cc}] + \frac{1}{2}\int_{x_i,\delta x(\tau_i)=0}^{x_f,\delta x(\tau_f)=0} \mathrm{d}\tau \left((\delta x_{,\tau})^2 + V''(x_{cc})(\delta x)^2 \right) + \cdots$$

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Fluctuations

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Positive EVs increase action

Negative EVs decrease action

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Positive EVs increase action

Negative EVs decrease action

Finding EVs is hard!

Which path matters?

Recall eigenvalue expression and nodal theorem:

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + V''(x_{cc})\right] \delta x_n = \omega_n \delta x_n$$

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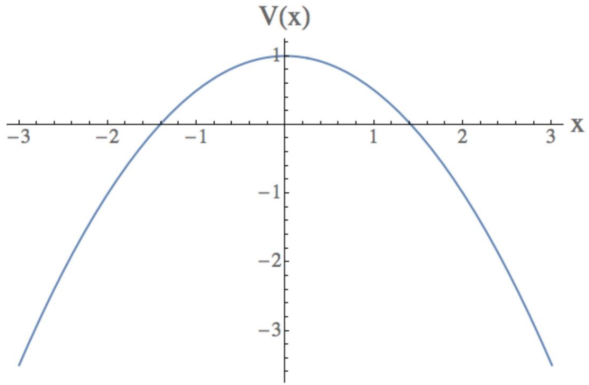
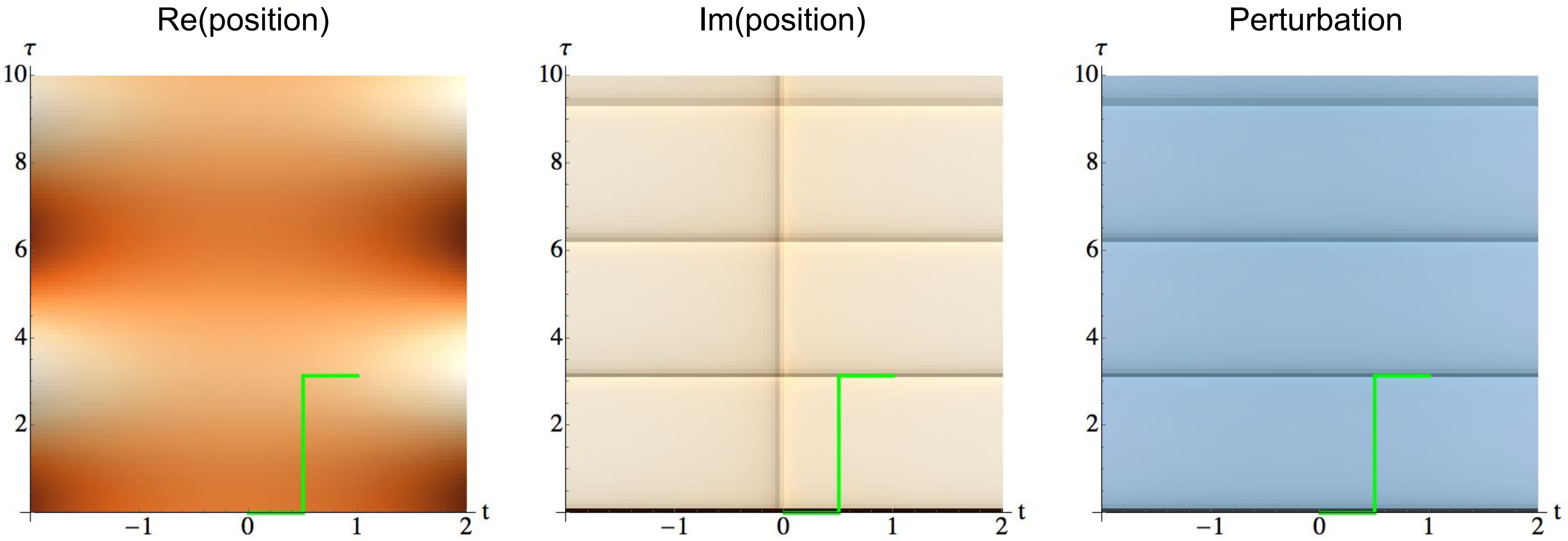
# of negative EVALs = # of nodes in the zero EVAL equation

Numerical Studies: Inverted Harmonic Oscillator

$$V(x) = -\frac{1}{2}\Omega^2 x^2 + V_0$$



Numerical Studies



Quantum Cosmology

Canonical quantization of gravity.

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right)$$

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Work within semi-classical “minisuperspace” setting.

$$ds^2 = -\tilde{N}^2(\lambda) d\lambda^2 + a^2(\lambda) d\Omega_3^2$$

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$$ds^2 = -\tilde{N}^2(\lambda) d\lambda^2 + a^2(\lambda) d\Omega_3^2$$

$$S = \frac{6\pi^2}{\kappa^2} \int d\lambda \tilde{N} \left(-a \frac{\dot{a}^2}{\tilde{N}^2} + a + \frac{\kappa^2 a^3}{3} \left(\frac{1}{2} \frac{\dot{\phi}^2}{\tilde{N}^2} - V \right) \right)$$

Inflation and Ekpyrosis

$$\epsilon = \frac{V_{,\phi}^2}{2V^2}$$

$\epsilon < 1$ — Inflation

$\epsilon > 3$ — Ekpyrosis

Inflation and Ekpyrosis

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Inflation

after initial singularity

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Inflation and Ekpyrosis

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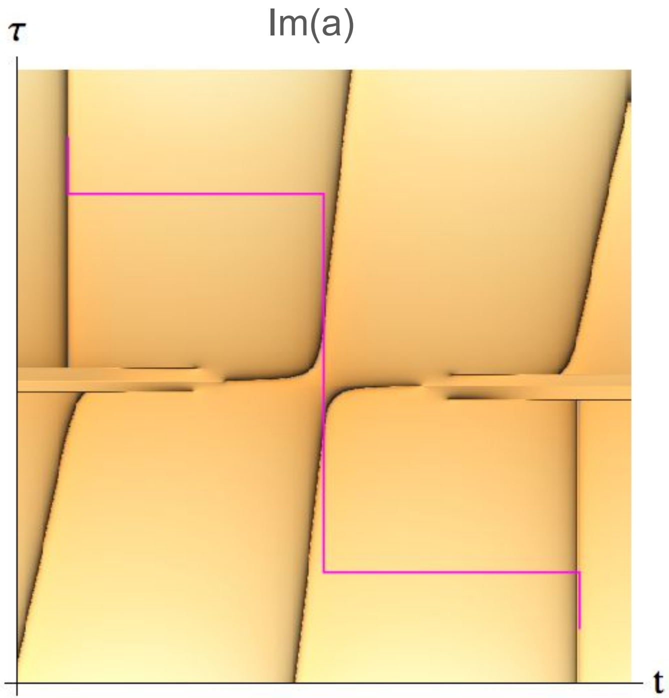
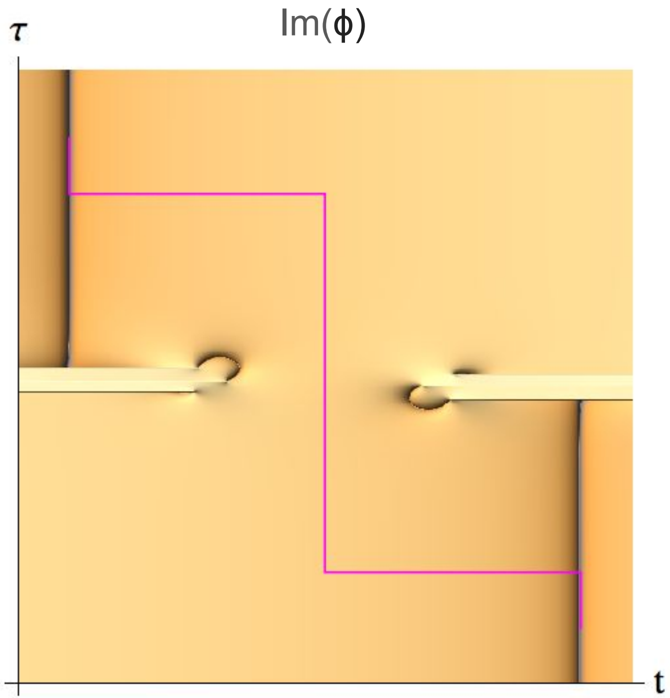
$\epsilon < 1$	$\epsilon > 3$
Inflation	Ekpyrosis
after initial singularity	leads to singularity
rapidly expanding universe	slowly contracting universe

Can we transition between two classical regions of the universe via a quantum transition?

Inflation to Inflation





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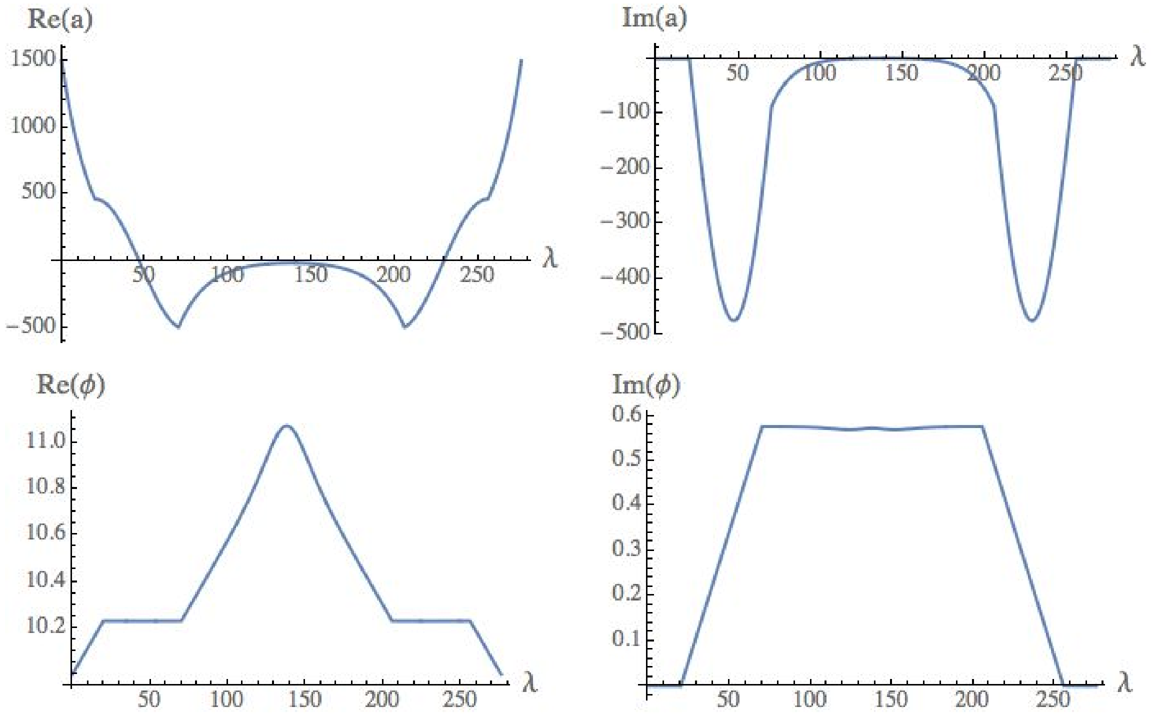
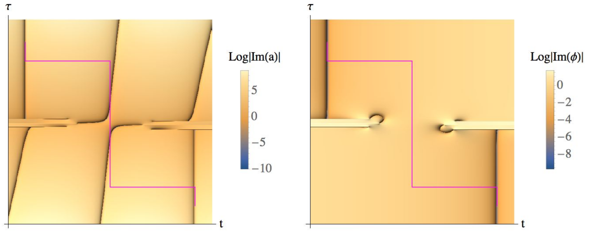
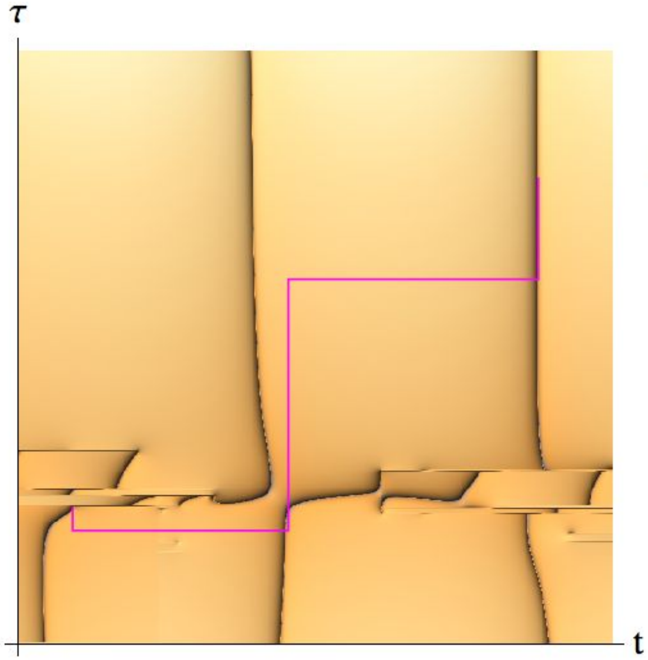


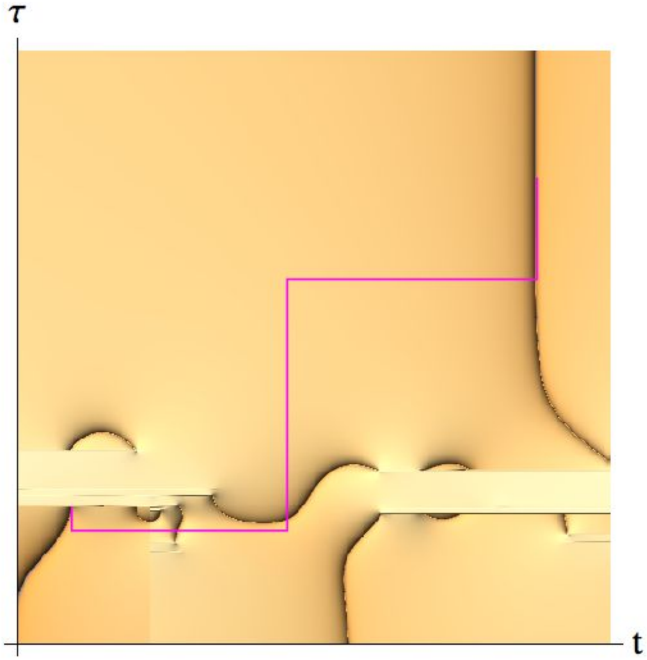


Ekpyrosis to Inflation

Im(a)



Im(ϕ)



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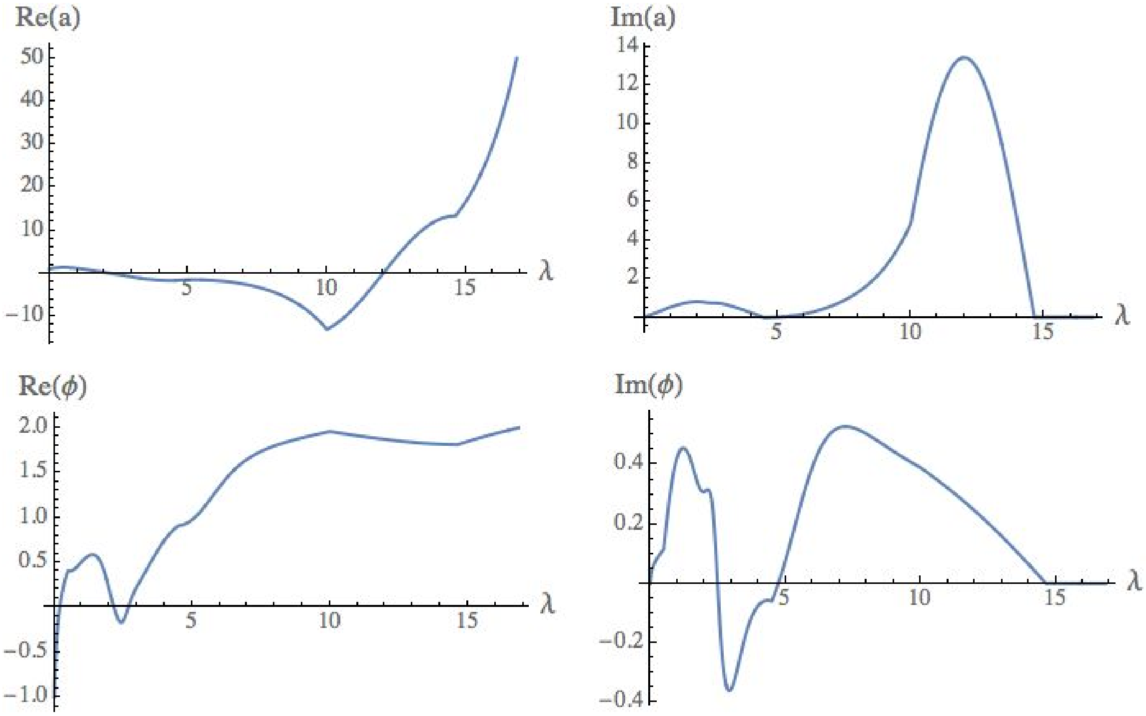

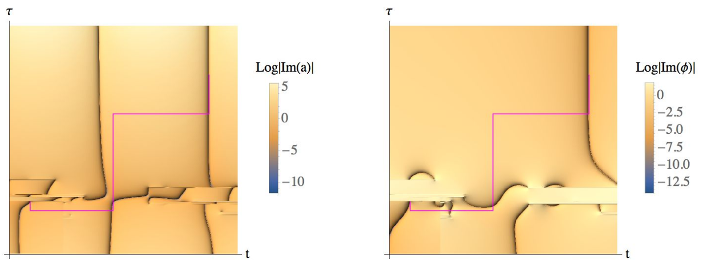

The Road So Far

Path integral description of 1D quantum tunneling using complex time paths

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Path integral description of 1D quantum tunneling using complex time paths

Identified relevant paths using perturbations

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + V''(x_{cc})\right] \delta x_n = \omega_n \delta x_n$$

The Road So Far

Path integral description of 1D quantum tunneling using complex time paths

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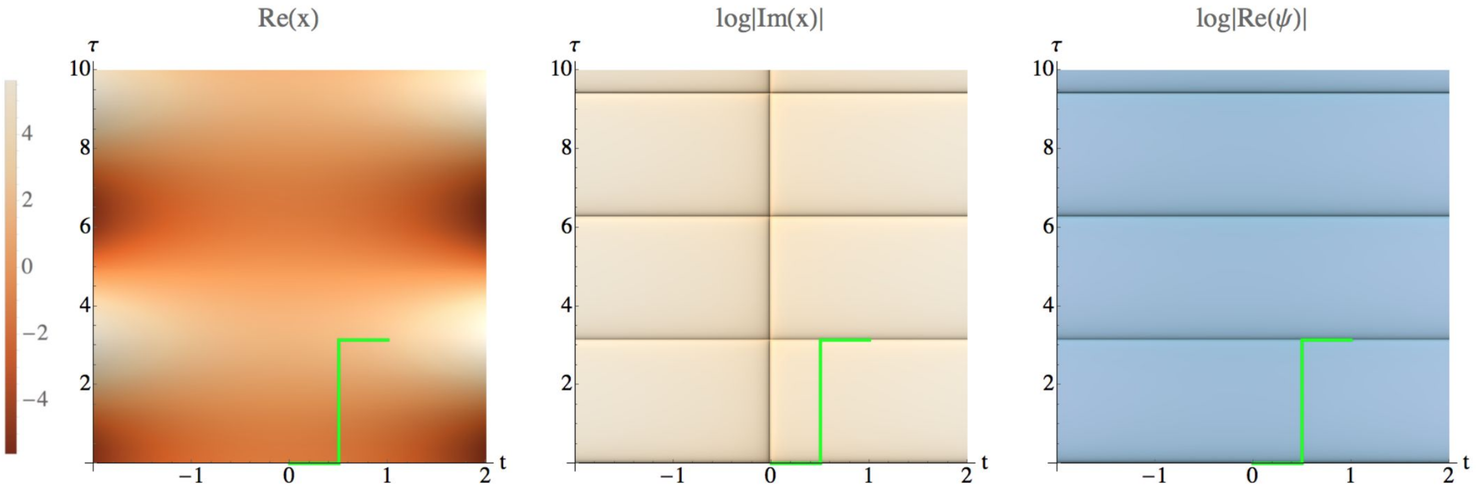
Developed numerical tools for analysis



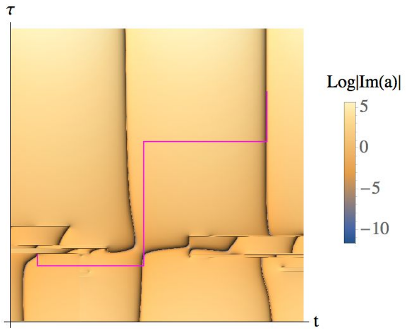
The Road So Far

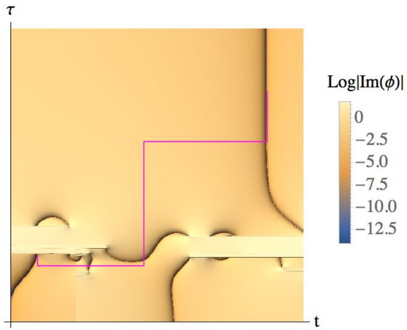
Path integral description of 1D quantum tunneling using complex time paths

Identified relevant paths using perturbations

Developed numerical tools for analysis

Found cosmological quantum transitions





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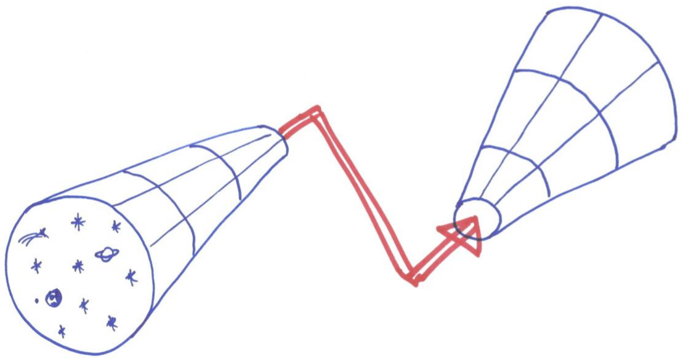
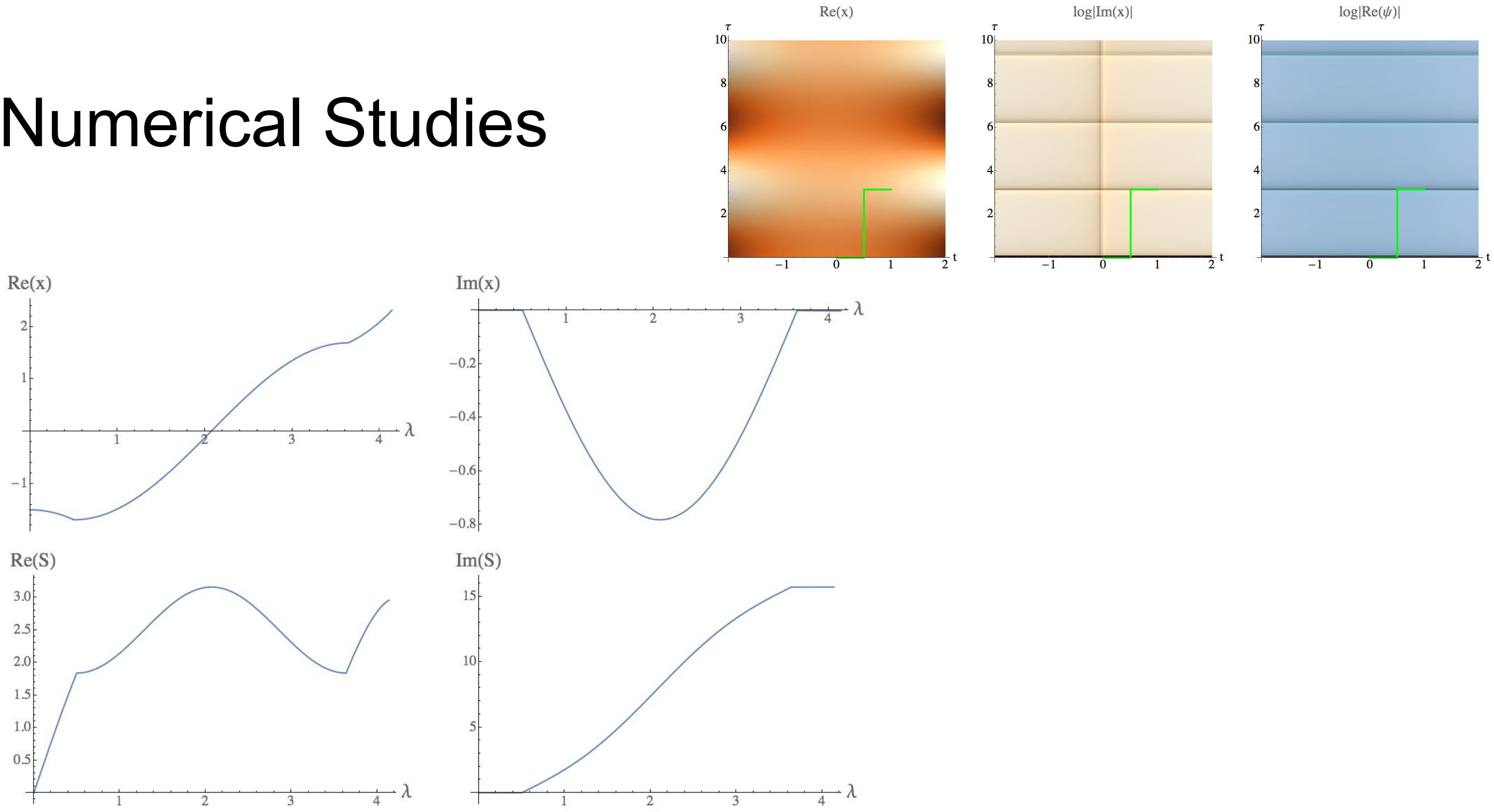


Numerical Studies



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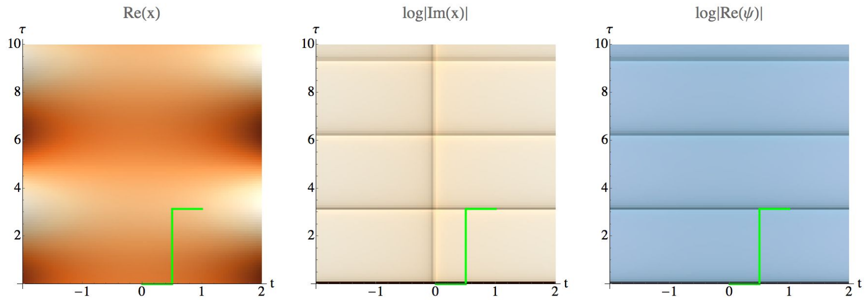
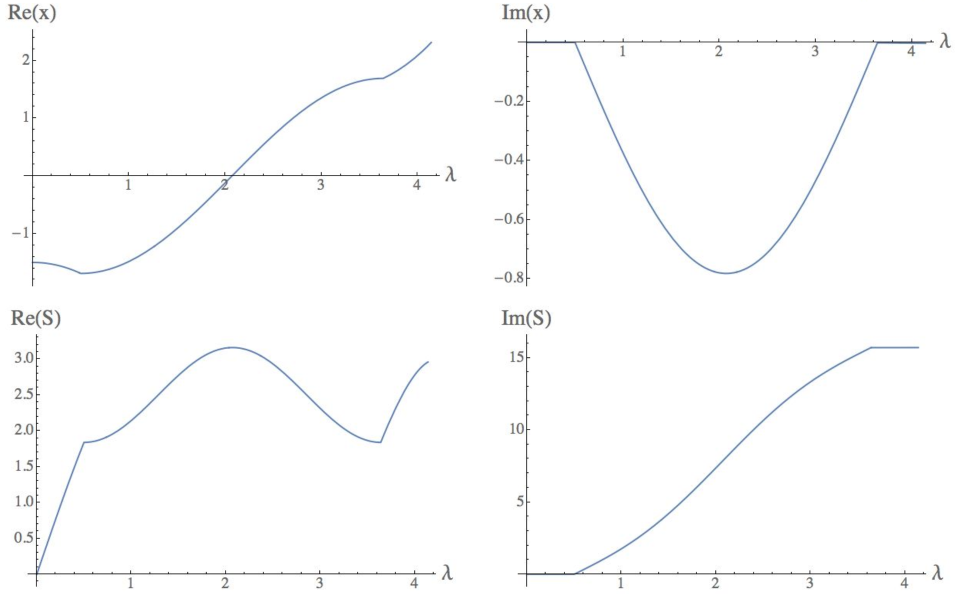
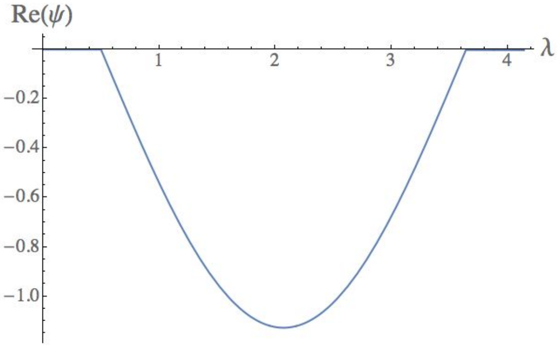
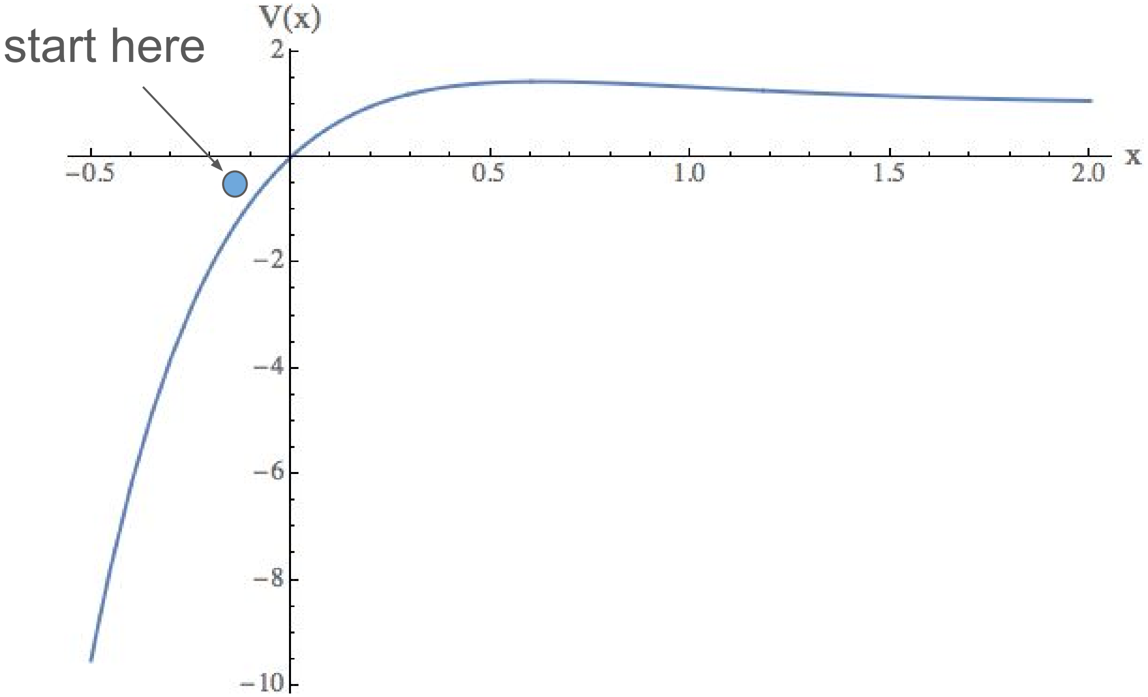






Ekpyrosis to Inflation



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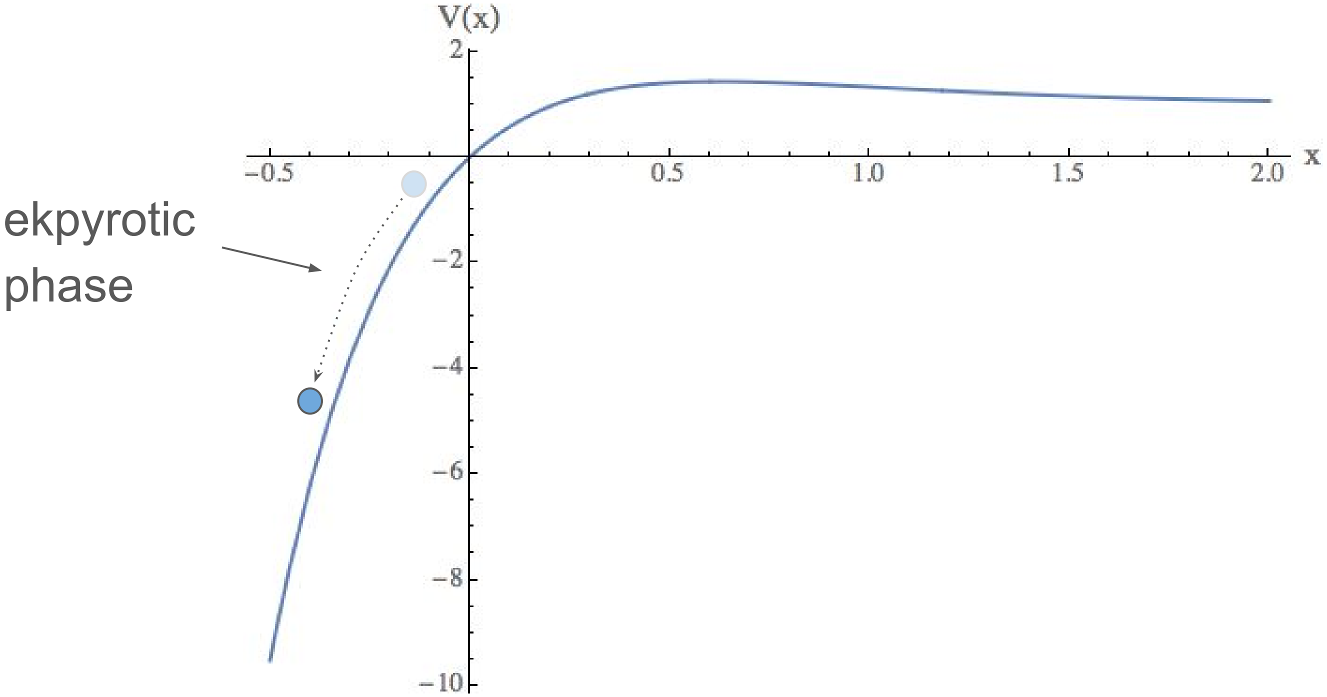

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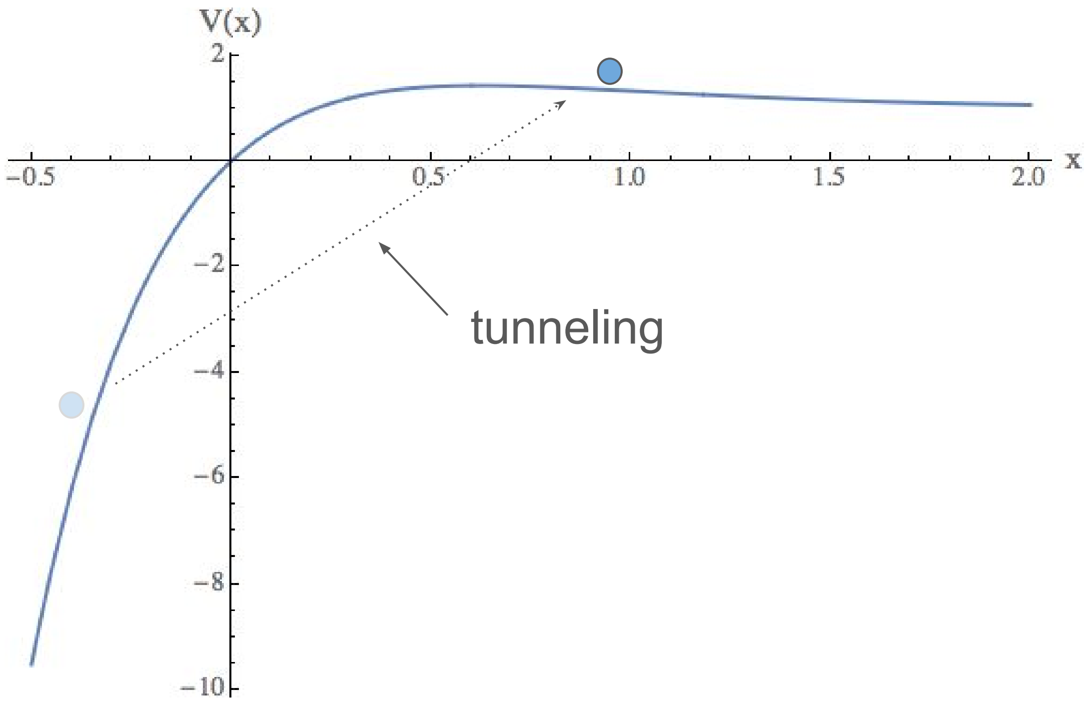

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