Axion dark matter minicluster

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Karlsruher Institut für Technologie

elusives
neutrinos, dark matter &

QCD Axion

review talk by A. Ringwald

- zero-T mass fixed by f_{PQ}

$$m(T=0) \approx 5.7 \times 10^{-5}\,\mathrm{eV}\,\frac{10^{11}\,\mathrm{GeV}}{f_{\mathrm{PQ}}}$$

- periodic potential

$$V(\theta, T) = m^2(T)(1 - \cos\theta)$$

QCD Axion

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- below I will adopt the harmonic approximation

$$V(\theta, T) \approx \frac{1}{2} m^2(T)\theta^2$$

Axion minicluster

Hogan, Rees, 1988; Kolb, Tkachev, 93, 94, 95

- PQ symmetry breaking after inflation: Axion field takes random values in different Hubble volumes
- O(1) density fluctuations when Axion mass switches on
- expect gravitationally bound objects with size ~ Hubble volume @ QCD PT

$$M \sim \tfrac{4\pi}{3} d_H^3(T_{\rm osc})\overline{\rho}(T_{\rm osc}) \qquad d_H \sim 1/H$$

$$M \sim 10^{-12} M_\odot (f_{\rm PQ}/10^{11}\,{\rm GeV})^2$$

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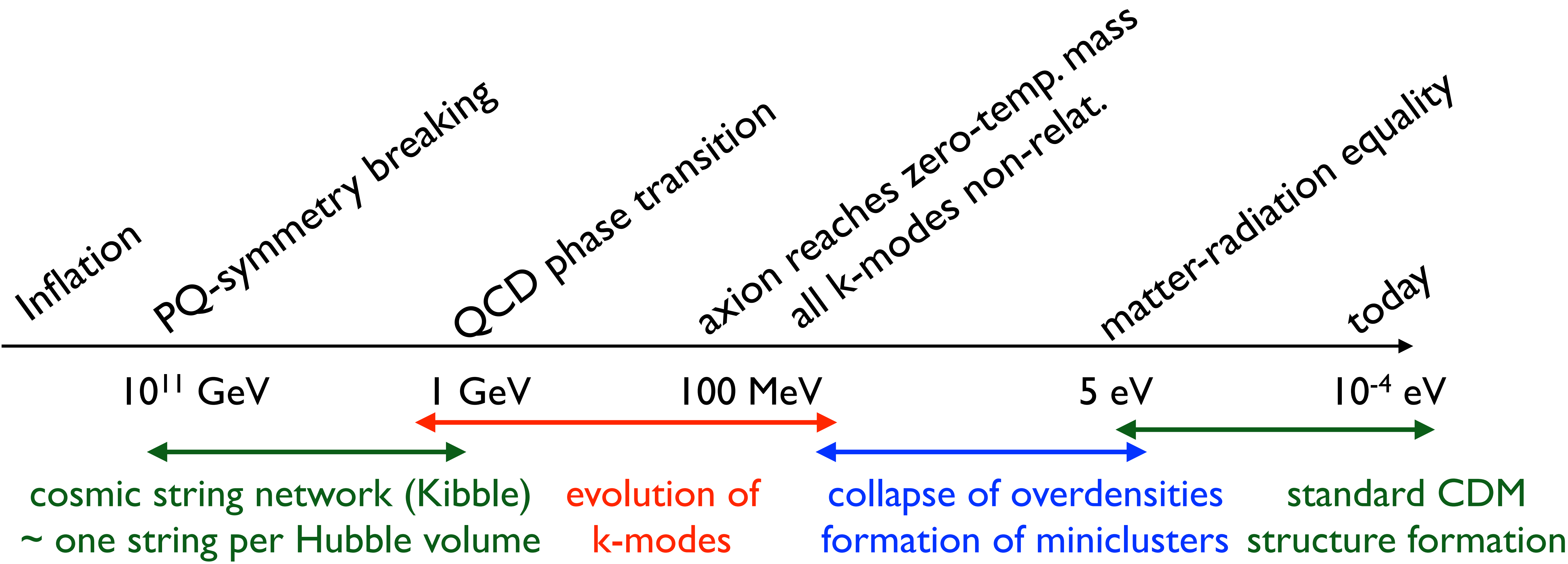
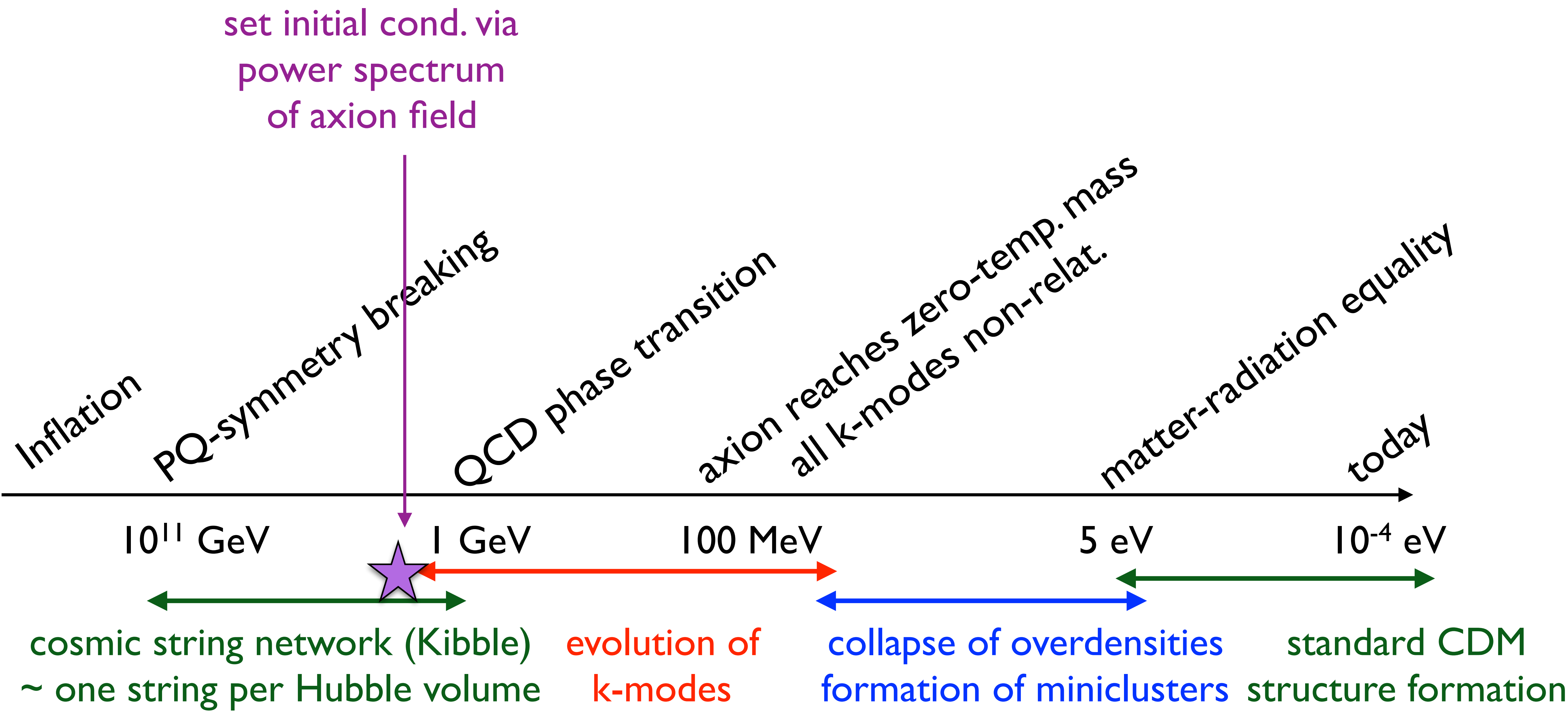
→ see later


Inflation
PQ-symmetry breaking
QCD phase transition
axion reaches zero-temp. mass
all k-modes non-relat.
matter-radiation equality
today
10^11 GeV
1 GeV
100 MeV
5 eV
10^-4 eV
cosmic string network (Kibble)
~ one string per Hubble volume
evolution of
k-modes
collapse of overdensities
formation of miniclusters
standard CDM
structure formation



set initial cond. via
power spectrum
of axion field
Inflation
PQ-symmetry breaking
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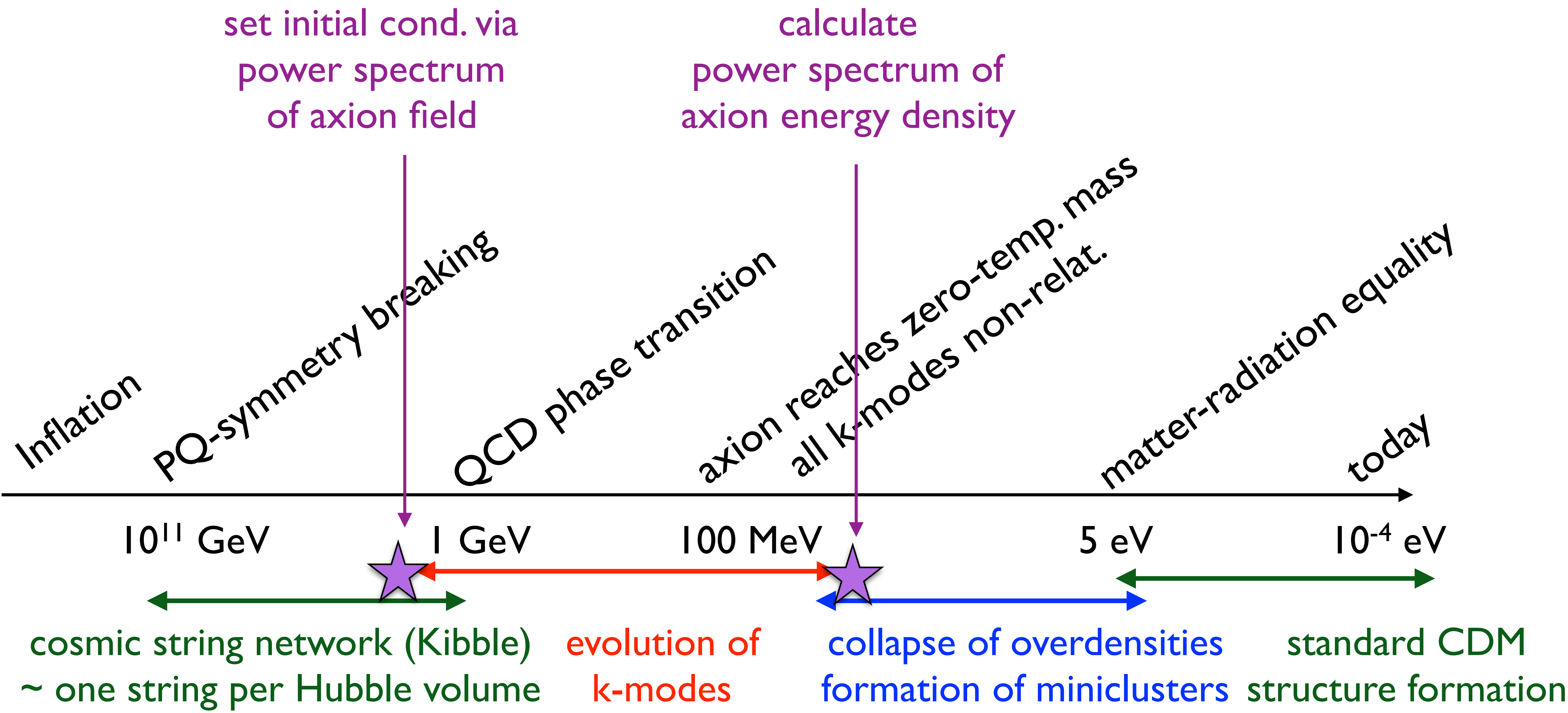
set initial cond. via power spectrum of axion field
calculate power spectrum of axion energy density
Inflation
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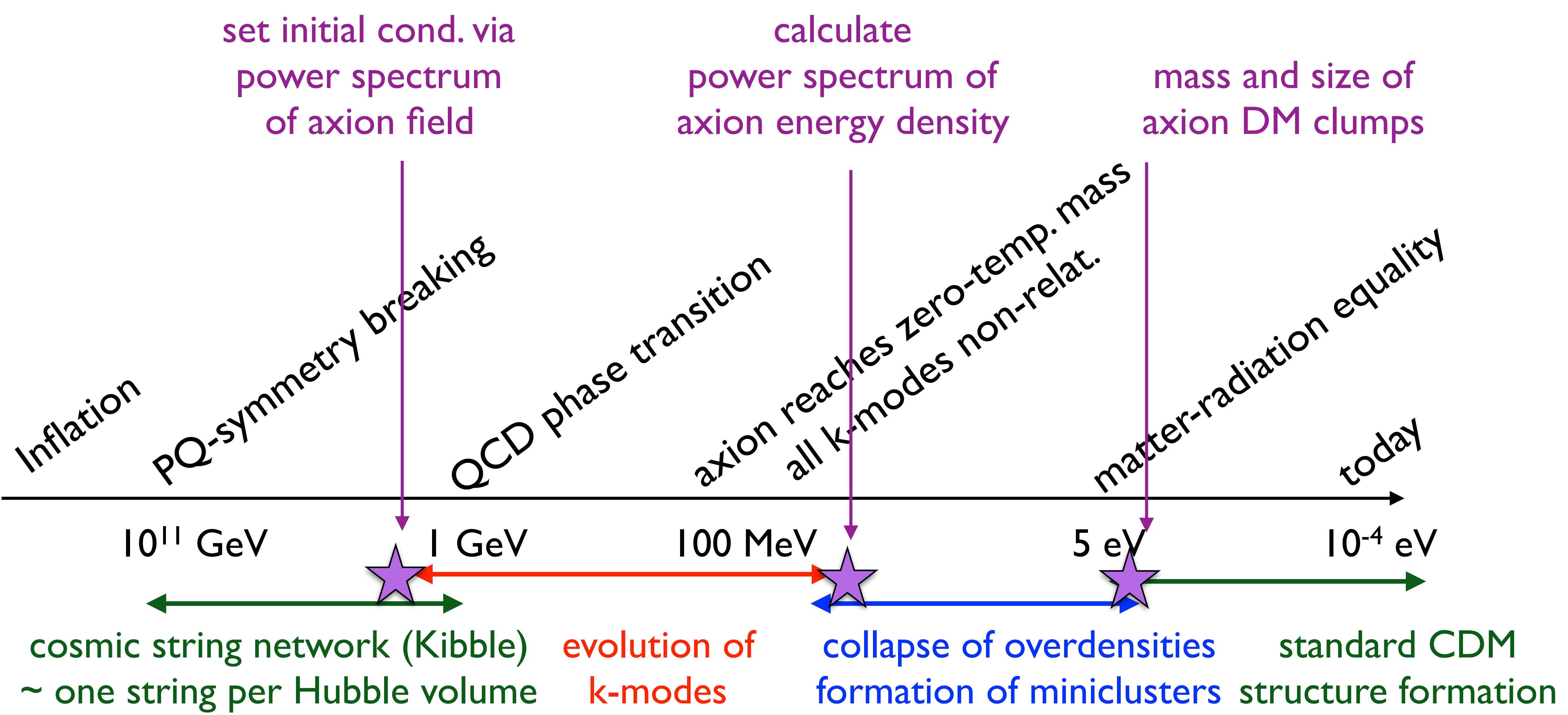

set initial cond. via
power spectrum
of axion field
calculate
power spectrum of
axion energy density
mass and size of
axion DM clumps
Inflation
PQ-symmetry breaking
QCD phase transition
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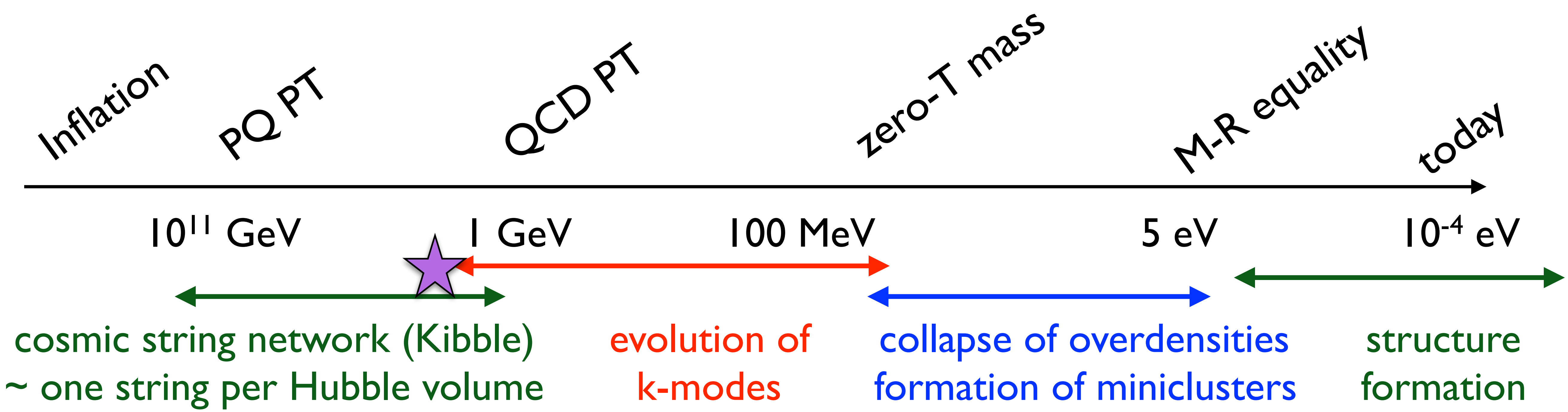
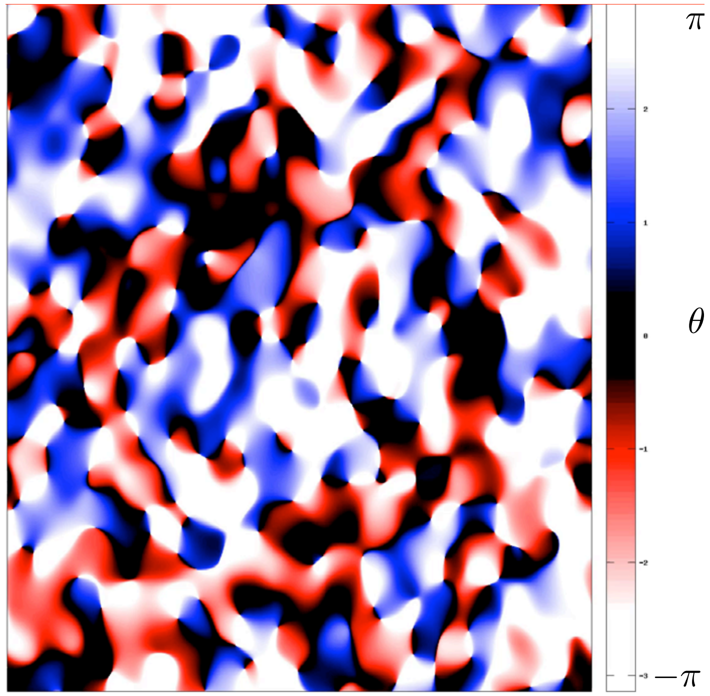
Initial condition



Initial condition

- axion field smooth on scales < horizon
  uncorrelated on scales > horizon
- assume power spectrum for axion field
  w Gaussian cut-off



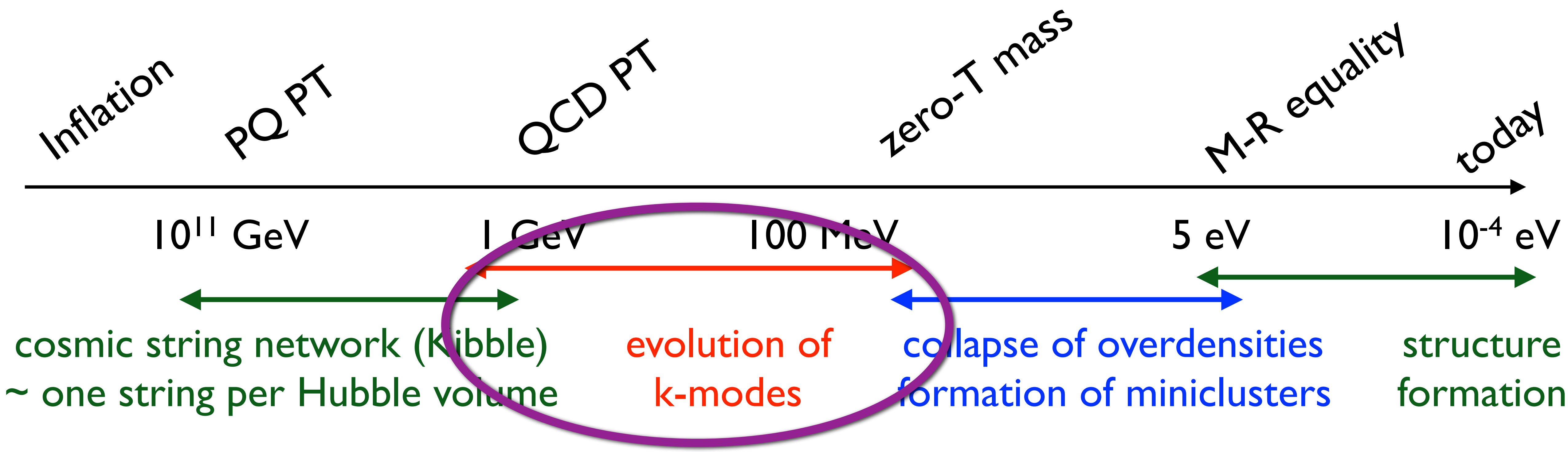
J. Redondo

$$\langle \theta_k \theta^*_{k'} \rangle = (2\pi)^3 \, \delta^3(\vec{k} - \vec{k}') P_\theta(k)$$

$$P_\theta^{\rm G}(k) = \frac{8\pi^4}{3\sqrt{\pi}K^3} \exp\left(-\frac{k^2}{K^2}\right)$$

- normalization: fixed by flat distribution $\langle \theta(\vec{x})^2 \rangle = \pi^2/3$
- cut-off: comoving horizon wave-number $K = a_i H_i$

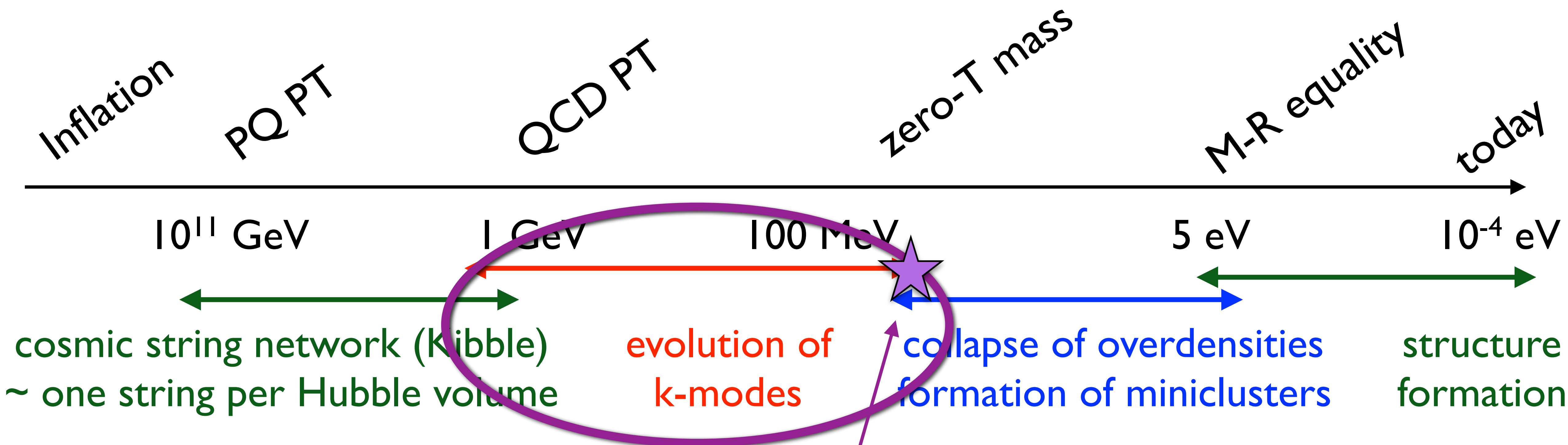
Axion field evolution



- harmonic approximation of axion potential
- equation of motion including gradient terms:

$$\ddot{\theta}_k + 3H(T)\dot{\theta}_k + \omega_k^2\theta_k = 0\,, \qquad \omega_k^2 \equiv \frac{k^2}{a^2} + m(T)^2$$

Axion field evolution



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- solve EoM to calculate axion energy density and density power spectrum

Axion energy density

$$\rho(\vec{x}) = \frac{f_{\rm PQ}^2}{2}\left[\dot{\theta}^2 - \frac{1}{a^2}(\vec{\nabla}\theta)^2 + m^2\left(T\right)\theta^2\right]$$

$$\theta_k(a) = \theta_k\, f_k(a)$$

- go to Fourier space for field

$$\rho(\vec{x}) = \frac{1}{(2\pi)^6}\frac{f_{\rm PQ}^2}{2}\int d^3k d^3k'\,\theta_k\theta^*_{k'}F(k,k')e^{-i\vec{x}(\vec{k}-\vec{k}')}$$

$$F(k,k') = \dot{f}_k\dot{f}_{k'} + \left(\frac{\vec{k}\cdot\vec{k}'}{a^2} + m^2\left(T\right)\right)f_k f_{k'}$$

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- average energy density (without string contribution):

$$\overline{\rho} \equiv \langle\rho(\vec{x})\rangle = \frac{1}{2\pi^2}\frac{f_{\rm PQ}^2}{2}\int_0^\infty dk\, k^2\, P_\theta(k) F(k,k)$$

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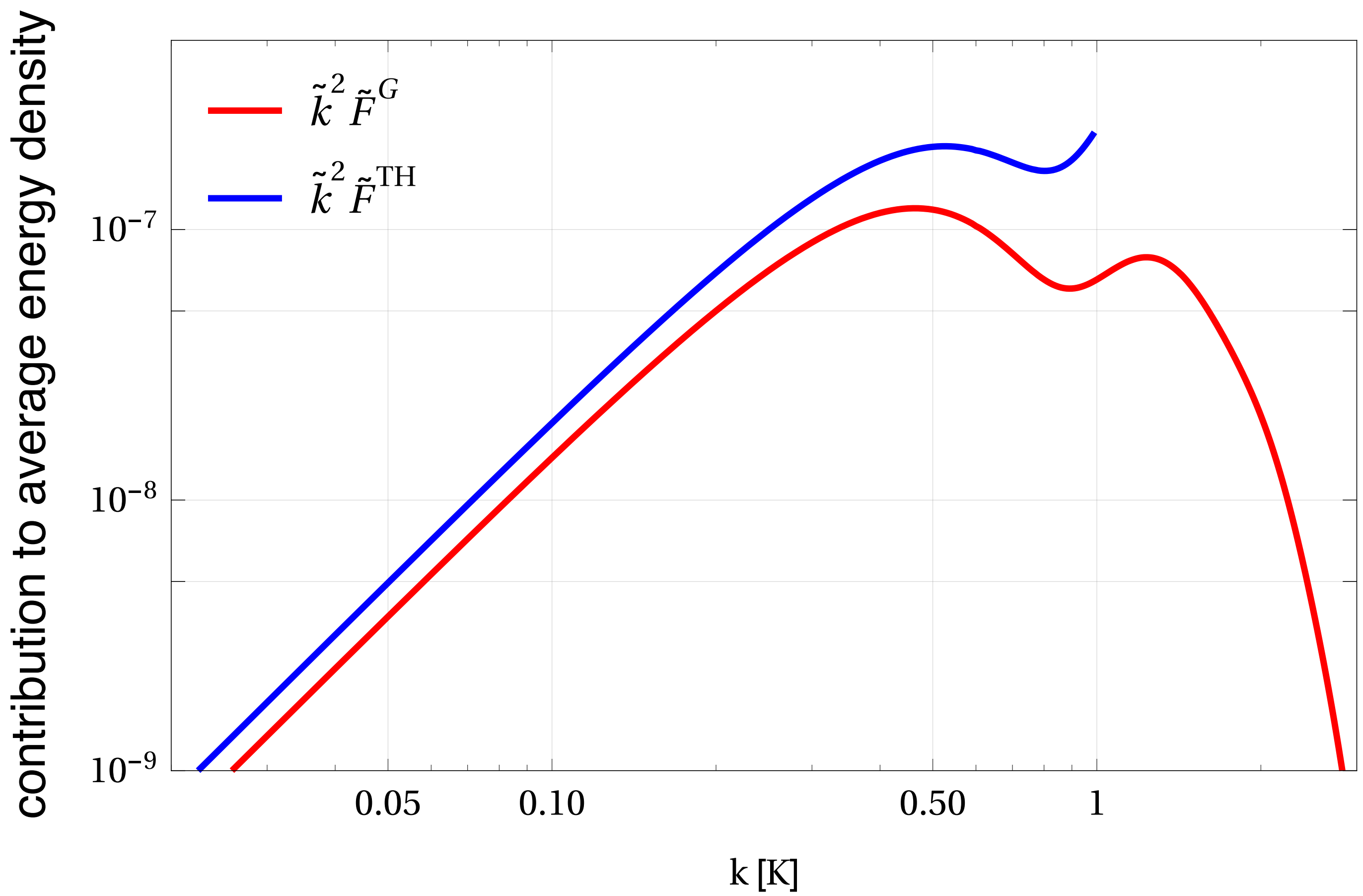
Axion energy density



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Density power spectrum

$$P(q) = \frac{1}{V}\frac{\langle|\rho_q|^2\rangle}{\overline{\rho}^2} = 2(2\pi)^3 \frac{\int d^3k\, P_\theta(|\vec{k}|) P_\theta(|\vec{k}-\vec{q}|)\, F(k,k-q)^2}{\left[\int d^3k\, P_\theta(k) F(k,k)\right]^2}$$

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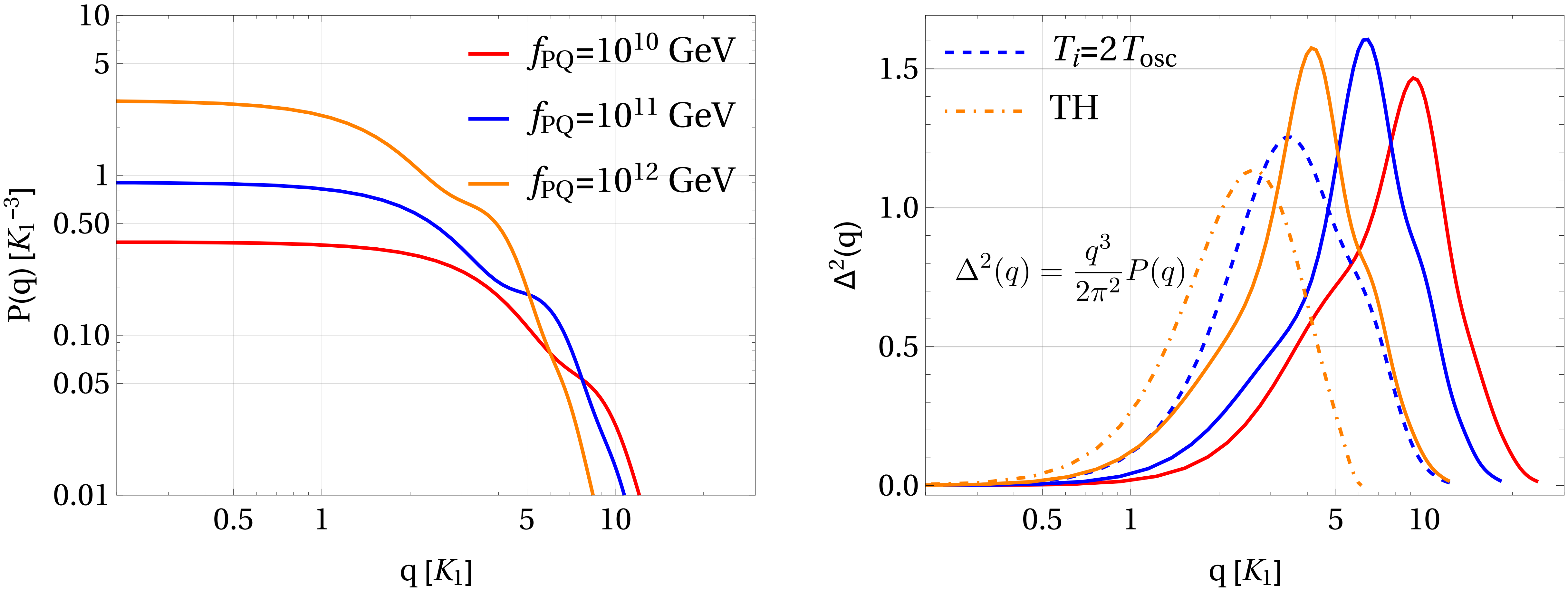
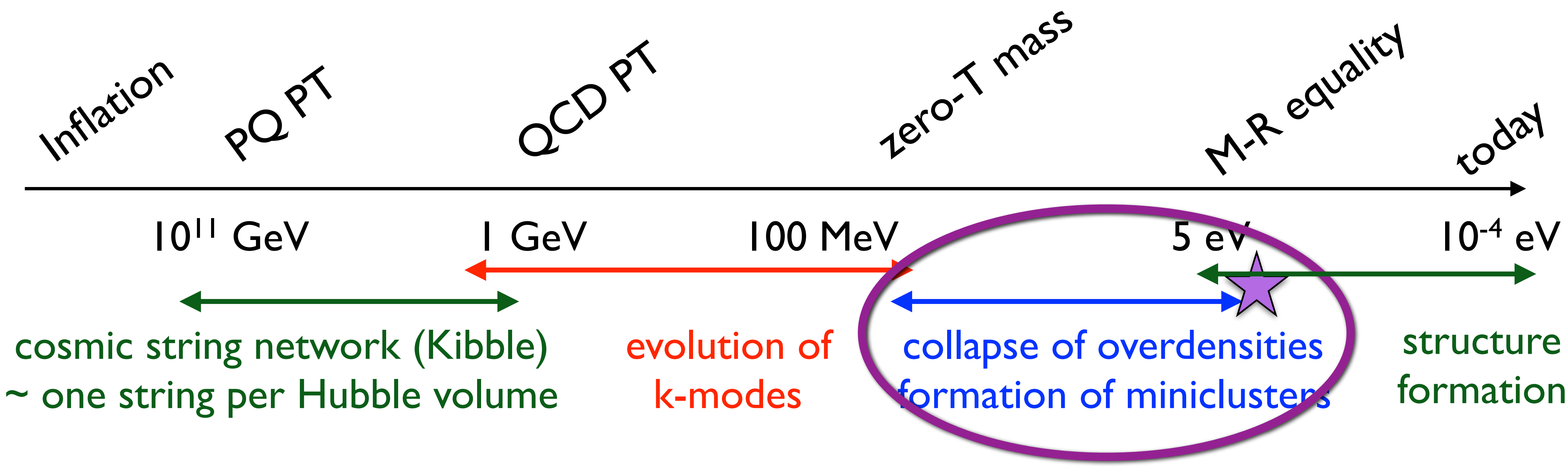


- density fluctuations of order one
- charact. size a few times smaller than horizon @ T_{osc}

Collapse of over-densities



- spherical collapse model for non-linear density fluctuations during radiation domination
  [Kolb, Tkachev, 94]

- modified Press-Schechter ansatz to calculate double-differential minicluster distribution in mass and size

Double-differential mass function

- variance of the smoothed density contrast

$$\sigma_R^2 \equiv \langle \delta_R(\vec{x})^2 \rangle = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2 P(k) \left| \tilde{W}_R(k) \right|^2$$

- Gaussian distribution for the smoothed contrast

$$f_{\rm sm}(\delta; R) = \frac{1}{\sqrt{2\pi}\sigma_R} \exp\left(-\frac{\delta^2}{2\sigma_R^2} \right) .$$

- derive distribution in δ and R:

$$f(\delta, R) = -\frac{1}{\sigma_0} \frac{d\sigma_R}{dR} \frac{\delta^2}{\sigma_R^2} f_{\rm sm}(\delta; R)$$

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- double-differential distribution in M and R using: $M = \frac{4\pi}{3} \overline{\rho}\, (1+\delta)\, r^3$

$$\frac{dn}{dM\, dR} = \frac{3}{2\pi M R^3} f(\delta, R) \Theta[\delta - \delta_c(x)]$$

Dimensionless double-differential mass function

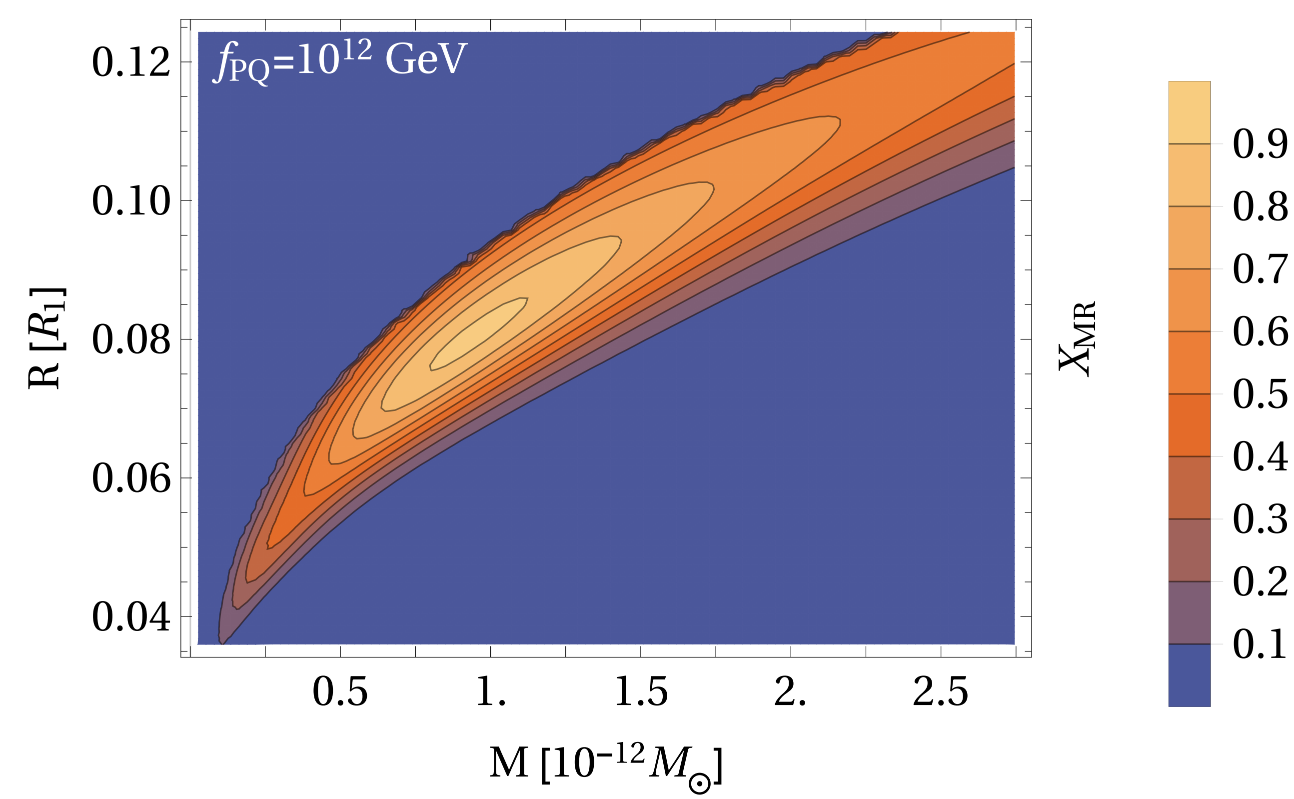
physical sizes at turn around

$f_{\rm PQ}$ [GeV]	$M_{\rm peak}$ $[M_\odot]$	M range $[M_\odot]$	$r_{\rm ta}^{\rm peak}$ [km]	$r_{\rm ta}$ range [km]
10^{10}	4×10^{-16}	$[2\times 10^{-17}, 1\times 10^{-14}]$	4×10^{4}	$[2\times 10^{4}, 2\times 10^{5}]$
10^{11}	2×10^{-14}	$[5\times 10^{-16}, 3\times 10^{-13}]$	2×10^{5}	$[4\times 10^{4}, 7\times 10^{5}]$
10^{12}	8×10^{-13}	$[6\times 10^{-14}, 2\times 10^{-11}]$	2×10^{6}	$[7\times 10^{5}, 7\times 10^{6}]$

- MC masses span 3 orders, sizes span 1 order of magn.
- peak-masses 2 orders of mag. smaller than naive estimates

  $$M \sim 10^{-12} M_\odot (f_{\rm PQ}/10^{11}\,{\rm GeV})^2$$

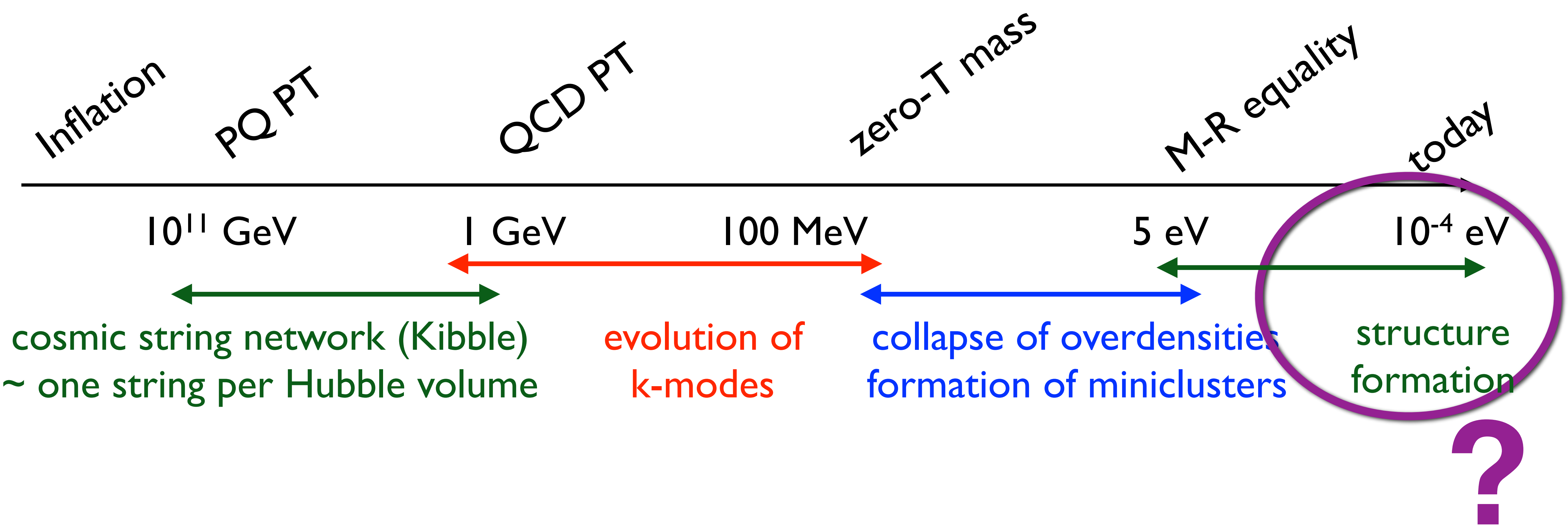
  (typical fluctuations smaller than horizon at $T_{\rm osc}$)



Outlook - approximations/assumptions

- initial power spectrum: should follow from evolution of string network (Kibble mechanism)
- harmonic approximation: anharmonic effects may lead to spikes in axion density [Kolb, Tkachev, 93]
- contribution from string/domain wall decays: likely to introduce additional energy density & fluctuations

Axion DM today - in our galaxy?



- Do minicluster survive non-linear structure formation?
- Do they collapse to dense Axion-stars?
  Are Axion-stars stable?

Axion DM today - in our galaxy?

- if a large fraction of the DM energy density is in MC-sized bound objects, the probability to meet one is very low (bad news for direct axion detection)

- depending on structure formation history, potentially interesting lensing signatures:
  femto-lensing Kolb, Tkachev, 95
  micro-lensing Fairbairn, Marsh et al, 17

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Thank you!

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