Quantum Groups behind the AdS/CFT S-Matrix

Maximally extended $\mathfrak{su}(2|2)$ and $3D$ κ-Poincaré

- as a Quantum Double.

  arXiv:1602.04988, Beisert, de Leeuw, RH.
- as a contraction limit.

  arXiv:1704.05093, Beisert, RH, Hoare.

Reimar Hecht, ETH Zurich

DESY, 27.09.2017

S-matrix in $\mathcal{N} = 4$ SYM

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Eigenstates

$|p_1, p_2\rangle + S(p_2, p_1)|p_2, p_1\rangle$

$$|p_1, p_2\rangle = \sum_{x_1<x_2} e^{ip_1x_1+ip_2x_2} |\cdots \underset{x_1}{\uparrow} \cdots \underset{x_2}{\uparrow} \cdots\rangle$$

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Integrability $S_{12}(p_1,p_2)S_{13}(p_1,p_3)S_{23}(p_2,p_3)=S_{23}(p_2,p_3)S_{13}(p_1,p_3)S_{12}(p_1,p_2)$

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S-matrix

$$S_{\mathfrak{psu}(2,2|4)} = S_{\mathfrak{psu}(2|2)} \otimes S_{\mathfrak{psu}(2|2)}$$

Fundamental $S_{\mathfrak{psu}(2|2)}$ determined by $\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3$

Algebraic Picture

Hopf algebra Algebra $\mathcal{H}$ with coproduct

$$\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}.$$

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Universal R-matrix

- an invertible element $\mathcal{R} \in \mathcal{H} \otimes \mathcal{H}$, s.t

$$\Delta^{\mathrm{op}}(a)\mathcal{R} = \mathcal{R}\Delta(a), \; \forall a \in \mathcal{H},$$

with opposite coproduct $\Delta^{\mathrm{op}} := \tau \circ \Delta, \quad \tau(a \otimes b) = b \otimes a.$

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- Given a representation $\rho : \mathcal{H} \to \mathrm{End}(\mathbb{V})$

$$R_\rho = (\rho \otimes \rho)\mathcal{R}, \qquad S_\rho = \tau R_\rho.$$

How to construct R-matrix?

Quantum Double $\mathcal{D}(\mathcal{H})$

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$$\mathcal{R} = \sum_{i \in I} e_i \otimes e_i^* \in \mathcal{D}(\mathcal{H}) \otimes \mathcal{D}(\mathcal{H}),$$

where $\{e_i\}_{i \in I} \subset \mathcal{H}$ and $\{e_i^*\}_{i \in I} \subset \mathcal{H}^{*\mathrm{op}}$ are dual bases.

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Let's do this for q-deformed $\mathcal{U}_q(\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3)$!

$\mathcal{U}_q(\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3)$

Chevalley-Serre generators $q = e^{\hbar}$,

$$[H_i, E_j] = a_{ij} E_j, \qquad [E_i, F_j] = \delta_{ij} \frac{q^{H_i} - q^{-H_i}}{q - q^{-1}},$$
$$[H_i, F_j] = -a_{ij} F_j,$$

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E_2, F_2 fermionic, and Cartan matrix

$$a_{ij} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 0 & +1 \\ 0 & +1 & -2 \end{pmatrix},$$

Note: Cartan matrix is degenerate

$H_C := H_1 + 2H_2 + H_3$ is central.

$\mathcal{U}_q(\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3)$

Serre relations:

Additional central elements E_C, F_C

$$\{[E_1, E_2]_q, [E_3, E_2]_q\} = E_C, \qquad \{[F_1, F_2]_q, [F_3, F_2]_q\} = F_C.$$

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$$E_C = F_C = 0 \;\Rightarrow\; \mathfrak{su}(2|2); \qquad H_C = 0 \Rightarrow \mathfrak{psu}(2|2).$$

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Coproduct

$$\begin{aligned} \Delta H_i &= H_i \otimes 1 + 1 \otimes H_i, & \Delta H_C &= H_C \otimes 1 + 1 \otimes H_C \\ \Delta E_i &= E_i \otimes 1 + q^{-H_i} \otimes E_i, & \Delta F_i &= F_i \otimes q^{H_i} + 1 \otimes F_i, \\ \Delta E_C &= E_C \otimes 1 + q^{-H_C} \otimes E_C, & \Delta F_C &= F_C \otimes q^{H_C} + 1 \otimes F_C \end{aligned}$$

Quantum Double

How it works for $\mathcal{U}_q(\mathfrak{g})$

[Drinfeld '86], [Burroughs '90],[Rosso '89], [Levendorskii, Soibelman '91], [Kirillov, Reshetikhin '90]

1.) positve Borel sub-algebra $\mathcal{U}_q(\mathfrak{b}^+)$:

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$$\widehat{H}_j \sim \frac{1}{\hbar} \sum_i a_{ij} H_i^*, \qquad F_j \sim E_i^*,$$

and thus $\quad \mathcal{U}_q(\mathfrak{g}) \cong \mathcal{D}(\mathcal{U}_q(\mathfrak{b}^+))/\langle \widehat{H} - H \rangle$

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5.) R-matrix $\qquad e_q^X := \sum_{n=0}^{\infty} \frac{X^n}{[n;q]!}, \quad [n;q] := \frac{1-q^n}{1-q}$

$$\mathcal{R} = e_{q^{-2}}^{(q-q^{-1})E_1\otimes F_1}\, e_{q^{-2}}^{(q-q^{-1})E_{12}\otimes F_{21}}\, e_{q^{-2}}^{(q-q^{-1})E_2\otimes F_2}\, q^{H_1\otimes(2H_1+H_2)/3}\, q^{H_2\otimes(2H_2+H_1)/3},$$

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Now for $\mathcal{U}_q(\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3)$

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$\Rightarrow$ **no chance to identify:** $\mathcal{U}_q(\mathfrak{b}^+)^{*\mathbf{op}} \ncong \mathcal{U}_q(\mathfrak{b}^-)$

Overview

	$\mathcal{U}_q(\mathfrak{b}^+)$	$\mathcal{U}_q(\mathfrak{b}^-)$
$\mathfrak{psu}(2\|2)$	$H_{1/3}$	$H_{1/3}$
	E_i	F_i
$\mathbb{R}^3$	H_C	H_C
	E_C	F_C

Overview

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\begin{tikzcd}
 & \mathcal{U}_q(\mathfrak{b}^+) & \mathcal{U}_q(\mathfrak{b}^+)^* & \mathcal{U}_q(\mathfrak{b}^-) \\
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 & E_i \arrow[r, "*"] & E_i^* \arrow[r, equal, "\sim"] & F_i \\
\mathbb{R}^3 & H_C \arrow[ddr, "*"] & & H_C \\
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Solution: Enlarge Hopf algebra with additional generators

$E_\mathrm{A}, H_\mathrm{A}, F_\mathrm{A}$

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\end{tikzcd}$$

Solution: Enlarge Hopf algebra with additional generators $E_{\mathrm{A}}, H_{\mathrm{A}}, F_{\mathrm{A}}$

$\Rightarrow$ **one-parameter** (ξ) family of consistent Hopf algebras

R-matrix

$$\mathcal{R} = \sum_{i \in I} e_i \otimes e_i^*$$

R-matrix

$$\begin{aligned}
\mathcal{R} &= \sum_{i \in I} e_i \otimes e_i^* \\
&= e^{\gamma E_2 \otimes F_2} e^{\gamma E_{12} \otimes F_{21}} \\
&\cdot \exp\left[\left(\frac{E_A \otimes F_C + E_C \otimes F_A}{\gamma E_C \otimes F_C} - \frac{\xi}{2\hbar}\right) \log(1 + \gamma^2 E_C \otimes F_C) - \frac{\xi}{4\hbar} \mathrm{Li}_2(-\gamma^2 E_C \otimes F_C)\right] \\
&\cdot e^{\gamma E_{32} \otimes F_{32}} e^{\gamma E_{132} \otimes F_{132}} e_{q^{-2}}^{\gamma E_1 \otimes F_1} e_{q^2}^{-\gamma E_3 \otimes F_3} \\
&\cdot q^{\frac{1}{2} H_1 \otimes H_1 - \frac{1}{2} H_3 \otimes H_3 + H_C \otimes H_A + H_A \otimes H_C + \xi H_C \otimes H_C}
\end{aligned}$$

$$\gamma = (q - q^{-1})$$

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- cannot write $\mathcal{U}_q(\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3)$ as a quantum double
- extended with $\mathfrak{sl}(2)$ automorphisms and found Hopf algebra
- $\mathcal{U}_{q,\xi}(\mathfrak{sl}(2) \ltimes \mathfrak{psu}(2|2) \ltimes \mathbb{R}^3)$ is a Quantum Double
- $\mathcal{U}_{q,\xi}(\mathfrak{sl}(2) \ltimes \mathbb{R}^3)$ with $\xi = 0$ known as 3D κ-Poincaré

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- appearance of free parameter ξ
- R-matrix does not factorize and appearance of log and Li_2

Can we understand their origin better?

Contraction Limit

Plan:

- Focus on $\mathcal{U}_{q,\xi}(\mathfrak{sl}(2) \ltimes \mathbb{R}^3)$
- re-derive it as a limit $\epsilon, \tilde{\epsilon} \to 0$ of $\mathcal{U}_{q^\epsilon}(\mathfrak{sl}(2)) \otimes \mathcal{U}_{q^{\tilde{\epsilon}}}(\mathfrak{sl}(2))$

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$\mathcal{U}_{q^\epsilon}(\mathfrak{sl}(2))$

$$[H, E] = 2E, \qquad [H, F] = -2F, \qquad [E, F] = \frac{q^{\epsilon H} - q^{-\epsilon H}}{q^\epsilon - q^{-\epsilon}}$$

$$\Delta E = E \otimes 1 + q^{-\epsilon H} \otimes E, \qquad \Delta F = F \otimes q^{\epsilon H} + 1 \otimes F, \qquad \Delta H = H \otimes 1 + 1 \otimes H.$$

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stay $\mathfrak{sl}(2)$, "loose" q, commute , "keep" q,

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- Take simultaneous limit $\epsilon, \tilde{\epsilon} \to 0$

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Recover $\mathcal{U}_{q,\xi}(\mathfrak{sl}(2) \ltimes \mathbb{R}^3)$

R-matrix

$$\mathcal{R} = e_{q^{-2\epsilon}}^{(q^{\epsilon}-q^{-\epsilon})\, E\otimes F}\, q^{\frac{1}{2}\epsilon\, H\otimes H}\; e_{q^{-2\tilde{\epsilon}}}^{(q^{\tilde{\epsilon}}-q^{-\tilde{\epsilon}})\, \tilde{E}\otimes \tilde{F}}\, q^{\frac{1}{2}\tilde{\epsilon}\, \tilde{H}\otimes \tilde{H}}$$

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$$= \exp_{q^{-2\epsilon}}\left[-\frac{1}{\epsilon}E_C \otimes F_C\right] \exp_{q^{-2\tilde{\epsilon}}}\left[\frac{\tilde{\epsilon}}{\epsilon^2}(E_C + \epsilon E_{\mathrm{A}}) \otimes (F_C + \epsilon F_{\mathrm{A}})\right]$$

$$\cdot\, q^{\frac{1}{2\epsilon} H_C\otimes H_C}\, q^{\frac{\tilde{\epsilon}}{2\epsilon^2}(H_C-\epsilon H_{\mathrm{A}})\otimes(H_C-\epsilon H_{\mathrm{A}})}$$

R-matrix

$$
\begin{aligned}
\mathcal{R} &= e_{q^{-2\epsilon}}^{(q^{\epsilon}-q^{-\epsilon})\,E\otimes F}\, q^{\frac{1}{2}\epsilon\, H\otimes H}\; e_{q^{-2\tilde{\epsilon}}}^{(q^{\tilde{\epsilon}}-q^{-\tilde{\epsilon}})\,\tilde{E}\otimes\tilde{F}}\, q^{\frac{1}{2}\tilde{\epsilon}\,\tilde{H}\otimes\tilde{H}} \\
&= \exp_{q^{-2\epsilon}}\left[-\frac{1}{\epsilon}E_C\otimes F_C\right]\exp_{q^{-2\tilde{\epsilon}}}\left[\frac{\tilde{\epsilon}}{\epsilon^2}(E_C+\epsilon E_{\mathrm{A}})\otimes(F_C+\epsilon F_{\mathrm{A}})\right] \\
&\quad\cdot q^{\frac{1}{2\epsilon}H_C\otimes H_C}\, q^{\frac{\tilde{\epsilon}}{2\epsilon^2}(H_C-\epsilon H_{\mathrm{A}})\otimes(H_C-\epsilon H_{\mathrm{A}})}
\end{aligned}
$$

q-Dilogarithm: $\mathrm{Li}_2[X;q] := \log\exp_q[X]$

[Faddeev, Kashaev '94]
[Kirillov '95]

$$
\mathrm{Li}_2\left[\frac{X(\epsilon)}{\epsilon};q^{\epsilon}\right] = -\frac{1}{\hbar\epsilon}\,\mathrm{Li}_2\big(-\hbar X(\epsilon)\big)+\mathcal{O}(\epsilon).
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R-matrix

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$$= \exp_{q^{-2\epsilon}}\left[-\frac{1}{\epsilon}E_C\otimes F_C\right]\exp_{q^{-2\tilde{\epsilon}}}\left[\frac{\tilde{\epsilon}}{\epsilon^2}(E_C+\epsilon E_{\mathrm{A}})\otimes(F_C+\epsilon F_{\mathrm{A}})\right]$$

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$$\mathrm{Li}_2\left[\frac{X(\epsilon)}{\epsilon};q^{\epsilon}\right] = -\frac{1}{\hbar\epsilon}\,\mathrm{Li}_2\big(-\hbar X(\epsilon)\big)+\mathcal{O}(\epsilon).$$

$\Rightarrow$ R-matrix of $\mathcal{U}_{q,\xi}(\mathfrak{sl}(2)\ltimes\mathbb{R}^3)$ reproduced

Summary

- "undeformed" $\mathfrak{sl}(2)$ and plain $\hbar$ from $q^\epsilon = e^{\epsilon\hbar}$
- free parameter ξ stems from $\tilde{\epsilon}(\epsilon) = -\epsilon + \xi\epsilon^2$
- dilogarithm in R-matrix from $\log \exp_q$

Summary

- "undeformed" $\mathfrak{sl}(2)$ and plain $\hbar$ from $q^\epsilon = e^{\epsilon\hbar}$
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- dilogarithm in R-matrix from $\log \exp_q$

- To include $\mathfrak{psu}(2|2)$ start with $\mathcal{U}_q(\mathfrak{d}(2,1;\epsilon)) \otimes \mathcal{U}_{q^{\tilde{\epsilon}}}(\mathfrak{sl}(2))$
- Also possible $\mathcal{U}_q(\mathfrak{d}(2,1;\epsilon)) \otimes \mathcal{U}_q(\mathfrak{d}(2,1;\tilde{\epsilon}))$
  $\rightarrow \mathcal{U}_{q,\xi}(\mathfrak{sl}(2) \ltimes (\mathfrak{psu}(2|2) \oplus \mathfrak{psu}(2|2)) \ltimes \mathbb{R}^3)$

Thank you!

Hopf structure of $\mathfrak{sl}(2)$ automorphism

$\mathfrak{sl}(2)$

$$[H_\mathrm{A}, E_\mathrm{A}] = 2E_\mathrm{A}, \quad [H_\mathrm{A}, F_\mathrm{A}] = -2F_\mathrm{A}, \quad [E_\mathrm{A}, F_\mathrm{A}] = (H_\mathrm{A} + \xi H_C)\frac{q^{H_C} + q^{-H_C}}{2} - \xi\frac{q^{H_C} - q^{-H_C}}{4\hbar},$$

acting on central extensions

$$[H_\mathrm{A}, E_C] = 2E_C, \qquad [H_\mathrm{A}, H_C] = 0, \qquad [H_\mathrm{A}, F_C] = -2F_C,$$

$$[E_\mathrm{A}, E_C] = 0, \qquad [E_\mathrm{A}, H_C] = \frac{q - q^{-1}}{\hbar}E_C, \qquad [E_\mathrm{A}, F_C] = \frac{q^{H_C} - q^{-H_C}}{q - q^{-1}},$$

$$[F_\mathrm{A}, E_C] = -\frac{q^{H_C} - q^{-H_C}}{q - q^{-1}}, \qquad [F_\mathrm{A}, H_C] = -\frac{q - q^{-1}}{\hbar}F_C, \qquad [F_\mathrm{A}, F_C] = 0,$$

acting on $\mathfrak{psl}(2|2)$

$$[H_\mathrm{A}, E_j] = \delta_{j2}E_j, \quad [H_\mathrm{A}, F_j] = -\delta_{j2}F_j$$

$$[E_\mathrm{A}, E_j] = (q - q^{-1})(\delta_{j2}\tfrac{1}{2}E_2E_C + \delta_{j3}E_{32}E_{132}),$$

$$[E_\mathrm{A}, F_j] = \delta_{j2}(E_{132} + (q - q^{-1})E_{32}E_1)q^{-H_2} + \delta_{j3}(q - q^{-1})E_2E_{12}q^{H_3},$$

$$E \leftrightarrow F, \; E_C \leftrightarrow F_C,$$

Hopf structure of $\mathfrak{sl}(2)$ automorphisms

coproduct

$$\Delta H_{\mathrm{A}} = H_{\mathrm{A}} \otimes 1 + 1 \otimes H_{\mathrm{A}}$$

$$\begin{aligned}
\Delta E_{\mathrm{A}} = {} & E_{\mathrm{A}} \otimes 1 + q^{-2H_C} \otimes E_{\mathrm{A}} + \frac{q-q^{-1}}{2}(H_{\mathrm{A}} + \xi H_C) q^{-2H_C} \otimes E_C \\
& + (q-q^{-1})(E_{132} + (q-q^{-1}) E_{32} E_1) q^{-H_2} \otimes E_2 \\
& - (q-q^{-1}) E_{32} q^{-H_1 - H_2} \otimes E_{12} - (q-q^{-1})^2 E_3 q^{-H_1 - 2H_2} \otimes E_2 E_{12}
\end{aligned}$$

$$\Delta F_{\mathrm{A}} = \text{analogously}$$