The Toric SO(10) F-theory Landscape Part II: Transitions and Superconformal Sectors

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based on: arXiv 1709.06609 & *M. Dierigl's talk*
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Outline

1. **Motivation**
2. **Toric construction of SO(10) theories**
3. **Matter Charges and Multiplicities**
4. **Transitions to superconformal points**
5. **Anomalies of superconformal points**
6. **Conclusion and Outlook**

Motivation

We have **36 different types** of SO(10) F-theory fibrations with some**additional gauge symmetries** (non-Abelian,(discrete-)Abelian)

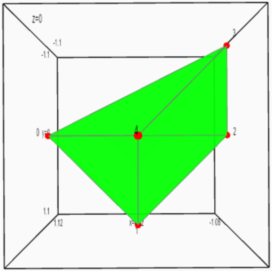
What to do with that?

- Good starting point for a **pheno survey** (see Markus' talk)

What else can we learn?

- Are there **relations** among the 36 different fibrations?
- What are **generic features?**
  $\sim 80\%$ percent of the models have **superconformal points**
- Can we understand those models (spectra & anomalies) as well?

Toric construction of SO(10) theories



Strategy to engineer SO(10) theories

- Start with 1/16 reflexive polytopes (dP_1) as an **ambient space** and consider its anticanonical hypersurface (this is **a torus**) [Klevers, Mayorga, Piragua, O., Reuter'14]
- make coefficients s_i sections of the **base** $\rightarrow$ fibration
- Fibration with a generic gauge group (Mordell-Weil group of rank 1: U(1))
- Enhance the ambient space with a top [Candelas,Font; Bouchard,Skarke]

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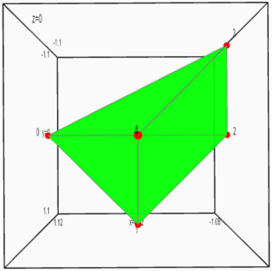

$$p = s_1 u^3 e_1^2 + s_2 u^2 v e_1^2 + s_3 u v^2 e_1^2 + s_4 v^3 e_1^2 + s_5 u^2 w e_1 + s_6 u v w e_1 + s_7 v^2 w e_1 + s_8 u w^2 + s_9 v w^2$$

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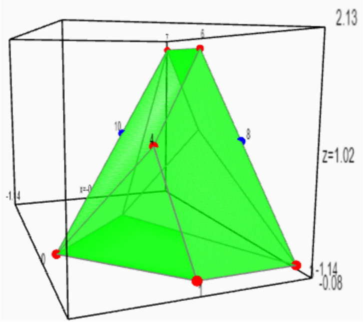


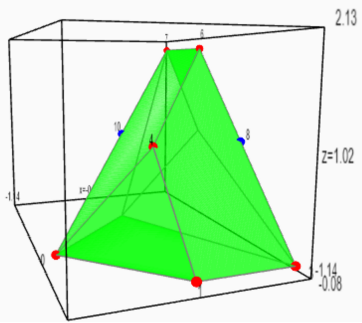
$$\begin{aligned} \mathrm{p} = \quad & s_1 e_1^2 f_0 f_2^2 f_4 f_1 u^3 + s_2 e_1^2 f_0^2 f_2^2 f_3 f_4 f_1^2 f_5 u^2 v + s_3 e_1^2 f_0^2 f_2 f_3 f_1 u v^2 + s_4 e_1^2 f_0^3 f_2 f_3^2 f_1^2 f_5 v^3 \\ & + s_5 e_1 f_2 f_4 u^2 w + s_6 e_1 f_0 f_2 f_3 f_4 f_1 f_5 uvw + s_7 e_1 f_0 f_3 v^2 w + s_8 f_2 f_3 f_4^2 f_1 f_5^2 u w^2 + s_9 f_3 f_4 f_5 v w^2 \ \mathrm{r} \end{aligned}$$

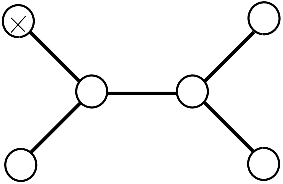
Strategy to engineer SO(10) theories

- Enhance the ambient space with a top [Candelas,Font; Bouchard,Skarke]
  - A top is a **half**-polytope over the ambient of the generic fiber
  - The $z > 0$ points: ADE resolution divisors D_{f_i} that restrict to base divisor $\mathcal{Z}$

$$\mathcal{S}_{ADE}(B) : \mathcal{Z} = 0 \qquad D_{f_i} = \pi^{-1}(\mathcal{Z})$$

Toric construction of SO(10) theories





Strategy to engineer SO(10) theories

- Enhance the ambient space with a top [Candelas,Font; Bouchard,Skarke]
    - z=0, the generic fiber: U(1) theory
    - z=1, the outer roots of SO(10)
    - z=2, the roots with Dynkin label 2

    - Resolution divisors intersect as the affine Dynkin Diagram of SO(10)

$$D_{f_i} \cdot \mathbb{P}^1_j = -\hat{G}^{i,j}_{\mathrm{SO}(10)} \tag{1}$$

Computation of the Spectrum

The Weierstass Form

Any elliptic curve **birationally equivalent** to the a (singular) **Weierstrass form**:

$$P = y^2 - x^3 + f x + g$$
$$\Delta = 4f^3 + 27g^2$$

Vanishing orders of f, g and Δ hint at the matter types via Kodaira's classification [Kodaira'62]

$$\begin{aligned} f =& \mathcal{Z}^2 \left(\mathcal{B}^2\mathcal{C}^2 + \mathcal{C}\mathcal{Z}R_1 + \mathcal{O}(\mathcal{Z}^2)\right) \,, \\ g =& \mathcal{Z}^3 \left(\mathcal{B}^3\mathcal{C}^3 + \mathcal{C}^2\mathcal{Z}R_2 + \mathcal{C}\mathcal{Z}^2R_3 + \mathcal{Z}^3R_4 + \mathcal{O}(\mathcal{Z}^4)\right) \,, \\ \Delta =& \mathcal{Z}^7 \left(\mathcal{A}^2\mathcal{B}^3\mathcal{C}^5 + \mathcal{C}^4\mathcal{Z}R_5 + \cdots + \mathcal{Z}^5R_9 + \mathcal{O}(\mathcal{Z})\right) \,, \end{aligned}$$

Computation of the Spectrum

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\begin{aligned}
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\end{aligned}
$$

Kodaira classification

	(f, g, Δ)	fiber type	rep	multiplicity
$\mathcal{Z} = 0$	$(2,3,7)$	I_1^*	$SO(10)$	–
$\mathcal{Z} = \mathcal{A} = 0$	$(2,3,8)$	I_2^*	**10**	$[\mathcal{A}]\mathcal{Z}$
$\mathcal{Z} = \mathcal{B} = 0$	$(3,4,8)$	III	**16**	$[\mathcal{B}]\mathcal{Z}$
$\mathcal{Z} = \mathcal{C} = 0$	$(4,6,12)$	non-min	SCP	$[\mathcal{C}]\mathcal{Z}$

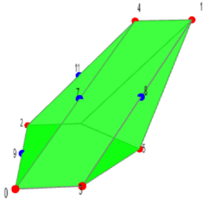
- **6d Multiplicities:** Intersections of two base divisor classes
- **matter charges:**Impose the loci for the **resolved** fibration

A full spectrum

- **Full spectrum** fixed by intersections of 4 divisor classes $\mathcal{Z}, \mathcal{S}_7, \mathcal{S}_9, K_B^{-1}$
- **All Singlets**, (i.e. complex structure moduli) included
- **non-minimal** $Van(f, g, \Delta) = (4, 6, 12)$ points: superconformal points (SPCs)

$SO(10) \times U(1)^2$ *Rep.*	multiplicity
$\mathbf{16}_{1/4,0}$	$(2K_B^{-1} - \mathcal{S}_7)\mathcal{Z}$
$\mathbf{10}_{1/2,0}$	$(2K_B^{-1} - \mathcal{S}_9 - \mathcal{Z})\mathcal{Z}$
$\mathbf{10}_{1/2,1}$	$(K_B^{-1} - \mathcal{S}_7 + \mathcal{S}_9)\mathcal{Z}$
SCP	$(\mathcal{S}_7 - \mathcal{Z})\mathcal{Z}$
$\mathbf{1}_{1,-1}$	$(K_B^{-1} + \mathcal{S}_7 - \mathcal{S}_9 - 2\mathcal{Z})(\mathcal{S}_7 - \mathcal{Z})$
$\mathbf{1}_{1,2}$	$(K_B^{-1} - \mathcal{S}_7 + \mathcal{S}_9)(\mathcal{S}_9 - \mathcal{Z})$
$\mathbf{1}_{0,2}$	$(\mathcal{S}_9 - \mathcal{Z})(\mathcal{S}_7 - \mathcal{Z})$
$\mathbf{1}_{-1,-1}$	$6(K_B^{-1})^2 + \mathcal{S}_7^2 - 2\mathcal{S}_9^2 + 2\mathcal{Z}^2 - 2\mathcal{S}_9\mathcal{Z}$ $+K_B^{-1}(-5\mathcal{S}_7 + 4\mathcal{S}_9 - 4\mathcal{Z}) + \mathcal{S}_7\mathcal{S}_9$
$\mathbf{1}_{1,0}$	$6(K_B^{-1})^2 - 2\mathcal{S}_7^2 + \mathcal{S}_9^2 + 3\mathcal{Z}^2 + 2\mathcal{S}_9\mathcal{Z}$ $+K_B^{-1}(4\mathcal{S}_7 - 5\mathcal{S}_9 - 11\mathcal{Z}) + \mathcal{S}_7(\mathcal{S}_9 + \mathcal{Z})$
$\mathbf{1}_{0,1}$	$6(K_B^{-1})^2 - 2\mathcal{S}_7^2 - 2\mathcal{S}_9^2 + 4\mathcal{Z}^2 - \mathcal{S}_9\mathcal{Z}$ $+K_B^{-1}(4\mathcal{S}_7 + 4\mathcal{S}_9 - 13\mathcal{Z}) + \mathcal{S}_7\mathcal{Z}$
$\mathbf{1}_{0,0}$	$19 + 11(K_B^{-1})^2 + 2\mathcal{S}_7^2 + 2\mathcal{S}_9^2 + 2\mathcal{S}_9\mathcal{Z} + 7\mathcal{Z}^2$ $-\mathcal{S}_7(\mathcal{S}_9 + 2\mathcal{Z}) - 4K_B^{-1}(\mathcal{S}_7 + \mathcal{S}_9 + 3\mathcal{Z})$

Transitions of Theories



Organization of models by transitions

Theories are related by a **conifold transitions** (toric blow-ups/downs)

- Blow-up at at height 0:

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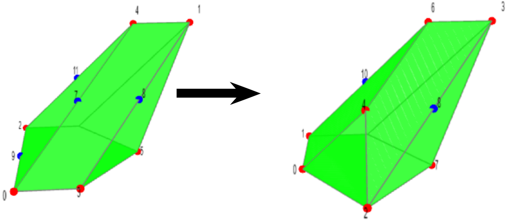

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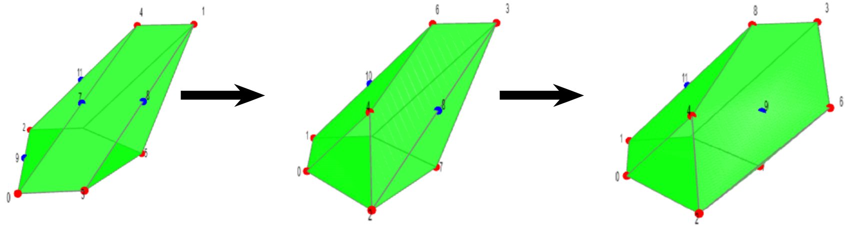

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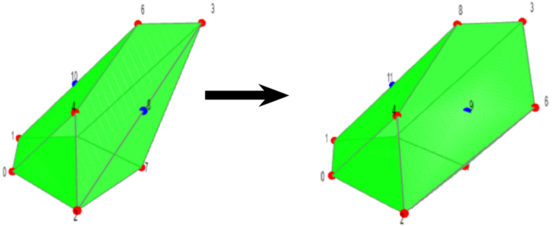
- Blow-up at at height: 1 (one point interior of a facet)
  Superconformal transition: $SO(10) \times [U(1)^2]$ + superconformal points
  $\rightarrow$Tensionless string states appearing [Bershadsky,Johanson'94]

Transitions in theories



Field Theory perspective:

The transitions induce a specific change of the spectrum: [Anderson,Gray,Rhaguram,Taylor]

- **Loose:** $[\mathbf{16}_{-1/4,-1/2} + \mathbf{10}_{1/2,0} + \mathbf{1}_{-1,-1} + \mathbf{1}_{0,1} + \mathbf{1}_{0,0}] \times n_{scp} = 29 \times n_{scp}$

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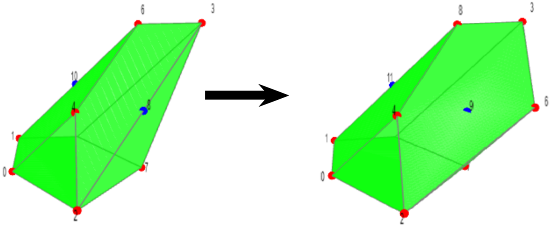

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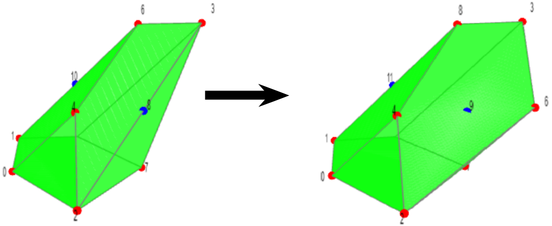

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- **Gain:** $[\mathcal{Z}][\mathcal{C}] = n_{scp}$ superconformal points
- Due to the **missing hypers** we can not satisfy**anomalies** !
- Hint: Superconformal points are **tensor** multiplets, not located in the base

Non-flat fibers and superconformal points

Appearance of non-flat fibers

The point $\widetilde{f_2}$ is associated to the point in the top facet with hypersurface:

$$\begin{aligned} P = \quad & s_1 e_1^2 e_2^2 f_0^2 f_1^3 \widetilde{f_2} f_3^2 g_1^3 g_2^2 u^3 + s_2 e_1 e_2^2 f_0^2 f_1^2 \widetilde{f_2} f_3 g_1^2 g_2 u^2 v + s_3 e_2^2 f_0^2 f_1 \widetilde{f_2} g_1 u v^2 + s_{scp} e_1^2 e_2 f_1 f_3 u^2 w \\ & + s_6 e_1 e_2 f_0 f_1 \widetilde{f_2} f_3 f_4 g_1 g_2 u v w + s_7 e_2 f_0 \widetilde{f_2} f_4 v^2 w + s_8 e_1^2 f_1 \widetilde{f_2} f_3^2 f_4^2 g_1 g_2^2 u w^2 + s_9 e_1 \widetilde{f_2} f_3 f_4^2 g_2 v w^2 \end{aligned}$$

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$$+ s_6 e_1 e_2 f_0 f_1 \widetilde{f_2} f_3 f_4 g_1 g_2 uvw + s_7 e_2 f_0 \widetilde{f_2} f_4 v^2 w + s_8 e_1^2 f_1 \widetilde{f_2} f_3^2 f_4^2 g_1 g_2^2 u w^2 + s_9 e_1 \widetilde{f_2} f_3 f_4^2 g_2 v w^2$$

- over $s_{scp} = 0$ the fiber becomes reducible
- over $\mathcal{Z} = \pi_B(f_1 f_2 f_3 f_4 g_1 g_2) = 0$, D_{f_2} becomes an additional curve in the fiber $\rightarrow$ **non-flat fiber**: the fiber dimension jumps
- D_{f_2} is a non-toric divisor that intersects the Calabi-Yau $[s_{scp}][\mathcal{Z}] = n_{scp}$ times

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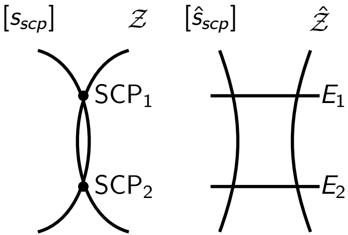
- D_{f_2} is a non-toric divisor that intersects the Calabi-Yau $[s_{scp}][\mathcal{Z}] = n_{scp}$ times
- Therefore it contributes n_{scp} additional **non-toric** Kahler deformations that are **not located** in the base
- From a base-independent computation of the**Euler numbers** for a generic base, we have to modify the Kahler moduli as:

$$h^{1,1}(X) = rank(G) + 2 + h^{1,1}(B) + n_{scp}$$

Anomalies of SCP theories

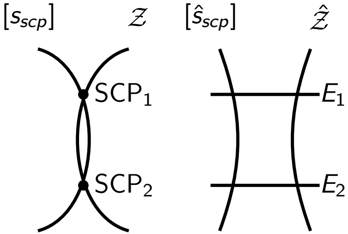


Physical picture of superconformal points

The super conformal points have an interpretation as **tensionles string modes** [Witten; Seiberg '96] :

- Obtain SUGRA by a **blow-up in the base** via exceptional divisors E_i in the base with Kahler moduli a_i
- The **Kahler moduli** a_i are also the **vev** of the additional 6d **tensor** multiplet
- The **tensors couple to strings** with **tension** a_i and **coupling strength** $g_s = 1/a_i$
- **SCP**: Blow-down $a_i \to 0$, **tensionless string**, **strongly coupled** to a tensor

Anomalies of SCP theories



Blow-up the super conformal points **in the base** introduce: [Bershadsky, Johnson '97]

- **Exceptional divisors:** E_i with $E_i E_j = -\delta_{i,j}$, $i, j = 1..n_{scp}$
- **Blow-down map:** $\beta_* : H_2(\hat{B}, \mathbb{Z}) \to H_2(B, \mathbb{Z})$
- **Shift base divisor classes**

$$K_{\hat{B}}^{-1} \to (K_b^{-1})^* - \sum_i E_i \,, \qquad \mathcal{Z}^* = \hat{\mathcal{Z}} + \sum_i n_{scp,i} E_i$$

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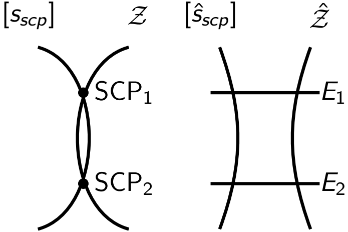

We have **no superconformal points** anymore and obtain a **well defined 6d SUGRA** description, where all anomalies are satisfied

Factorization of anomalies

$$\begin{aligned} Grav^4 Anomaly: \quad & T := 9 - (K_B^{-1})^2 \to T^* + n_{scp} \\ & H - V + 29(T^* + n_{scp}) - 273 = 0 \checkmark \end{aligned}$$

- Indeed, all other (gauge and mixed) anomalies **factorize now**
- This procedure can again be applied for a generic base

Summary and Outlook

1. We made a **complete classification** of torically resolved SO(10) fibrations in F-theory
2. We computed the **full spectrum** and 6d multiplicities **base independently**
3. Checked **full anomaly cancellation** (also in theories with superconformal points)

What is that good for ?

1. **Phenomenological explorations** for SO(10) models in 6d
2. **Playground to study** theories with **superconformal points** and their **transitions**

Outlook

- Further **exploration of superconformal points** (M-theory limit of non-flat fibers, 6d anomaly lattice)
- Study the 6d $\rightarrow$ 4d **orbifold flux compactification** of the pheno models

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Thank you very much!