BMS4 at Spatial infinity

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Asymptotic symmetries

In gauge theories, part of the relevant symmetries are asymptotic symmetries:

Asymptotic symmetries are gauge symmetries with a non-trivial action on the boundary of the system.

This notion allows for the definition of non-trivial conserved charges associated to a sub-algebra of the gauge transformations.

Asymptotically flat space-times: ADM mass, angular momentum, ...

AdS3

One of the most know example is asymptotically AdS3 space-times:

- expected algebra $so(2,2)$
- algebra found is the infinite dimensional 2D conformal algebra.

$\Rightarrow$ 3D gravity with negative cosmological constant is a 2D CFT.

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First example of enhancement (Bondi-Metzner-Sachs):

asymptotic symmetries of 4d asymptotically flat space-times at null infinity.

$$\text{Poincaré} \quad \rightarrow \quad BMS4.$$

How relevant is this algebra? Can it provide us control on the theory?

BMS4 algebra I

Null infinity: reach infinity along null rays

- retarded time : u,
- coordinates on the sphere : $x^A = (\theta, \phi)$.

$$\text{BMS4} = \text{Supertranslations } T(x^A) \quad \oplus_\sigma \quad \text{Lorentz } Y^A.$$

BMS4 infinitesimal transformations can be represented by

$$\xi = \left(T + \frac{u}{2} D_A Y^A\right) \partial_u + Y^A \partial_A,$$
$$\mathcal{L}_Y \gamma_{AB} = D_C Y^C \gamma_{AB}.$$

where γ_{AB} and D are the metric and covariant derivative on the sphere.

BMS4 algebra II

Poincaré is a sub-algebra of BMS4.

- Translations $\subset$ Supertranslations

$$T = \sum_{lm} T_{lm} Y^l_m = T_{00} Y^0_0 + \sum_m T_{1m} Y^1_m + \dots$$

- Y^A conformal Killing vector on the sphere

Only 6 globally well-defined Killing vectors
$\Rightarrow$ they form a Lorentz algebra.

Strominger's interpretation (2013)

Null infinity: reach infinity along null rays

$\Rightarrow$ end points of the trajectories of massless particles.

The action of BMS4 algebra can be extended to act on massive particles at infinite late times.

When considering isolated 4D gravitational systems with $\Lambda = 0$, BMS4 acts on the space of out-states of evolution.

Supertranslations generators

They can be written as a weighted integral of the energy flux at infinity

$$\mathcal{Q}_T = \frac{1}{4\pi G} \int du \oint_{S_2^\infty} d^2\Omega \, T(x^A) \, \Phi(u, x^A).$$

Infinitely many charges encoding the angular profile of the energy-flux:

- total energy:

$$\mathcal{M} = \frac{1}{4\pi G} \int du \oint_{S_2^\infty} d^2\Omega \, \Phi(u, x^A),$$

- total momenta:

$$\mathcal{P}_m = \frac{1}{4\pi G} \int du \oint_{S_2^\infty} d^2\Omega \, Y_m^1(x^A) \, \Phi(u, x^A).$$

The scattering problem

$BMS4^+$ acts on the out-states of gravitational evolution $< out|$.

The same analysis can be done with in-states:

$BMS4^-$ acts on the in-states of gravitational evolution $|in>$.

A priori, the two algebras $BMS4^+$ and $BMS4^-$ are independent.

In a well-defined scattering problem, we expect to two structures to be related. For instance, we expect only one Poincaré algebra.

Strominger's assumptions

- The two BMS4 algebras are identified using the following matching conditions

$$T^+(x) = T^-(-x), \qquad Y^{+A}(x) = P\left(Y^{-A}(x)\right),$$

where $P(x^A) = -x^A$ is the antipodal map.

- The associated charges are conserved:

$$\mathcal{Q}^+_{T^+(x)} = \mathcal{Q}^-_{T^-(x)}.$$

With these assumptions, BMS4 is a symmetry of gravitational evolution in asymptotically flat space-times.

Global BMS4

Consequences

- infinity of conserved charges,
- ward identities give soft theorems,
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However,

- Its description exists only in the infinite past and in the infinite future.
- The conservation of the associated charges is an assumption.

Question

Can we define these symmetries at finite times?

Spatial infinity - Hamiltonian formalism

Spatial infinity in Cartesian coordinates:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{r} h_{\mu\nu} + ...$$

Using the hamiltonian formalism, these read

$$g_{ij} = \delta_{ij} + \frac{1}{r} h_{ij} + ..., \quad \pi^{ij} = \frac{1}{r^2} \bar{\pi}^{ij} + ...$$

ADM mass - Generator of time translations

$$\xi = \partial_t \quad \Rightarrow \quad M_{ADM} = \int d^3x \mathcal{H}_\perp + \oint d^2\Omega (\partial_i h^{ir} - \partial^r h^i_i).$$

Supertranslations

Claim

The original ADM description can be adapted to describe the BMS4 supertranslations.

BMS4 supertranslations:

$$\xi = \left(T_0 + \sum_{m=-1}^{1} T_m Y_m^1 \right) \partial_u + \ldots \quad \rightarrow \quad \xi = T(x^A)\partial_u + \ldots .$$

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- time translations

$$\xi = T_0 \partial_t + \ldots \quad \rightarrow \quad \xi = f(x^A)\partial_t + \ldots,$$

- spatial translations

$$\xi = \sum_{m=-1}^{1} T_m Y_m^1 \partial_r + \ldots \quad \rightarrow \quad \xi = W(x^A)\partial_r + \ldots$$

Supertranslations II

Generators of the transformations (assuming $g_{rA} = o(1)$)

$$\xi = f\partial_t + \dots \quad \Rightarrow \quad G_f = \int d^3x \xi \mathcal{H}_\perp + 2\oint_{S_2} d^2\Omega\, f\, h_{rr}.$$

$$\xi = W\partial_r + \dots \quad \Rightarrow \quad G_W = \int d^3x \xi^i \mathcal{H}_i + 2\oint_{S_2} d^2x\, W\, (\bar{\pi}^{rr} - \partial_A \bar{\pi}^{rA}).$$

These generators commute with the hamiltonian, they are conserved

$$d_t G_f \approx 0, \quad d_t G_W \approx 0.$$

What fails with the original Hamiltonian analysis?

In the original analysis, the simple asymptotics are not enough to define finite Lorentz charges.

$$g_{ij} = \delta_{ij} + \frac{1}{r} h_{ij} + \ldots, \quad \pi^{ij} = \frac{1}{r^2} \bar{\pi}^{ij} + \ldots$$

This a priori leads to infinite Lorentz charges:

$$G_Y = \lim_{r \to \infty} r \oint_{S_2} Y_A \bar{\pi}^{rA} + finite.$$

Original solution:

Impose extra constraints on the asymptotics
$\Rightarrow$ this kills all extra supertranslations.

Solution

In the original analysis, the simple asymptotics are not enough to define finite Lorentz charges.

$$g_{ij} = \delta_{ij} + \frac{1}{r} h_{ij} + \dots, \quad \pi^{ij} = \frac{1}{r^2} \bar{\pi}^{ij} + \dots$$

This a priori leads to infinite Lorentz charges:

$$G_Y = \lim_{r \to \infty} r \oint_{S_2} Y_A \bar{\pi}^{r}{}_{A} + \textit{finite}.$$

However, nothing extra needed. The divergent contribution is proportional to the constraints:

$$G_Y = \lim_{r \to \infty} r \oint_{S_2} Y^A \mathcal{H}_A + \textit{finite} \approx \textit{finite}.$$

BMS4 at spatial infinity

Total symmetry algebra is a semi-direct sum

$$\text{Supertranslations } (f, W) \quad \oplus_s \quad \text{Lorentz.}$$

One can show that the BMS4 algebra defined at null infinity is a subalgebra with:

$$f^{even} \leftrightarrow T^{even} \quad \text{and} \quad W^{odd} \leftrightarrow T^{odd}.$$

Conclusions

The symmetry algebra of asymptotically flat space-times in 4D is an infinite dimensional algebra containing Poincaré.

- The symmetry algebra of asymptotically flat space-times at spatial infinity in 4D contains BMS4.
- Their associated charges are conserved.
- The extra assumptions introduced by Strominger in the matching between $\mathscr{I}^+$ and $\mathscr{I}^-$ are consequences.