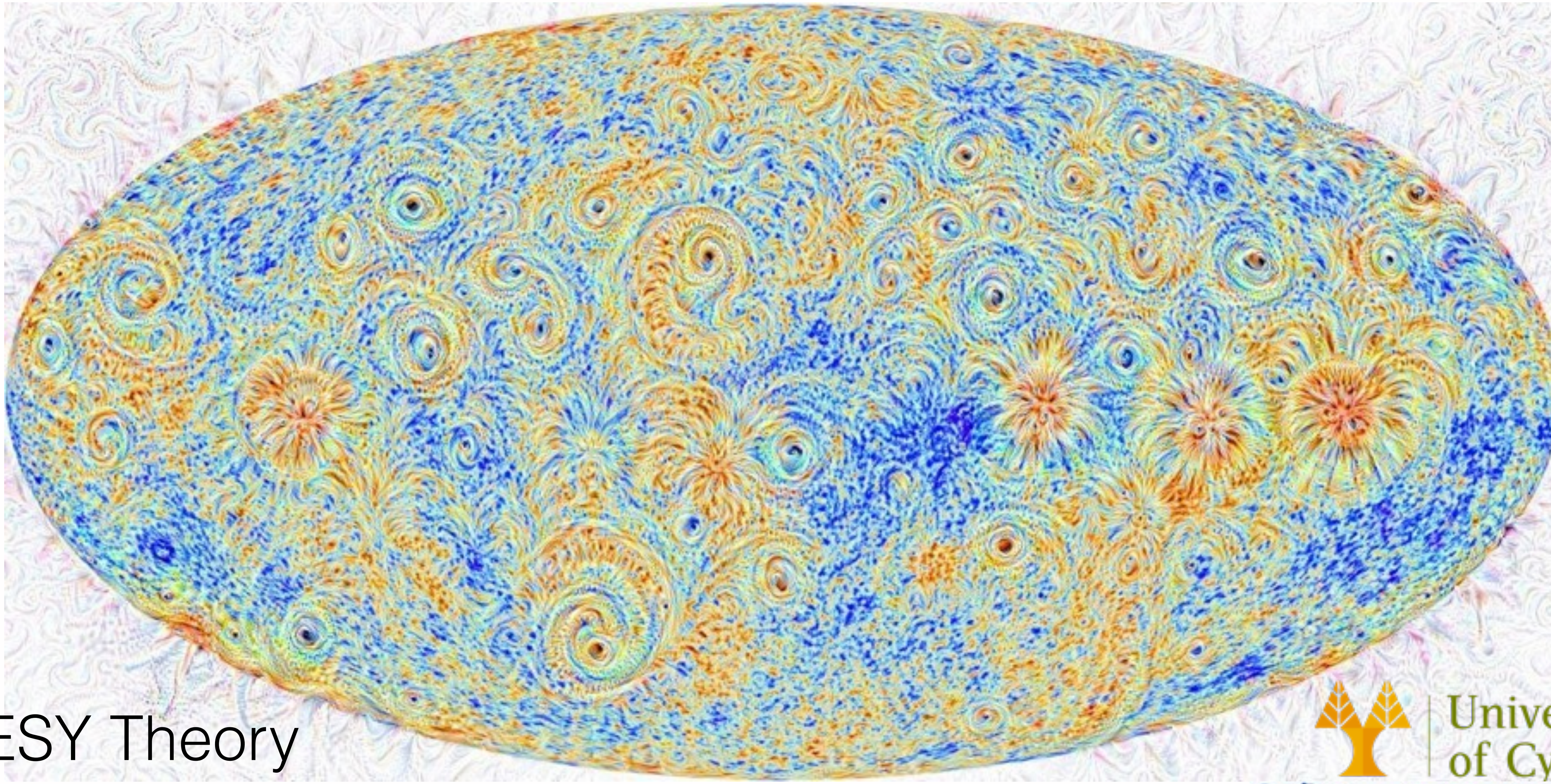
Testing the CDM paradigm with the CMB



DESY Theory Workshop 2017

University of Cyprus

Fyzikální ústav Akademie věd ČR, v. v. i.

Michael Kopp

in collaboration with Dan Thomas and Costas Skordis

Thomas et al, **1601.05097** Kopp et al, **1605.00649**

+3 papers soon!

Outline

(1) Generalised dark matter (**GDM**),

- 3 new parameters
- CMB with Λ-GDM and constraints from Planck. ***New***: time varying DM equation state. Halo model, neutrinos...
- Everything consistent with ΛCDM. No hints for "beyond CDM"

(2) Parametrized Post Friedmann (**PPF**) frame work,

- 2 new parameters. No degree of freedom associated with DM.
- Bad fit to CMB. We need extra degree of freedom.

Believing in dark matter

- Extensions of the standard model of particle physics (SM)
- Cold dark matter (CDM) gives concordant picture within General Relativity (GR)
- Modified gravity?:
  - ✦ GR's success for a century
  - ✦ Lack of working alternatives

Axions
Wimps
Sterile ν
} natural extensions of SM

Internal galaxy and cluster dynamics
Expansion history
Large scale structure
Lensing CMB BBN

no modified gravity theory yet to explain all those phenomena

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$$T^{\mu}{}_{\nu} = T_{\rm cdm}{}^{\mu}{}_{\nu} + T_{\Lambda}{}^{\mu}{}_{\nu} + T_{\rm SM}{}^{\mu}{}_{\nu} = (8\pi\, G_{\rm N})^{-1} \quad G^{\mu}{}_{\nu}$$

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$$T^{\mu}{}_{\nu} = T_{\mathrm{g}}{}^{\mu}{}_{\nu} + T_{\Lambda}{}^{\mu}{}_{\nu} + T_{\mathrm{SM}}{}^{\mu}{}_{\nu} = (8\pi\, G_{\mathrm{N}})^{-1} \quad G^{\mu}{}_{\nu}$$

g for GDM

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PPF

$$T^{\mu}{}_{\nu} = \quad T_{\Lambda}{}^{\mu}{}_{\nu} + T_{\mathrm{SM}}{}^{\mu}{}_{\nu} = (8\pi\, G_{\mathrm{N}})^{-1} \left(G^{\mu}{}_{\nu} + \boxed{U^{\mu}{}_{\nu}} \right)$$

Dark matter fluid

$$T_{\mu\nu} = \rho u_\mu u_\nu + P(g_{\mu\nu} + u_\mu u_\nu) + \Sigma_{\mu\nu}$$

- Cold and Collisionless particles in the continuum limit are described before shell crossing as pressureless perfect fluid (=CDM in camb and CLASS)

Planck 2015 XX, 1502.02114 $\beta_{c,\mathrm{iso}} < 0.038$
Planck 2015 XIII, 1502.01589 $\omega_c = 0.12 \pm 0.0027$

- A general fluid can have pressure P=P(ρ,S,$\nabla_\mu u^\mu$,...). We allow for non-adiabatic, but exclude bulk viscosity

- Shear $\Sigma_{\mu\nu}$(ρ,$\nabla_\mu u_\nu$, $g_{\mu\nu}$,...), spatial and traceless

➡ More DM properties that we can potentially measure!

Generalised Dark Matter

Linear scalar perturbation

- GDM parameters $w(\tau)$ $c_s^2(k,\tau)$ $c_{\rm vis}^2(k,\tau)$ $c_a^2 = \frac{\dot{\bar{P}}_g}{\dot{\bar{\rho}}_g} = w - \frac{\dot{w}}{3\mathcal{H}(1+w)}$

  'equation of state' 'sound speed' 'viscosity' 'adiabatic sound speed'

- GDM closure relations made-up by W. Hu 1998 ApJ 506

$$\Pi_g = c_a^2\delta_g + (c_s^2 - c_a^2)\hat{\Delta}_g$$

rest frame

$$\dot{\Sigma}_g = -3\mathcal{H}\Sigma_g + \frac{4}{(1+w)}c_{\rm vis}^2\hat{\Theta}_g$$

Newtonian

non-adiabatic pressure $\Pi_{\rm nad}$ vanishes if P=P(ρ)

derived for neutrinos: Blas et al 2011 JCAP 7

- Perturbed stress-energy-momentum tensor

$$T_{\mu\nu} = \rho u_\mu u_\nu + P(g_{\mu\nu} + u_\mu u_\nu) + \Sigma_{\mu\nu}$$

$$\delta_g = \delta\rho_g/\bar{\rho}_g \qquad w = \bar{P}_g/\bar{\rho}_g$$

$$u_{g\,i} = -a\,\partial_i\theta_g \qquad \Pi_g = \delta P_g/\bar{\rho}_g$$

g for GDM

$$\Sigma^i_{g\,j} = \bar{\rho}_g(1+w)(\partial_i\partial_j\Sigma_g)^{\rm tracefree}$$

- $\nabla_\mu T_g{}^\mu{}_\nu = 0$ perturbed conservation equation close

Rough estimates

$$w \simeq c_s^2 \simeq c_{\rm vis}^2$$

GDM with constant parameters

Thomas et al, 1601.05097

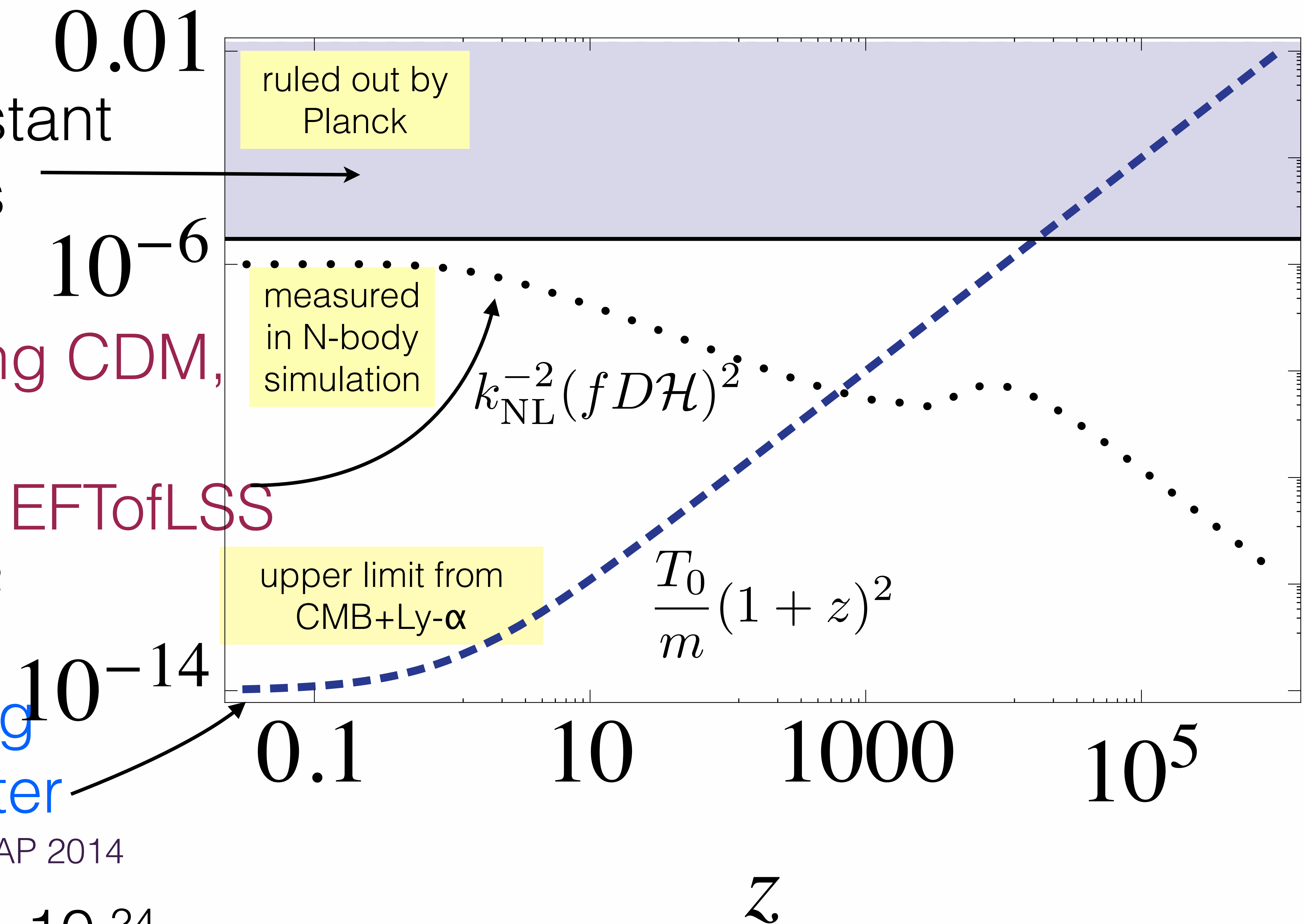
Freely streaming CDM, warmed-up by non-linearities, EFTofLSS

Baumann et al, JCAP 2012

Freely streaming warm dark matter

Armendariz-Picon et al, JCAP 2014

CDM neutralino 10^{-24}



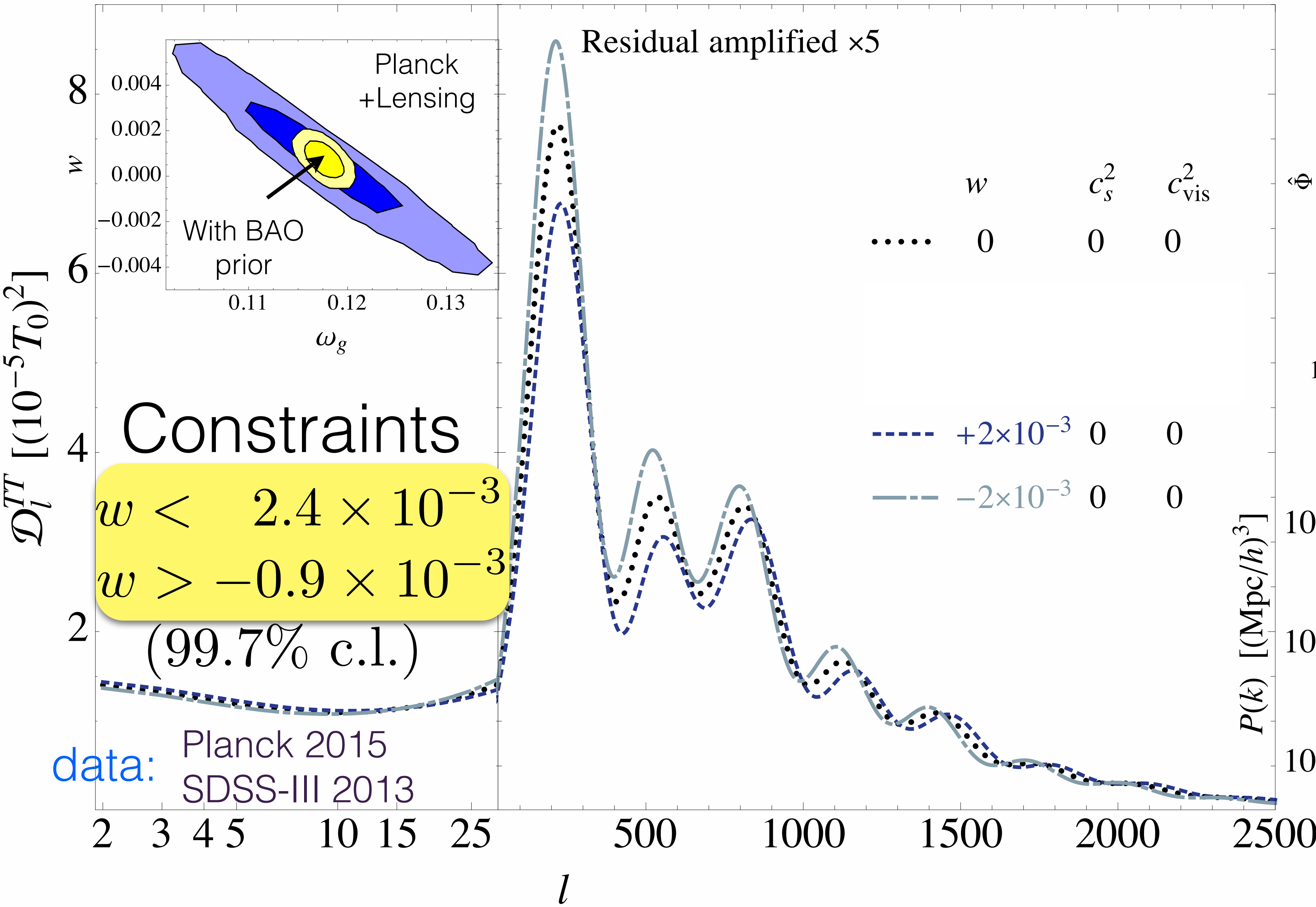
Extending ΛCDM into Λ-GDM: imprints on the CMB and constraints from Planck

$$a^3 \rho_g \propto \omega_g (1 + 3w \ln(1+z))$$

$$k_{\mathrm{decay}}^{-1} \mathcal{H} \simeq \sqrt{c_s^2 + 0.5\, c_{\mathrm{vis}}^2}$$

CMB and constraints

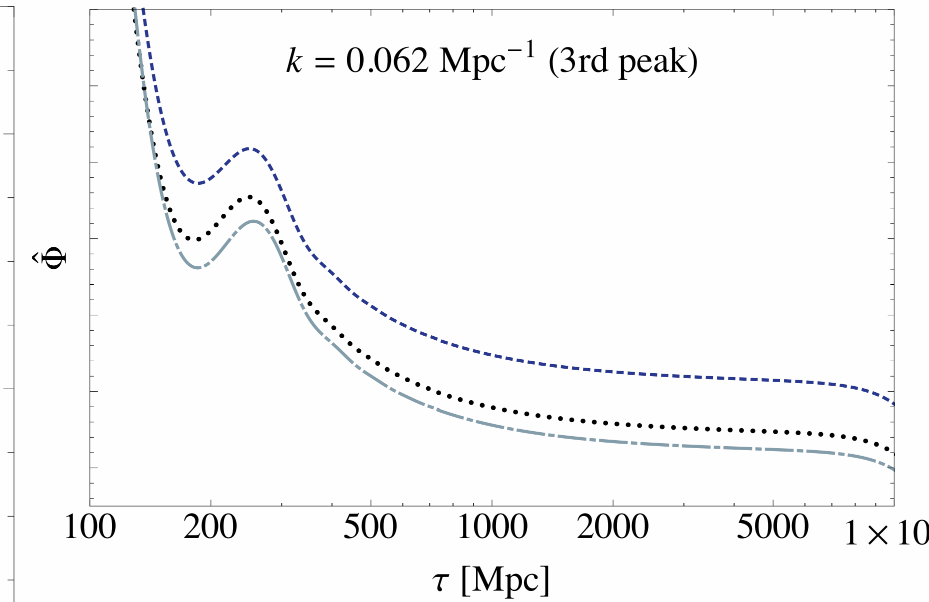
Based on a modified CLASS code Lesgourgues 2011 , MCMC 6+3 params with montepython

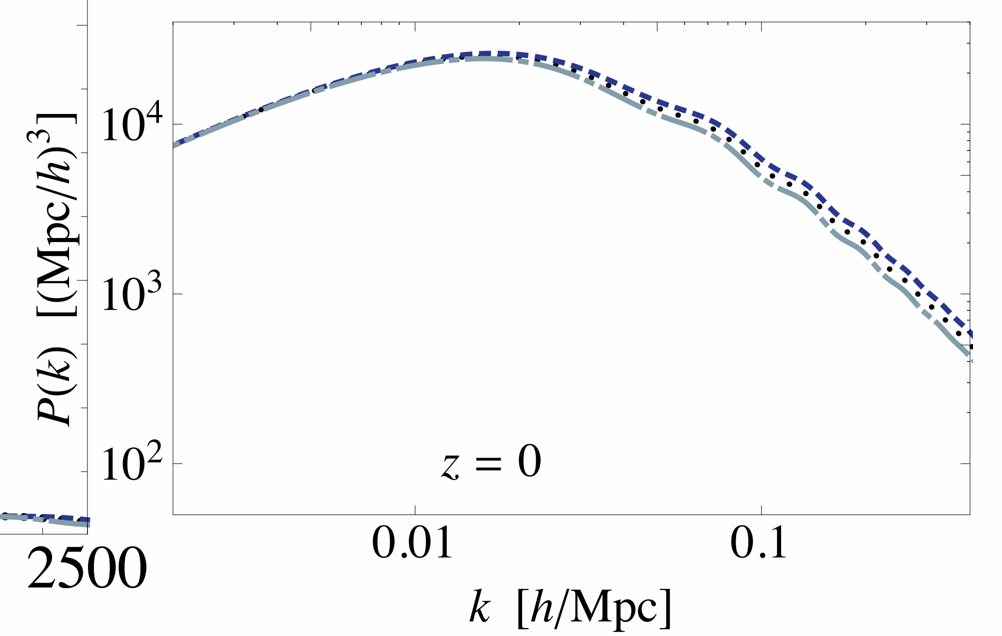


Constraints

$$w < 2.4 \times 10^{-3}$$
$$w > -0.9 \times 10^{-3}$$
(99.7% c.l.)

data: Planck 2015
SDSS-III 2013





- w ↗ means ρ_g ↗ and ρ_{rad}/ρ_{matter} ↘ during recombination
- w is anticorrelated with ω_g , correlated with H_0
- w is not correlated with c_s^2, c_{vis}^2

Xu, Chang, PRD 88, 2013
Calabrese et al, PRD 80 2009
Thomas et al, 1601.05097
Kopp et al, 1605.00649

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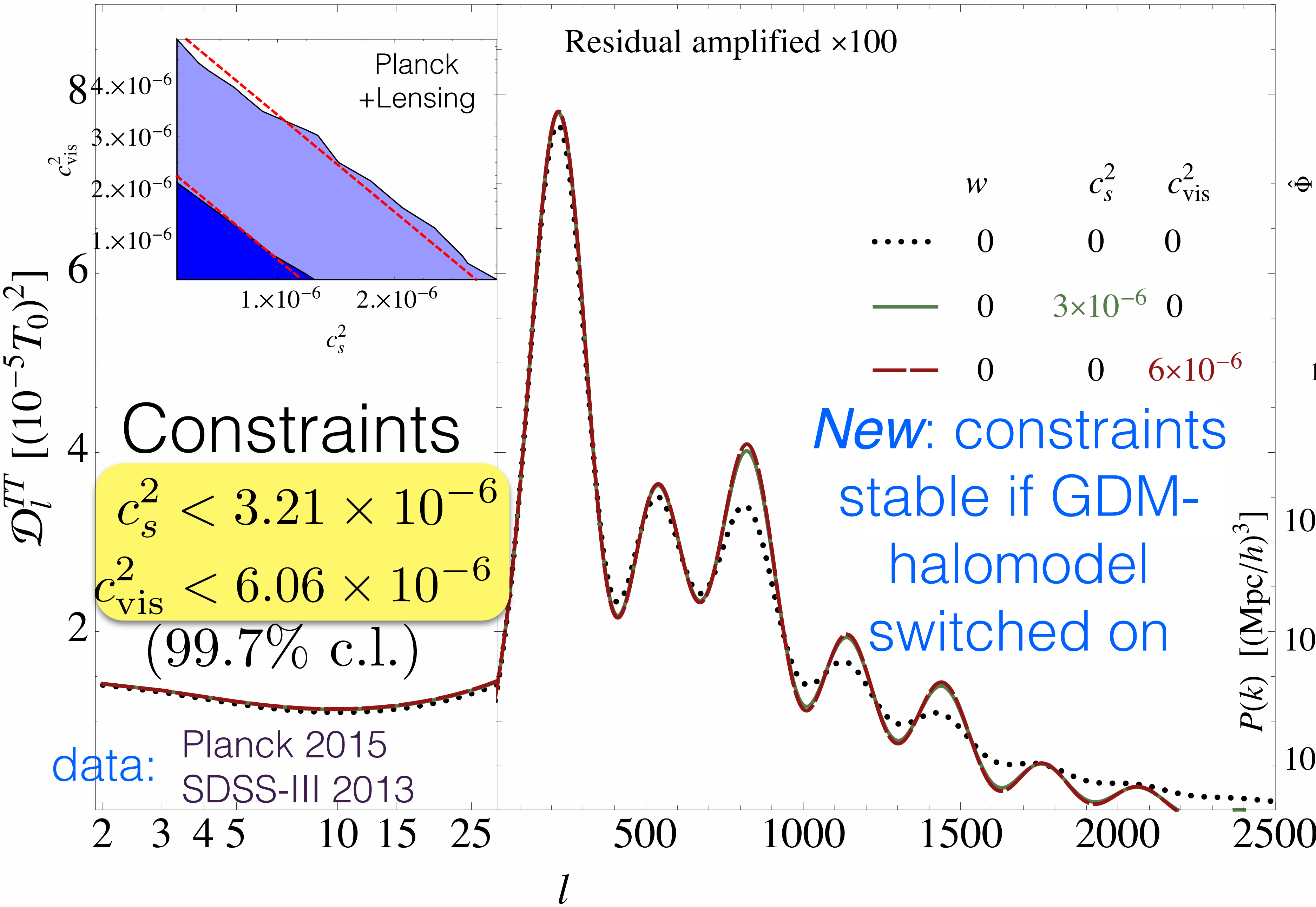


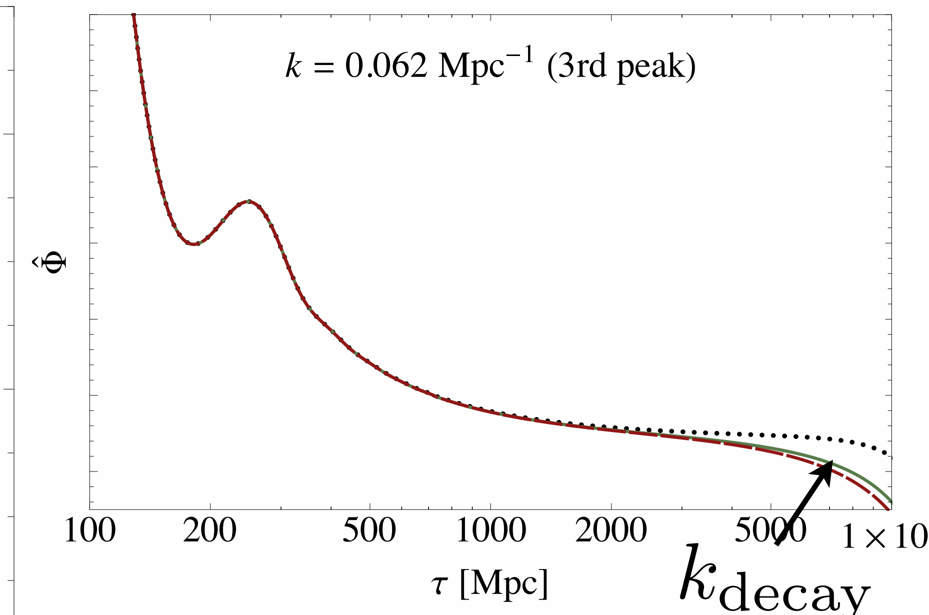
Constraints

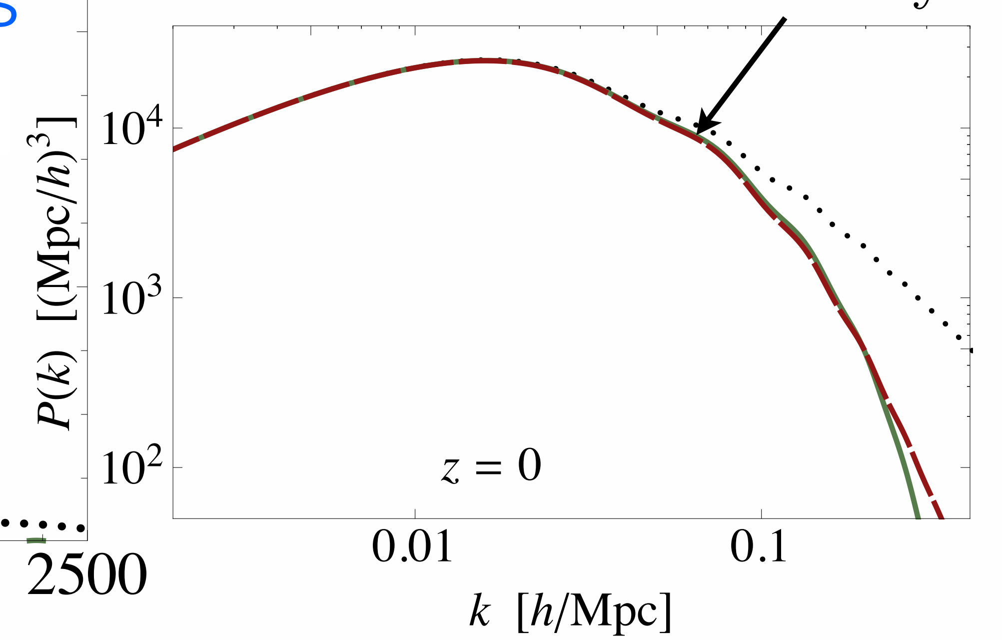
$c_s^2 < 3.21 \times 10^{-6}$

$c_{vis}^2 < 6.06 \times 10^{-6}$

(99.7% c.l.)

New: constraints stable if GDM-halomodel switched on



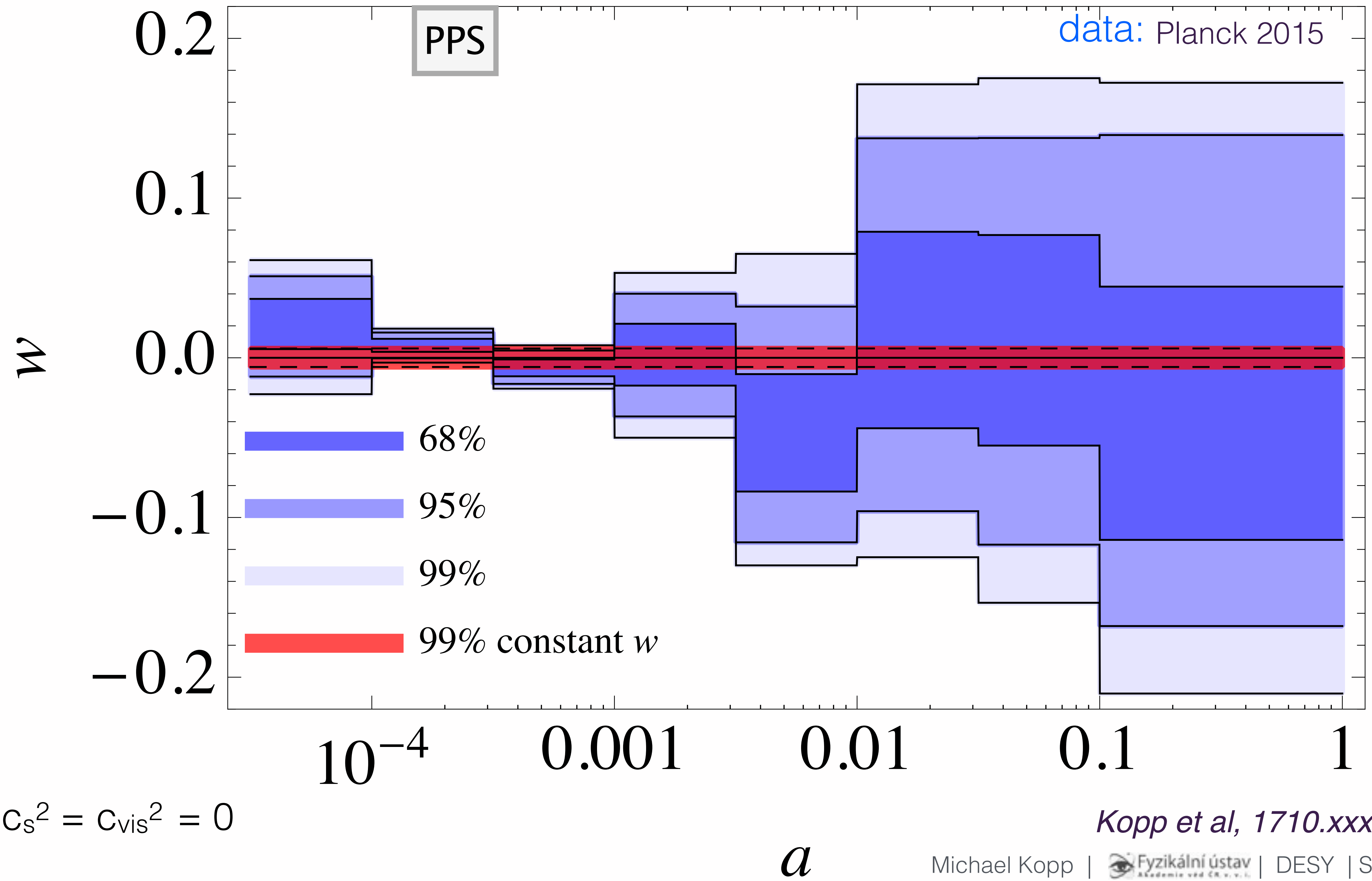


- c^2 ↗ means Φ ↘ decays at late times: *less lensing*
- c_s^2 is anticorrelated with c_{vis}^2 because $k_{decay}^{-1} = \tau\sqrt{c_s^2 + 0.5\, c_{vis}^2}$
- Not correlated with primary parameters but with Σm_ν

Mueller, PRD 71 2005
Calabrese et al, PRD 80 2009
Kunz et al, 1604.05701
Thomas et al, 1601.05097

GDM: w(z)

Simultaneous constraints on 8 piecewise constant w-bins.



$c_s^2 = c_{vis}^2 = 0$

Kopp et al, 1710.xxxxx

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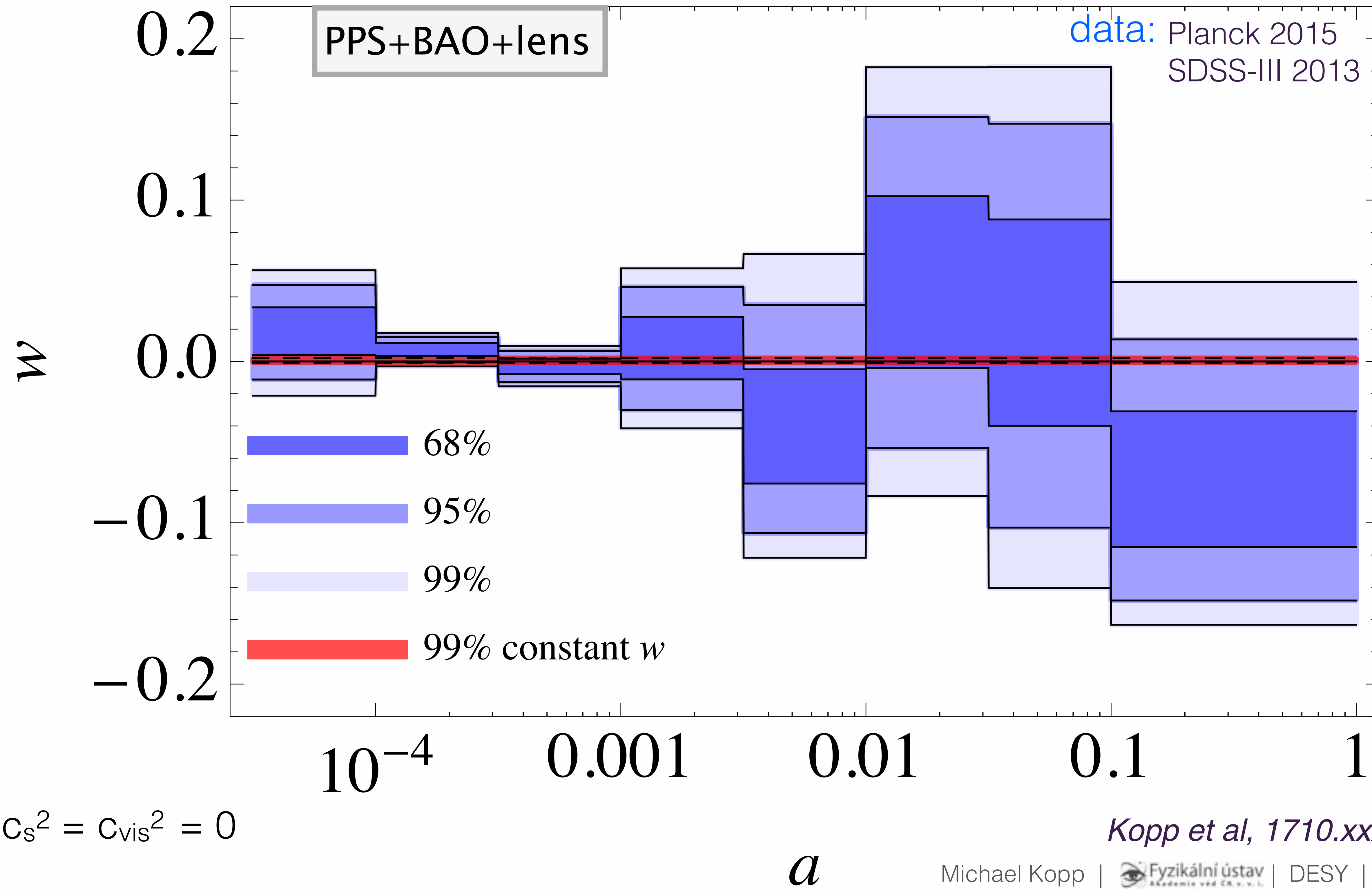

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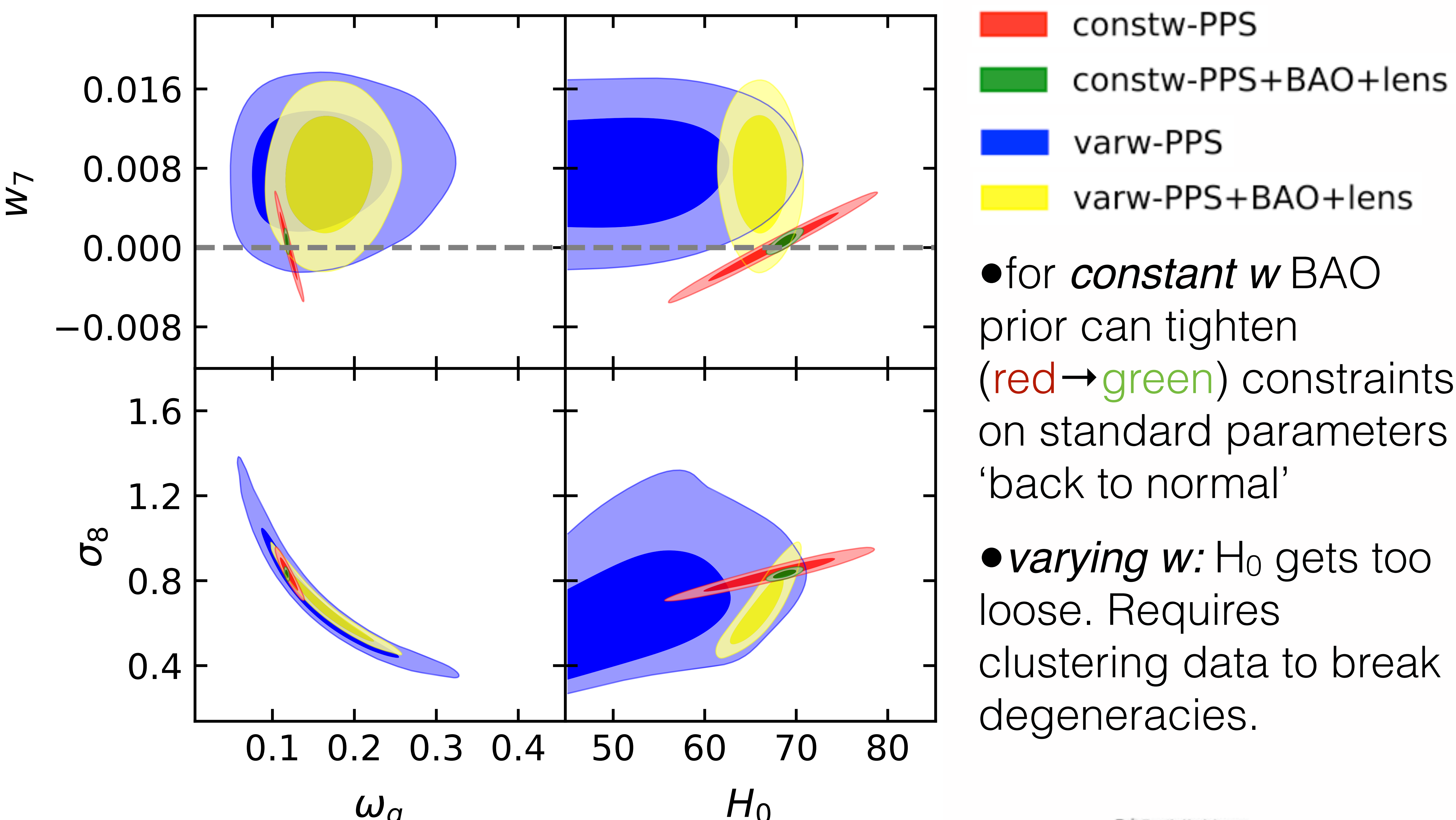
Kopp et al, 1710.xxxxx

GDM: w(z)

Loss of constraining power despite BAO prior: 7th w-bin, w_7



- for *constant w* BAO prior can tighten (red→green) constraints on standard parameters 'back to normal'
- ***varying w:*** H_0 gets too loose. Requires clustering data to break degeneracies.

Parametrized Post Friedmann

Linear scalar perturbations

Skordis, PRD 79, 2009

1. Correct number of time derivatives in constraint Eqs.

2. Modification should lead to gauge invariant field Eqs.

3. Satisfy linearized Bianchi identity (such that $\nabla_\mu T^\mu{}_\nu = 0$)

   - Full control of number of propagating dof (connection to non-perturbative parent theory)
   - Consistently implement numerics in any gauge
   - Reduces number of free parameters

PPF Details

$$ds^2 = a^2 \left\{ -(1+2\Psi)d\tau^2 - 2\vec{\nabla}_i \zeta d\tau dx^i + \left[(1+\frac{1}{3}h)\gamma_{ij} + D_{ij}\nu \right] dx^i dx^j \right\}$$

- Einstein tensor: special linear combination

$$E_\Delta = -a^2 \delta G^0{}_0 \qquad E_P = a^2 \delta G^i{}_i$$

$$\vec{\nabla}_i E_\Theta = -a^2 \delta G^0{}_i \qquad D^i{}_j E_\Sigma = a^2 \left[\delta G^i{}_j - \frac{1}{3} \delta G^k{}_k \delta^i{}_j \right]$$

$$E_\Delta = -6\mathcal{H}^2\Psi + \mathcal{H}\dot{h} - 2\left(k^2 - 3\kappa\right)\eta - 2\mathcal{H}k^2\zeta \qquad \eta = \frac{1}{6}\left[\vec{\nabla}^2\nu - h\right]$$

$$E_\Theta = 2\dot{\eta} + 2\mathcal{H}\Psi + \kappa\left(\dot{\nu} + 2\zeta\right)$$

$$E_P = -\ddot{h} - 2\mathcal{H}\dot{h} + 6\mathcal{H}\dot{\Psi} + 6\left(\mathcal{H}^2 + 2\dot{\mathcal{H}}\right)\Psi - 2k^2\left(\Psi - \eta - \dot{\zeta} - 2\mathcal{H}\zeta\right) - 6\kappa\eta$$

$$E_\Sigma = \frac{1}{2}\ddot{\nu} + \dot{\zeta} + \mathcal{H}\left(\dot{\nu} + 2\zeta\right) + \eta - \Psi$$

- Modified Einstein equations: *no extra dof*

$$E_\Delta(ds^2) = 8\pi G\, T_\Delta(ds^2, \text{baryons}, \text{radiation}, \Lambda) + U_\Delta(ds^2)$$

PPF Details

$U_\Delta, U_\Theta, U_P, U_\Sigma$

ansatz: linear combination of metric perturbation and their first and 2nd derivatives: **48** free functions A_0 ,A_1,...

$$U_\Delta = A_0 \Psi + A_1 h + \ldots$$

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2x(4x2) + 2x(4x3) = **40** free functions

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2 free functions
$P_0(k,\tau), P_1(k,\tau)$

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spatially non-local parent theory

2 free functions
$P_0(k,\tau), P_1(k,\tau)$

With extra dof: mapping of PPF to covariant parent theories Baker et al, 1209.2117

PPF Application

CDM Background

- We assume a CDM type background Y=0 ⇒ X = X_0/a^3 ,

  where X (or Y) is a free function specifying the PPF background $\bar{U}^{\mu}{}_{\nu} = \mathrm{diag}(-X, Y, Y, Y)$ via Bianchi identity $\dot{X} = -3\mathcal{H}(X + Y)$

- *Question*: can we chose $P_0(k,\tau), P_1(k,\tau)$ to exactly mimic CDM perturbatively?

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- *Question*: can we chose $P_0(k,\tau), P_1(k,\tau)$ to exactly mimic CDM perturbatively? *Yes we can!*

$$P_0(\tau, k) = -\frac{9\Omega_X}{2}\frac{\mathcal{H}\hat{\Theta}^{\text{Newtonian}}_{\text{cdm}}}{\hat{\Phi}}\Big|_{\text{GR+cdm}}$$

$$P_1(\tau, k) = -\frac{3\Omega_X}{2}\frac{\hat{\Delta}^{\text{rest frame}}_{\text{cdm}}}{\hat{\Phi}}\Big|_{\text{GR+cdm}}$$

used in GR+PPF **without** CDM

determined from GR+CDM **without** PPF

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"Parameters"?

$$P_1(\tau,k) = -\frac{3\Omega_X}{2}\frac{\hat{\Delta}^{\text{rest frame}}_{\text{cdm}}}{\hat{\Phi}}\Bigg|_{\text{GR+cdm}}$$

used in GR+PPF **without** CDM

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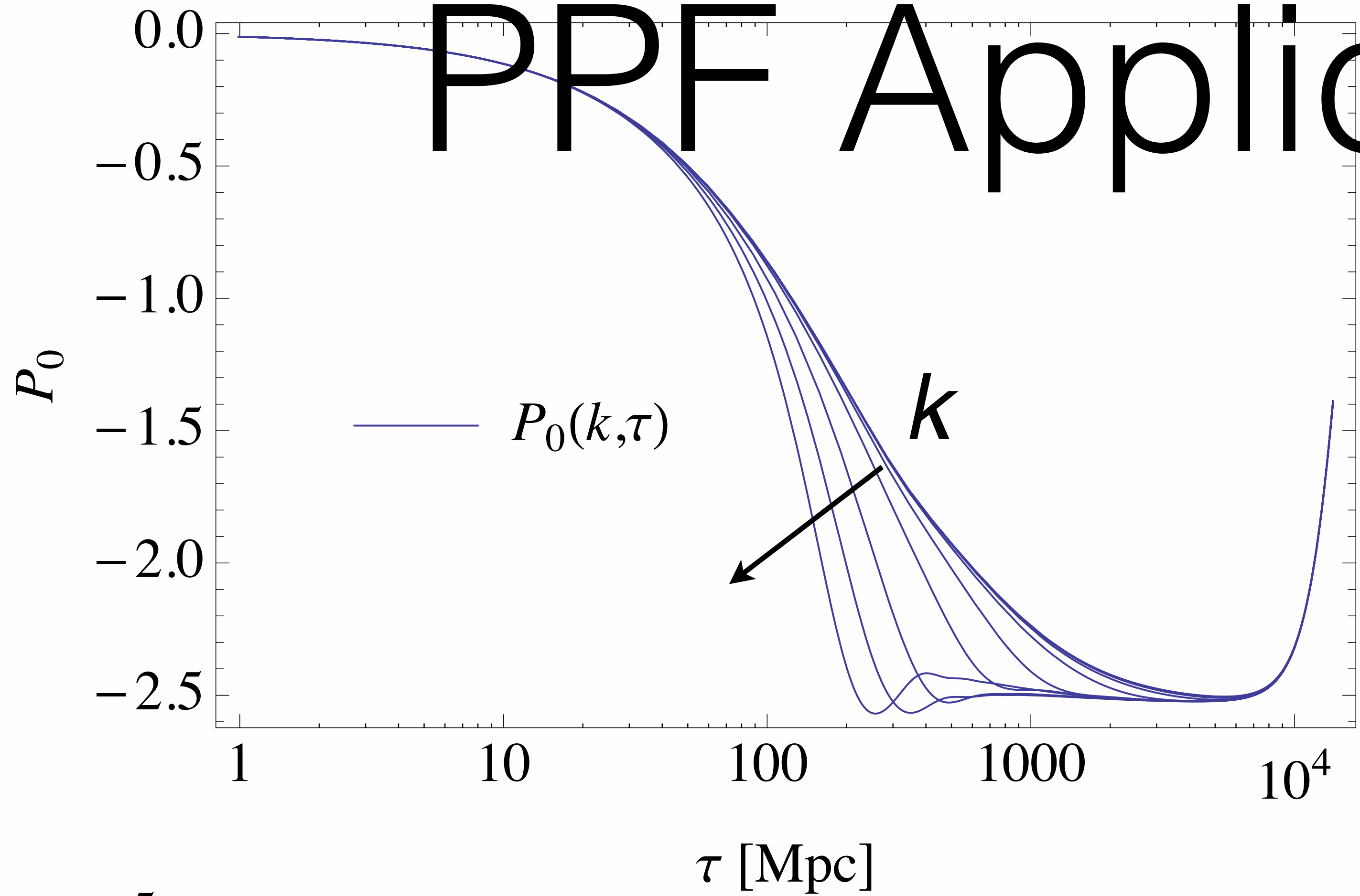
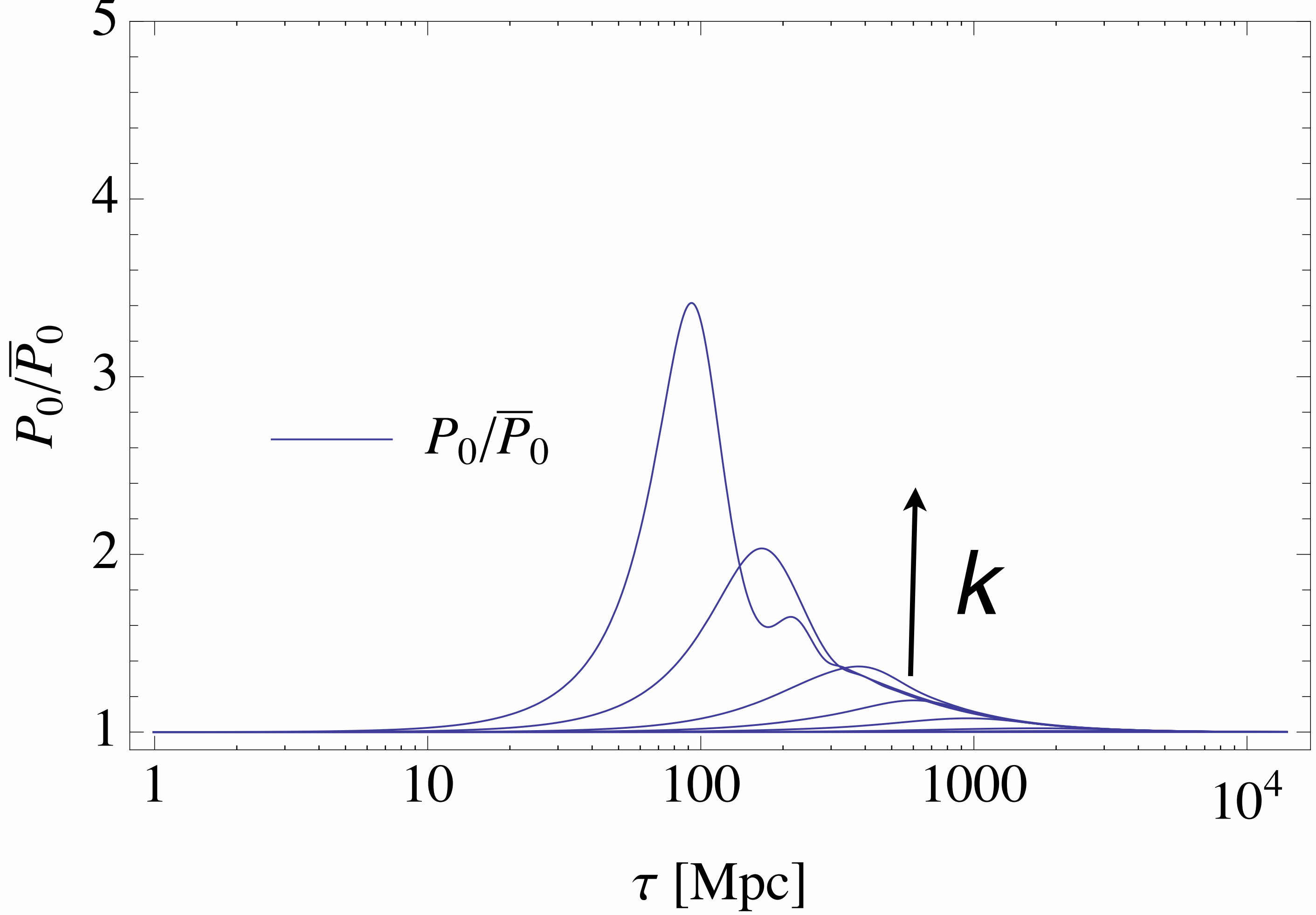


$$\bar{P}_0(\tau) := P_0(k \to 0)$$



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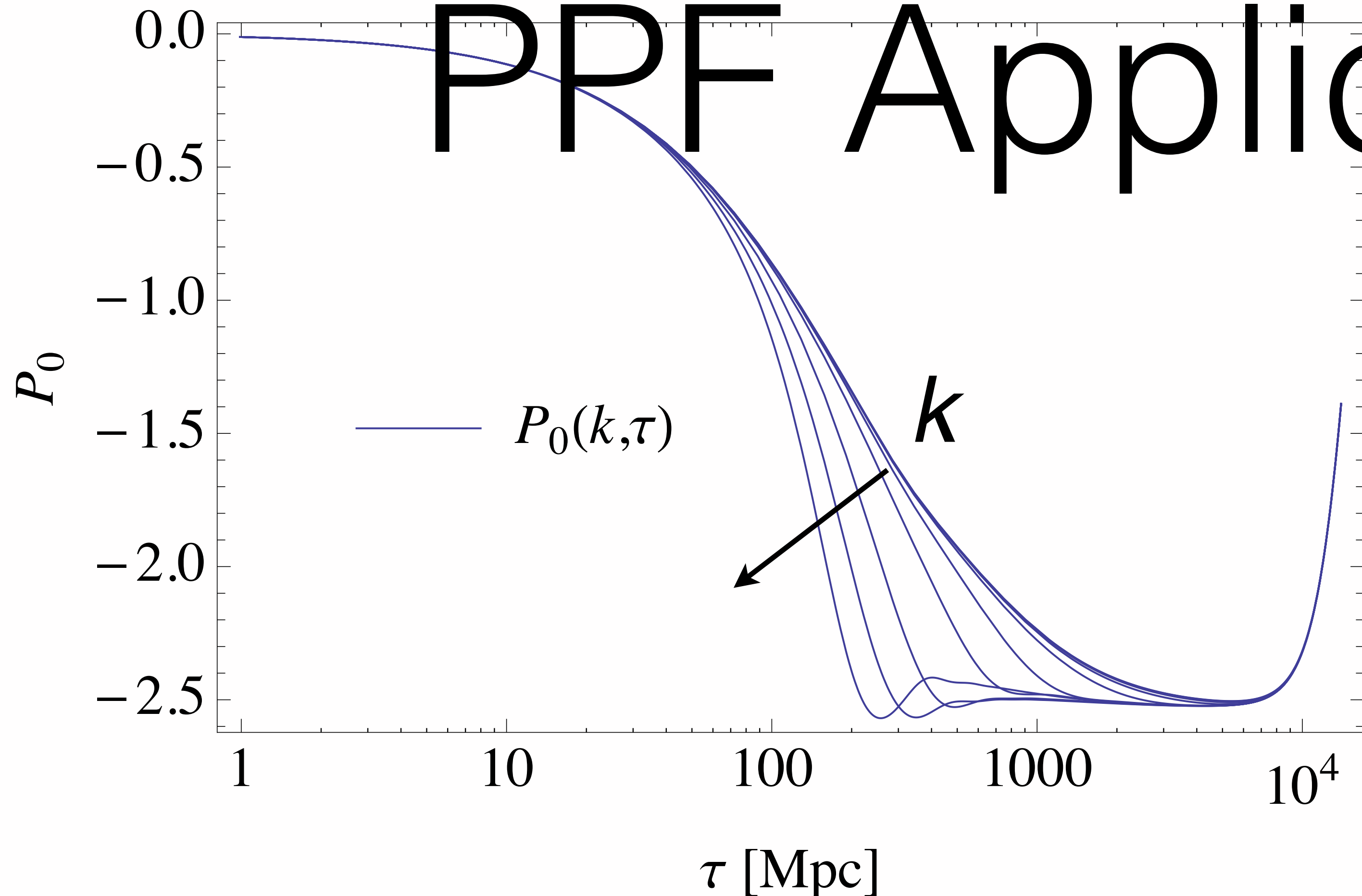
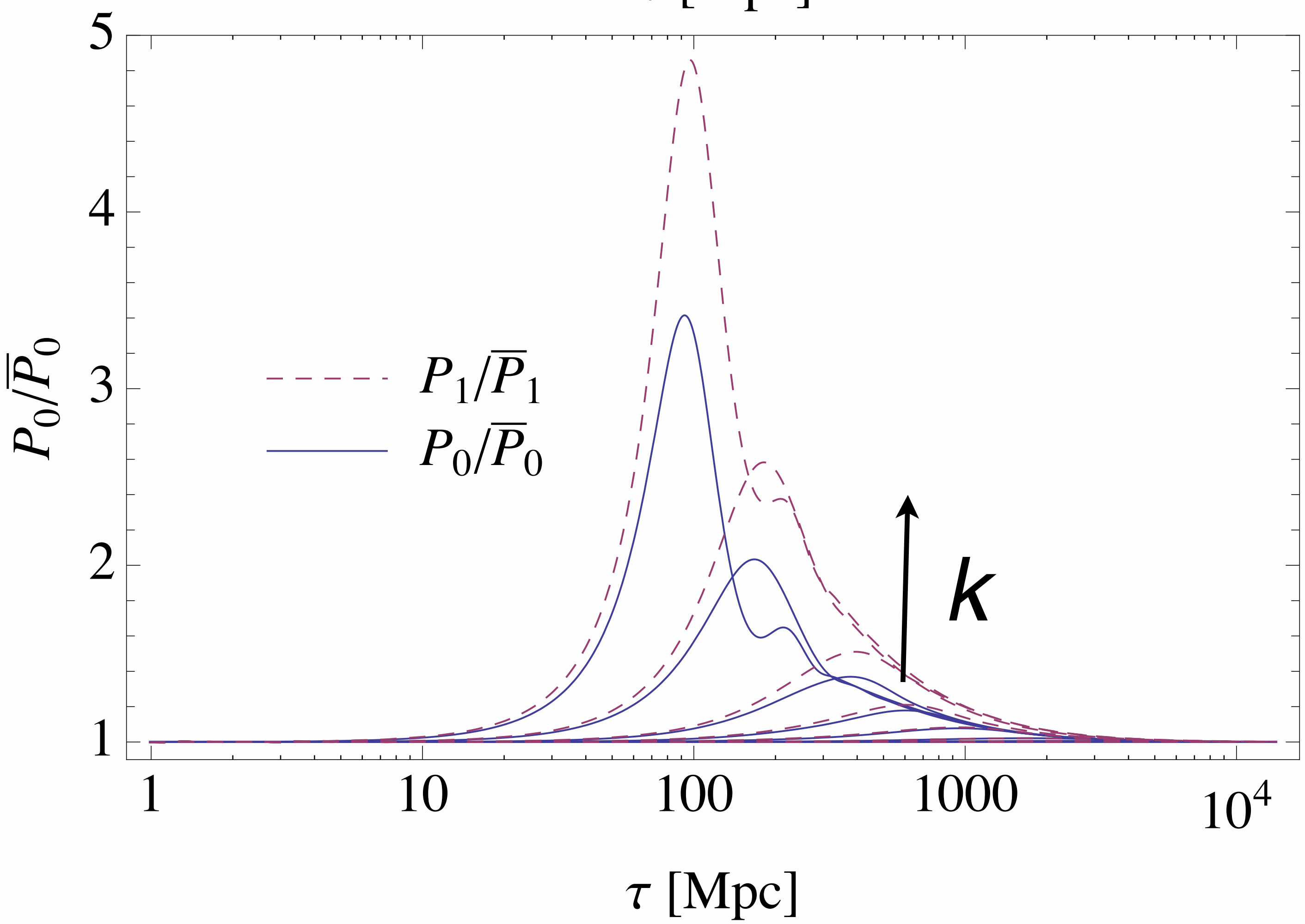
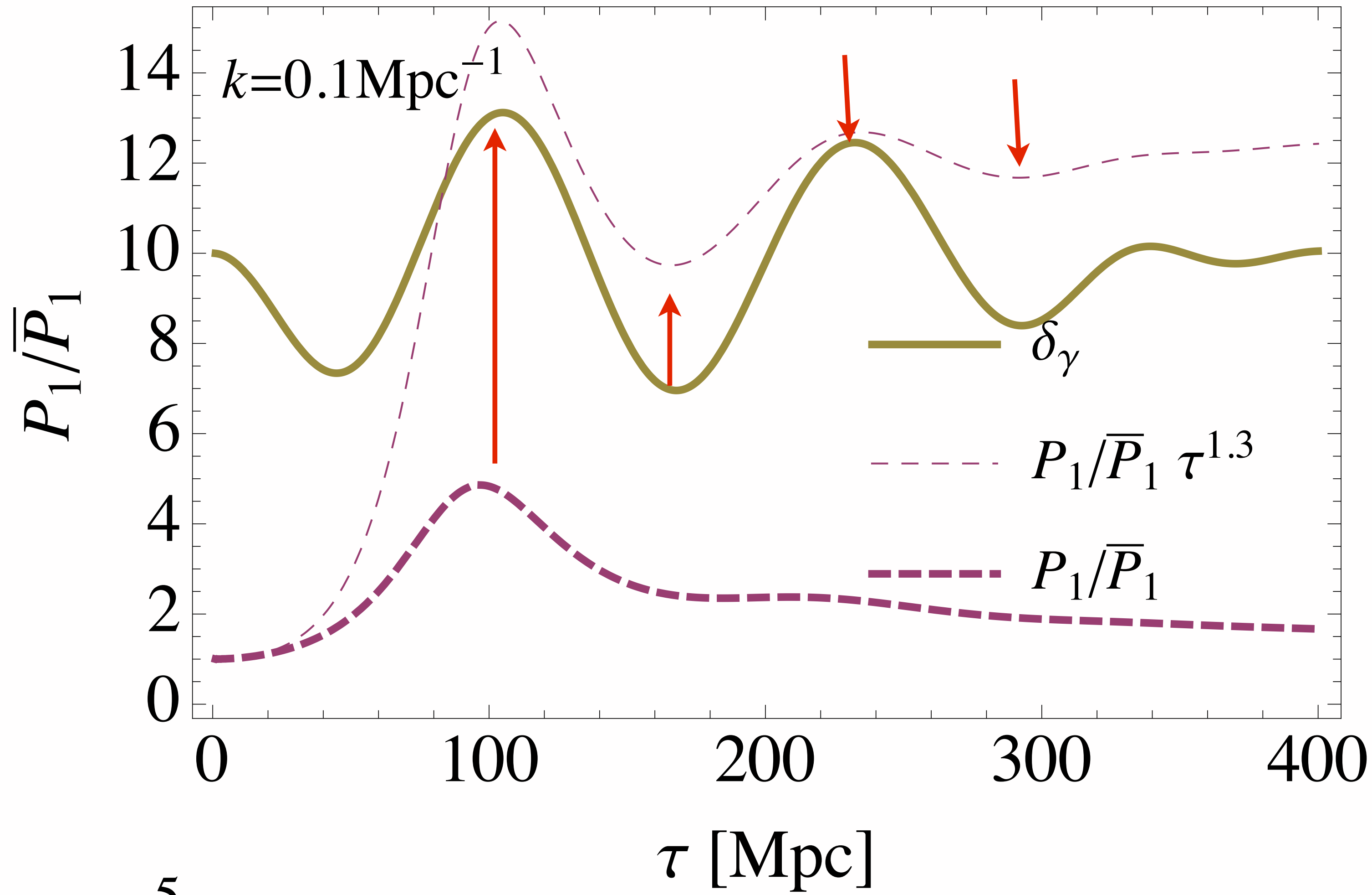


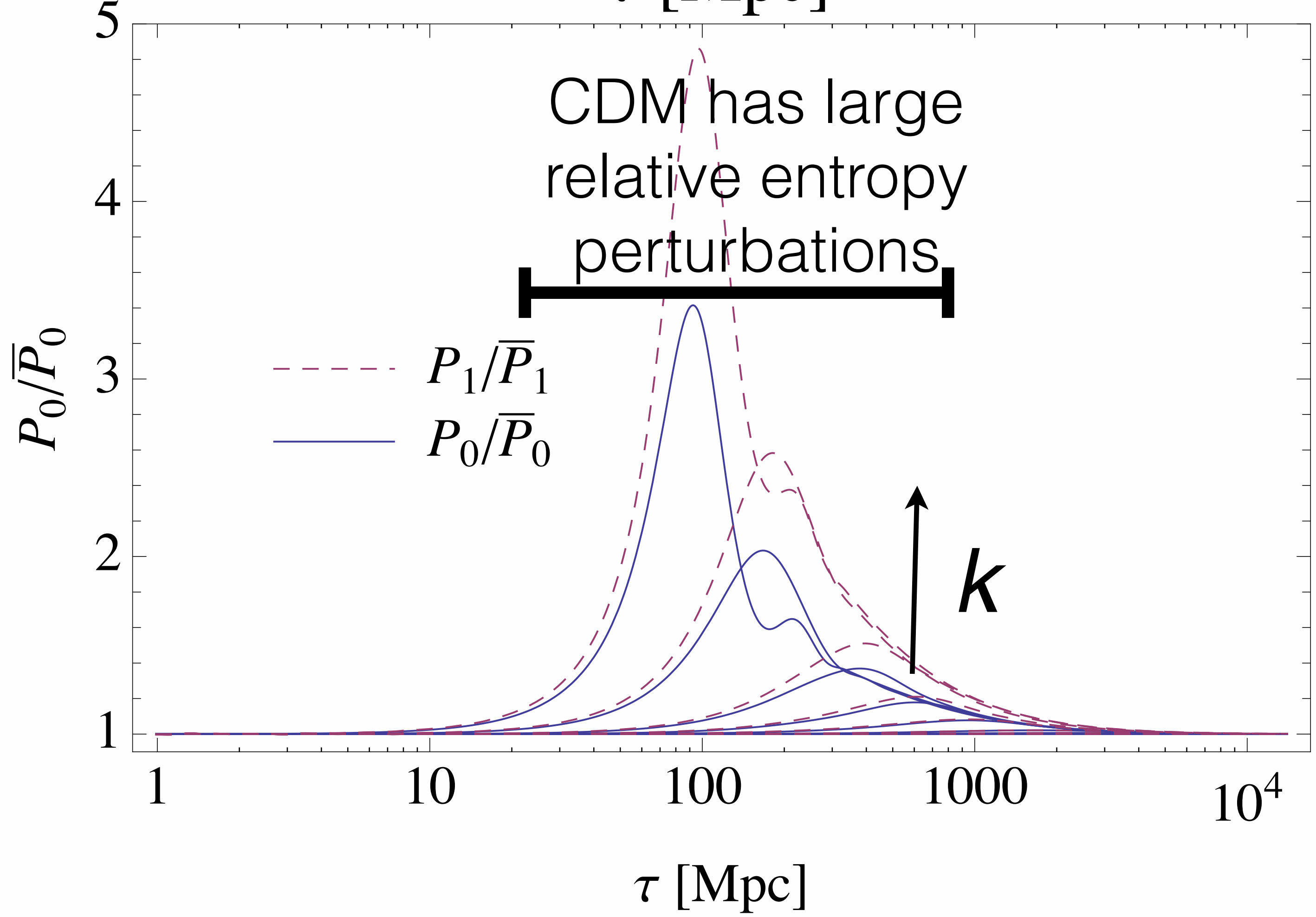
$$\bar{P}_1(\tau, k^2) := k^2 \lim_{k\to 0} \frac{P_1(k)}{k^2}$$

$$\bar{P}_0(\tau) := P_0(k \to 0)$$





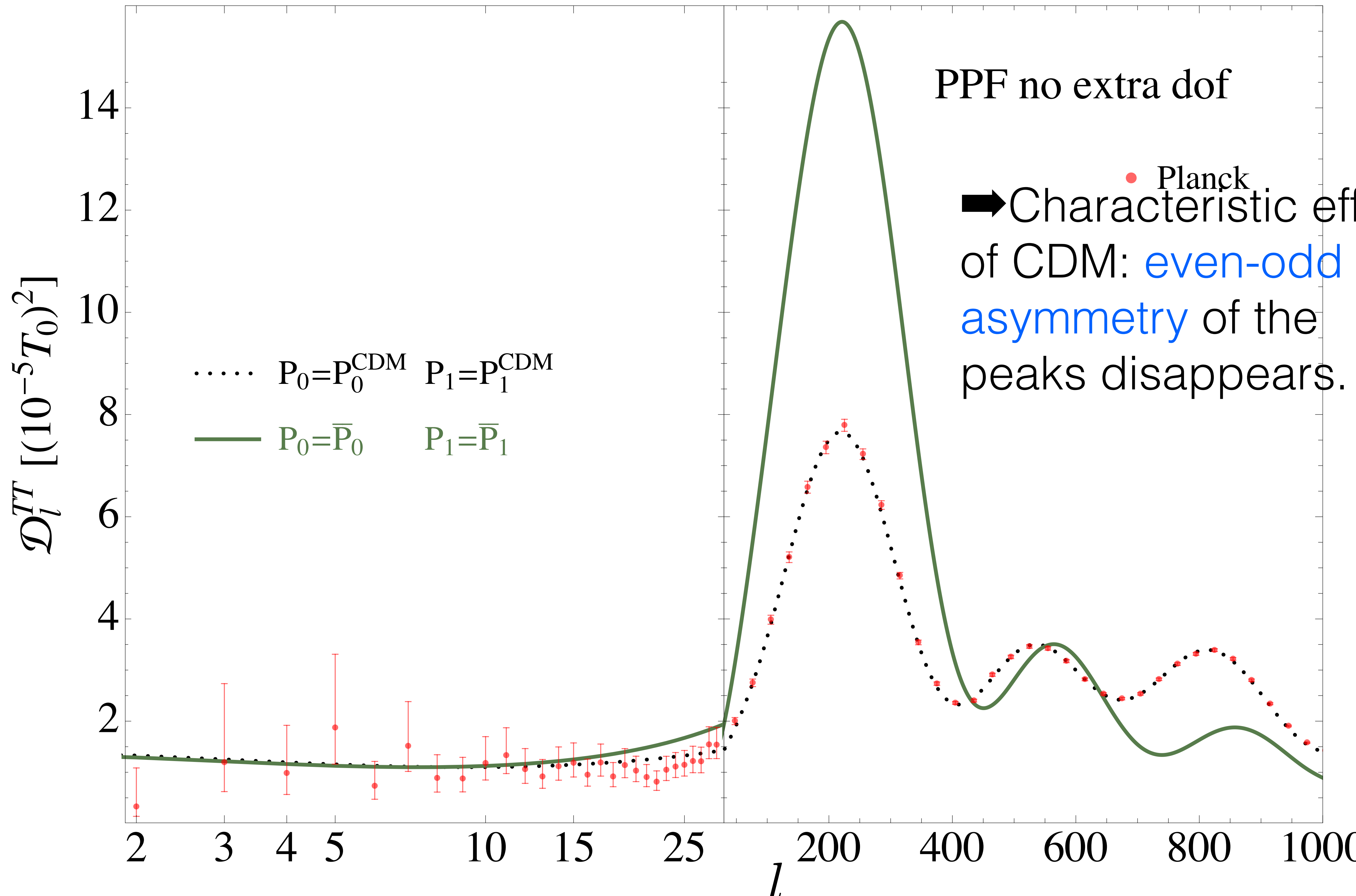
Deviations from the trivial k-dependence are ***strongly correlated*** with the amplitudes and phase of photon density perturbations.



Get rid of the correlated part and check the CMB!

PPF: CMB

Based on a modified CLASS code Lesgourges 2011



➡Characteristic effect of CDM: even-odd asymmetry of the peaks disappears.

Kopp et al, 1710.xxxxx

PPF: CMB

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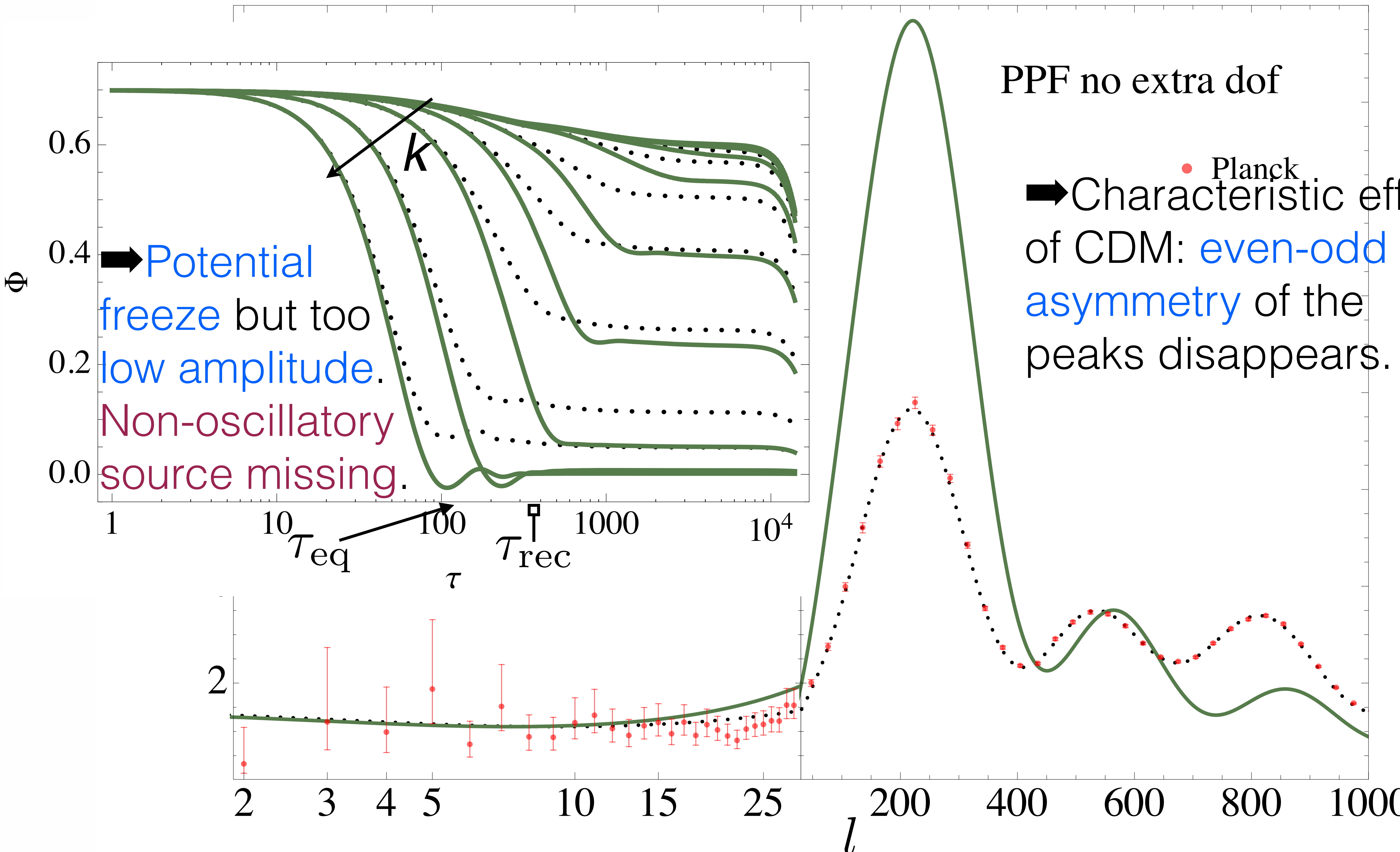

➡ Potential freeze but too low amplitude. Non-oscillatory source missing.

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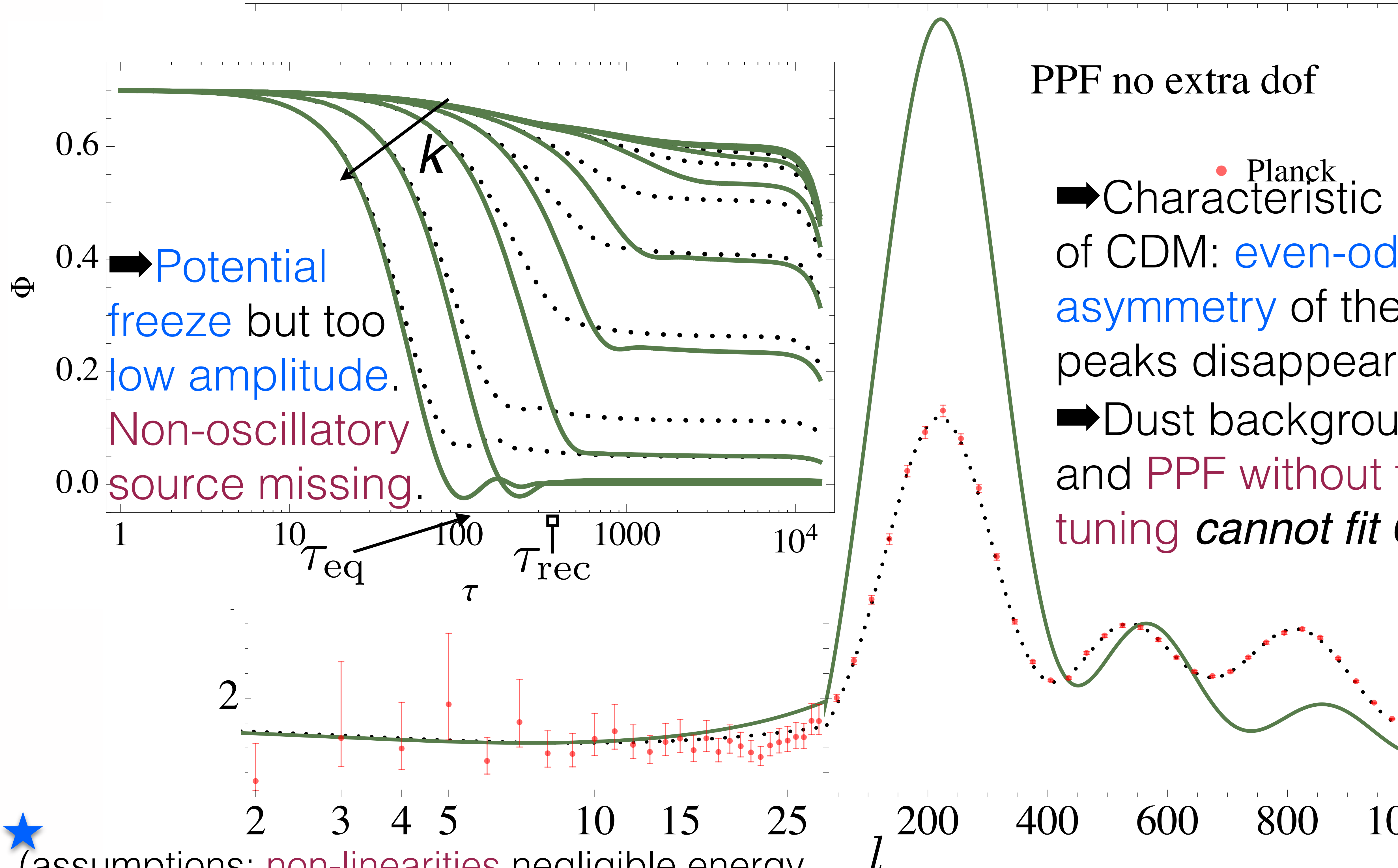

➡Potential freeze but too low amplitude. Non-oscillatory source missing.

➡Characteristic effect of CDM: even-odd asymmetry of the peaks disappears.

➡Dust background and PPF without fine tuning *cannot fit CMB*

(assumptions: non-linearities negligible,energy conservation of matter, adiabatic initial conditions)

Kopp et al, 1710.xxxxx

Summary GDM

Thomas et al, 1601.05097

- 'Generalized dark matter' GDM with 3 new parameters offers the possibility to **test dark matter properties** in a wide class of models: warm + free streaming, interacting, condensate, mimetic dark matter, EFTofLSS...

- GDM parameter estimation with **Planck** likelihood + BAO prior: strong constraints and **consistent with ΛCDM**. Halo model construction shows that **constraints are robust** and not expected to weaken in a fully non-linear extension of GDM.

Kopp et al, 1712.xxxxx

- **Constrained general time dependence** of the equation of state DM in 8 redshift bins: **consistent with ΛCDM**

Thomas et al, 1711.xxxxx

Ongoing

- Exploring nonlinear regime with GDM halo model and fit to large scale structures data: WiggleZ, CFHTlenS, Ly-α
- Constrain scale and time dependent sound speed and viscosity. Constrain isocurvature modes.

Summary PPF

Kopp et al, 1710.xxxxx

- ‘Parametrized Post Friedmann’ PPF is a model-independent framework that allows the possibility to **test the dark matter paradigm** beyond the fluid picture. It replaces DM in linear perturbation theory.

- Without new degrees of freedom beyond the spin 2 of GR, **modified gravity cannot fit the CMB** without fine tuning.

Ongoing

- Allow 1 extra scalar dof, but impose spatial locality: 5 free time dependent functions.

- Find useful parametrizations and constrain with CMB.

Backup

GDM from...

Kopp et al, 1605.00649

Particles (Boltzmann equation)

- Freely streaming warm dark matter

Armendariz-Picon, Neelakanta, JCAP 2014

- Specific models, like self interacting massive neutrinos and dark atoms + dark photons

Oldengott et al JCAP 2015

Cyr-Racine, Sigurdson, PRD 2013

Fields (effective or fundamental)

- Axion condensates.

Sikivie, Yang, PRL 2009
Hlozek, et al, PRD 2015

- Effective theory of large scale structure: Landau-Lifshitz type energy momentum tensor for CDM due to small scale nonlinearities

Baumann et al, JCAP 2012

- Mimetic dark matter and more general constrained-norm scalar field theories.

Mirzagholi, Vikman, arXiv 2015
Ballesteros, JCAP 2015

Initial conditions

Bucher et al 2000 PRD 62

Ansatz for $x = k\tau \ll 1$

$$\eta = \eta_0 + \eta_1 x \qquad + \eta_2 x^2 + \qquad \dots$$

$$\mathcal{I}_0 = \{\eta_0, \delta_{b,0}, \delta_{c,0}, \delta_{\gamma,0}, v_{\gamma,0}, \delta_{\nu,0}, v_{\nu,0}, \sigma_{\nu,0}, \qquad \}$$

Initial conditions

Bucher et al 2000 PRD 62

Ansatz for $x = k\tau \ll 1$

$$\eta = \eta_0 + \eta_1 x + \varepsilon(\eta_1^{(\varepsilon)} x + \eta_1^{(\ln,\varepsilon)} x \ln x) + \eta_2 x^2 + \varepsilon(\eta_2^{(\varepsilon)} x^2 + \eta_2^{(\ln,\varepsilon)} x^2 \ln x)...$$

$$\delta_g = \delta_{g,0} + \varepsilon \delta_{g,0}^{(\ln,\varepsilon)} \ln x + \delta_{g,1} x + \varepsilon(\delta_{c,1}^{(\varepsilon)} x + \delta_{g,1}^{(\ln,\varepsilon)} x \ln x) + ...$$

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$$\delta_g = \delta_{g,0} + \varepsilon \delta_{g,0}^{(\ln,\varepsilon)} \ln x + \delta_{g,1} x + \varepsilon(\delta_{c,1}^{(\varepsilon)} x + \delta_{g,1}^{(\ln,\varepsilon)} x \ln x) + ...$$

$$\mathcal{I}_0 = \{\eta_0, \delta_{b,0}, \delta_{c,0}, \delta_{\gamma,0}, v_{\gamma,0}, \delta_{\nu,0}, v_{\nu,0}, \sigma_{\nu,0}, \delta_{g,0}, v_{g,0}, \sigma_{g,0}\}$$

11 1st order ODEs

Initial conditions

Bucher et al 2000 PRD 62

Ansatz for $x = k\tau \ll 1$

$$\eta = \eta_0 + \eta_1 x + \varepsilon(\eta_1^{(\varepsilon)} x + \eta_1^{(\ln,\varepsilon)} x \ln x) + \eta_2 x^2 + \varepsilon(\eta_2^{(\varepsilon)} x^2 + \eta_2^{(\ln,\varepsilon)} x^2 \ln x)...$$

$$\delta_g = \delta_{g,0} + \varepsilon \delta_{g,0}^{(\ln,\varepsilon)} \ln x + \delta_{g,1} x + \varepsilon(\delta_{c,1}^{(\varepsilon)} x + \delta_{g,1}^{(\ln,\varepsilon)} x \ln x) + ...$$

$$\mathcal{I}_0 = \{\eta_0, \delta_{b,0}, \delta_{c,0}, \delta_{\gamma,0}, v_{\gamma,0}, \delta_{\nu,0}, v_{\nu,0}, \sigma_{\nu,0}, \delta_{g,0}, v_{g,0}, \sigma_{g,0}\}$$

11 1st order ODEs

Task 1: find the maximal subset I_{modes} of I_0 that can be chosen independently

Initial conditions

Bucher et al 2000 PRD 62

Ansatz for $x = k\tau \ll 1$

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$$\mathcal{I}_0 = \{\eta_0, \delta_{b,0}, \delta_{c,0}, \delta_{\gamma,0}, v_{\gamma,0}, \delta_{\nu,0}, v_{\nu,0}, \sigma_{\nu,0}, \delta_{g,0}, v_{g,0}, \sigma_{g,0}\}$$

11 1st order ODEs

Task 1: find the maximal subset I_{modes} of I_0 that can be chosen independently

Brute force method: simply considered all possible subsets of I_0 found:

$$\mathcal{I}_{\text{modes}} = \{\eta_0, \delta_{g,0}, \delta_{b,0}, \delta_{c,0}, \delta_{\nu,0}, v_{\nu,0}\}$$

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Bucher et al 2000 PRD 62

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$$\mathcal{I}_0 = \{\eta_0, \delta_{b,0}, \delta_{c,0}, \delta_{\gamma,0}, v_{\gamma,0}, \delta_{\nu,0}, v_{\nu,0}, \sigma_{\nu,0}, \delta_{g,0}, v_{g,0}, \sigma_{g,0}\}$$

11 1st order ODEs

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11 1st order ODEs

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Test 1: Taking the ansatz up to x^n, the two unused trace and traceless Einstein equations are solved up to x^{n-1}.

Test 2: Exact numerical solution tracks analytical solution

Initial conditions

Bucher et al 2000 PRD 62

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$$\eta = \eta_0 + \eta_1 x + \varepsilon(\eta_1^{(\varepsilon)} x + \eta_1^{(\ln,\varepsilon)} x \ln x) + \eta_2 x^2 + \varepsilon(\eta_2^{(\varepsilon)} x^2 + \eta_2^{(\ln,\varepsilon)} x^2 \ln x) \ldots$$

$$\delta_g = \delta_{g,0} + \varepsilon \delta_{g,0}^{(\ln,\varepsilon)} \ln x + \delta_{g,1} x + \varepsilon(\delta_{c,1}^{(\varepsilon)} x + \delta_{g,1}^{(\ln,\varepsilon)} x \ln x) + \ldots$$

$$\mathcal{I}_0 = \{\eta_0, \delta_{b,0}, \delta_{c,0}, \delta_{\gamma,0}, v_{\gamma,0}, \delta_{\nu,0}, v_{\nu,0}, \sigma_{\nu,0}, \delta_{g,0}, v_{g,0}, \sigma_{g,0}\}$$

11 1st order ODEs

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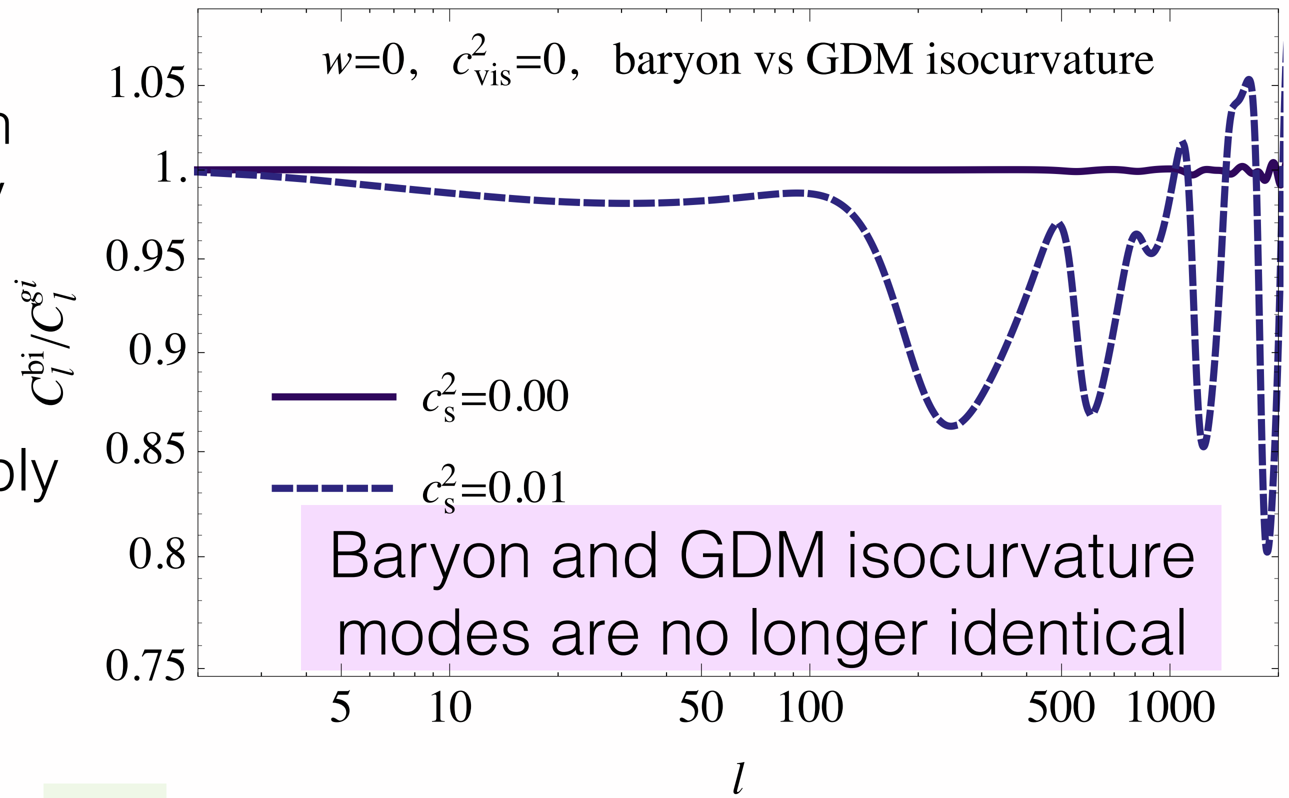

Baryon and GDM isocurvature modes are no longer identical

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GDM mapping

Linear scalar perturbations

Non-equil. thermodynamics (Landau and Lifshitz, Vol.6 1987)		Effective theory of fluids (Ballesteros, JCAP 2015)
• Principle: thermodynamics		volume-preserving 3D-diffeos
4 free functions	$p, \zeta, \eta, \kappa\,(\rho, S)$	$F, m^2, \alpha, \gamma\,(b)$
• no bulk viscosity	$\zeta = 0$	$m^2 = 0$
• equation of state	$w = \frac{\bar{p}}{\bar{\rho}}$	$w = -1 + \frac{d\ln(-\bar{F})}{d\ln b}$
• non-adiabatic Π_{nad}	κ	γ
algebraic function of	$\hat{\Delta}_g$	$\hat{\Delta}_g$
if ▶ $\kappa = 0$ ▶ $\kappa \to \infty$		always
$c_s^2 - c_a^2 = 0$	$c_s^2 - c_a^2 \propto \partial_S p\|_\rho$	$c_s^2 - c_a^2 \propto (\bar{\gamma} - 1)k^2$
• shear Σ_g	$c_{\mathrm{vis}}^2 \propto \eta$	$c_{\mathrm{vis}}^2 \propto \bar{\alpha} - 1,\ \dot{\bar{\alpha}}$
algebraic function of	$\hat{\Theta}_g$ ✓	✗ $\hat{\Theta}_g, \hat{\Delta}_g, \hat{\Psi}$

GDM mapping

Linear scalar perturbations

Non-equil. thermodynamics Landau and Lifshitz, Vol.6 1987		Effective theory of fluids Ballesteros, JCAP 2015
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algebraic function of	$\hat{\Theta}_g$ ✓	✗ $\hat{\Theta}_g, \hat{\Delta}_g, \hat{\Psi}$

Hu, et al, PRL 85, 2000
same for axions

Extended pressure

$$\Pi_{\rm nad}^{\rm extended} = (c_s^2 - c_a^2)\left\{ (1 - C_1 - C_2)\,\hat{\Delta}_g + C_1 \hat{\Delta}_g^{\rm Newtonian} + C_2 \hat{\Delta}_g^{\rm flat} \right\}$$

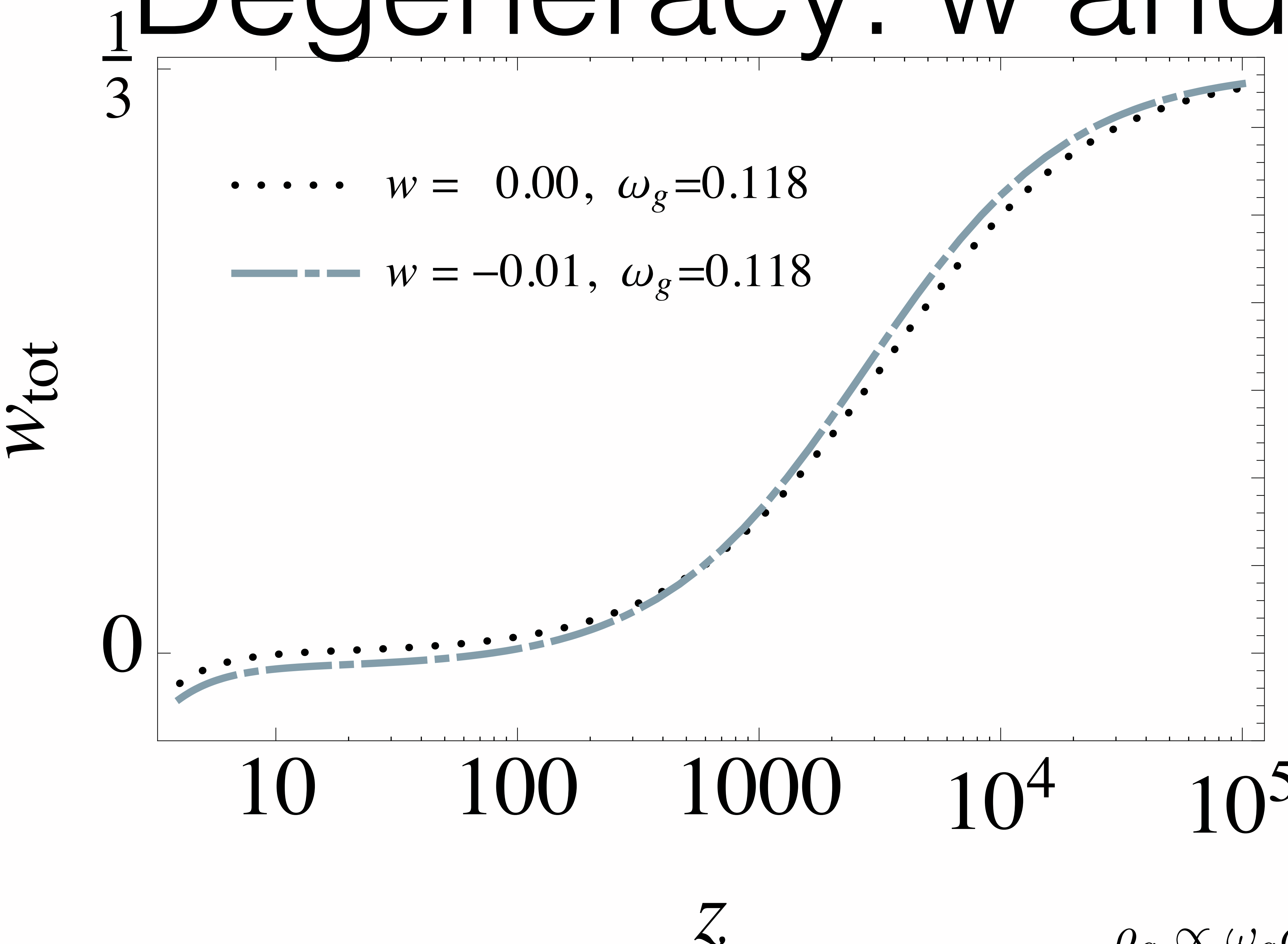
- GDM dominated universe:

$$\mathcal{H}^{-1}\dot{\hat{\Phi}} = \left(\frac{3}{2}(1+w) + \frac{12}{5}c_{\rm vis}^2\right)(\mathcal{R} - \hat{\Phi}) - \hat{\Phi}$$

$$\mathcal{H}^{-1}\dot{\mathcal{R}} = -\frac{2}{3(1+w)}\left(\frac{k}{\mathcal{H}}\right)^2 \left[c_s^2\hat{\Phi} + \frac{4}{5}c_{\rm vis}^2(\mathcal{R} - \hat{\Phi})\right] + 3(c_a^2 - c_s^2)\left[(C_1 + C_2)(\mathcal{R} - \hat{\Phi}) + C_2\hat{\Phi}\right]$$

- GDM has the only simple non-adiabatic pressure
  - that leads to a modification of sound speed without spoiling conservation of superhorizon modes
  - that maximally de-correlates w and c_s^2

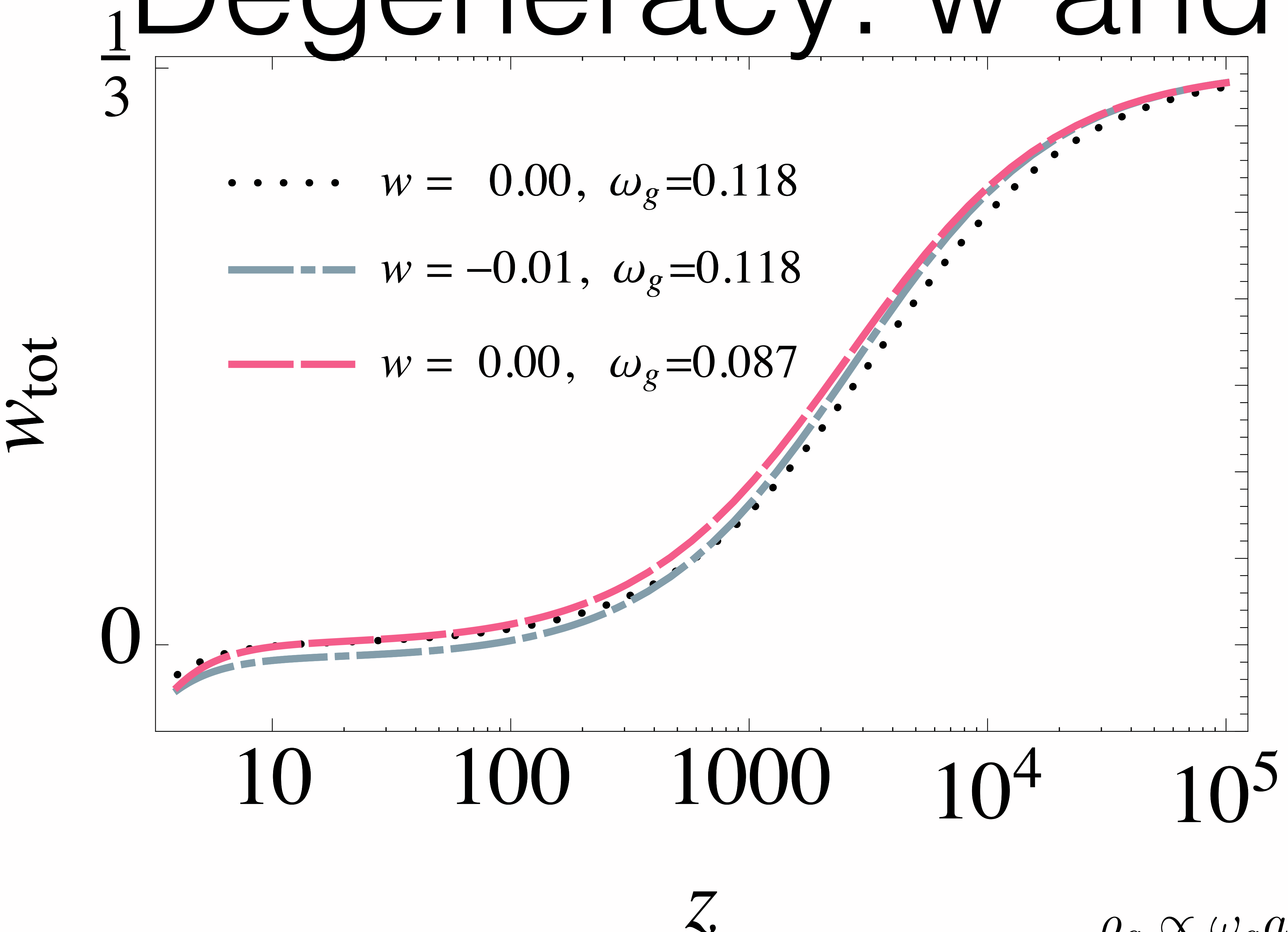
Degeneracy: w and ω_g



- w ↘ means ρ_g ↘ during decoupling and w_{tot} ↗
- w is anticorrelated with ω_g Calabrese et al, PRD 80 2009

$$\rho_g \propto \omega_g a^{-3(1+w)}$$
$$\propto \omega_g (1 + 3w \ln(1+z))$$

Degeneracy: w and ω_g



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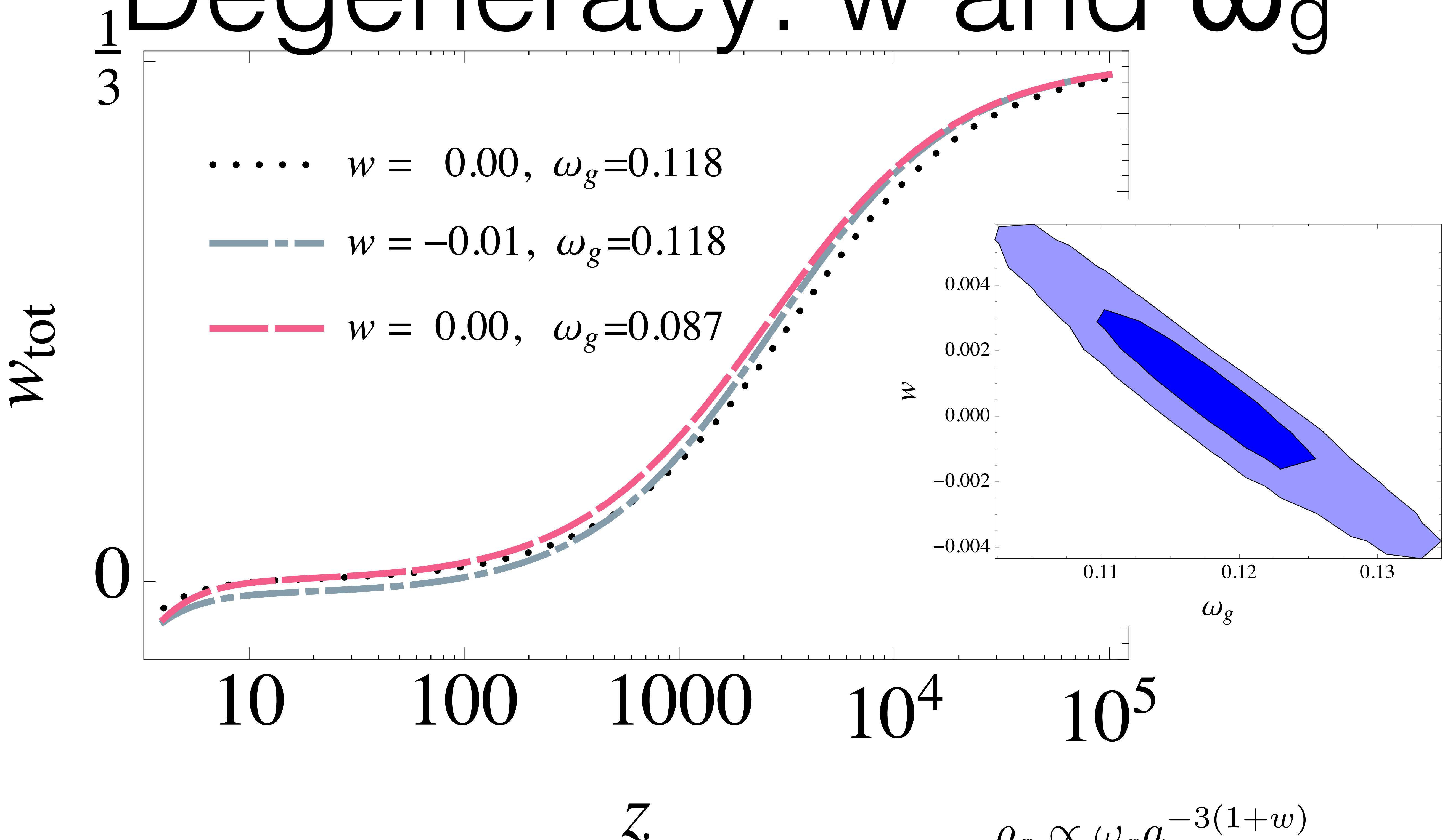

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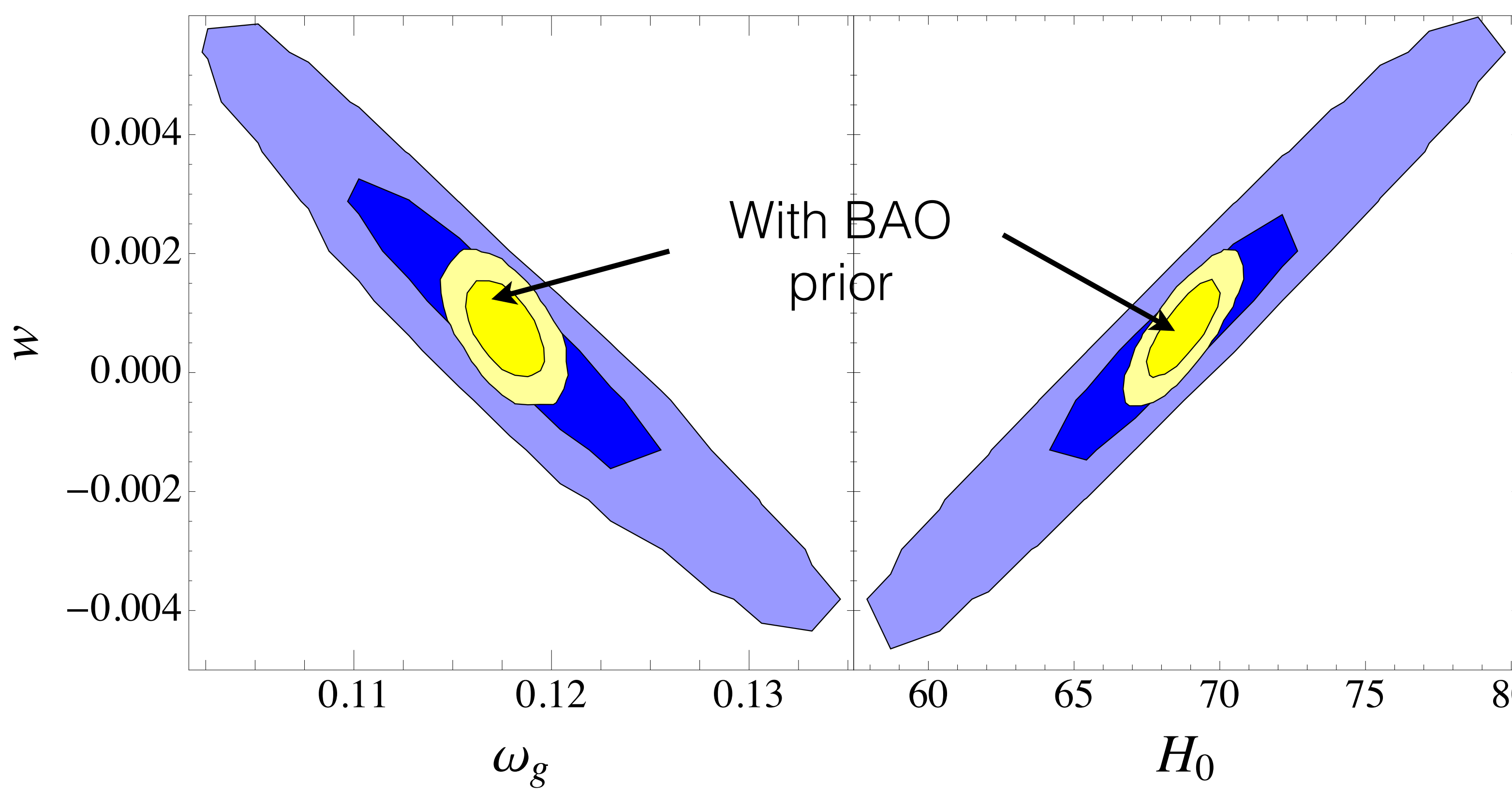
$$\propto \omega_g (1 + 3w \ln(1+z))$$

GDM constraints

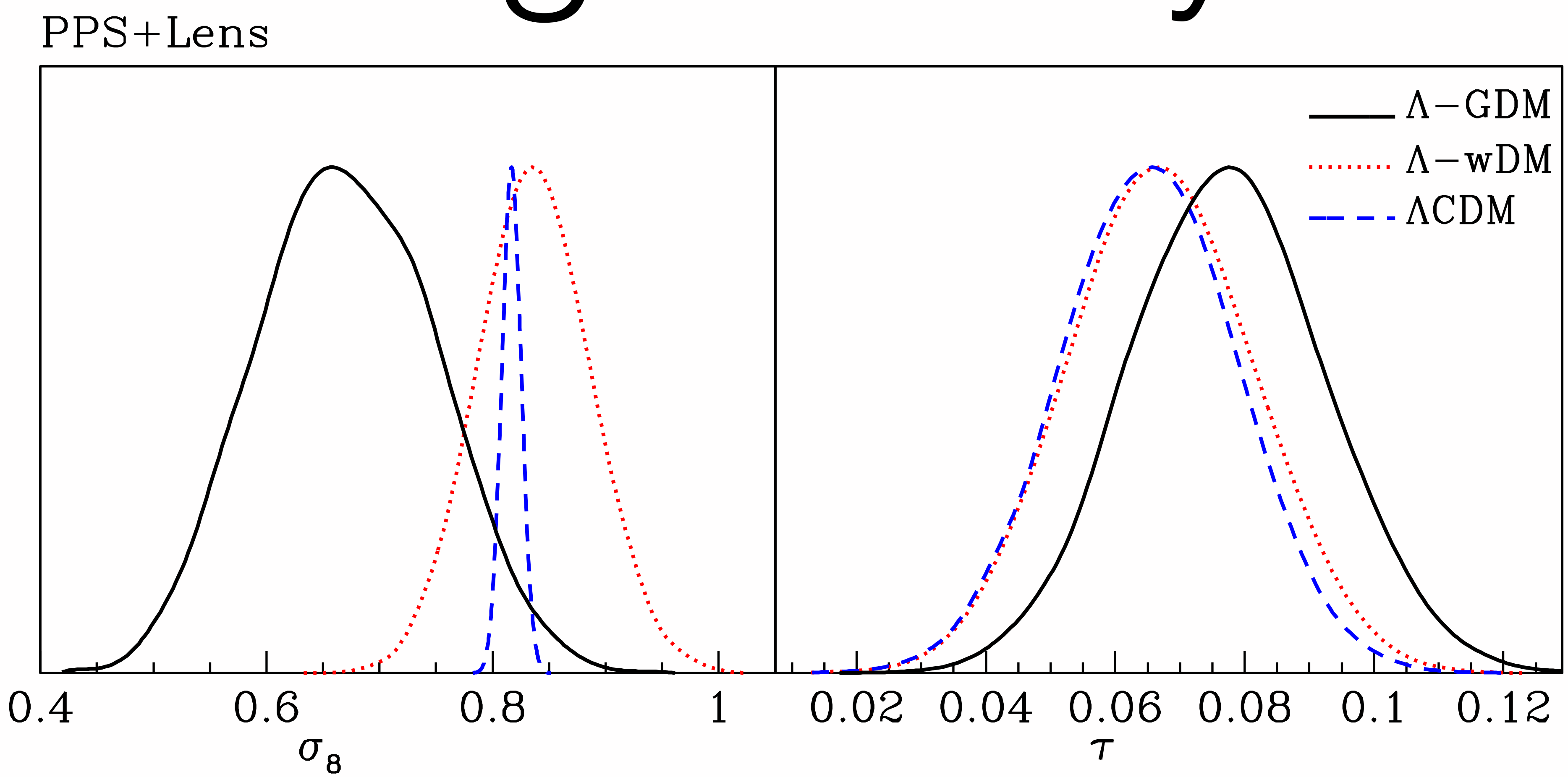
Likelihoods	Λ-wDM $10^2 w$ 95%	Λ-wDM $10^2 w$ 99.7%	Λ-GDM $10^2 w$ 95%	Λ-GDM $10^2 w$ 99.7%	Λ-GDM $10^6 c_s^2$, upper bound 95%	Λ-GDM $10^6 c_s^2$, upper bound 99.7%	Λ-GDM $10^6 c_{\rm vis}^2$, upper bound 95%	Λ-GDM $10^6 c_{\rm vis}^2$, upper bound 99.7%
PPS	$0.007^{+0.463}_{-0.466}$	$0.007^{+0.676}_{-0.673}$	$-0.040^{+0.473}_{-0.468}$	$-0.040^{+0.700}_{-0.701}$	3.31	6.31	5.70	11.3
PPS + Lens	$0.087^{+0.439}_{-0.448}$	$0.087^{+0.662}_{-0.648}$	$0.066^{+0.434}_{-0.427}$	$0.066^{+0.654}_{-0.642}$	1.92	3.44	3.27	5.99
PPS + Lens + HST	$0.256^{+0.217}_{-0.217}$	$0.256^{+0.322}_{-0.323}$	$0.259^{+0.216}_{-0.218}$	$0.259^{+0.321}_{-0.326}$	1.87	3.38	3.11	5.56
PPS + Lens + BAO	$0.063^{+0.108}_{-0.112}$	$0.063^{+0.163}_{-0.164}$	$0.074^{+0.111}_{-0.110}$	$0.074^{+0.164}_{-0.163}$	1.91	3.21	3.30	6.06

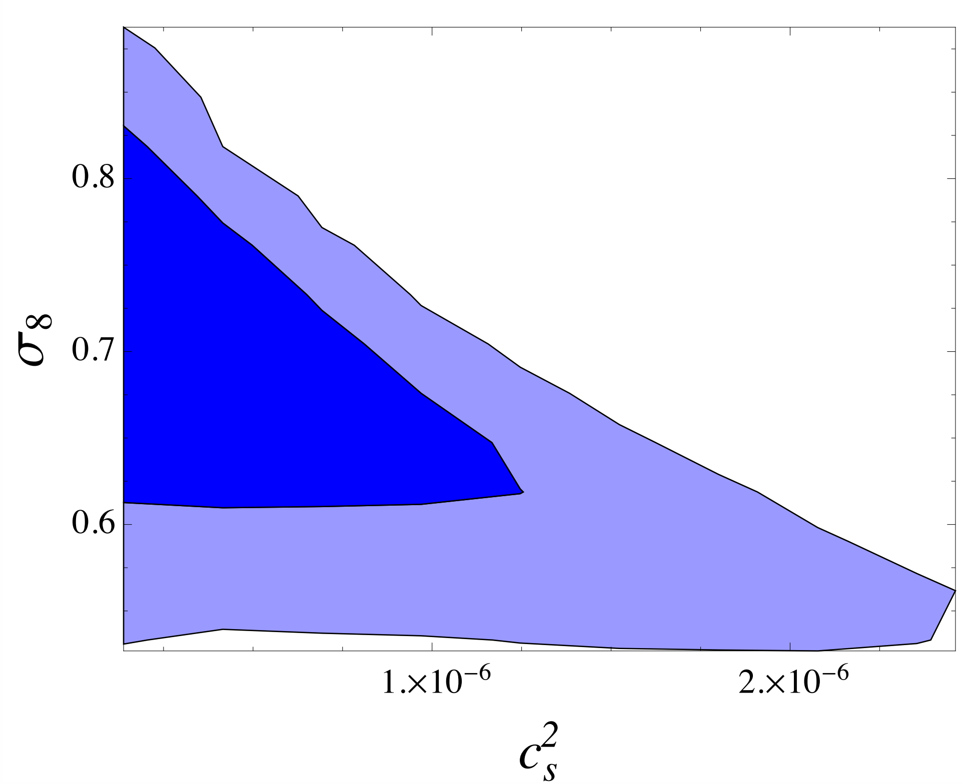
HST-Planck tension alleviated by weakening constraints on H_0 from Planck. H_0-w degeneracy moves w

Very close to the late time value of EFTofLSS



Degeneracy with σ_8





positive GDM parameters bias σ_8 because potential can only decay.

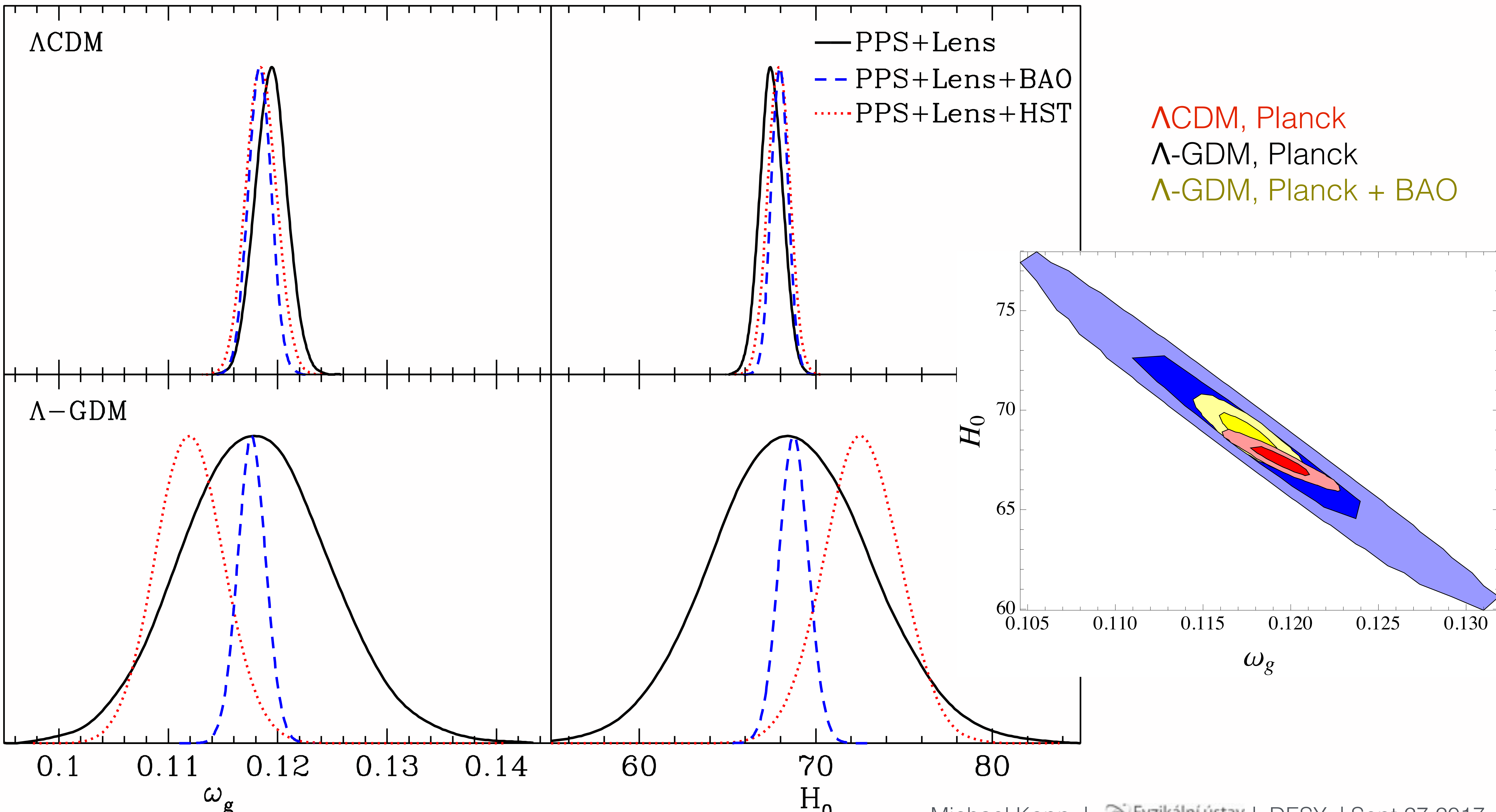
Standard params

Models	Likelihoods / Parameters	PPS+Lens 2σ	PPS+Lens 3σ	PPS+Lens+HST 2σ	PPS+Lens+HST 3σ	PPS+Lens+BAO 2σ	PPS+Lens+BAO 3σ
ΛCDM	$100\omega_b$	$0.02225^{+0.00032}_{-0.00032}$	$0.02225^{+0.00049}_{-0.00048}$	$0.02233^{+0.00032}_{-0.00031}$	$0.02233^{+0.00047}_{-0.00045}$	$0.02233^{+0.00029}_{-0.00029}$	$0.02233^{+0.00042}_{-0.00043}$
	ω_g	$0.1194^{+0.0030}_{-0.0029}$	$0.1194^{+0.0045}_{-0.0042}$	$0.1185^{+0.0028}_{-0.0028}$	$0.1185^{+0.0042}_{-0.0041}$	$0.1184^{+0.0022}_{-0.0022}$	$0.1184^{+0.0032}_{-0.0032}$
	$100\theta_s$	$1.04184^{+0.00061}_{-0.00061}$	$1.04184^{+0.00089}_{-0.00090}$	$1.04193^{+0.00060}_{-0.00060}$	$1.04193^{+0.00089}_{-0.00087}$	$1.04194^{+0.00059}_{-0.00059}$	$1.04194^{+0.00087}_{-0.00088}$
	$\ln 10^{10}A_s$	$3.064^{+0.051}_{-0.051}$	$3.064^{+0.076}_{-0.076}$	$3.074^{+0.051}_{-0.052}$	$3.074^{+0.075}_{-0.077}$	$3.075^{+0.046}_{-0.047}$	$3.075^{+0.068}_{-0.069}$
	n_s	$0.9646^{+0.0099}_{-0.0097}$	$0.9646^{+0.015}_{-0.014}$	$0.9670^{+0.0097}_{-0.0096}$	$0.9670^{+0.014}_{-0.014}$	$0.9673^{+0.0084}_{-0.0084}$	$0.9673^{+0.013}_{-0.012}$
	τ	$0.065^{+0.028}_{-0.028}$	$0.065^{+0.042}_{-0.042}$	$0.0713^{+0.028}_{-0.028}$	$0.0713^{+0.041}_{-0.042}$	$0.072^{+0.025}_{-0.025}$	$0.072^{+0.037}_{-0.037}$
	Ω_Λ	$0.687^{+0.018}_{-0.018}$	$0.687^{+0.026}_{-0.028}$	$0.693^{+0.017}_{-0.017}$	$0.693^{+0.024}_{-0.026}$	$0.694^{+0.013}_{-0.013}$	$0.694^{+0.019}_{-0.019}$
	H_0	$67.5^{+1.3}_{-1.3}$	$67.5^{+2.0}_{-2.0}$	$67.9^{+1.3}_{-1.3}$	$67.9^{+1.9}_{-1.9}$	$70.0^{+1.0}_{-1.0}$	$68.0^{+1.5}_{-1.5}$
	σ_8	$0.817^{+0.018}_{-0.018}$	$0.817^{+0.027}_{-0.026}$	$0.818^{+0.018}_{-0.018}$	$0.818^{+0.027}_{-0.027}$	$0.819^{+0.017}_{-0.018}$	$0.819^{+0.026}_{-0.027}$
Λ-wDM	$100\omega_b$	$0.02223^{+0.00034}_{-0.00033}$	$0.02223^{+0.00051}_{-0.00049}$	$0.02221^{+0.00033}_{-0.00033}$	$0.02221^{+0.00049}_{-0.00048}$	$0.02224^{+0.00033}_{-0.00033}$	$0.02224^{+0.00047}_{-0.00049}$
	ω_g	$0.1170^{+0.0138}_{-0.0134}$	$0.1170^{+0.0212}_{-0.0189}$	$0.1117^{+0.0064}_{-0.0062}$	$0.1117^{+0.0095}_{-0.0092}$	$0.1175^{+0.0027}_{-0.0026}$	$0.1175^{+0.0040}_{-0.0039}$
	$100\theta_s$	$1.04189^{+0.00065}_{-0.00067}$	$1.04189^{+0.00099}_{-0.00099}$	$1.04201^{+0.00062}_{-0.00061}$	$1.04201^{+0.00093}_{-0.00091}$	$1.04188^{+0.00060}_{-0.00060}$	$1.04188^{+0.00088}_{-0.00089}$
	$\ln 10^{10}A_s$	$3.068^{+0.057}_{-0.056}$	$3.068^{+0.082}_{-0.083}$	$3.080^{+0.051}_{-0.051}$	$3.080^{+0.075}_{-0.077}$	$3.067^{+0.048}_{-0.049}$	$3.067^{+0.072}_{-0.075}$
	n_s	$0.9665^{+0.0139}_{-0.0140}$	$0.9665^{+0.0207}_{-0.0204}$	$0.9709^{+0.0102}_{-0.0101}$	$0.9709^{+0.0153}_{-0.0146}$	$0.9658^{+0.0087}_{-0.0086}$	$0.9658^{+0.0127}_{-0.0131}$
	τ	$0.067^{+0.030}_{-0.029}$	$0.067^{+0.044}_{-0.043}$	$0.072^{+0.028}_{-0.027}$	$0.072^{+0.041}_{-0.041}$	$0.067^{+0.027}_{-0.027}$	$0.067^{+0.040}_{-0.040}$
	Ω_Λ	$0.703^{+0.102}_{-0.112}$	$0.703^{+0.128}_{-0.187}$	$0.745^{+0.042}_{-0.043}$	$0.745^{+0.058}_{-0.069}$	$0.703^{+0.020}_{-0.020}$	$0.703^{+0.029}_{-0.031}$
	H_0	$69.3^{+9.0}_{-9.1}$	$69.3^{+13.4}_{-13.4}$	$72.8^{+4.3}_{-4.4}$	$72.8^{+6.5}_{-6.4}$	$68.8^{+1.7}_{-1.7}$	$68.8^{+2.5}_{-2.5}$
	σ_8	$0.837^{+0.103}_{-0.102}$	$0.837^{+0.155}_{-0.149}$	$0.876^{+0.053}_{-0.052}$	$0.876^{+0.079}_{-0.077}$	$0.831^{+0.028}_{-0.028}$	$0.831^{+0.043}_{-0.041}$
Λ-GDM	$100\omega_b$	$0.02219^{+0.00033}_{-0.00033}$	$0.02219^{+0.00049}_{-0.00049}$	$0.02216^{+0.00034}_{-0.00033}$	$0.02216^{+0.00052}_{-0.00049}$	$0.02218^{+0.00033}_{-0.00033}$	$0.02218^{+0.00048}_{-0.00049}$
	ω_g	$0.1180^{+0.0135}_{-0.0134}$	$0.1180^{+0.0205}_{-0.0194}$	$0.1120^{+0.0062}_{-0.0062}$	$0.1120^{+0.0095}_{-0.0092}$	$0.1176^{+0.0027}_{-0.0027}$	$0.1176^{+0.0041}_{-0.0040}$
	$100\theta_s$	$1.04186^{+0.00065}_{-0.00064}$	$1.04186^{+0.00096}_{-0.00097}$	$1.04199^{+0.00060}_{-0.00060}$	$1.04199^{+0.00089}_{-0.00088}$	$1.04186^{+0.00059}_{-0.00059}$	$1.04186^{+0.00090}_{-0.00091}$
	$\ln 10^{10}A_s$	$3.090^{+0.060}_{-0.059}$	$3.090^{+0.090}_{-0.087}$	$3.100^{+0.057}_{-0.056}$	$3.100^{+0.084}_{-0.083}$	$3.089^{+0.057}_{-0.055}$	$3.089^{+0.084}_{-0.080}$
	n_s	$0.9651^{+0.0140}_{-0.0136}$	$0.9651^{+0.0203}_{-0.0205}$	$0.9698^{+0.0103}_{-0.0104}$	$0.9698^{+0.0156}_{-0.0151}$	$0.9651^{+0.0087}_{-0.0087}$	$0.9651^{+0.0131}_{-0.0123}$
	τ	$0.0773^{+0.0310}_{-0.0303}$	$0.0773^{+0.0456}_{-0.0445}$	$0.0815^{+0.0298}_{-0.0295}$	$0.0815^{+0.0450}_{-0.0440}$	$0.0769^{+0.0297}_{-0.0293}$	$0.0769^{+0.0441}_{-0.0433}$
	Ω_Λ	$0.695^{+0.102}_{-0.112}$	$0.695^{+0.134}_{-0.197}$	$0.743^{+0.041}_{-0.044}$	$0.743^{+0.059}_{-0.070}$	$0.703^{+0.020}_{-0.020}$	$0.703^{+0.029}_{-0.031}$
	H_0	$68.6^{+8.9}_{-8.8}$	$68.6^{+13.6}_{-12.7}$	$72.6^{+4.3}_{-4.3}$	$72.6^{+6.4}_{-6.5}$	$68.8^{+1.7}_{-1.7}$	$68.8^{+2.6}_{-2.6}$
	σ_8	$0.671^{+0.155}_{-0.155}$	$0.671^{+0.226}_{-0.213}$	$0.702^{+0.154}_{-0.156}$	$0.702^{+0.197}_{-0.216}$	$0.672^{+0.134}_{-0.143}$	$0.672^{+0.167}_{-0.191}$

constraints on background quantities can be strengthened "back to normal" by BAO prior

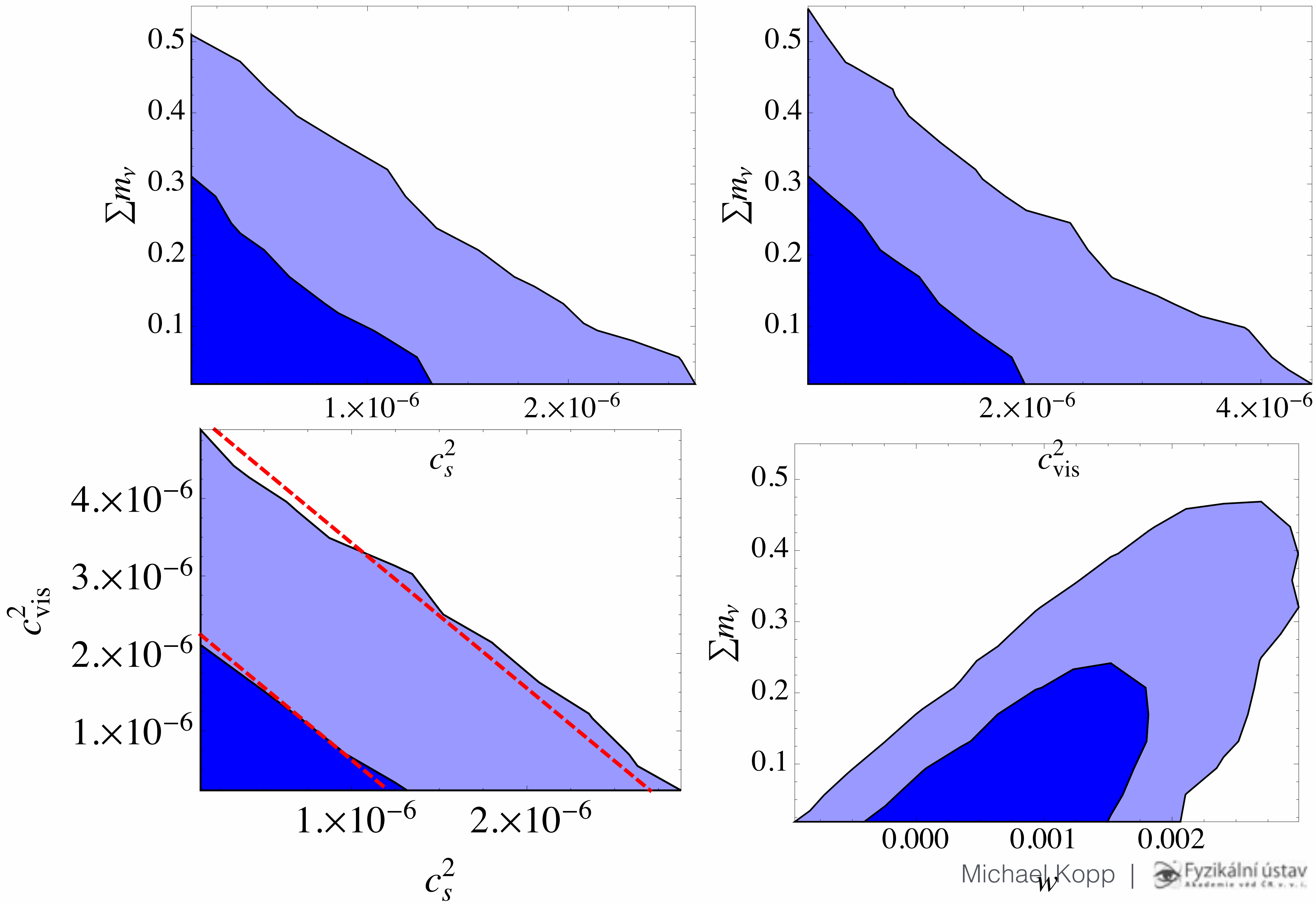
constraints on σ_8 can be also fixed for Λ-wDM but not for Λ-GDM. ***Need LSS data:*** Ly-α, WiggleZ, CFHTlenS

Weakening of constraints on standard parameters

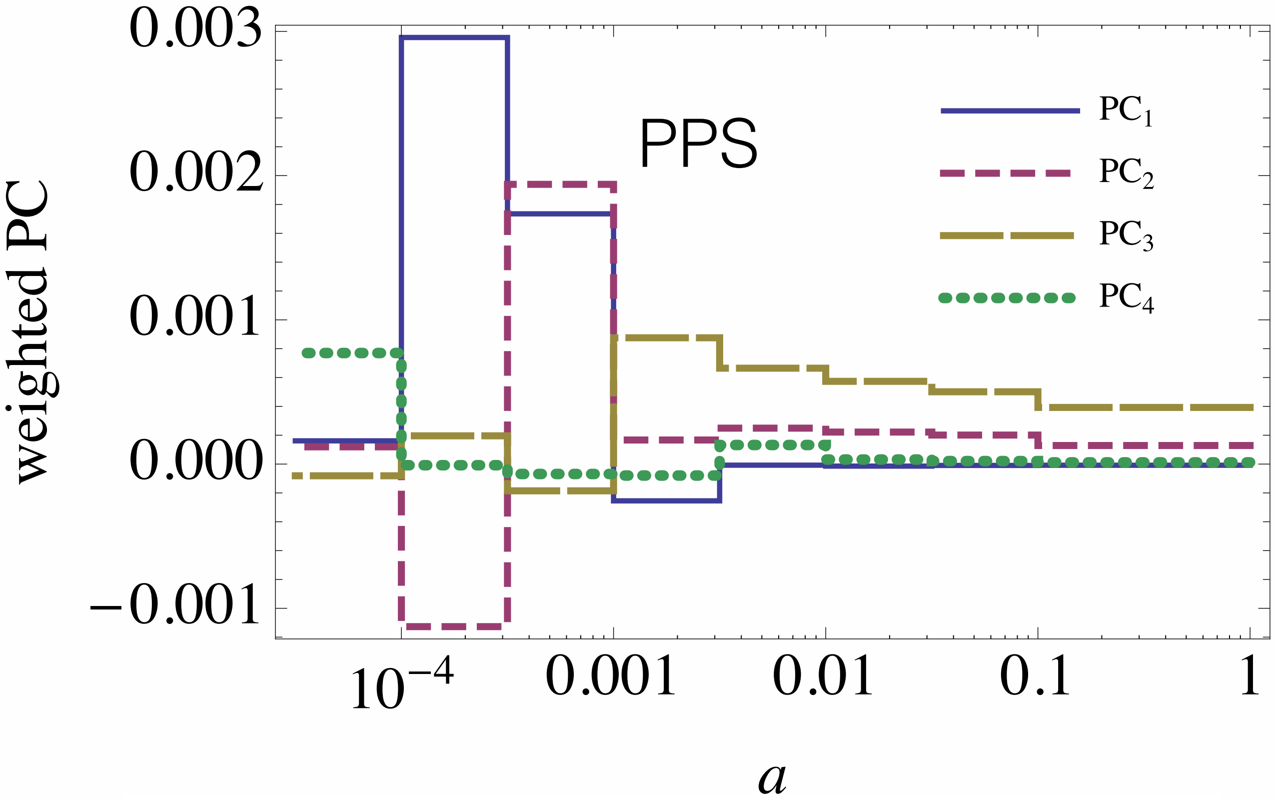


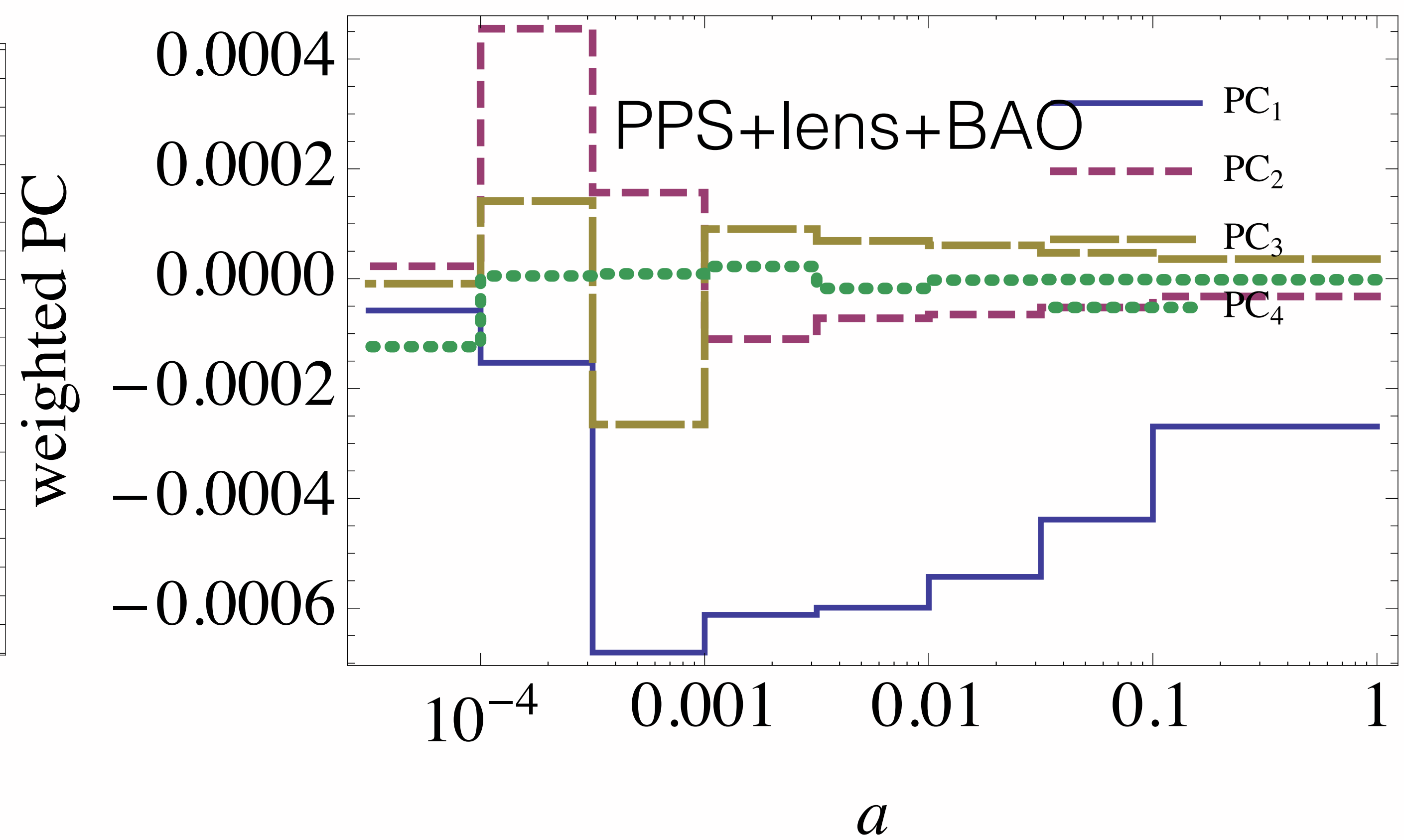
Neutrino masses

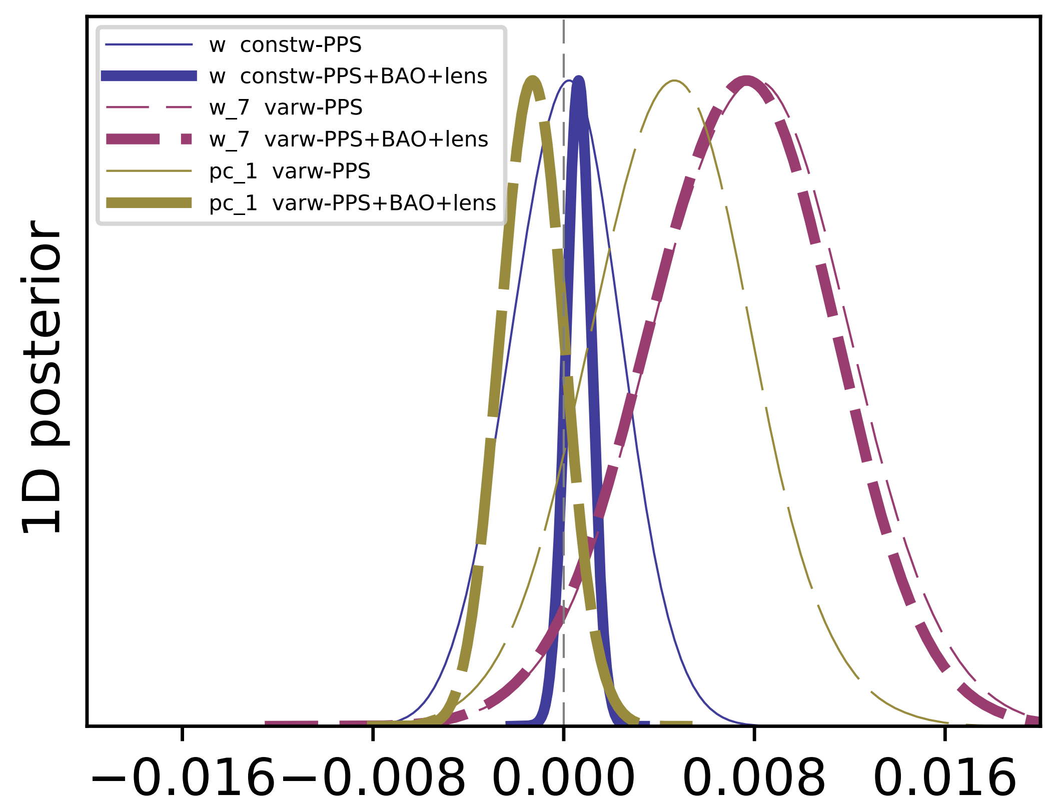
Based on a modified `class` code and
MCMC 7+3 params with `montepython`



PCA of w(a)







- Adding BAO changes strongest PC, PC_1 from recombination-supported to basically constant after recombination.
- Explains why constraints on const w improved so much when BAO was added, whereas the improvement on w-pixels was much weaker.

Evolution of Phi

Bardeen potentials

$$\hat{\Phi} \equiv \eta + \frac{1}{2}\mathcal{H}(\dot{\nu} + 2\zeta)$$

$$\hat{\Psi} \equiv \Psi - \frac{1}{2a}[a(\dot{\nu} + 2\zeta)]^{\cdot}$$

Comoving curvature perturbation

$$\mathcal{R} \equiv \hat{\Phi} + \frac{2}{3}\frac{\dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi}}{\mathcal{H}(1 + w_{\rm tot})}$$

velocity orthogonal

$$\hat{\Delta}_g \equiv \delta_g + 3(1+w)\mathcal{H}\theta_g$$

Newtonian

$$\hat{\Theta}_g \equiv \theta_g - \zeta - \tfrac{1}{2}\dot{\nu}$$

traceless Einstein eq

$$\hat{\Phi} - \hat{\Psi} = 8\pi G a^2 \bar{\rho}(1 + w_{\rm tot})\Sigma$$

trace Einstein eq

$$\dot{\hat{\Phi}} = \mathcal{H}(1 + w_{\rm tot})\left(-\left(\frac{3}{2} + \frac{1}{(1 + w_{\rm tot})}\right)\hat{\Phi} + \frac{3}{2}\mathcal{R} + 8\pi G\bar{\rho}a^2\Sigma\right)$$

$$\dot{\mathcal{R}} = \frac{2}{3\mathcal{H}(1 + w_{\rm tot})}\left(4\pi G\bar{\rho}a^2\Pi_{\rm nad,tot} - k^2 c^2_{a,\rm tot}\hat{\Phi}\right)$$

total non-adiabatic pressure for fluids without energy exchange + non-adiabatic pressure of GDM

$$\Pi_{\rm nad,tot} = \frac{1}{2\bar{\rho}^2(1 + w_{\rm tot})}\sum_{I,J}\bar{\rho}_I\bar{\rho}_J(1 + w_I)(1 + w_J)(c^2_{aI} - c^2_{aJ})\left(\frac{\delta_I}{1 + w_I} - \frac{\delta_J}{1 + w_J}\right) + \Pi_{\rm nad}$$

$$ds^2 = a^2\left\{-(1 + 2\Psi)d\tau^2 - 2\vec{\nabla}_i\zeta d\tau dx^i + \left[(1 + \frac{1}{3}h)\gamma_{ij} + D_{ij}\nu\right]dx^i dx^j\right\} = g_{\mu\nu}dx^\mu dx^\nu$$

PPF Details

- Already imposed gauge invariance and number of time derivatives

$$E_\Delta = 8\pi G a^2 \rho\delta + U_\Delta = 8\pi G a^2 \rho\delta + A_0\hat{\Phi} + F_0\hat{\Gamma} - \frac{3\mathcal{H}}{2}a^2\left(X+Y\right)\left(\dot{\nu}+2\zeta\right)$$

$$E_\Theta = 8\pi G a^2\,\rho(1+w)\theta + U_\Theta = 8\pi G a^2\,\rho(1+w)\theta + B_0\hat{\Phi} + I_0\hat{\Gamma} + \frac{1}{2}a^2(X+Y)(\dot{\nu}+2\zeta)$$

$$E_P = 24\pi G a^2\,\rho\Pi + U_P = 24\pi G a^2\,\rho\Pi + C_0\hat{\Phi} + C_1\dot{\hat{\Phi}} + J_0\hat{\Gamma} + J_1\dot{\hat{\Gamma}} + \frac{3}{2}a^2\dot{Y}(\dot{\nu}+2\zeta)$$

$$E_\Sigma = 8\pi G a^2\,\rho(1+w)\Sigma + U_\Sigma = 8\pi G a^2\,\rho(1+w)\Sigma + D_0\hat{\Phi} + D_1\dot{\hat{\Phi}} + K_0\hat{\Gamma} + K_1\dot{\hat{\Gamma}}$$

$$\hat{\Gamma} \equiv \dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi} = \dot{\eta} + \mathcal{H}\Psi + \frac{1}{2}(\dot{\mathcal{H}} - \mathcal{H}^2)(\dot{\nu}+2\zeta)$$

← combination has maximally one time derivative on perturbations

- Now impose dust background and Bianchi identity

$$U_\Delta = 2\mathcal{H}^2(P_0 - P_1)\hat{\Phi} - \tfrac{3}{2}X_0 a^{-1}\mathcal{H}(\dot{\nu}+2\zeta)$$

$$U_\Theta = -\tfrac{2}{3}\mathcal{H}P_0\hat{\Phi} + \tfrac{1}{2}X_0 a^{-1}(\dot{\nu}+2\zeta)$$

$U_P, U_\Sigma \ldots$ don't fit on this slide. Contain: $\dot{P}_0, \dot{P}_1$

PPF CDM mapping

We impose the equality ${T_{\rm cdm}}^{\mu}{}_{\nu} \overset{!}{=} U^{\mu}{}_{\nu}$ for background $8\pi G_{\rm N}\,\rho_{\rm cdm} \overset{!}{=} X$ and perturbations:

$$U_{\Delta} \overset{!}{=} 8\pi G a^2 \rho_{\rm cdm}\delta_{\rm cdm}$$

$$U_{\Theta} \overset{!}{=} 8\pi G a^2 \rho_{\rm cdm}\theta_{\rm cdm}$$

} solve for P_0, P_1

$$U_P \overset{!}{=} 0$$

$$U_{\Sigma} \overset{!}{=} 0$$

} evaluate $\dot{P}_0, \dot{P}_1$ and find that expressions vanish identically