The Toric SO(10) F-Theory Landscape

based on: [1709.06609]

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DESY Theory Workshop 2017

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DESY, Hamburg

September 28, 2017

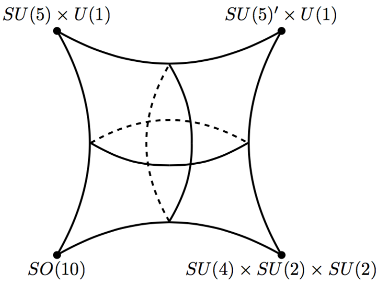
Outline

- **Motivation:**
  - SO(10)$\times$U(1) grand unified theory (GUT) in six dimensions
  - Supersymmetry broken by **Abelian flux** (D-term breaking)
  - Matter generations (charged **16**-plets) as fermionic zeromodes
  - Higgs (uncharged **10**-plets) in split multiplets
- **F-theory** construction of **global toric** SO(10) models
- **Base-independent** analysis
- **Scan** of possible realizations

Motivation: SO(10) GUT

Supersymmetric SO(10) GUT in 6d compactified on $T^2/\mathbb{Z}_2$
[Asaka, Buchmüller, Covi '01], [Buchmüller, MD, Ruehle, Schweizer '15]

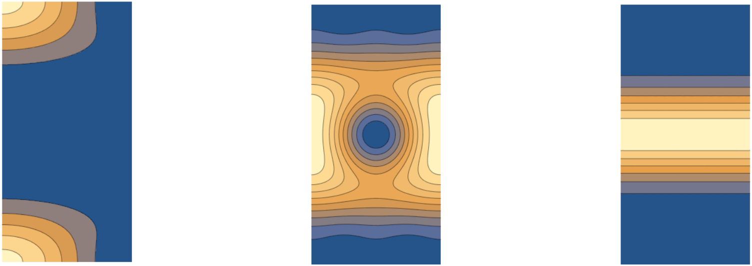
- Wilson line breaking: SO(10) $\rightarrow$ SU(3)$\times$SU(2)$\times$U(1)$\times$U(1)$_{B-L}$
- Full matter generation (and right-handed neutrino) in **16**-plet
- Higgs doublets in **10**-plets
- **Abelian flux background** (SUSY broken $M_{\text{SUSY}} \propto 1/L^2$)



Motivation: Matter generations

Index theorem dictates presence of **fermion zeromodes** for charged states (**16**-plets containing matter fields) [Bachas '95][Buchmüller, MD, Ruehle, Schweizer '15]

- **Multiplicity** $\leftrightarrow$ number of **flux quanta**
- **Localization** of field profiles (boundary conditions) [Cremades, Ibanez, Marchesano '04]
- **Bosonic** superpartners **heavy** $m^2 \sim 1/L^2$
- Excited states have "Landau level" masses $m_n^2 \propto n$



Motivation: Higgs fields

Uncharged states **not affected by flux at tree-level** (**10**-plets containing two EW Higgs doublets, **16**-plets containing fields breaking $U(1)_{B-L}$)

- **Tree-level supersymmetry**
- **Split multiplets** due to Wilson lines ⇒ solution to **doublet-triplet splitting**
- **Cancellation** of **quantum corrections** in charged states [Buchmüller, MD, Dudas, Schweizer '16]

Spectrum has to contain (3 generations):

≥ 2 neutral **10**-plets

≥ 2 neutral **16**-plets

charged **16**-plets	flux quanta
$\mathbf{16}_q$	3
$3 \times \mathbf{16}_q$	1
$\mathbf{16}_q, \mathbf{16}_{2q}$	1

Motivation: Anomalies

In **6d** there are **very strong anomaly constraints** ($\tilde{F}$: SO(10), F: U(1))

$$\mathcal{I}_8 = -\frac{1}{5760}(H - V + 29T - 273)\left(\mathrm{tr}R^4 + \tfrac{5}{4}(\mathrm{tr}R^2)^2\right) - \frac{1}{128}(9-T)(\mathrm{tr}R^2)^2$$
$$- \frac{1}{96}\mathrm{tr}R^2\left(\mathrm{Tr}\tilde{F}^2 - \sum_I n[\mathbf{R}_I]\mathrm{tr}_{\mathbf{R}_I}\tilde{F}^2\right) + \frac{1}{24}\left(\mathrm{Tr}\tilde{F}^4 - \sum_I n[\mathbf{R}_I]\mathrm{tr}_{\mathbf{R}_I}\tilde{F}^4\right)$$
$$+ \frac{1}{96}\sum_I \mathcal{M}_I q_I^2 \mathrm{tr}R^2 F^2 - \frac{1}{4}\sum_I n[\mathbf{R}_I] q_I^2 (\mathrm{tr}_{\mathbf{R}_I}\tilde{F}^2)F^2 - \frac{1}{24}\sum_I \mathcal{M}_I q_I^4 F^4$$

- Irreducible part has to vanish
- Reducible part cancelled by Green-Schwarz mechanism

Consistent for:

representation	multiplicity	representation	multiplicity
$\mathbf{16}_{-1}$	1	$\mathbf{1}_1$	80
$\mathbf{16}_0$	3	$\mathbf{1}_0$	86
$\mathbf{10}_0$	6		

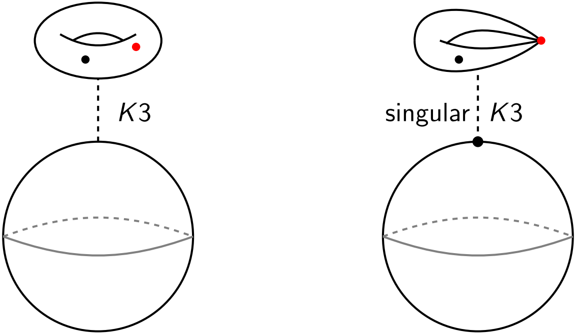
Motivation: Applications

The compactification in **Abelian background flux** has many interesting applications:

- **Split symmetries** (SUSY for charged, SO(10) for neutral sector)
  [Buchmüller, MD, Ruehle, Schweizer '15]
- Possibility for **de Sitter vacua** (with non-perturb. superpotential)
  [Buchmüller, MD, Ruehle, Schweizer '16]
- **Cancellation** of **quantum corrections** (gauge-Higgs-unification)
  [Buchmüller, MD, Dudas, Schweizer '16]

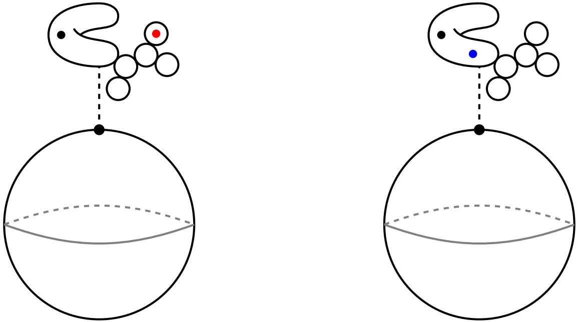
Realization in string theory?

F-Theory [Vafa '96]



- Non-perturbative type IIB string theory
- SL(2,$\mathbb{Z}$) invariance captured by additional torus
- Compactification on **elliptically fibered** CYs
- **Singularities** (in codim-1) indicate presence of (p,q)-branes

F-Theory: Gauge group

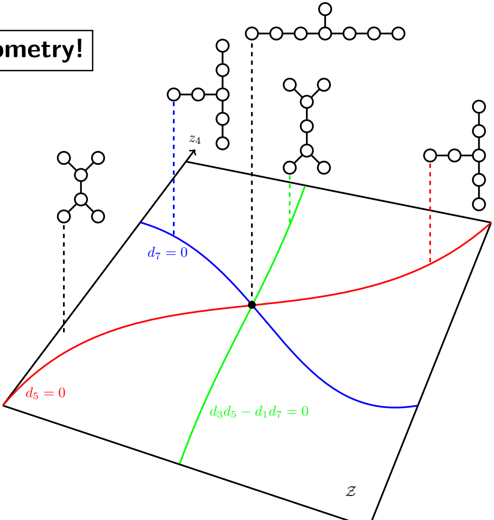


- **Resolution** of codim-1 singularities → **non-Abelian** gauge group
- Appearance of affine Dynkin diagrams as resolution divisors (Kodaira classification [Kodaira '63])
- **Abelian** gauge group given by (free part of) **Mordell-Weil** group
- Orthogonalization of Abelian and Cartan generators (Shioda map) [Morrison, Park '12]

F-theory: Matter and charges

- **Matter** located on **curves** over **codim-2 singularities** (CY_3)
- SO(10) representation as intersection with Cartan divisors
- U(1) charges as intersection with Shioda map

All geometry!



F-Theory: Toric construction

Nice description of $\mathbf{CY}_3$ as **hypersurfaces** in **toric** (ambient) **spaces**

$$\mathbb{A}_4 = (\mathbb{C}^{4+k} - Z)/(\mathbb{C}^*)^k$$

Cut out the CY_3 as **anticanonical hypersurface** in $\mathbb{A}_4$

Elliptically fibered $\mathbf{CY}_3$:

- T^2 as CY_1 in $\mathbb{A}_2$ (16 inequivalent ways to do this)
- Fiber $\mathbb{A}_2$ over 2-dim base B
- Make sure that CY condition is still fulfilled
- Often has generic non-Abelian gauge group
- Classification is available
  [Klevers, Pena, Oehlmann, Piragua, Reuter '14]

F-Theory: Base independence

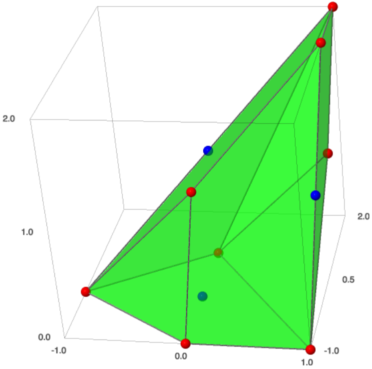
Once we use the toric description above:

- Use $\mathbb{C}^*$-scalings in $\mathbb{A}_2$ to **simplify the base dependence**
- Everything depends only on **two base divisor classes** $\mathcal{S}_7$ and $\mathcal{S}_9$ and the **anticanonical class** of the base K_B^{-1} [Cvetic, Klevers, Piragua '13]
- $\mathcal{S}_7$ and $\mathcal{S}_9$ are specified for chosen base as elements in $H_2(B, \mathbb{Z})$
- Allows **base-independent analysis** of:
  - **Charged matter** representations
  - Euler number $\Rightarrow$ **neutral matter**
  - **Anomaly** cancellation/coefficients

F-Theory: SO(10)

Engineering of **non-Abelian gauge group:**

- We know **intersection pattern** of resolution divisors
- can be **encoded** in the **toric ambient space** [Candelas, Font '96] [Perevalov, Skarke '97]
- Construction by "**Tops**" $\Rightarrow$ **new base divisor** $\mathcal{Z}$ (GUT-divisor)



- T^2 as polygon at $z = 0$
- Top at $z \geq 1$
- Dynkin diagram as intersection pattern of top
- Leads to resolved CY_3
- Singular limit $\rightarrow$ non-Abelian gauge group

Analysis and classification

Classification of all tops available [Bouchard, Skarke '03]

36 different SO(10) models in classification

We **analyzed** them **base-independently** (in terms of $\mathcal{Z}$, $\mathcal{S}_7$, $\mathcal{S}_9$, K_B^{-1}):
[Buchmüller, MD, Oehlmann, Ruehle '17]

- **Global gauge group** (fiber and top)
- **Non-Abelian matter** and **U(1) charges**
- **Charged singlets**
- **Euler number** and **neutral singlets**
- **Anomaly cancellation** and coefficients

A single model remains

We need:

- Neutral **16**- and **10**-plets (breaking of EW and $U(1)_{B-L}$)
- Charged **16**-plet(s) for matter generations

A single model satisfies these criteria: (F_3, top 4)

representation	multiplicity
$\mathbf{16}_0$	$(2K_B^{-1} - \mathcal{S}_9 - \mathcal{Z})\mathcal{Z}$
$\mathbf{10}_0$	$(\mathcal{S}_7 - \mathcal{Z})\mathcal{Z}$
$\mathbf{10}_1$	$(3K_B^{-1} - \mathcal{S}_7 - \mathcal{Z})\mathcal{Z}$
$\mathbf{16}_{-1}$	$\mathcal{S}_9\mathcal{Z}$
$\mathbf{1}_3$	$(K_B^{-1} - \mathcal{S}_7 + \mathcal{S}_9)\mathcal{S}_9$
$\mathbf{1}_2$	$6(K_B^{-1})^2 + \mathcal{S}_7^2 - 2\mathcal{S}_9^2 + \mathcal{Z}^2 - 4\mathcal{S}_9\mathcal{Z}$ $+K_B^{-1}(-5\mathcal{S}_7 + 4\mathcal{S}_9 - 5\mathcal{Z}) + 2\mathcal{S}_7(\mathcal{S}_9 + \mathcal{Z})$
$\mathbf{1}_1$	$12(K_B^{-1})^2 - 4\mathcal{S}_7^2 - \mathcal{S}_9^2 + 6\mathcal{Z}^2$ $+K_B^{-1}(8\mathcal{S}_7 - \mathcal{S}_9 - 22\mathcal{Z}) + \mathcal{S}_7(\mathcal{S}_9 + 2\mathcal{Z})$
$\mathbf{1}_0$	$18 + 11(K_B^{-1})^2 + 3\mathcal{S}_7^2 + 2\mathcal{S}_9^2 + 9\mathcal{Z}^2 + 4\mathcal{S}_9\mathcal{Z}$ $-K_B^{-1}(3\mathcal{S}_7 + 4\mathcal{S}_9 + 15\mathcal{Z}) - 4\mathcal{S}_7\mathcal{Z} - 2\mathcal{S}_7\mathcal{S}_9$

Scan through base spaces

To find our model **we demand**:

- **Lagrangian description** $\Rightarrow$ **single tensor field**

  Hirzebruch surfaces $\mathbb{F}_n$ [Morrison, Taylor '12]

- **Absence** of **non-Higgsable clusters**

  $n \in \{0, 1, 2\}$

- **Effectiveness of sections** (positive volume of relevant base divisors)
- Matter spectrum above

We find:

- **8 models** with $3 \times \mathbf{16}_q$
- **25 models** with $1 \times \mathbf{16}_q$
- **BUT: Not** the concrete model discussed above

Results and Outlook

- **Phenomenologically viable** SO(10) GUT models **realizable in F-theory** (non-perturbative string theory)
- **Not all** anomaly-free models can be realized **in toric geometry** (other consistency conditions?)
- **Complete, base-independent analysis** of SO(10) models
- **Many more** interesting aspects:
  - Superconformal matter points
  - Higgs and superconformal matter transition
  - Base-independent treatment