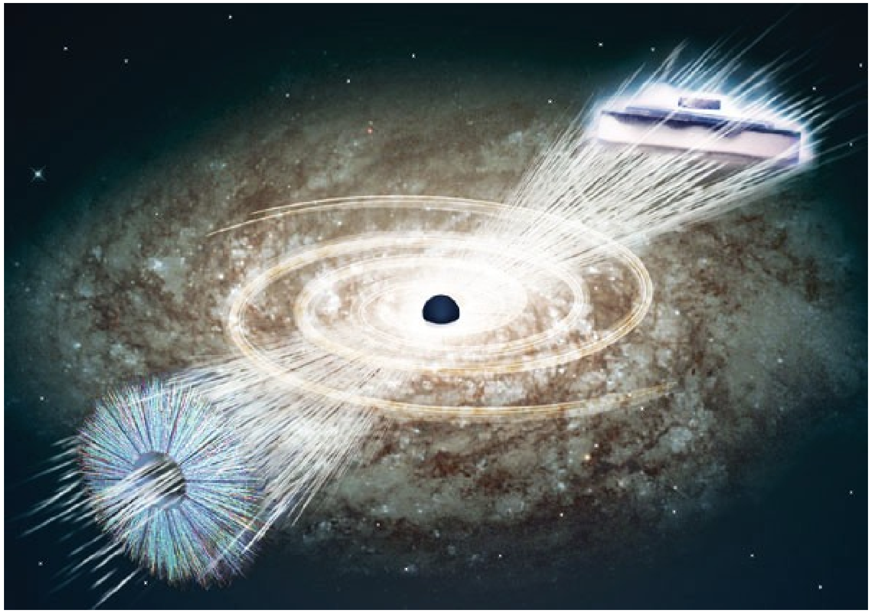
How general is holography?

Flat space limit and soft hairs in higher spin gravity



DESY Theory Workshop 2017

Martin Ammon

Friedrich Schiller Universität Jena
seit 1558

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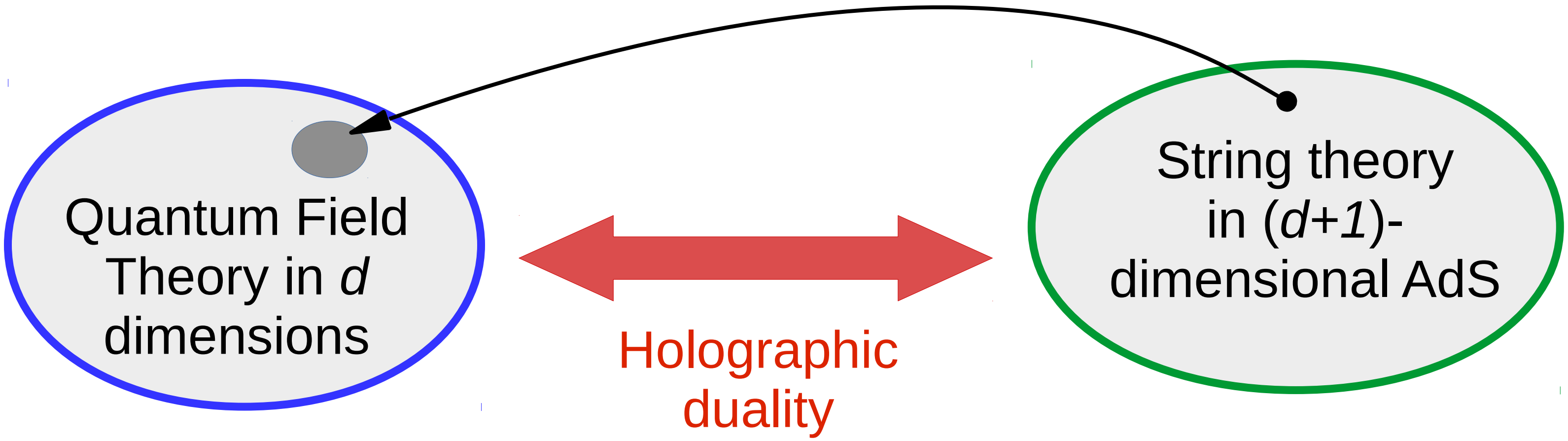
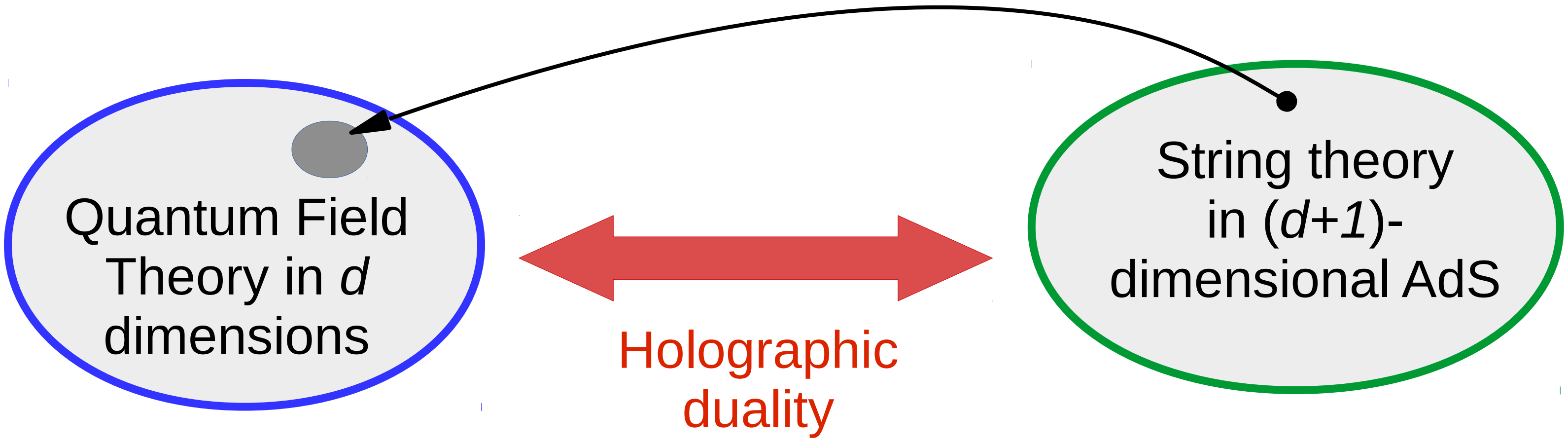
Applications of holography



Applications of holography



Applications of holography

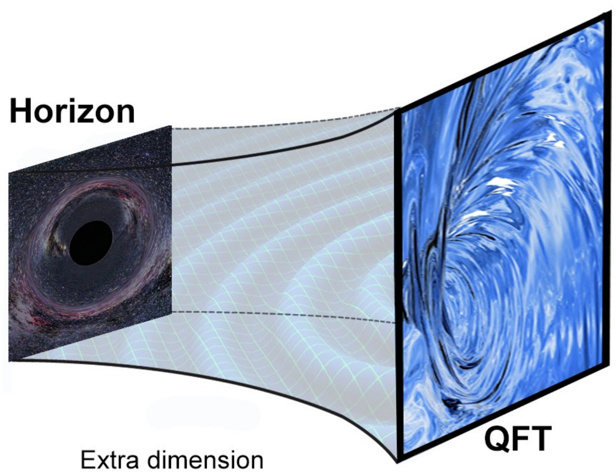
Study strongly coupled quantum field theories

Quantum Field Theory in *d* dimensions

Holographic duality

String theory in (*d*+*1*)-dimensional AdS

e.g. phase diagram, transport coefficients, non-equilibrium dynamics



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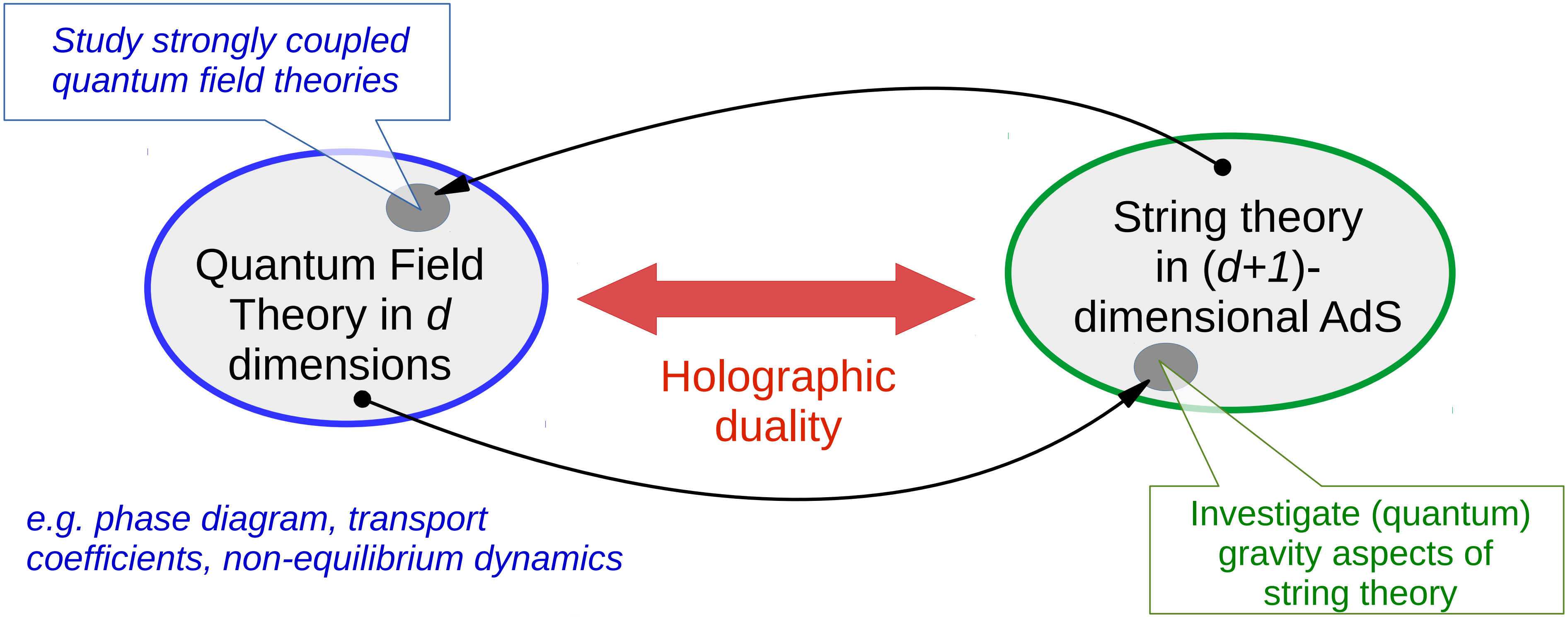
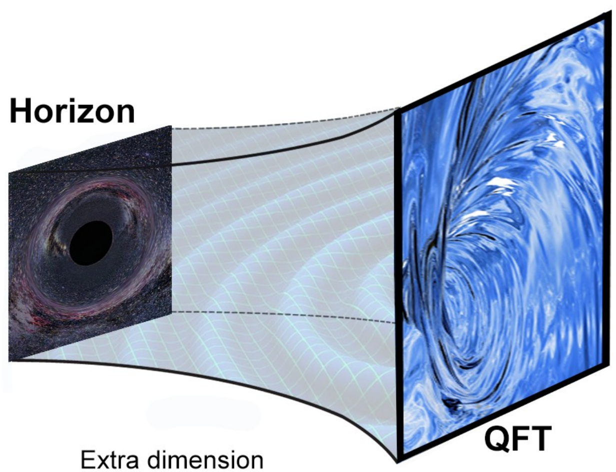
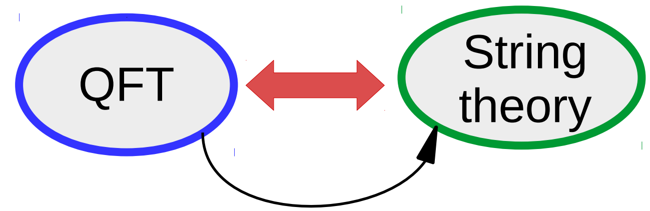




Investigate quantum gravity aspects of string theory



Insights into

– notion of background independence

– spacetime structure of string theory

– symmetries of string theory

– black holes in string theory

singularity resolution, firewalls, soft hairs

– resolution of black hole paradox

Long term goal:

use holography

What is the dual description of a *simple* QFT?

Simple = Free or Solvable ...

Higher Spin AdS/CFT Dualities

What is the dual description of a *simple* QFT?

(2+1)-dimensional
O(N) vector models

dual to

Higher spin gravity
in AdS4

(1+1)-dimensional
W_N minimal models

dual to

Higher spin gravity
in AdS3

Higher spin gravity (Vasiliev)

Infinite set of massless gauge fields with spin s=2,3,4,..., N, N+1, ...

+ scalar fields

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Why is higher spin gravity interesting?

- Dual description of "simple" QFTs
- Higher spin gravity may be sub sector of tension-less string theory

Massive string states get massless for $\alpha' \to \infty$

$$m^2 \sim \frac{n}{\alpha'} \to 0$$

New gauge symmetries present due to higher spin fields

→ Hidden symmetries within string theory?

Higher Spin Gravity in 3D AdS spacetimes

Interacting higher spin gravity in AdS possible:

Higher spin gravity (Vasiliev)

Infinite set of massless gauge fields with spin s=2,3,4,..., N, N+1, ...

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Higher Spin Gravity in 3D AdS spacetimes

Interacting higher spin gravity in AdS possible:

Higher spin gravity (Vasiliev)

Infinite set of massless gauge fields with spin s=2,3,4,..., N, ~~N+1, ...~~

~~+ scalar fields~~

- In 3D: truncation to finitely many higher spin fields possible
- Formulation in terms of Chern – Simons theory

Spin-Two Gravity as Chern – Simons theory

Action with *negative* cosmological constant

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}\left(R + \frac{2}{L^2}\right)$$

Metric may be written in terms of a Vielbein $g_{\mu\nu} = e_\mu^a \, e_\nu^b \, \eta_{ab}$

(Dual) spin connection $\omega_\mu^a = \frac{1}{2}\epsilon^{abc}\,\omega_{\mu bc}$

- Package into gauge fields $A, \bar{A} \in sl(2,\mathbb{R})$

$$A_\mu = (\omega_\mu^a + e_\mu^a)\,L_a\,, \qquad \overline{A}_\mu = (\omega_\mu^a - e_\mu^a)\,L_a$$

- L_a are the generators of $sl(2,\mathbb{R})$

$$[L_a, L_b] = (a-b)L_{a+b}\,, \qquad \mathrm{Tr}(L_a L_b) = \frac{1}{2}\eta_{ab}$$

Spin-Two Gravity as Chern – Simons theory

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Spin-Two Gravity as Chern – Simons theory II

Following actions are classically equivalent: [Witten, '88]

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}\left(R + \frac{2}{L^2}\right)$$

$$S = S_{CS}[A] - S_{CS}[\overline{A}] \qquad S_{CS}[A] = \frac{k}{4\pi}\int \mathrm{Tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$$

where $A, \bar{A} \in sl(2,\mathbb{R})$ $\qquad A = (\omega_\mu + e_\mu)\; dx^\mu\,, \qquad \overline{A} = (\omega_\mu - e_\mu)\, dx^\mu$

Metric $\quad g_{\mu\nu} = \frac{1}{2}\mathrm{Tr}(e_\mu\, e_\nu)$

Equations of motion

$$F = dA + A\wedge A = 0\,, \qquad \overline{F} = d\overline{A} + \overline{A}\wedge\overline{A} = 0$$

Higher Spin Gravity as Chern – Simons theory
(spin-two and spin-three fields)

Following actions are classically equivalent:

?????

$$S = S_{CS}[A] - S_{CS}[\overline{A}] \qquad S_{CS}[A] = \frac{k}{4\pi}\int \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

where $A, \bar{A} \in sl(3, \mathbb{R})$ $\qquad A = (\omega_\mu + e_\mu)\, dx^\mu\,, \qquad \overline{A} = (\omega_\mu - e_\mu)\, dx^\mu$

Metric $g_{\mu\nu} = \frac{1}{2}\mathrm{Tr}(e_\mu\, e_\nu)$ $\qquad$ Spin-3 field $\varphi_{\mu\nu\rho} = \frac{1}{6}\mathrm{Tr}(e_{(\mu} e_\nu e_{\rho)})$

Equations of motion

$$F = dA + A \wedge A = 0\,, \qquad \overline{F} = d\overline{A} + \overline{A} \wedge \overline{A} = 0$$

Higher Spin Gravity as Chern – Simons theory II

Metric-like formalism:

action only known to quadratic order in spin-3 field [Fredenhagen, '14,'15]

Generalizations with more higher spin fields possible

field cont.	gauge algebra	asymptotic symmetry algebra
$s = 2$	$sl(2, R) \oplus sl(2, R)$	$Vir \oplus Vir$
$s = 2, 3$	$sl(3, R) \oplus sl(3, R)$	$W_3 \oplus W_3$
$s = 2, \ldots, N$	$sl(N, R) \oplus sl(N, R)$	$W_N \oplus W_N$
$s = 2, \ldots, \infty$	$hs(\lambda) \oplus hs(\lambda)$	$W_\infty(\lambda) \oplus W_\infty(\lambda)$

[Brown, Henneaux '86;
Campeleoni, Fredenhagen, Pfenninger, Theisen, '10, '11
Henneaux, Rey, '10]

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$s = 2, \ldots, \infty$	$hs(\lambda) \oplus hs(\lambda)$	$W_\infty(\lambda) \oplus W_\infty(\lambda)$

Examples of extended symmetries for higher spin gravity

Novel black hole solutions

Generalization of BTZ black hole carrying higher spin charge

Black holes within higher spin gravity

Novel black hole solutions

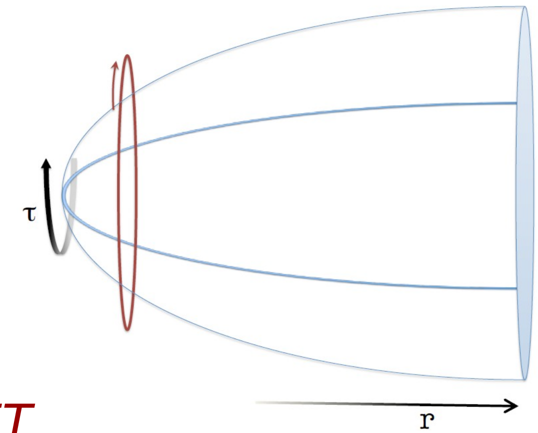
[MA, Gutperle,Kraus, Perlmutter, '11]

Generalization of BTZ black hole carrying higher spin charge

Gauge-invariant characterization

Trivial holonomies

$$\mathcal{P}\exp\left(\oint A_\tau d\tau\right)$$



Partition and correlation functions agree with CFT

[Kraus, Perlmutter, '11; Gaberdiel et al '12,'13]

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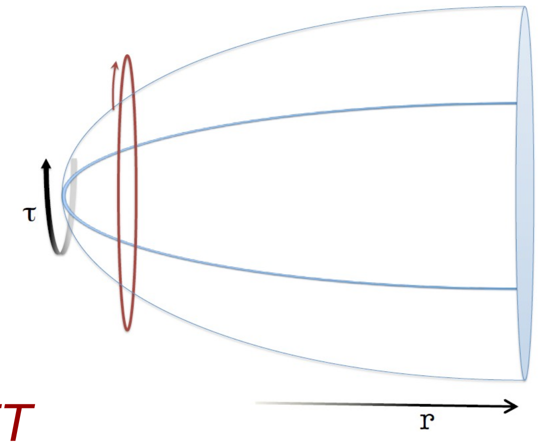

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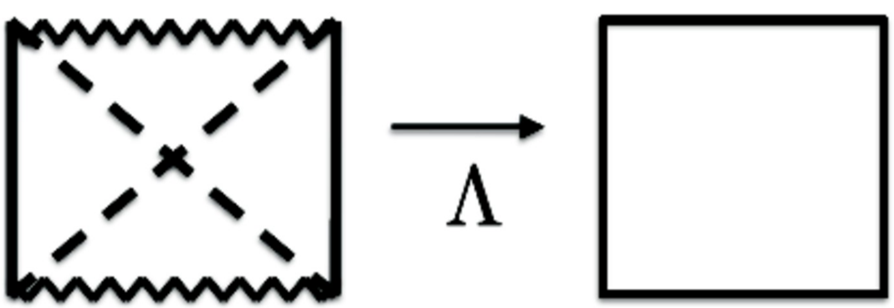
Causal structure not gauge-invariant



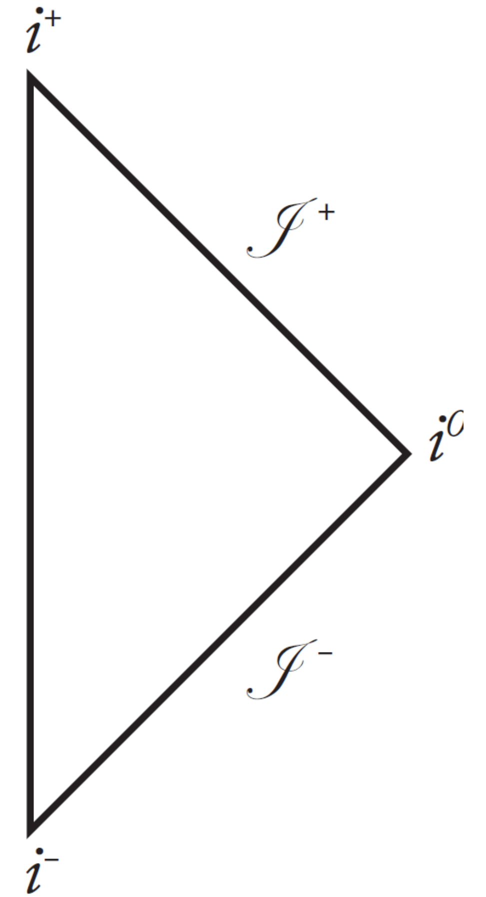
Notion of spacetime?

→ Entanglement entropy

Flat space limit of higher spin gravity

So far: Higher Spin Gravity in AdS / CFT

Similar duality for asymptotically flat spacetimes?



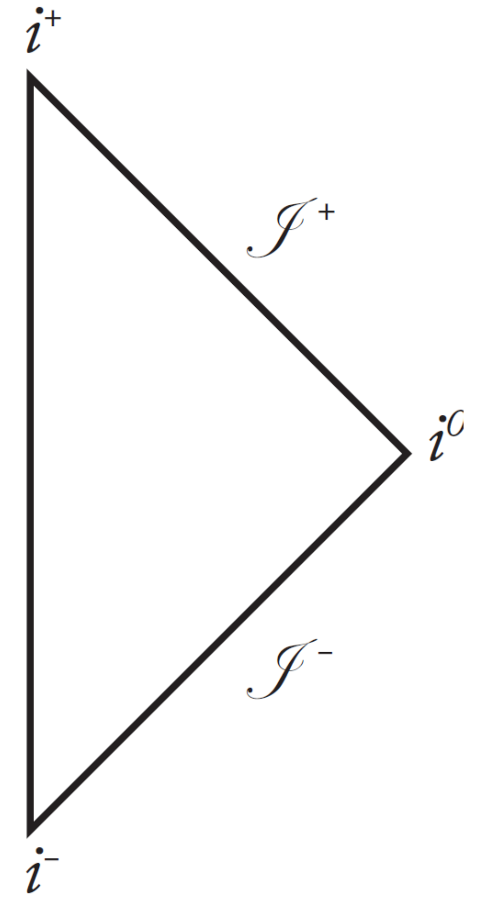
Flat space limit of higher spin gravity

So far: Higher Spin Gravity in AdS / CFT

Similar duality for asymptotically flat spacetimes?

- Naïve approach: take limit $L \to \infty$

  cosmological constant $\Lambda = -\frac{1}{L^2} \to 0$



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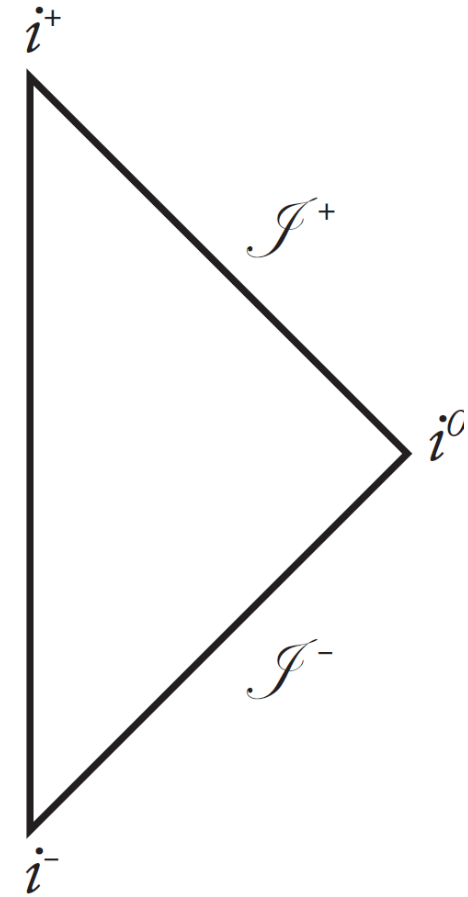
Similar duality for asymptotically flat spacetimes?

- Naïve approach: take limit $L \to \infty$

  cosmological constant $\Lambda = -\frac{1}{L^2} \to 0$

- Works straightforwardly *sometimes*, otherwise fails

  No No-Go-Theorems for Topological sector of Higher Spin gravity



Higher spin gravity (Vasiliev)

Infinite set of massless gauge fields with spin s=2,3,4,..., N, N+1, ...

+ ~~scalar fields~~

Flat space limit of higher spin gravity II

Does the naïve limit from AdS to flat spacetime work?

- Contraction of asymptotic symmetry algebra works!

Take linear combination of Virasoro generators

$$L_n = \mathcal{L}_n - \mathcal{L}_{-n} \qquad M_n = \frac{1}{L}\left(\mathcal{L}_n + \mathcal{L}_{-n}\right)$$

→ Wigner-Inönü contraction gives BMS algebra (or GCA, URCA)

$$\begin{aligned}
[L_n, L_m] &= (n-m)L_{n+m} + \frac{c_L}{12}n(n^2-1)\delta_{n+m,0} \\
[L_n, M_m] &= (n-m)M_{n+m} + \frac{c_M}{12}n(n^2-1)\delta_{n+m,0} \\
[M_n, M_m] &= 0
\end{aligned}$$

Flat space limit of higher spin gravity III

Does the naïve limit from AdS to flat spacetime work?

- Chern – Simons formulation

$S_{CS}[\mathcal{A}]$ for connection $\mathcal{A} \in isl(2)$ for spin-2 case and $\mathcal{A} \in isl(3)$ for spin-3

in particular $\mathcal{A} = e^a T_a + \omega^a J_a$ $\quad T_a = (M_n, V_m) \quad J_a = (L_n, U_m)$

Flat space limit of higher spin gravity III

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in particular $\mathcal{A} = e^a T_a + \omega^a J_a \qquad T_a = (M_n, V_m) \quad J_a = (L_n, U_m)$

Boundary conditions $\mathcal{A} = b^{-1}(r)\,(a(v,\phi) + d)b(r) \qquad b(r) = \exp\left(\frac{r}{2}M_{-1}\right)$

with $$\mathrm{a}_\phi = L_1 - \frac{\mathcal{M}(\phi)}{4}L_{-1} - \frac{\mathcal{N}(\phi)}{2}M_{-1} + \frac{\mathcal{V}(\phi)}{2}U_{-2} + \mathcal{Z}(\phi)V_{-2}$$

$$\mathrm{a}_v = M_1 - \frac{\mathcal{M}(\phi)}{4}M_{-1} + \frac{\mathcal{V}(\phi)}{2}V_{-2} + \dots$$

- Spin-2 and spin-3 charges $\mathcal{Q} = \frac{k}{2\pi}\int d\phi\left(\epsilon\mathcal{N} + \frac{\tau}{2}\mathcal{M} + 8\alpha\mathcal{Z} + 4\kappa\mathcal{V}\right)$

giving rise to novel W-algebra extending BMS

[Gonzalez, Matulich, Pino, Troncoso, '13]

Flat space cosmological solutions

Analogue of Hawking-Page transition:

two different solutions with the same boundary conditions

- Hot flat spacetime (HFS)

$$ds^2 = d\tau^2 + dr^2 + r^2 d\phi^2$$

- Flat space cosmology (FSC)

$$ds^2 = \left(1 - \frac{r_0^2}{r^2}\right) d\tau^2 + \frac{r^2}{r^2 - r_0^2} dr^2 + r^2 \left(d\phi - \frac{r_0}{r^2} d\tau\right)^2$$

- Obey: same boundary conditions, no conical singularities, same temperature $T = \beta^{-1}$ and angular momentum Ω

$$(\tau, \phi) \sim (\tau, \phi + 2\pi) \sim (\tau + \beta, \phi + \beta\Omega)$$

Flat space cosmological solutions II

- Thermodynamics in the semi-classical limit:

  For $T > T_c$ with $T_c = \Omega/2\pi$ FSC thermodynamically preferred

- Satisfies first law

- Entropy of flat space cosmologies

$$S_{\mathrm{Th}} = 2\pi k \frac{N\left(2\mathcal{R} - 6 + 3\mathcal{P}\sqrt{\mathcal{R}}\right)}{8\sqrt{M}(\mathcal{R}-3)\sqrt{1-\frac{3}{4\mathcal{R}}}},$$

with
$$\frac{V}{2M^{\frac{3}{2}}} = \frac{\mathcal{R}-1}{\mathcal{R}^{\frac{3}{2}}}, \qquad \frac{Z}{N\sqrt{M}} = \mathcal{P}$$

Near horizon geometry and asymptotic symmetry

[MA, Grumiller, Prohazka, Riegler, Wutte, '17]

Near horizon geometry of the FSC is given by

$$\mathcal{A} = b^{-1}(a+d)b, \qquad b = \exp\left(\frac{1}{\mu_P}\,M_1\right)\,e^{\frac{r}{2}M_{-1}} \qquad \text{with} \qquad \mathrm{a} = \mathrm{a}_v dv + a_\phi d\phi$$

and

$$a_\phi = \mathcal{J}\,L_0 + \mathcal{P}\,M_0 + \mathcal{J}^{(3)}\,U_0 + \mathcal{P}^{(3)}\,V_0,$$

$$a_v = \mu_{\mathcal{P}}\,L_0 + \mu_{\mathcal{J}}\,M_0 + \mu_{\mathcal{P}}^{(3)}\,U_0 + \mu_{\mathcal{J}}^{(3)}\,V_0.$$

- Asymptotic symmetry algebra: four copies of u(1) Kac-Moody algebra

$$[J_n, P_m] = k\,n\,\delta_{n+m,0}, \qquad [J_n^{(3)}, P_m^{(3)}] = \frac{4k}{3}\,n\,\delta_{n+m,0}.$$

- “Vacuum definition”

$$J_n|0\rangle = P_n|0\rangle = J_n^{(3)}|0\rangle = P_n^{(3)}|0\rangle = 0, \qquad \forall\, n \geq 0.$$

Near horizon geometry and asymptotic symmetry II

- “Soft hairs” may form a highest weight representations

$$|\psi(\{n_i, n_i^{(3)}, m_i, m_i^{(3)}\})\rangle \propto \prod_{n_i>0} J_{-n_i} \prod_{n_i^{(3)}>0} J^{(3)}_{-n_i^{(3)}} \prod_{m_i>0} P_{-m_i} \prod_{m_i^{(3)}>0} P^{(3)}_{-m_i^{(3)}} |0\rangle$$

“Soft hairs” commutate with H

$$H = \left(\mu_{\mathcal{J}} J_0 + \mu_{\mathcal{P}} P_0 + \frac{4}{3} \mu_{\mathcal{J}}^{(3)} J_0^{(3)} + \frac{4}{3} \mu_{\mathcal{P}}^{(3)} P_0^{(3)} \right)$$

and hence have the same energy as the vacuum!

Near horizon geometry and asymptotic symmetry III

[MA, Grumiller, Prohazka, Riegler, Wutte, '17]

- We can relate both asymptotic symmetry algebras:

Higher Spin BMS algebra ⟷ Four U(1) Kac Moody algebras

$$L_n = \frac{1}{k} \sum_{p \in \mathbb{Z}} \left(J_{n-p} P_p + \frac{3}{4} J^{(3)}_{n-p} P^{(3)}_p \right) - inP_n$$

(Higher spin) BMS algebra is composite with respect to near horizon ASA

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(Higher spin) BMS algebra is composite with respect to near horizon ASA

- Higher Spin entropy in terms of BMS generators simplifies

$$S_{\mathrm{Th}} = 2\pi k\frac{N\left(2\mathcal{R}-6+3\mathcal{P}\sqrt{\mathcal{R}}\right)}{8\sqrt{M}(\mathcal{R}-3)\sqrt{1-\frac{3}{4\mathcal{R}}}}$$

$$S_{\mathrm{Th}} = 2\pi P_0$$

Conclusions & Summary

- Investigate quantum gravity aspects of holography / string theory

- Higher spin gravity a convenient playground

  Insights into symmetries and spacetime notion of string theory (tensionless limit)

- Flat space limit in subsector of higher spin gravity possible

  → *higher spin flat space cosmologies*

  → *soft hair proposal and near horizon asymptotic algebra*

  → *BMS3 is composite*

How general is holography?

Appendix: Flat space cosmological solutions

Black holes do not exist in 3d flat spacetimes!

instead flat space cosmological solutions

BTZ in AdS

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2\, L^2}\, dt^2 + \frac{r^2\, L^2\, dr^2}{(r^2 - r_+^2)(r^2 - r_-^2)} + r^2 \left(d\varphi - \frac{r_+ r_-}{L^2\, r^2} dt\right)^2$$

Consider region between the two horizons $r_- < r < r_+$ and take $L \to \infty$

Keep $\hat{r}_+$ fixed with $r_+ = L\hat{r}_+$

Result in Euclidean signature

$$ds^2 = r_+^2 \left(1 - \frac{r_0^2}{r^2}\right) d\tau^2 + \frac{r^2\, dr^2}{r_+^2\, (r^2 - r_0^2)} + r^2 \left(d\varphi - \frac{r_+\, r_0}{r^2} d\tau\right)^2$$

Note: *no conical singularity*, but asymptotic conical defect!