

DESY Theory Workshop 2017

Primordial Black Holes from Critical Higgs Inflation

Jose María EZQUIAGA

Based on:

arXiv 1705.04861 by JME, J. GARCÍA-BELLIDO and E. RUIZ MORALES



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HIGGS $\longrightarrow$ INFLATION

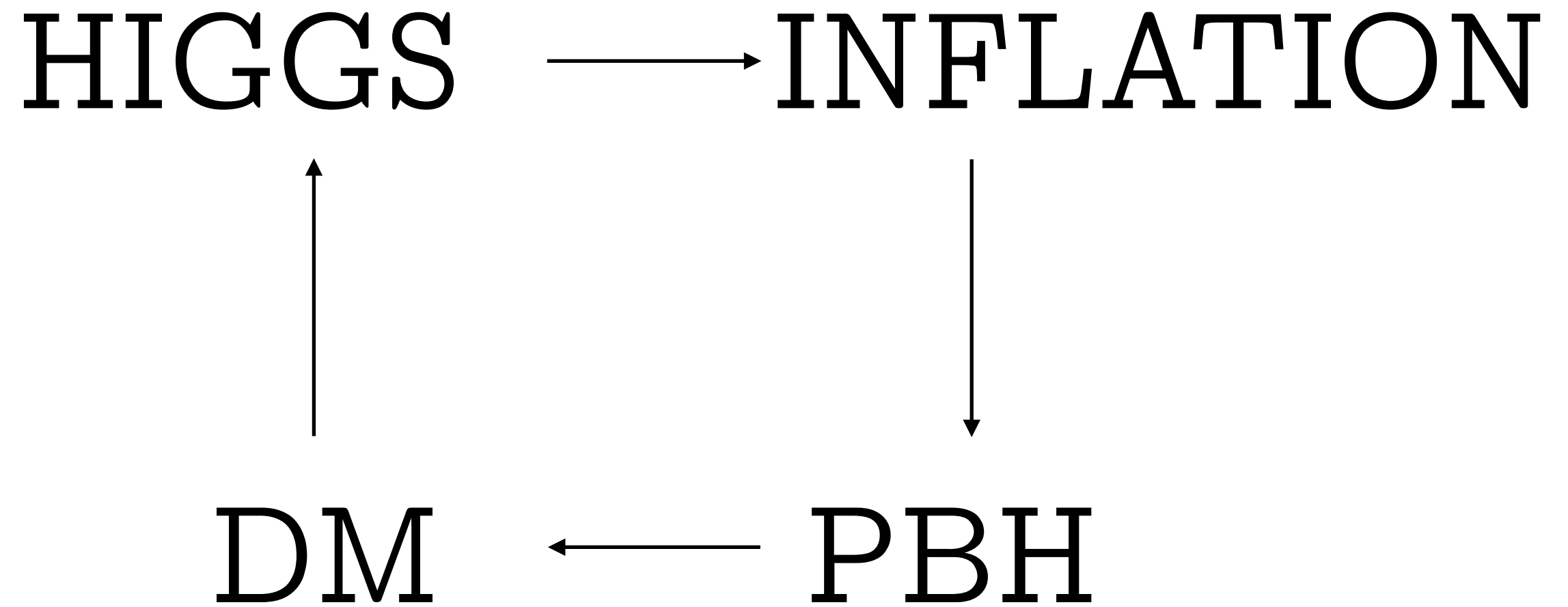
HIGGS $\longrightarrow$ INFLATION

DM $\longleftarrow$ PBH

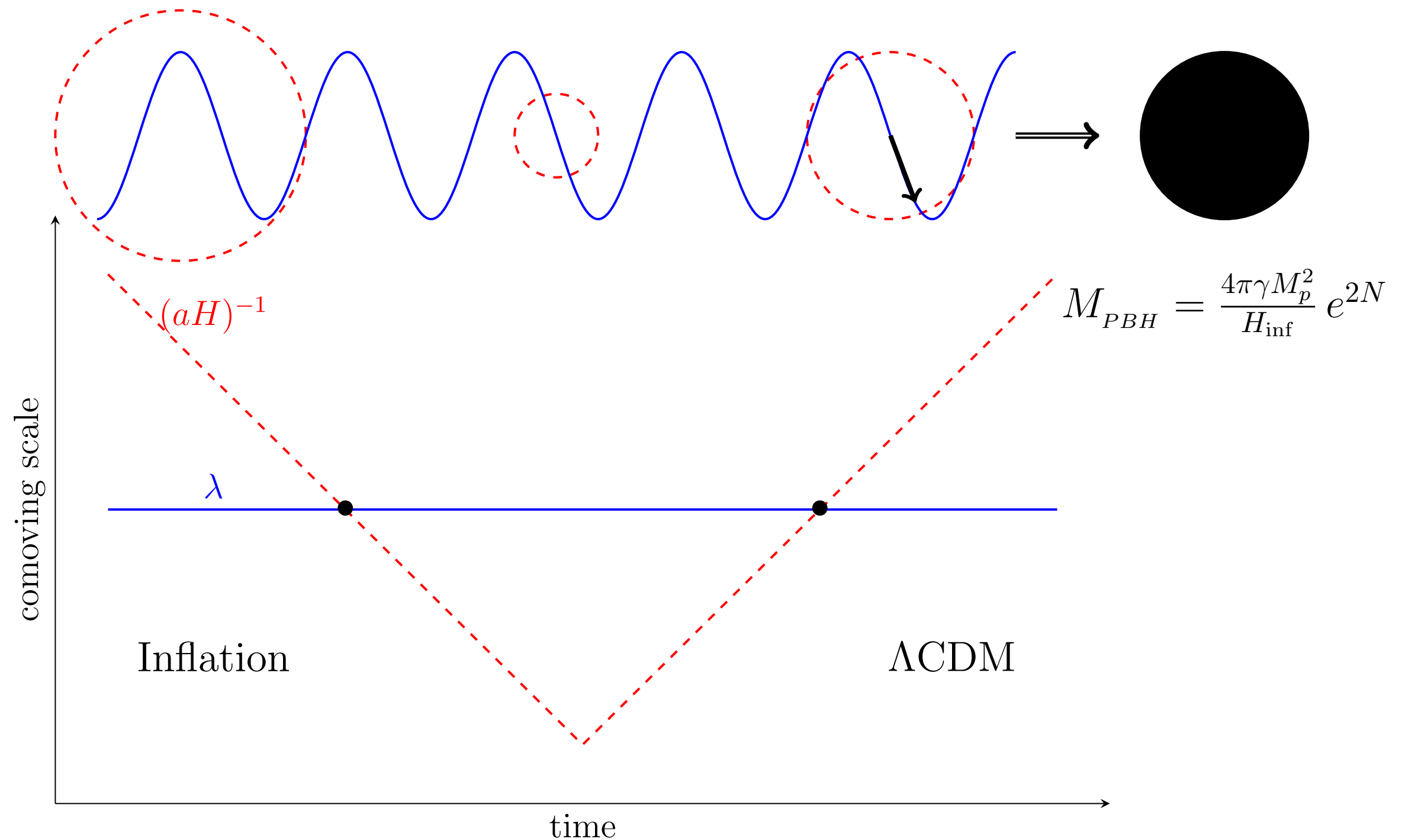
HIGGS → INFLATION



DM ← PBH

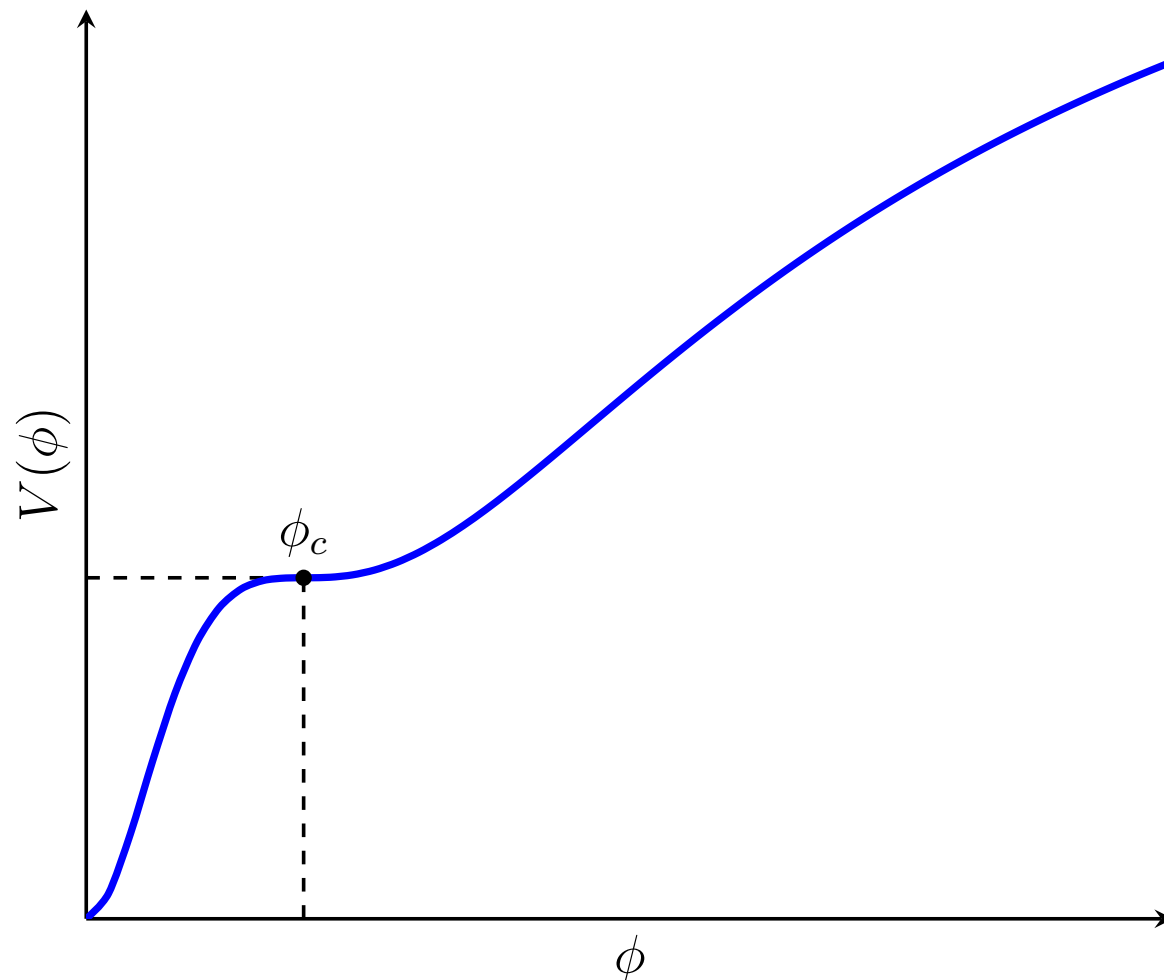


- PBHs form when large fluctuations reenter horizon



- PBH from Single-field Inflation

[García-Bellido and Ruiz Morales 2017]



- Large fluctuations are produced if there is a second plateau

$$\mathcal{P}_{\mathcal{R}}(N) \simeq \frac{\kappa^2}{2\epsilon} \left(\frac{H}{2\pi} \right)^2 \xrightarrow{\epsilon \rightarrow 0} \frac{1}{\epsilon}$$

- This must be a near-inflection point

$$V' \simeq V'' \simeq 0$$

- The difficulty here is to have consistent CMB observable at the same time that a copious production of PBH

Could the SM Higgs produce PBHs?

Higgs Inflation

[Bezrukov and Shaposhnikov 2008]

- The Higgs has a non-minimal coupling to gravity

$$S = \int d^4x \sqrt{-g} \left(\left(\frac{1}{2\kappa^2} + \frac{\xi}{2} \phi^2 \right) R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4} \lambda \phi^4 \right)$$

Higgs Inflation

[Bezrukov and Shaposhnikov 2008]

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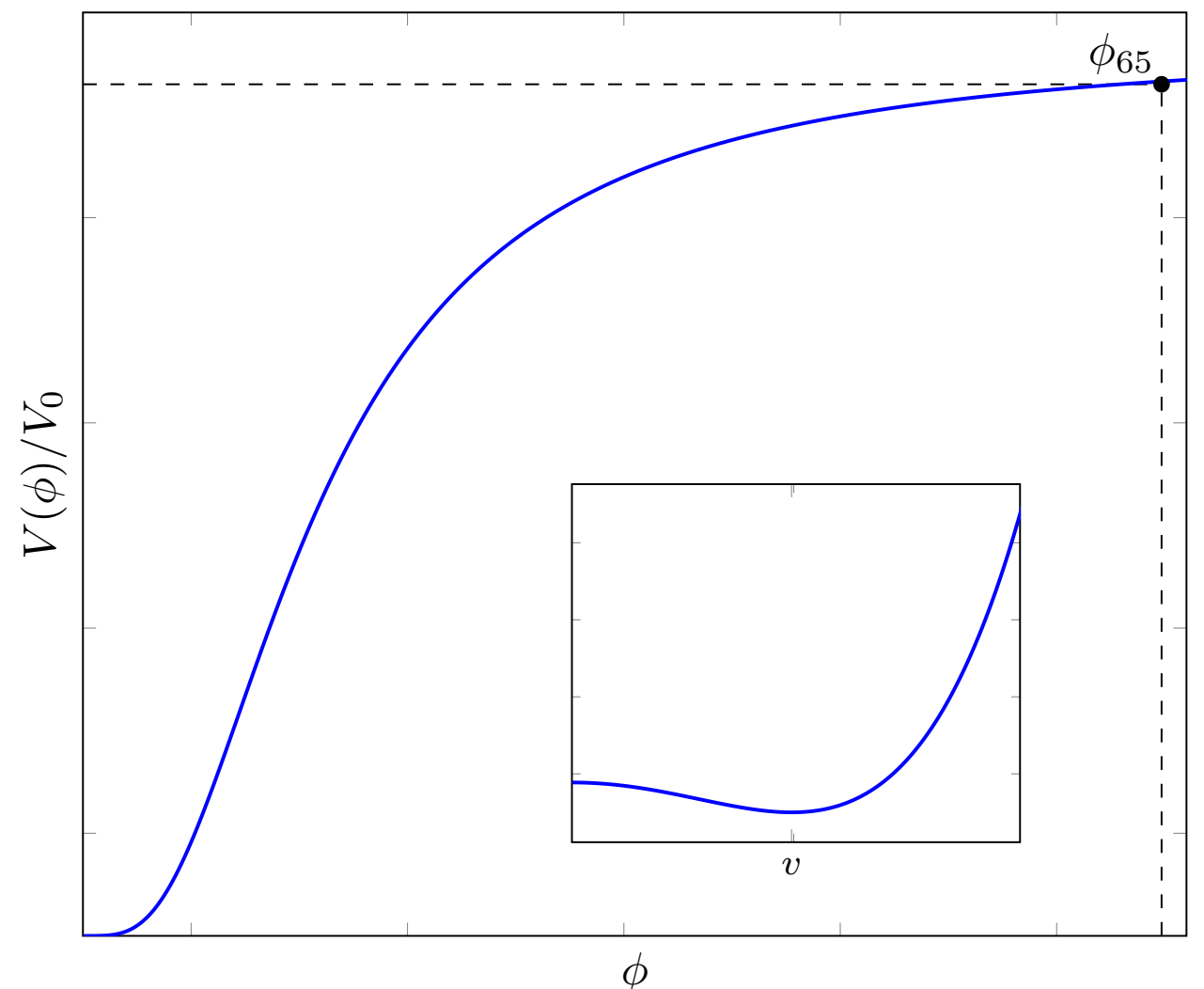
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- After a field redefinition

$$\Omega^2 = 1 + \xi \kappa^2 \phi^2$$

$$\frac{d\varphi}{d\phi} = \frac{\sqrt{1 + \xi(1 + 6\xi)\phi^2}}{1 + \xi\phi^2}$$

$$V(\phi) = \frac{\lambda \phi^4}{4(1 + \xi \phi^2)^2}$$



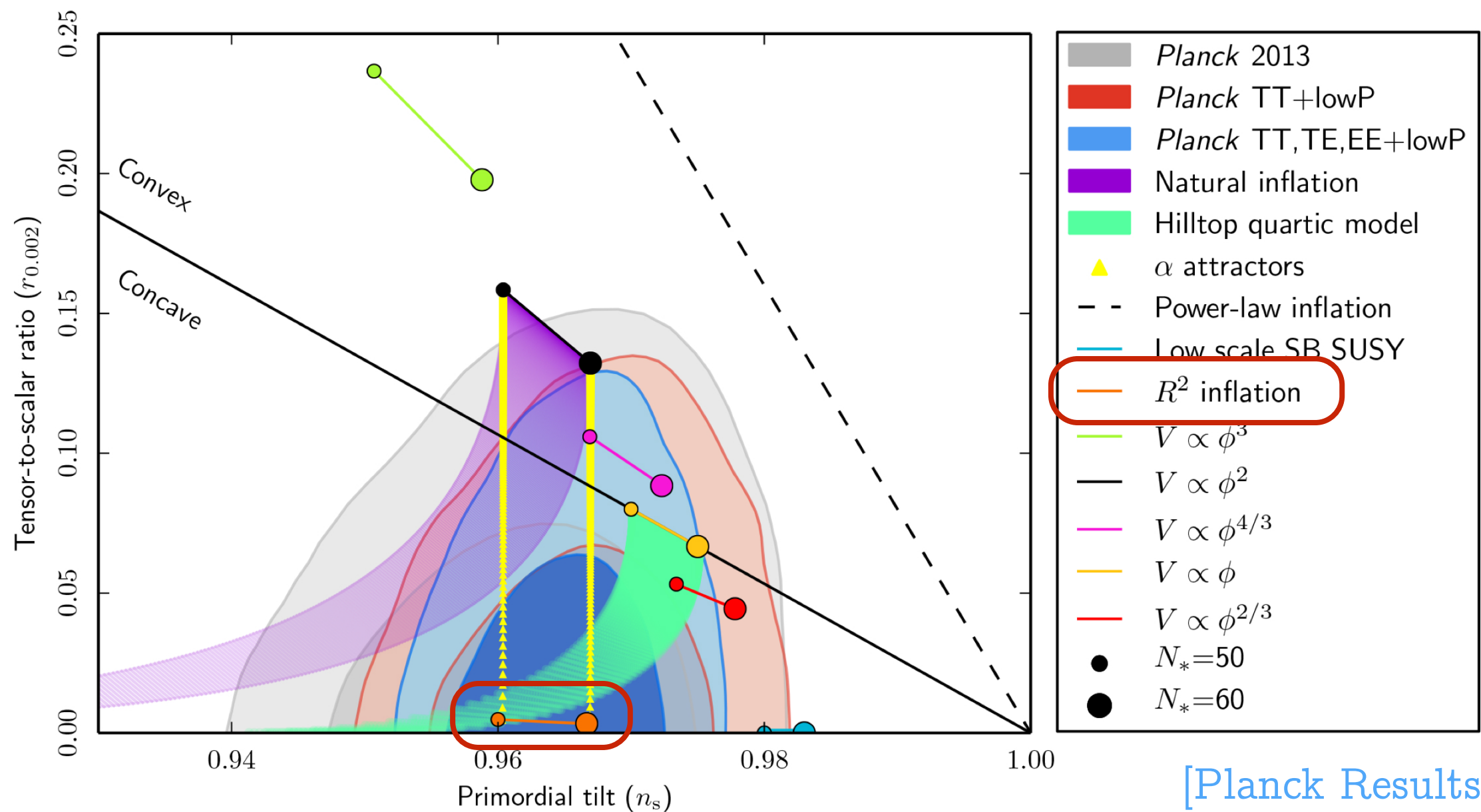
i) CMB determined by plateau

ii) Amplitude fixed by $\lambda \simeq 0.13$, $\xi \sim 10^5$

$$n_s = 1 - \frac{2}{N}$$

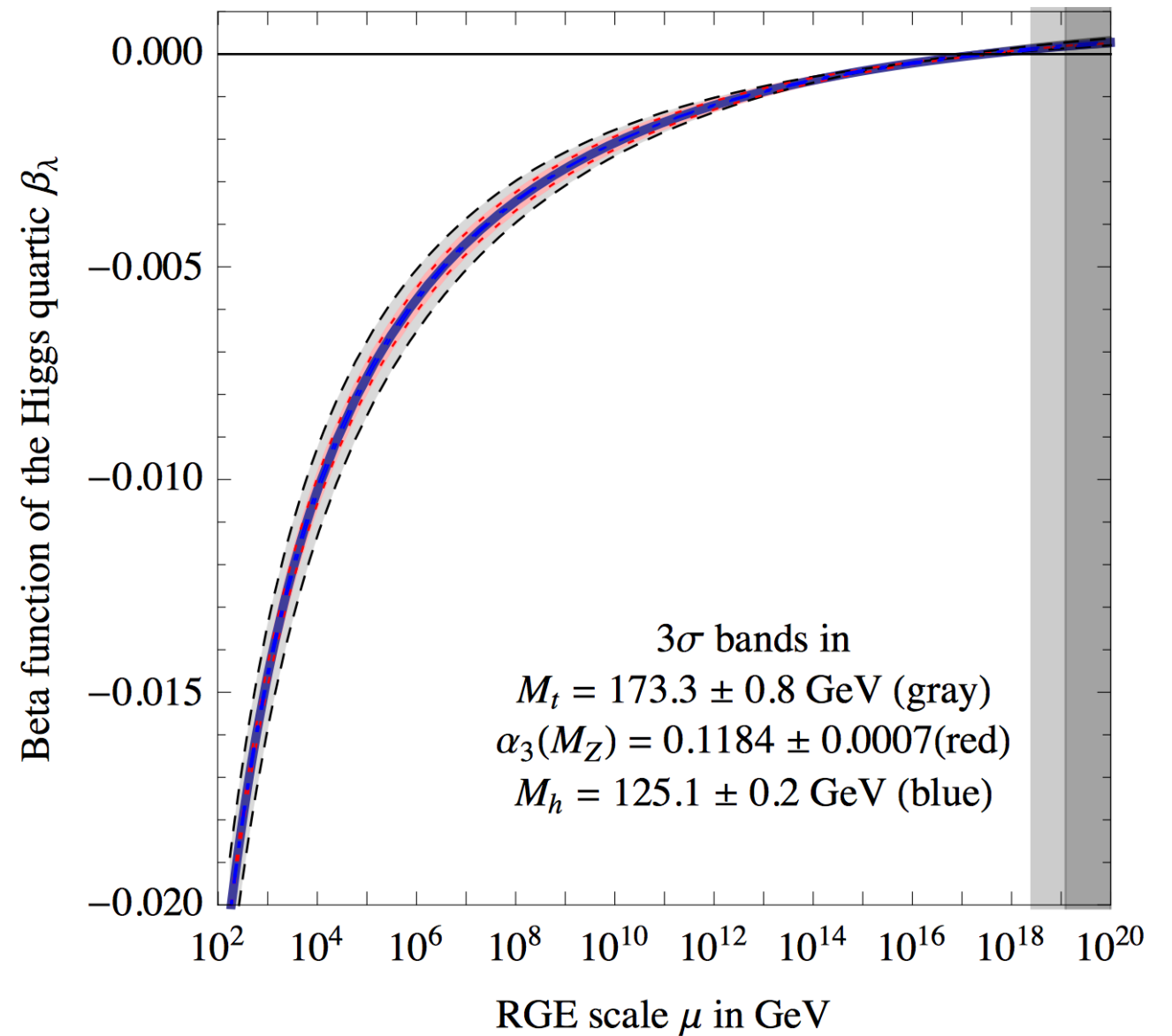
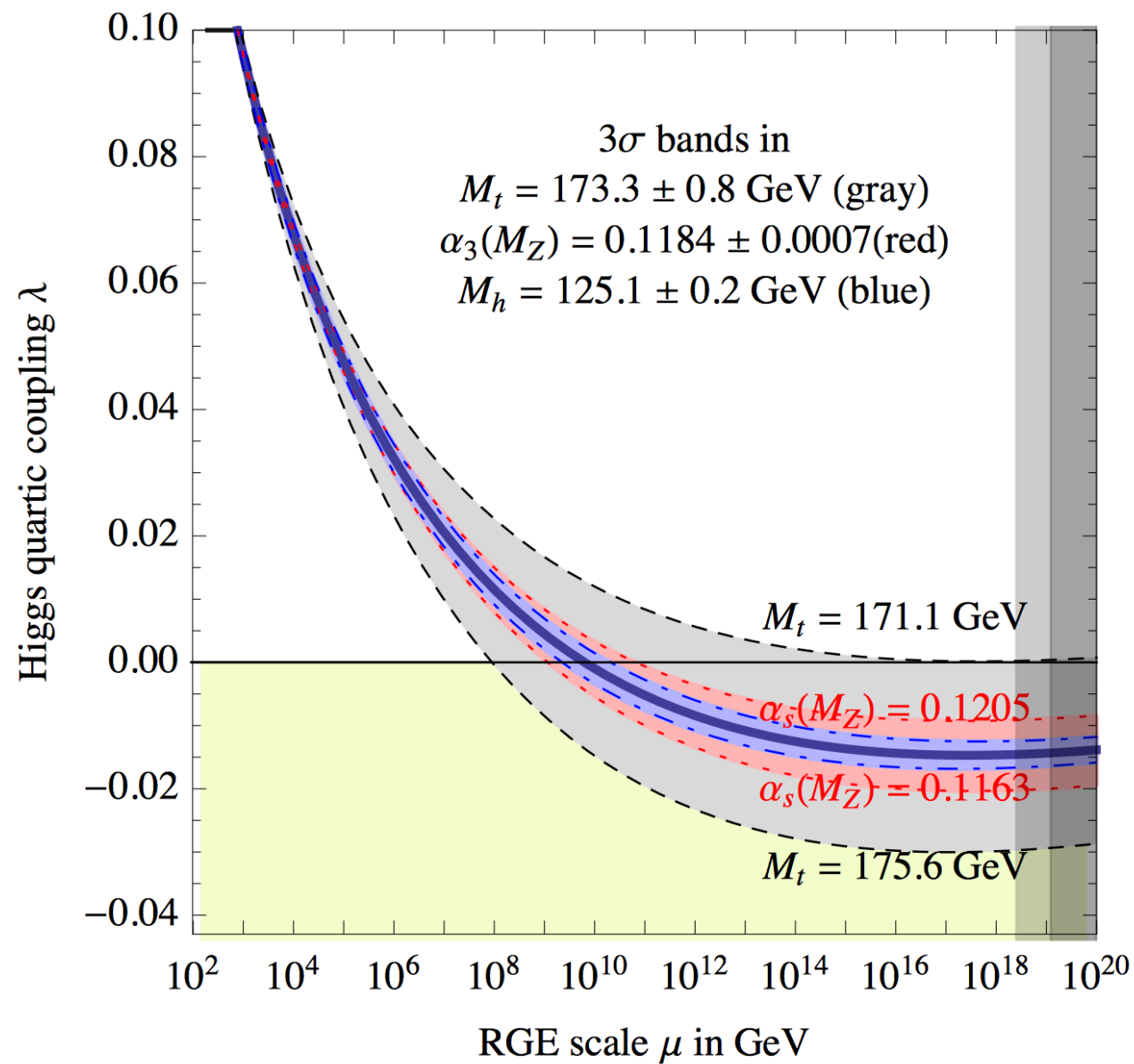
$$r = \frac{12}{N^2}$$

$$V(\phi) \rightarrow \lambda/\xi^2$$



[Planck Results 2015]

What if we include the RG flow?

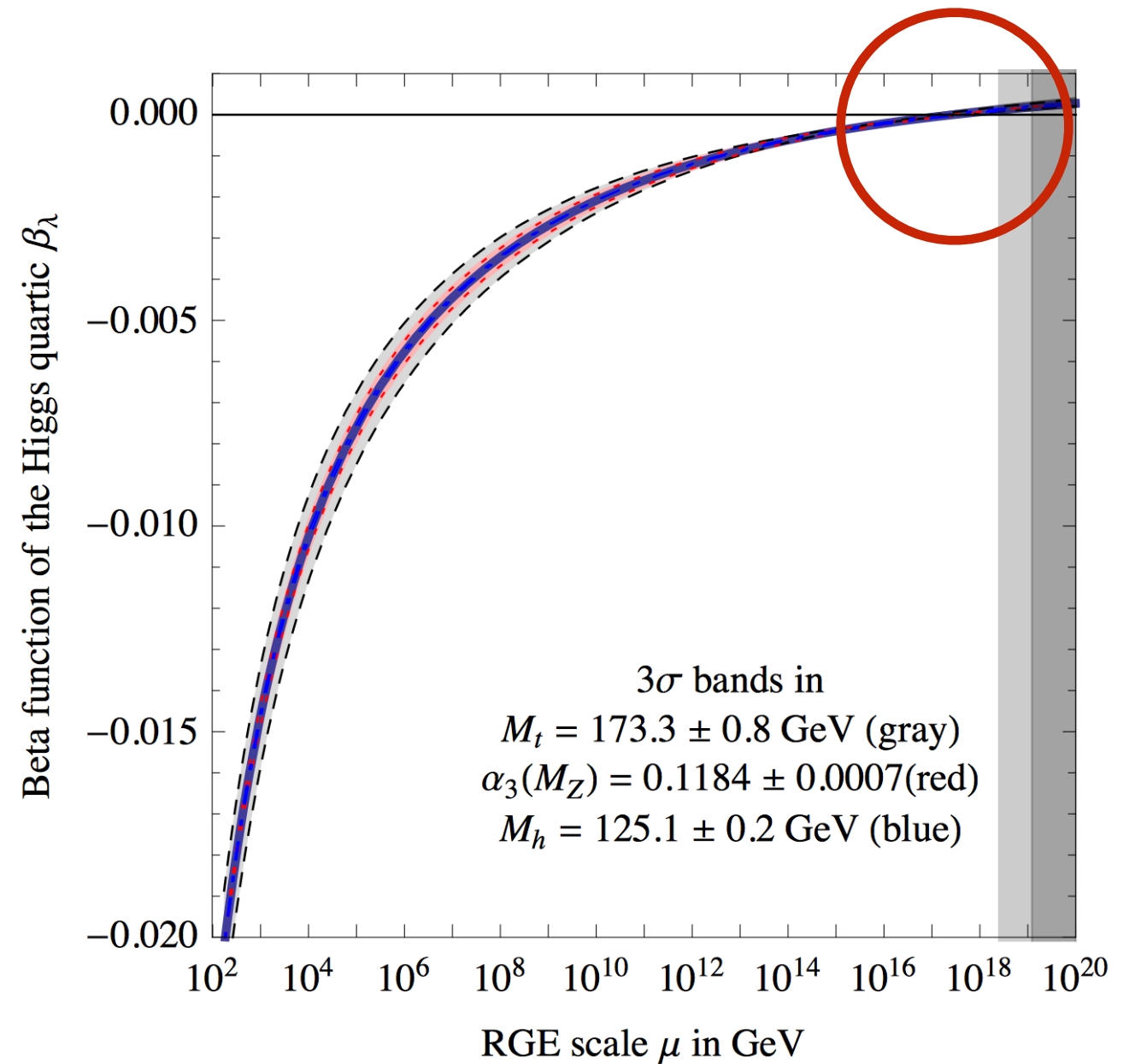
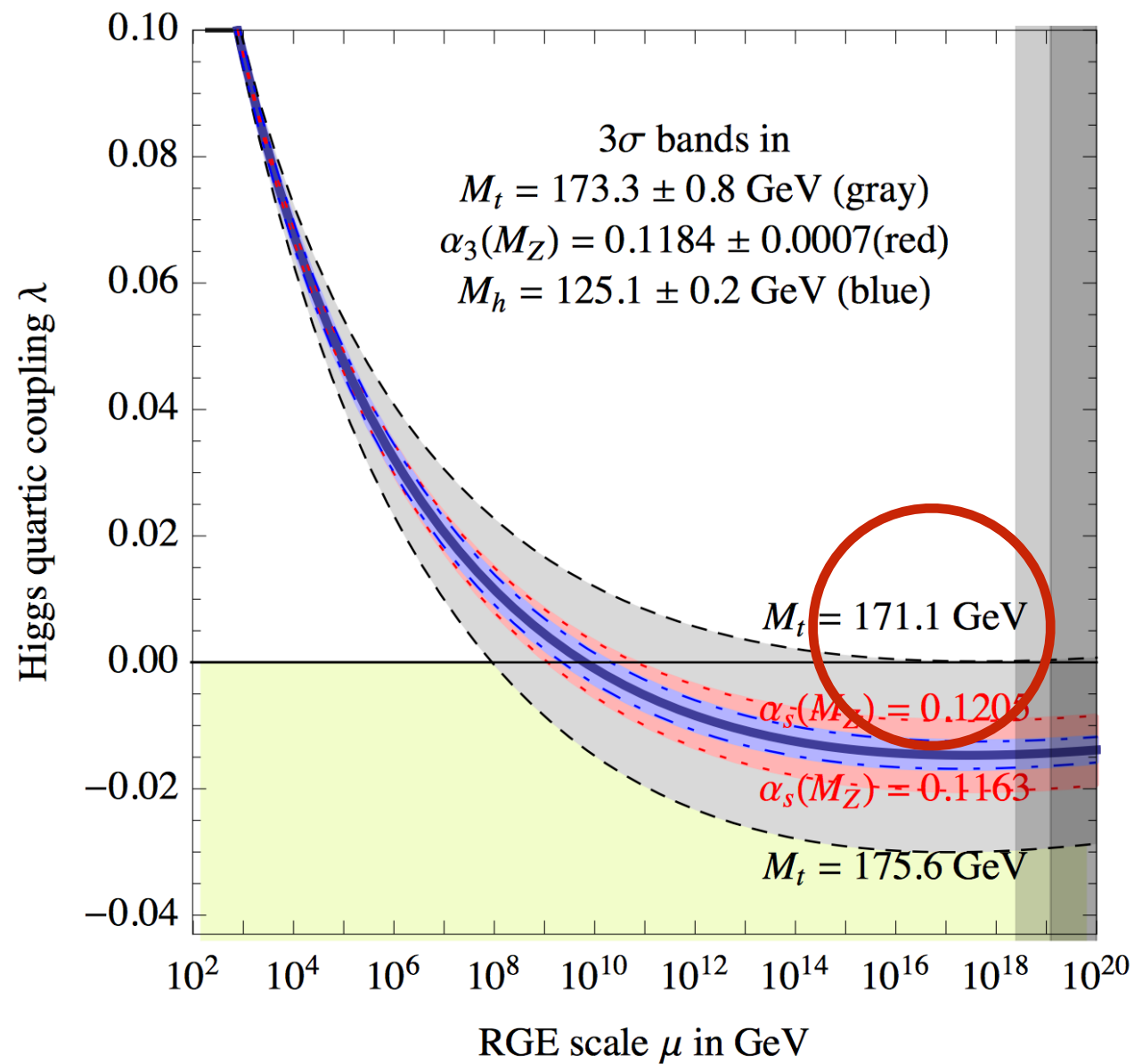


[Buttazzo *et al.* 2013]

Critical-point near
Planck scale

$$\lambda(\mu_c) \ll 1$$

$$\beta_\lambda(\mu_c) \simeq 0$$



[Buttazzo *et al.* 2013]

Critical Higgs Inflation

- We consider both the running of $\lambda(\phi)$ and $\xi(\phi)$

$$S = \int d^4x \sqrt{-g} \left(\left(\frac{1}{2\kappa^2} + \frac{\xi(\phi)}{2} \phi^2 \right) R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4} \lambda(\phi) \phi^4 \right)$$

where

$$\begin{aligned} \lambda(\phi) &= \lambda_0 + b_\lambda \ln^2(\phi/\mu) \\ \xi(\phi) &= \xi_0 + b_\xi \ln(\phi/\mu) \end{aligned}$$

Critical Higgs Inflation

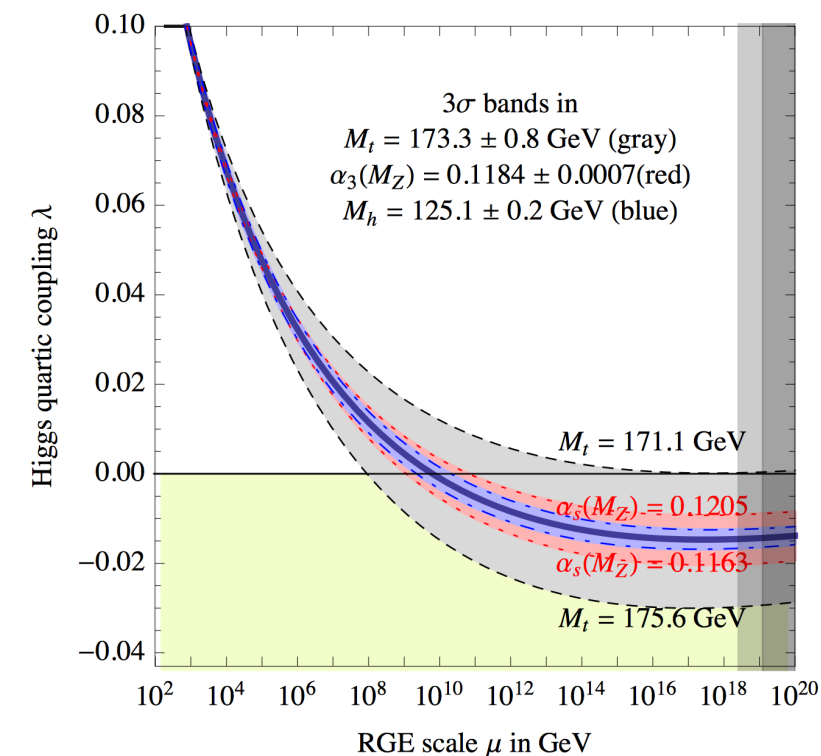
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where

$$\begin{aligned} \lambda(\phi) &= \lambda_0 + b_\lambda \ln^2(\phi/\mu) \\ \xi(\phi) &= \xi_0 + b_\xi \ln(\phi/\mu) \end{aligned}$$

- We are expanding the couplings around the near-critical point of the Higgs self-coupling



- We perform a standard metric and field redefinition

$$\Omega^2 = 1 + \xi \kappa^2 \phi^2$$

$$\frac{d\varphi}{d\phi} = \frac{\sqrt{1 + \phi^2(\xi(\phi) + 6(\xi(\phi) + \phi\xi(\phi)'/2)^2)}}{1 + \xi(\phi)\phi^2}$$

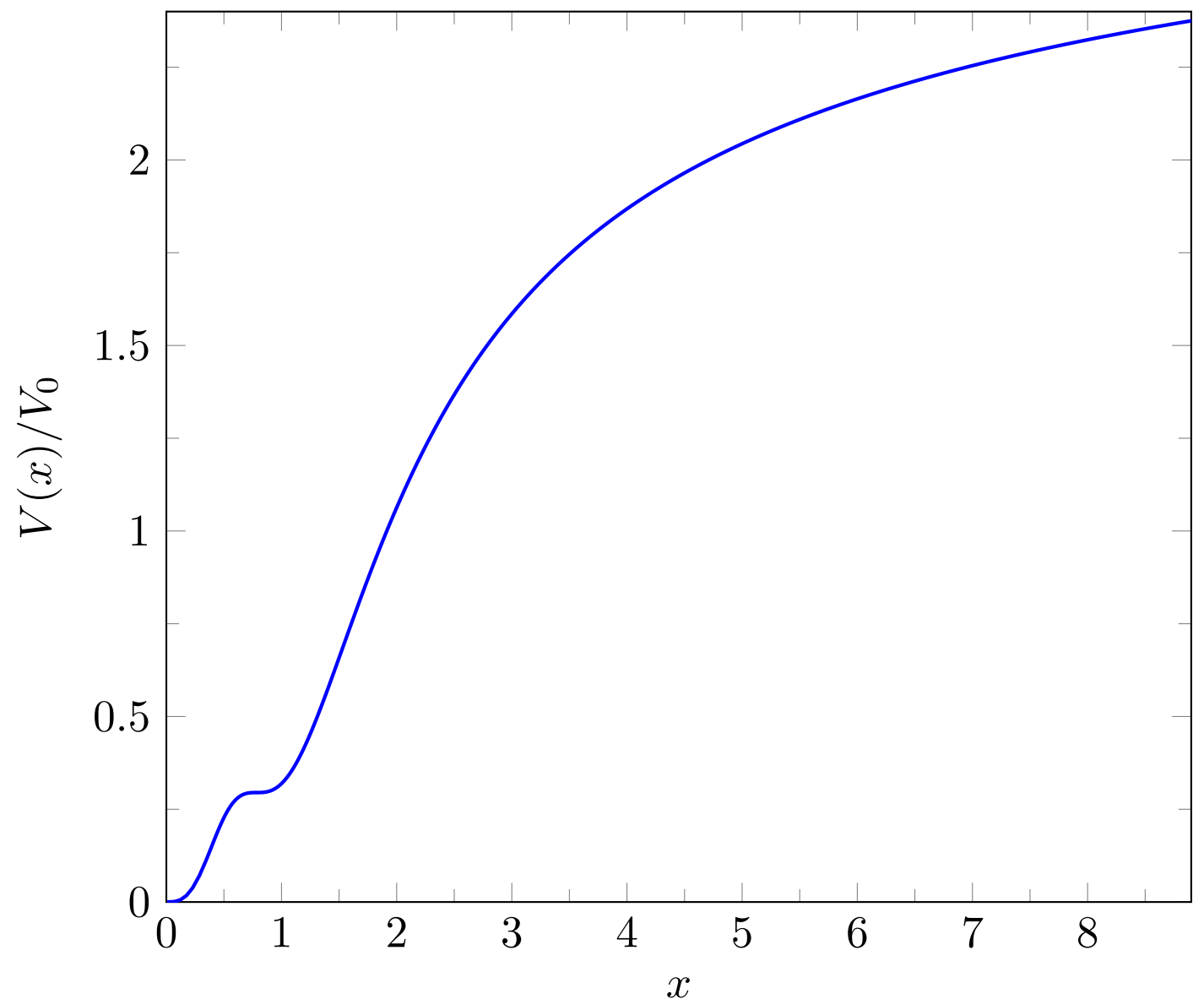
- The potential:

$$V(x) = \frac{V_0 (1 + a \ln^2 x) x^4}{(1 + c (1 + b \ln x) x^2)^2}$$

$$V_0 = \lambda_0 \mu^4 / 4$$

$$a = b_\lambda / \lambda_0 \quad b = b_\xi / \xi_0$$

$$c = \xi_0 \kappa^2 \mu^2 \quad x = \phi / \mu$$



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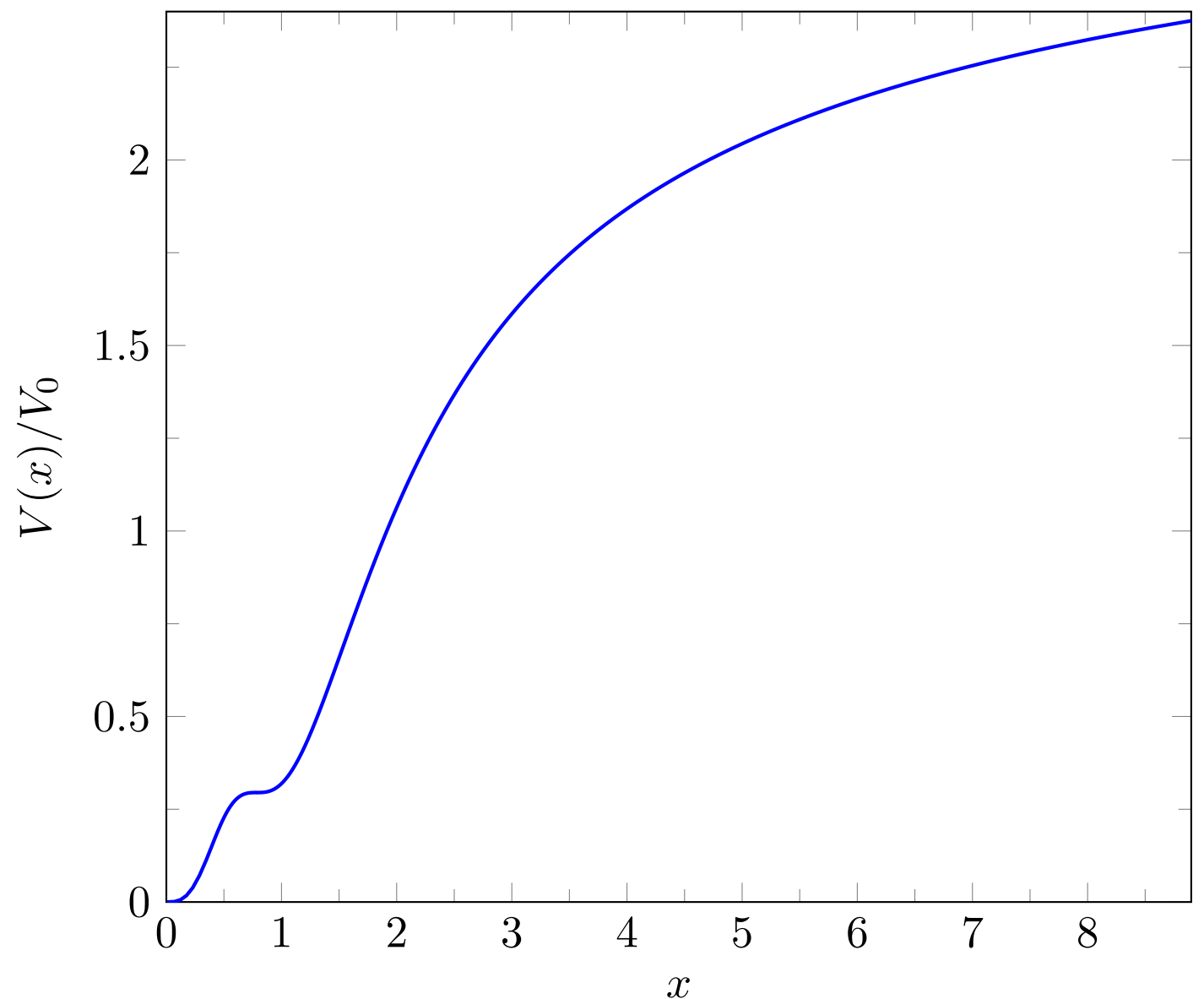
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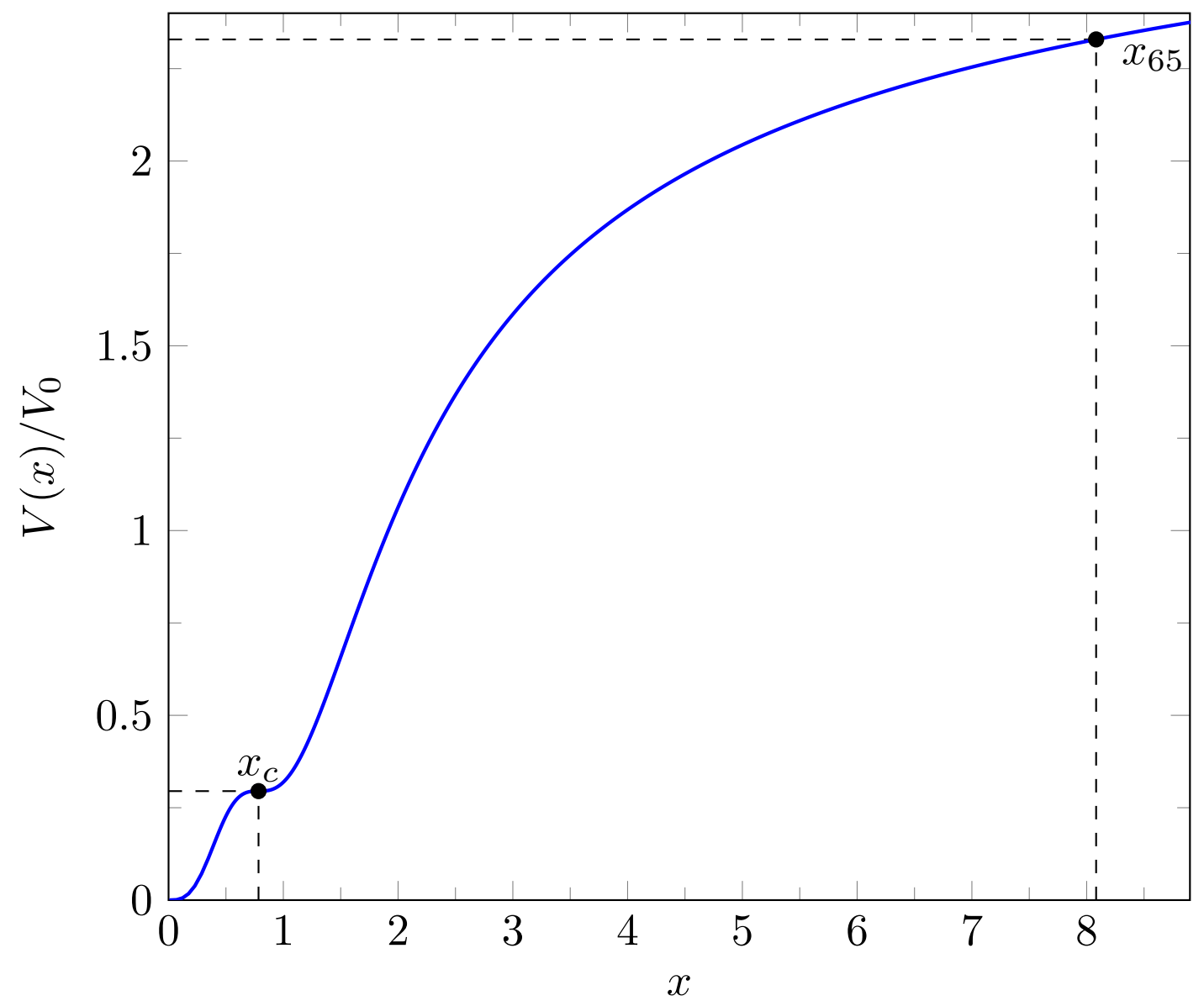
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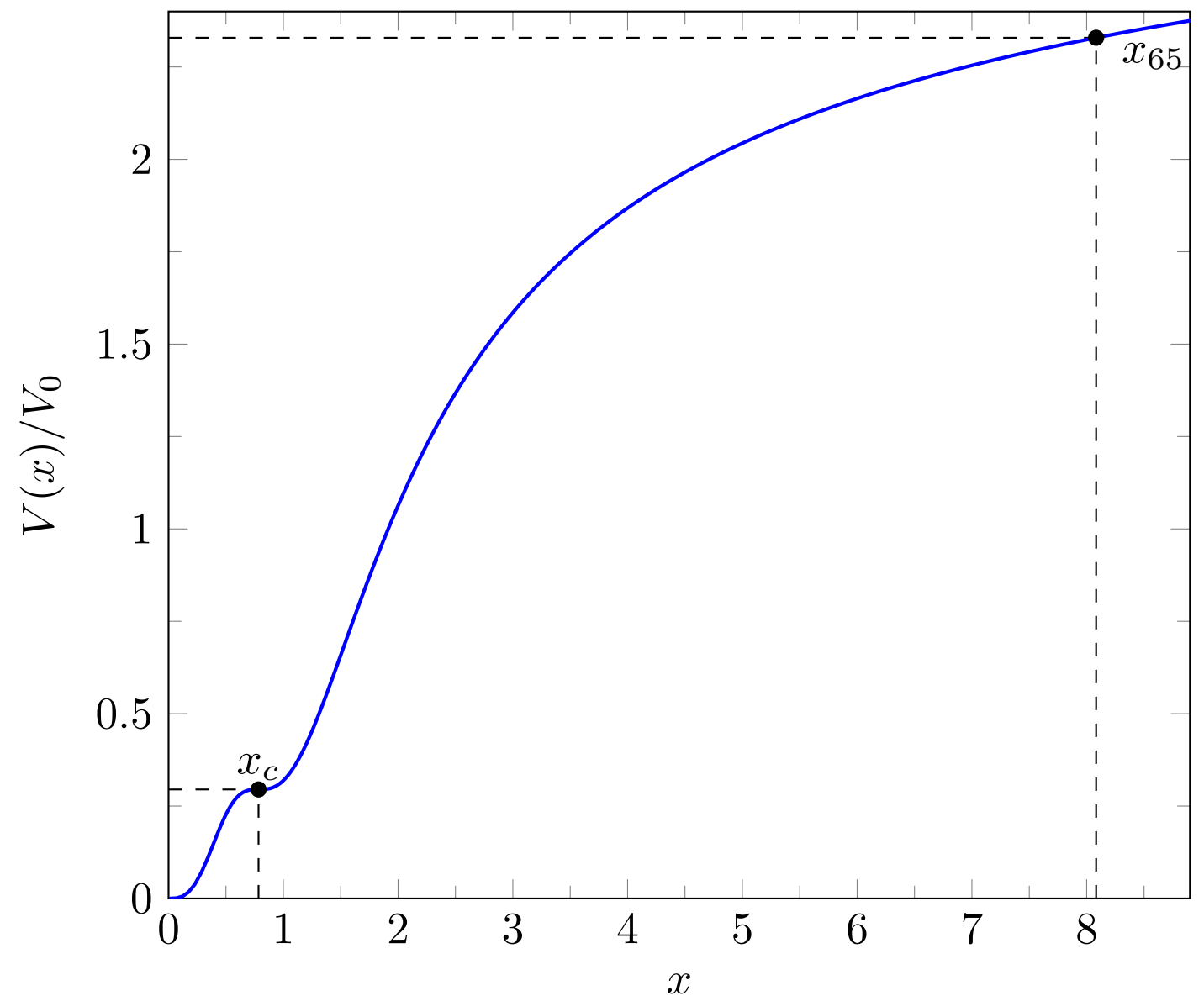
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- Large field values of the potential are dominated by the running

$$V(x \gg x_c) \simeq V_0 a/(b c)^2 \sim b_\lambda/b_\xi^2 \ll M_{pl}^4$$



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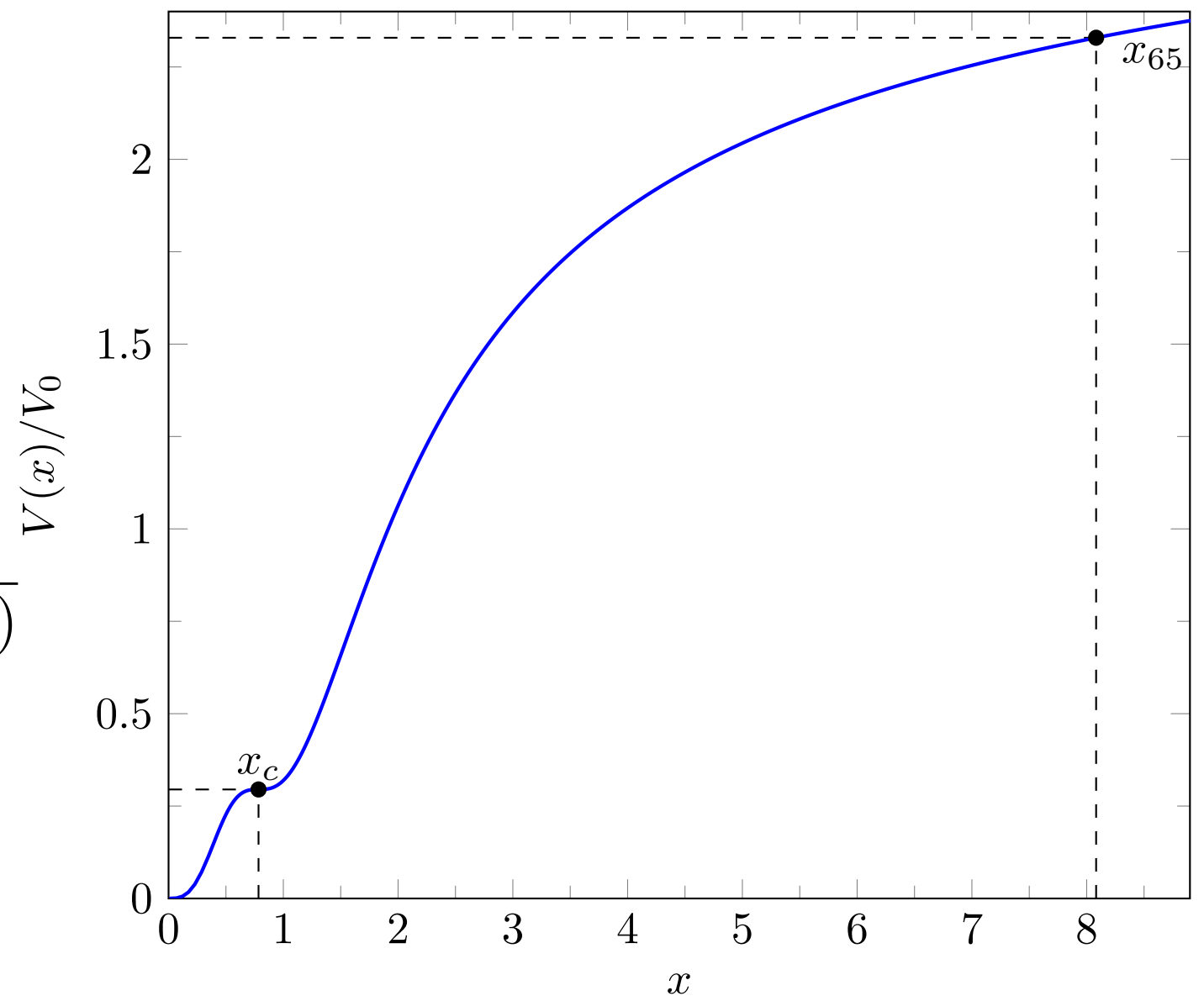
- The inflection point fixed by

$$V'(x_c) = V''(x_c) = 0$$

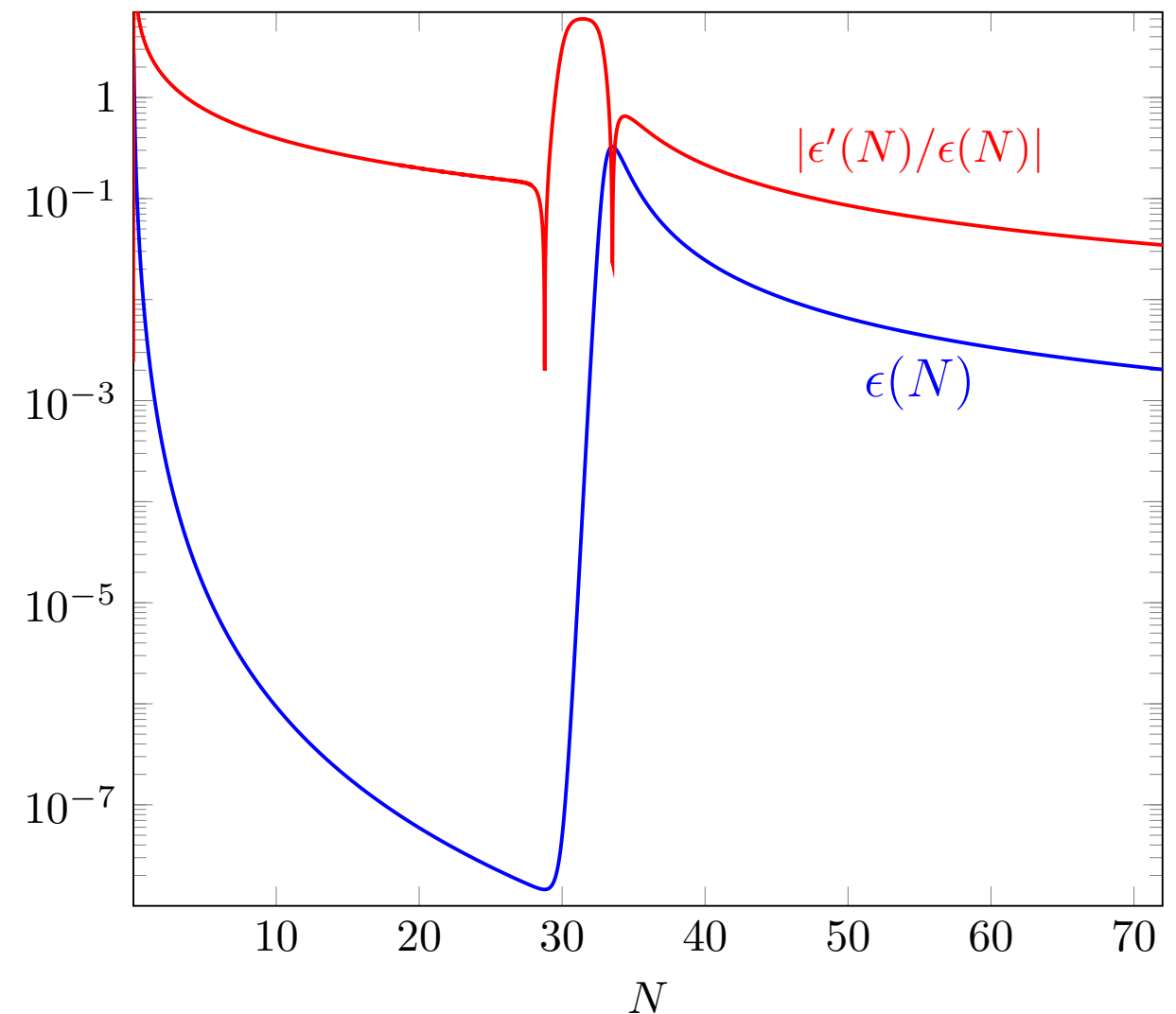
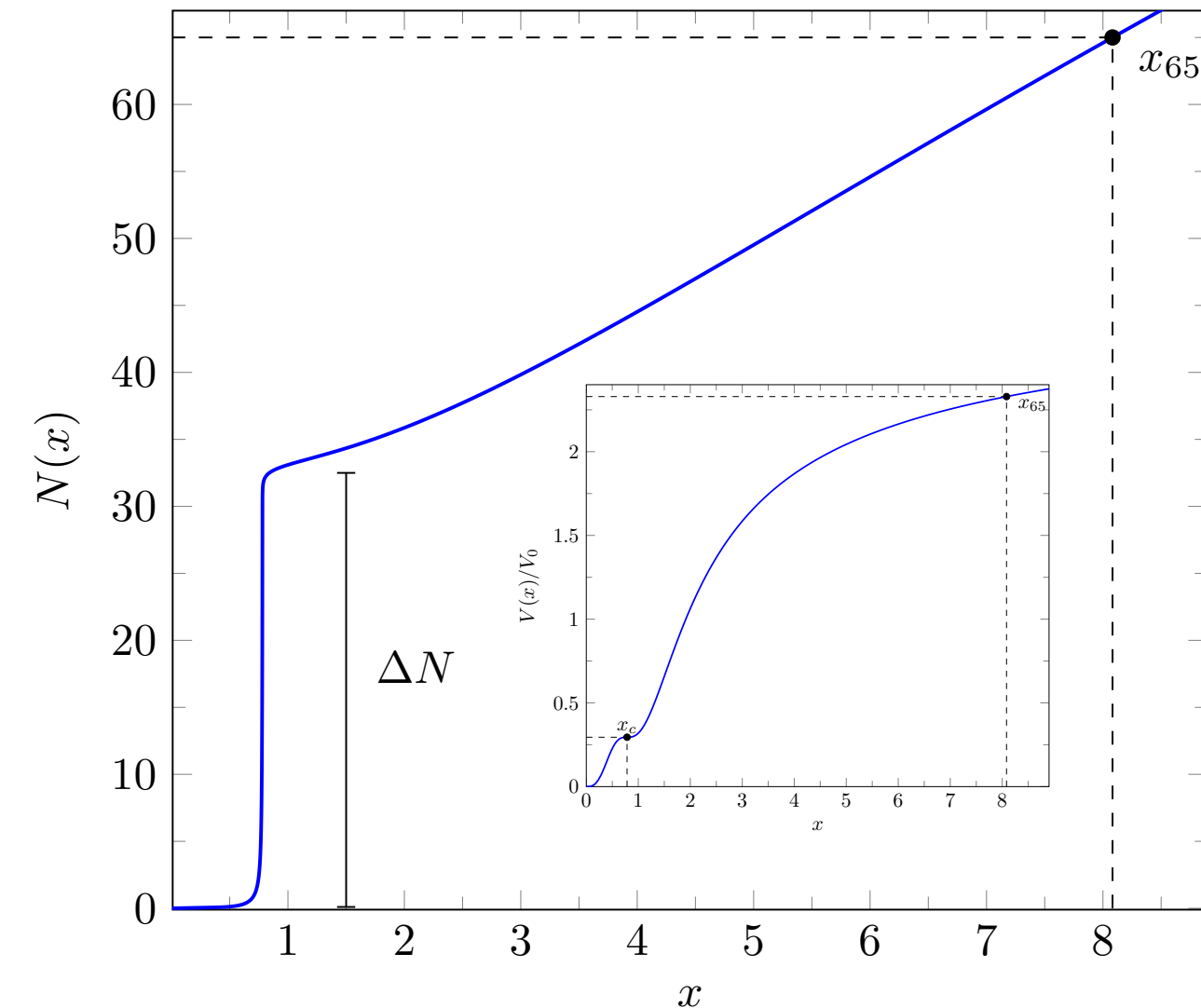
$$a(c, x_c) = \frac{4}{1 + c \cdot x_c^2 + 2z - 4z^2}$$

$$b(c, x_c) = \frac{2(1 + c \cdot x_c^2 + 4z - 4z^2)}{c \cdot x_c^2(1 + c \cdot x_c^2 + 2z - 4z^2)}$$

where $z \equiv \log x_c$



- The model can be characterized by the **width** and **peak** of the spectrum



- Near-inflection point condition:

$$\begin{aligned} a &\rightarrow a(c, x_c) \\ b &\rightarrow b(c, x_c)(1 - \beta) \end{aligned}$$

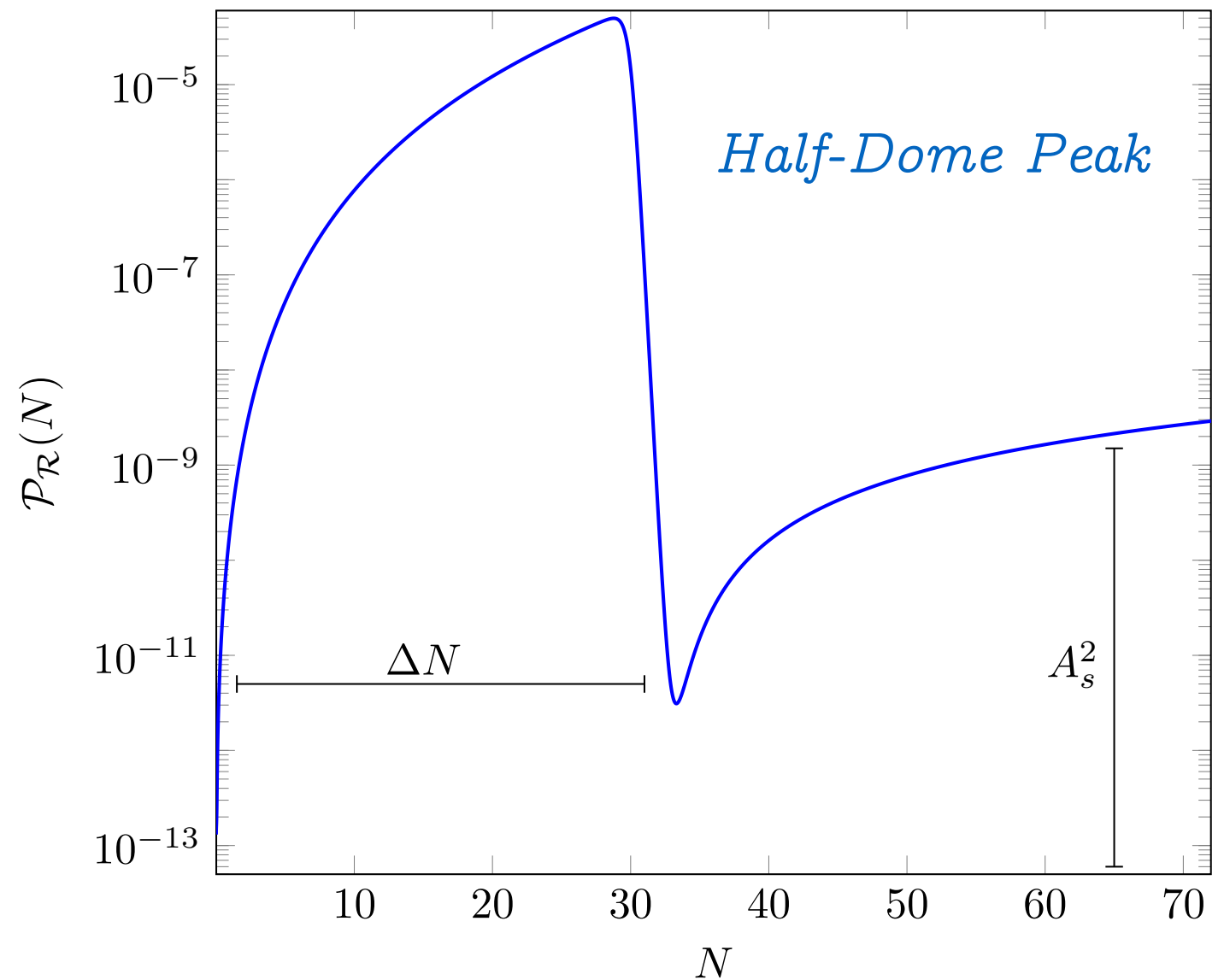
PBH from CHI

[Ezquiaga *et al.* 2017]

- Primordial Spectrum

$$\mathcal{P}_{\mathcal{R}}(N) \simeq \frac{\kappa^2}{2\epsilon} \left(\frac{H}{2\pi} \right)^2$$

$$H^2 = \kappa^2 \frac{V(\phi(N))}{3 - \epsilon(N)}$$



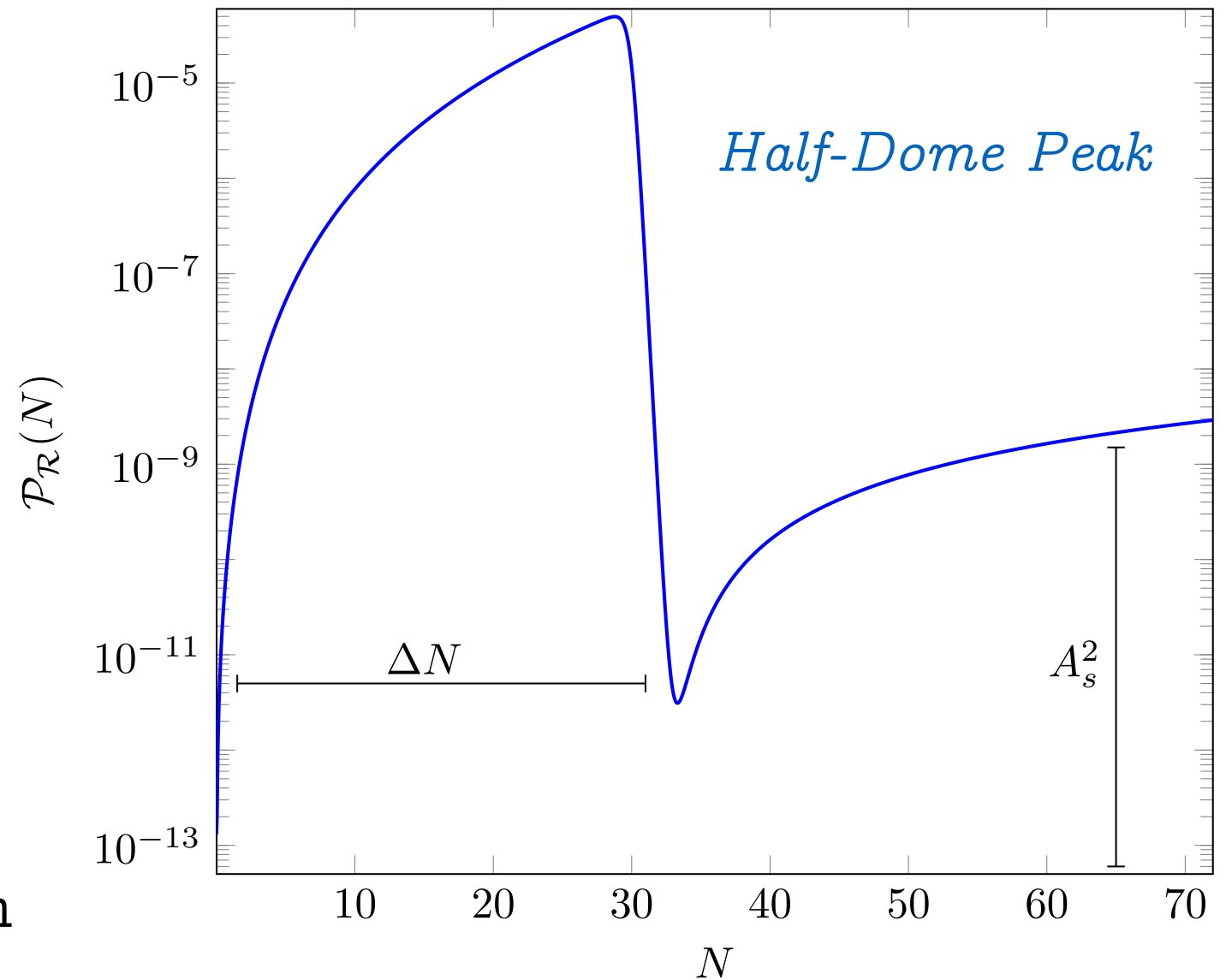
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- Press-Schechter formalism

$$\beta^{form}(M) \equiv \frac{\rho_{PBH}(M)}{\rho_T} \Big|_{t=t_M} = 2 \int_{\zeta_c}^{\infty} \frac{d\zeta}{\sqrt{2\pi}\sigma} e^{-\frac{\zeta^2}{2\sigma^2}} = \text{erfc} \left(\frac{\zeta_c}{\sqrt{2}\sigma} \right)$$

$$\sigma^2 = \mathcal{P}_{\mathcal{R}}(N)$$

- Evolution until matter-radiation equality:
 - Density of PBH grow faster than radiation

$$\beta^{eq}(M) = \frac{a_{eq}}{a(t_M)} \beta^{form}(M) \qquad \Omega_{PBH}^{eq} = \int_{M_{ev}}^{M_{eq}} \beta^{eq}(M) d \ln M$$

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- Evolution after matter-radiation equality:
 - Clustered PBHs: non-linear evolution+merging+accretion

[Chisholm 2006]

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- Evolution after matter-radiation equality:
 - Clustered PBHs: non-linear evolution+merging+accretion

[Chisholm 2006]

- Final distribution: approximately LogNormal

$$P(M) = \frac{A \mu_{PBH}}{M \sqrt{2\pi\sigma_{PBH}^2}} \exp \left(-\frac{\ln^2(M/\mu_{PBH})}{2\sigma_{PBH}^2} \right)$$

Results

[Ezquiaga *et al.* 2017]

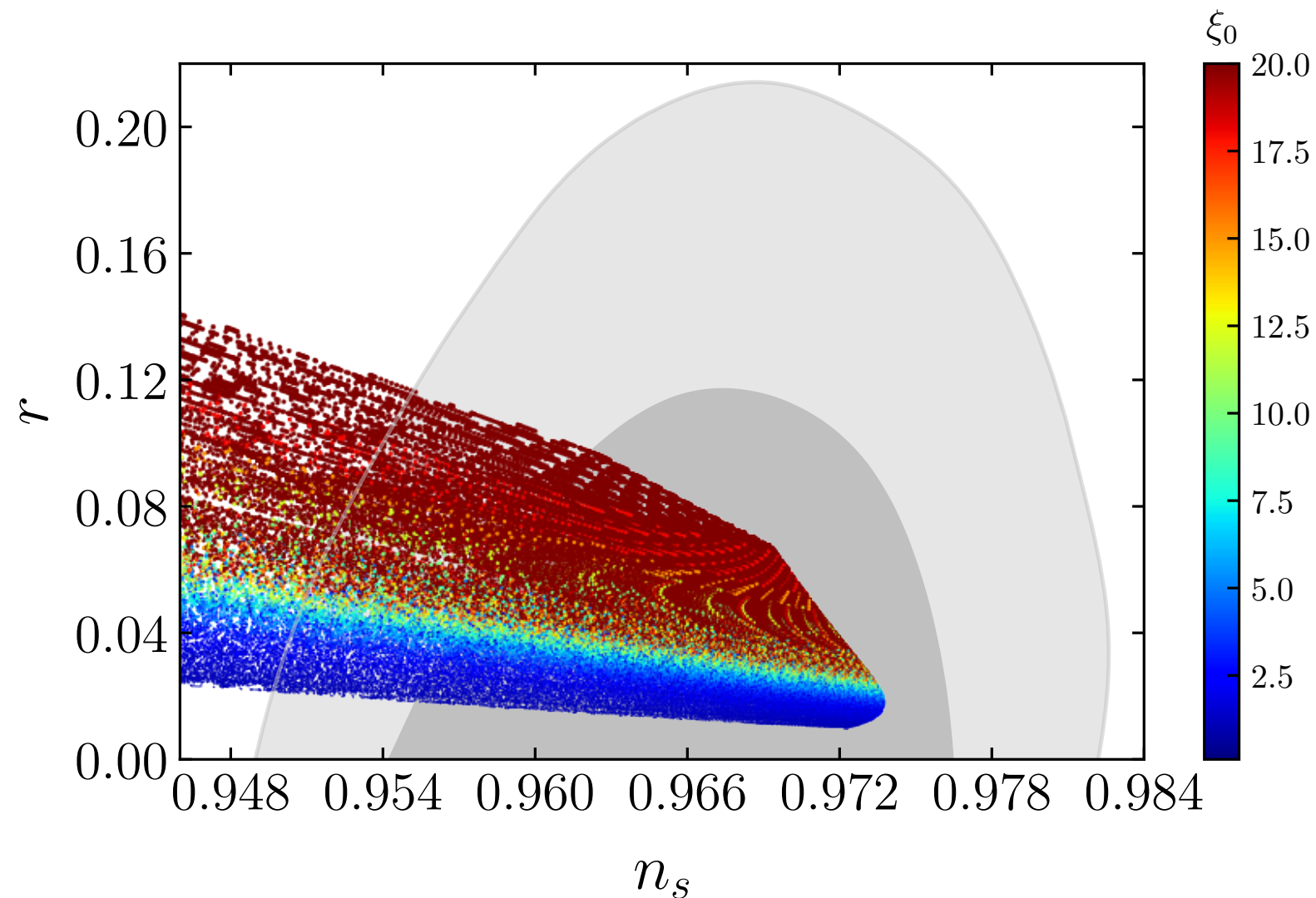
Results

[Ezquiaga *et al.* 2017]

- Large parameter space compatible with Planck 2015

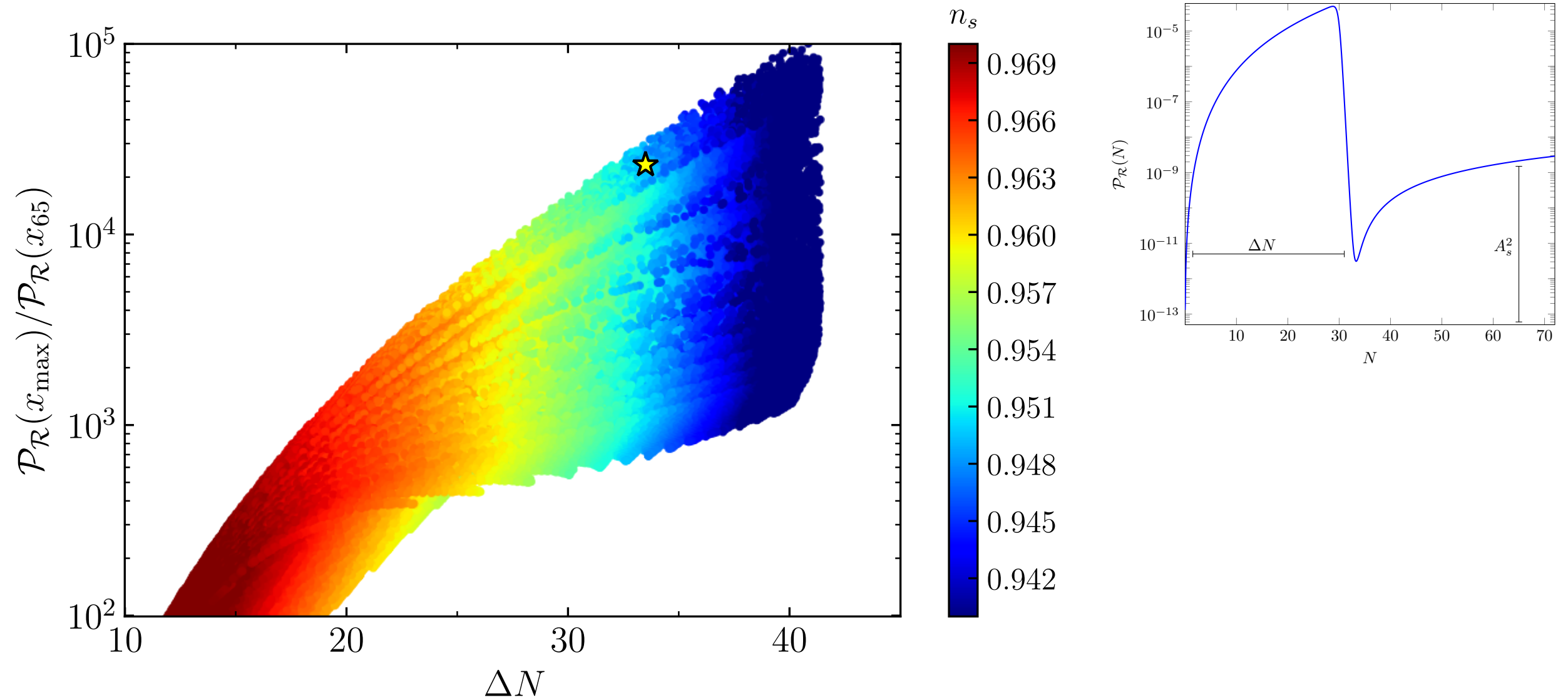
$$\beta \in (0.1 - 9) \times 10^{-4}$$

$$\Delta N \in (10 - 45)$$

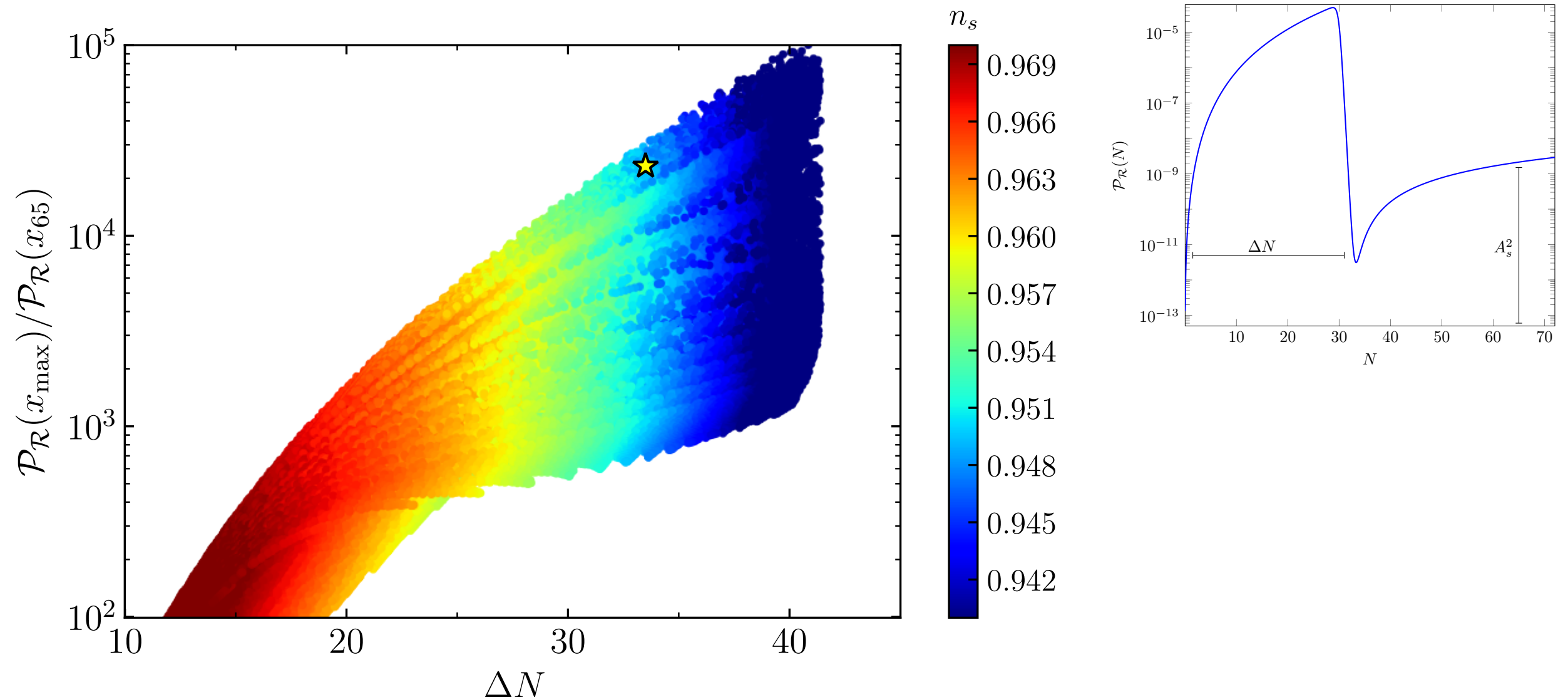


- CMB observables are sensitive to the width (ΔN) of the peak

- PBH production controlled by the **height** and **width** of the *half-dome*



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- Mean value of PBH distribution depends on the location of maximum

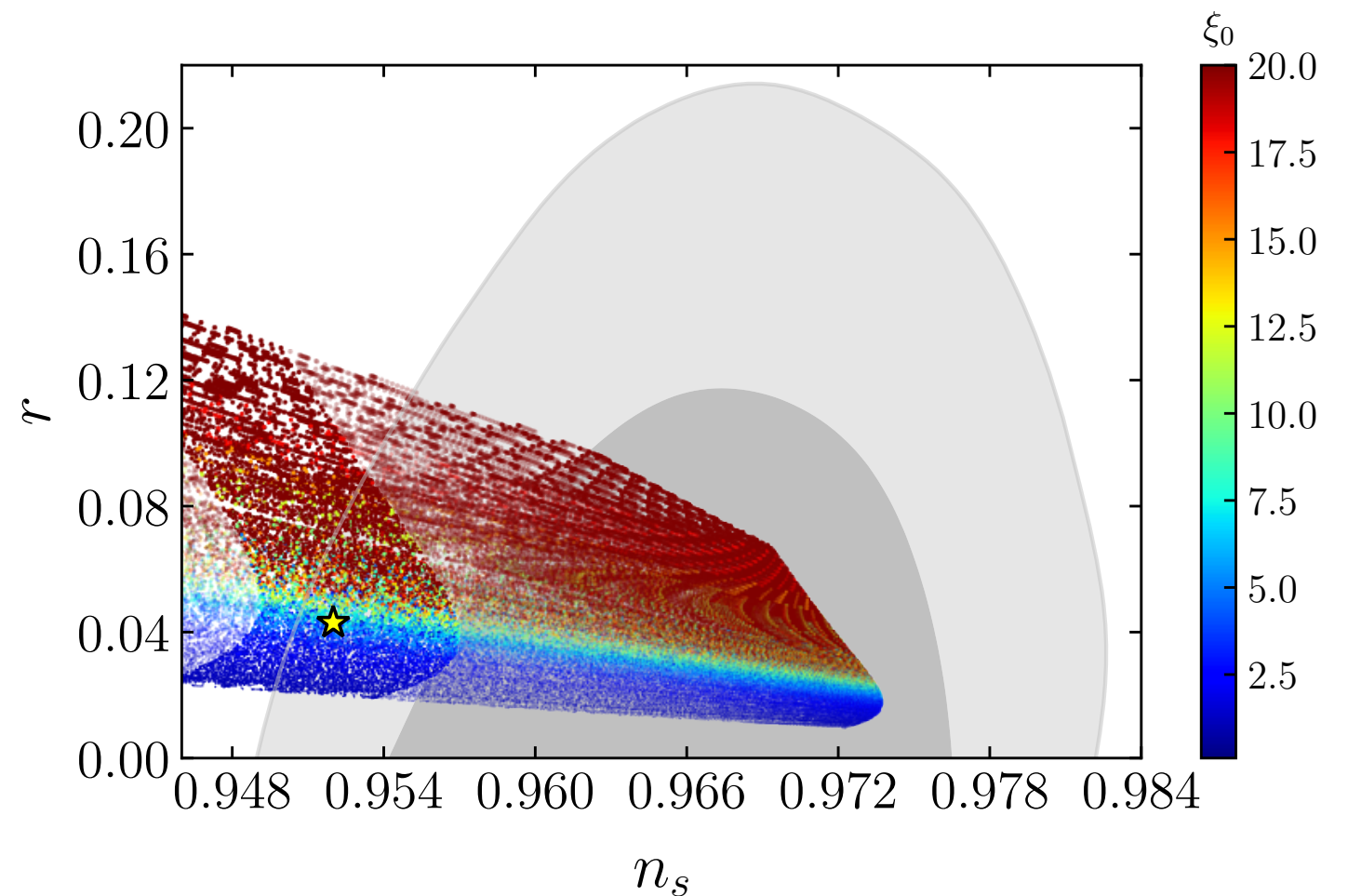
$$\mu_{\text{PBH}} \sim 10 M_{\odot} \cdot e^{2(N_{\text{peak}} - 28.8)}$$

★ $\mu_{\text{PBH}} \sim 11 M_{\odot}$

- CMB observables for reference case:

★ $n_s = 0.952$, $r = 0.043$, $\alpha_s = -0.0017$

$\lambda_0 = 2.23 \times 10^{-7}$, $\xi_0 = 7.55$, $\kappa^2 \mu^2 = 0.102$, $b_\lambda = 1.2 \times 10^{-6}$, $b_\xi = 11.5$



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- Region $\Delta N \in (30 - 35)$:

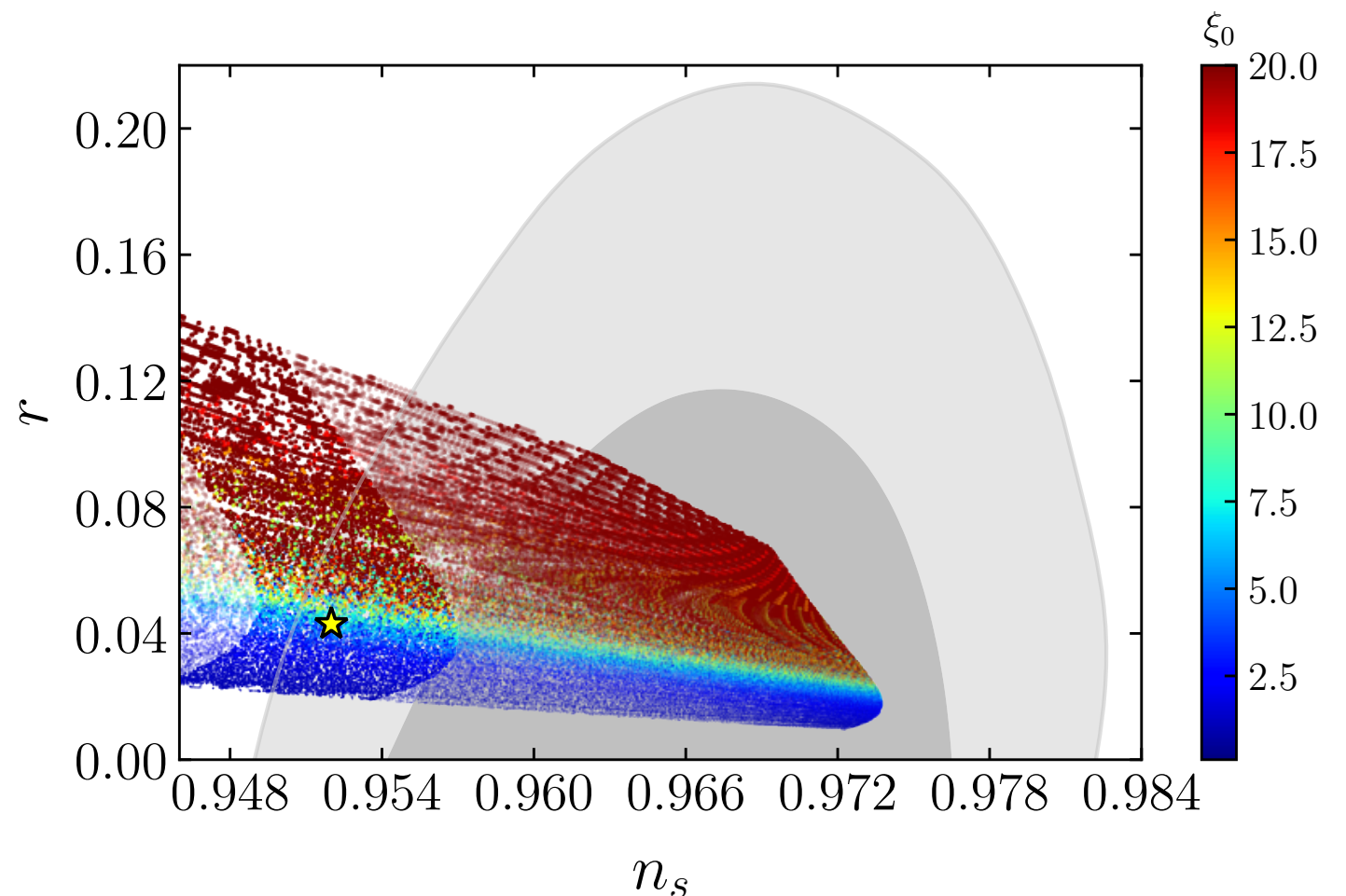
$$\lambda_0 \sim (0.01 - 8) \cdot 10^{-7}$$

$$\xi_0 \sim (0.5 - 15)$$

$$\kappa^2 \mu^2 \sim (0.05 - 1.2)$$

$$b_\lambda \sim (0.008 - 4) \cdot 10^{-6}$$

$$b_\xi \sim (1 - 18)$$



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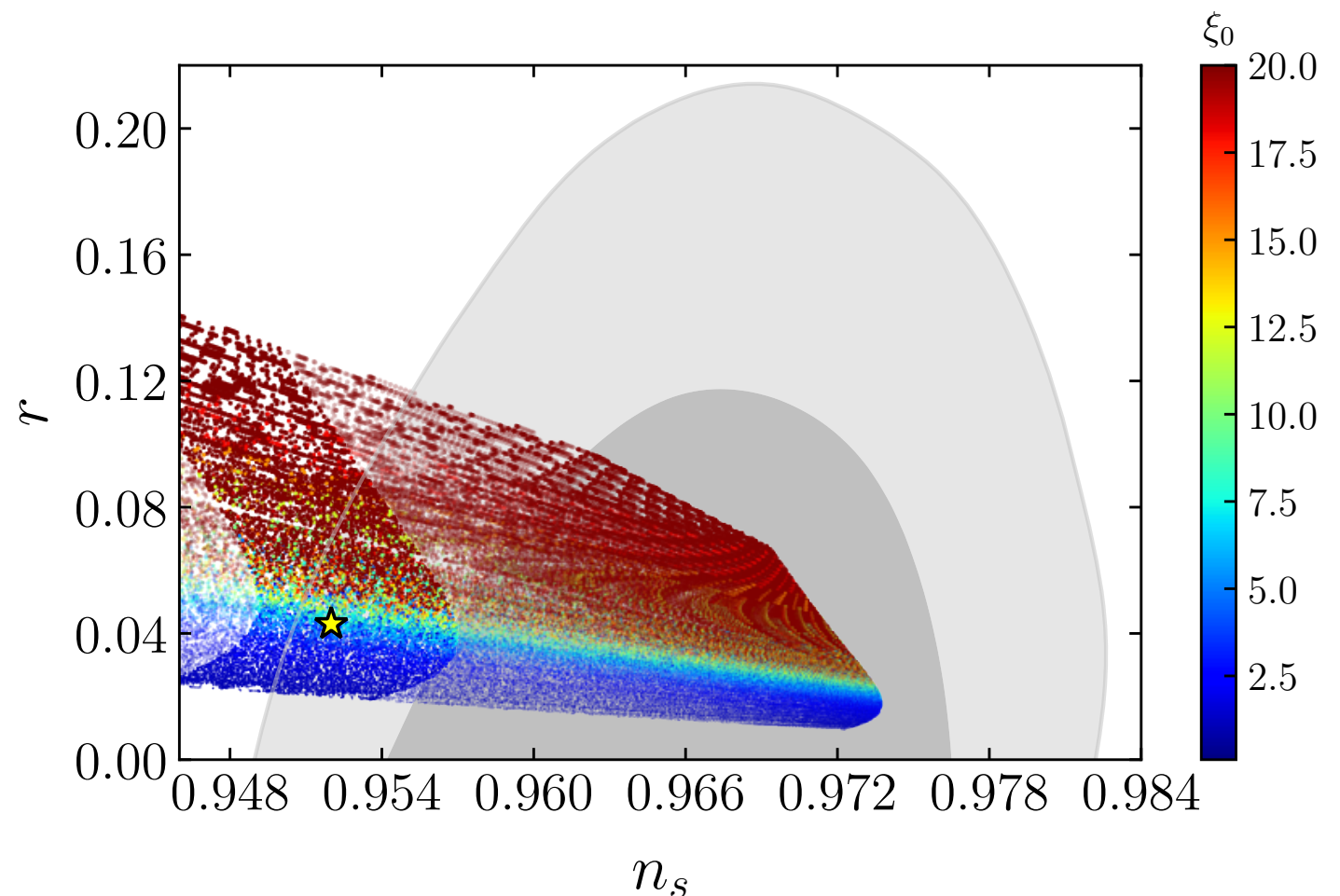
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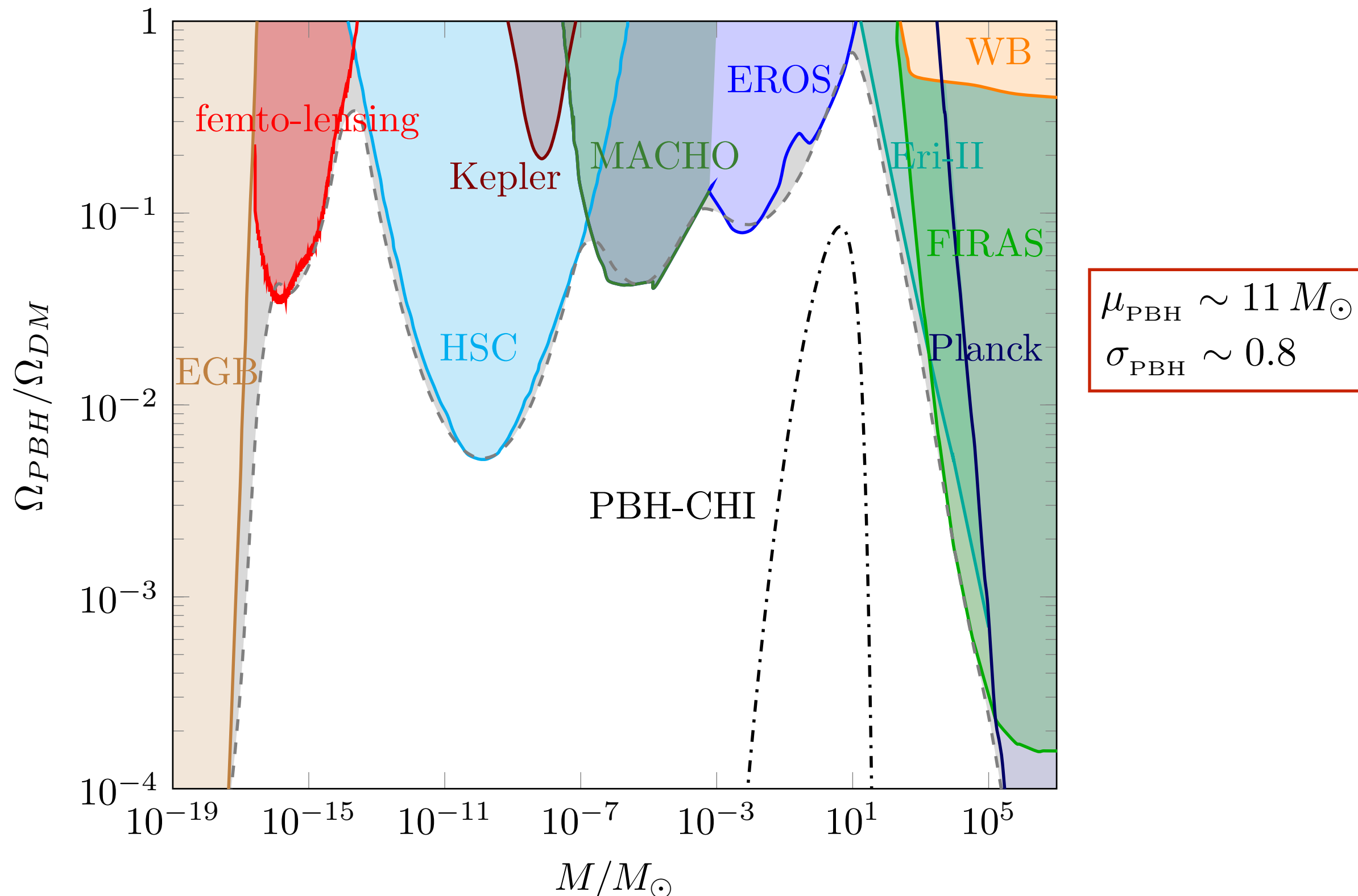
- PBH spectrum:

$$N_{\text{peak}} \sim (25 - 30)$$

$$\mu_{\text{PBH}} \sim (0.01 - 100) M_\odot$$

$$\sigma_{\text{PBH}} \sim (0.6 - 1)$$

- Constraints on the fraction of PBH (extended mass function)

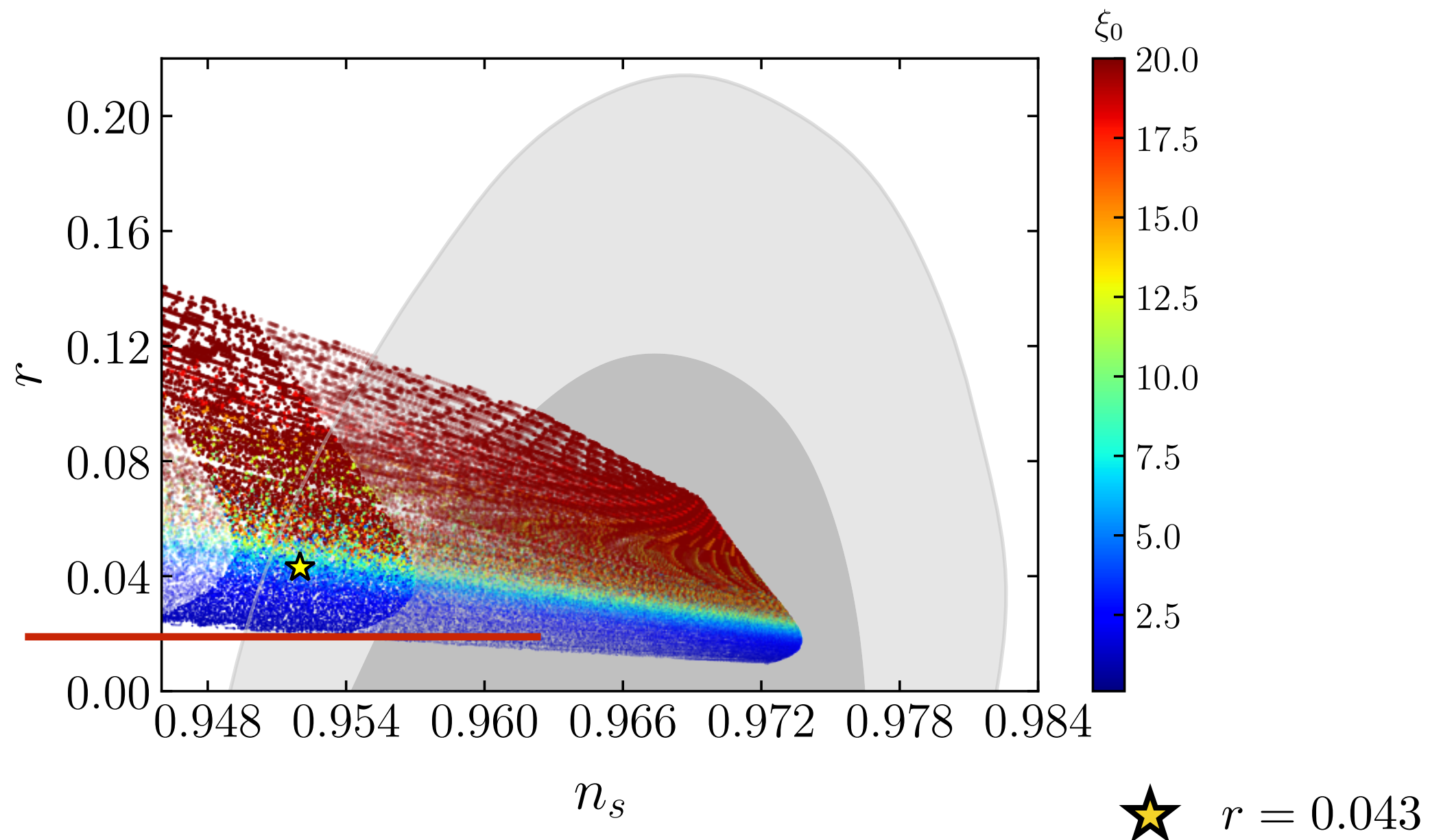


Signatures of PBH-CHI

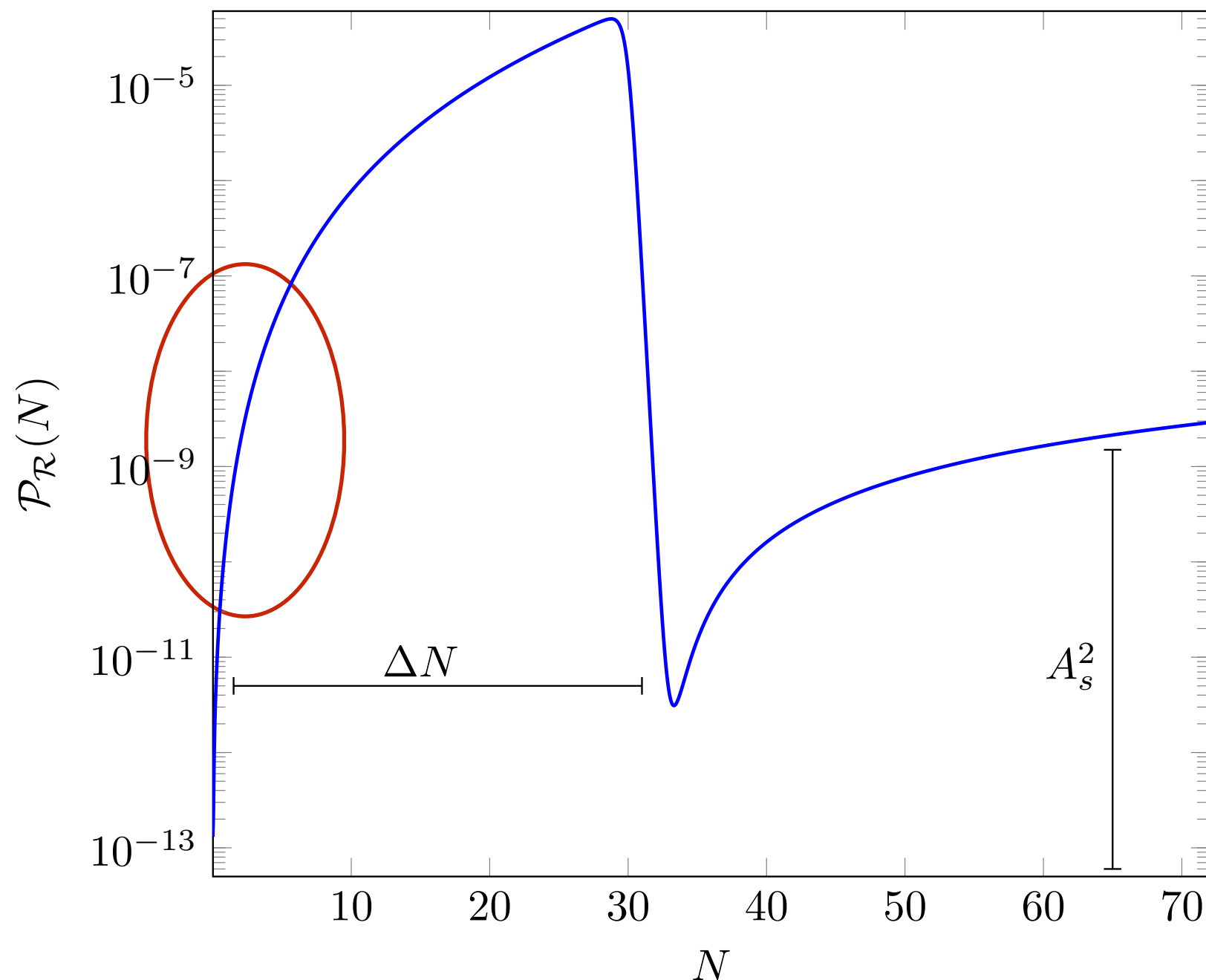
Signatures of PBH-CHI

[Early Universe]

- Relatively large tensor modes: $r > 0.019$ in $\Delta N \in (30 - 35)$



- Large fluctuations at the end of inflation: *inhomogeneous reheating*



$$\rho_{\text{end}} = 4 \times 10^{63} \text{ GeV}^4$$

$$T_{\text{rh}} = 3.2 \times 10^{15} \text{ GeV}$$

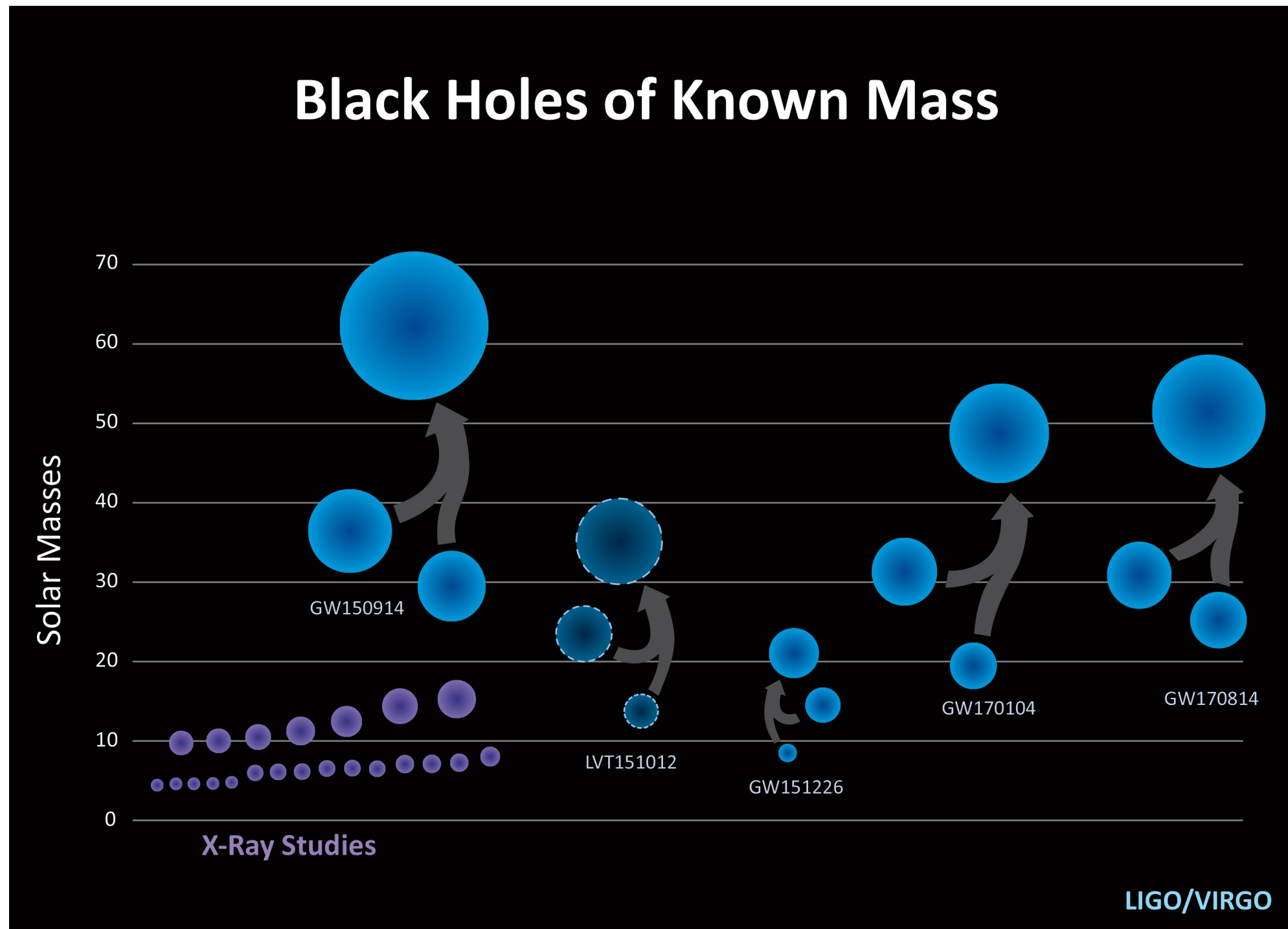
$$(\text{for } g_* = 106.75)$$

→ Possible production of GWs and more PBHs

[Late Universe]

[Ezquiaga *et al.* 2017]

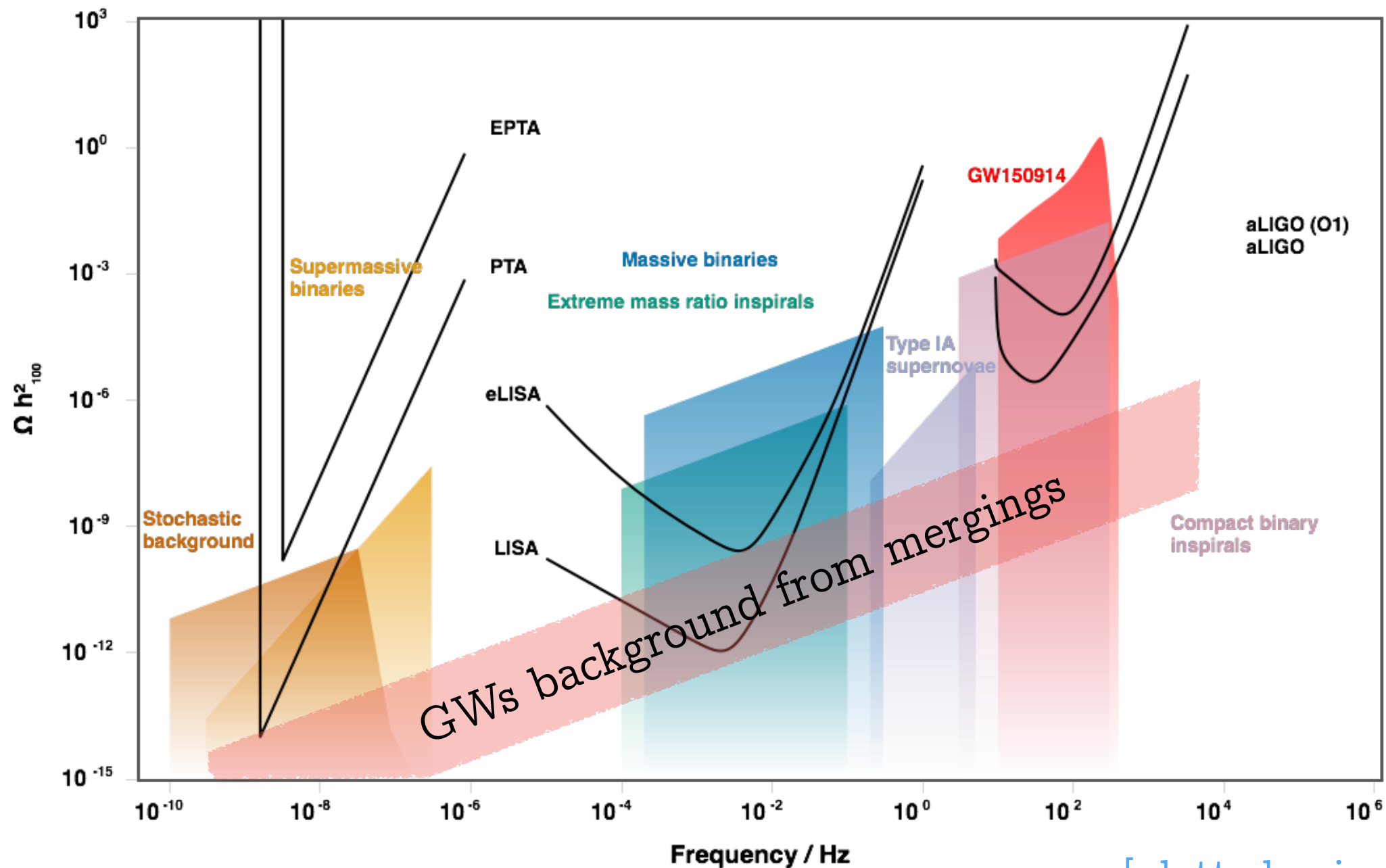
- LIGO-VIRGO events: $\mu_{\text{PBH}} \sim (0.01 - 100) M_{\odot}$



[yesterday's announcement!]

- Broad peak: clustering of PBH enhance GWs background

$$h^2 \Omega_{\text{GW}}(f) = 8 \times 10^{-15} \tau_m f^{2/3} (\text{Hz}) \mu^{5/3} (M_\odot) R(\sigma)$$



[plotted using GWplotter]

6 Discussion

[Ezquiaga *et al.* 2017]

- The PBH Critical Higgs Inflation is a successful inflationary model with distinctive signatures

i) $V(x \gg x_c) \sim b_\lambda/b_\xi^2 \ll M_{pl}^4$

ii) Large tensor modes $r \sim 0.04$

iii) Preheating with large fluctuations

- The production of PBH is associated to the near-criticality of the Higgs self-coupling

* PBH from CHI lie in the LIGO band

- This scenario could be tested by different means, such as CMB polarization, stochastic GWs backgrounds or microlensing

6 Future Prospects

- Further developments (in progress) are needed to test the model at the level of precision of the SM parameters
 - i) Theoretical:* full RGE evolution of SM non-minimally coupled to gravity from EW scales to inflationary scales.
Quantum diffusion effects may increase PBH production.
 - ii) Computational:* N-body simulations are required to determine the non-linear evolution of the mass spectrum in this clustered PBH scenario
- Constraints on PBHs should be carefully reexamined in the context of broad mass spectrum with clustered PBH



Find more details at

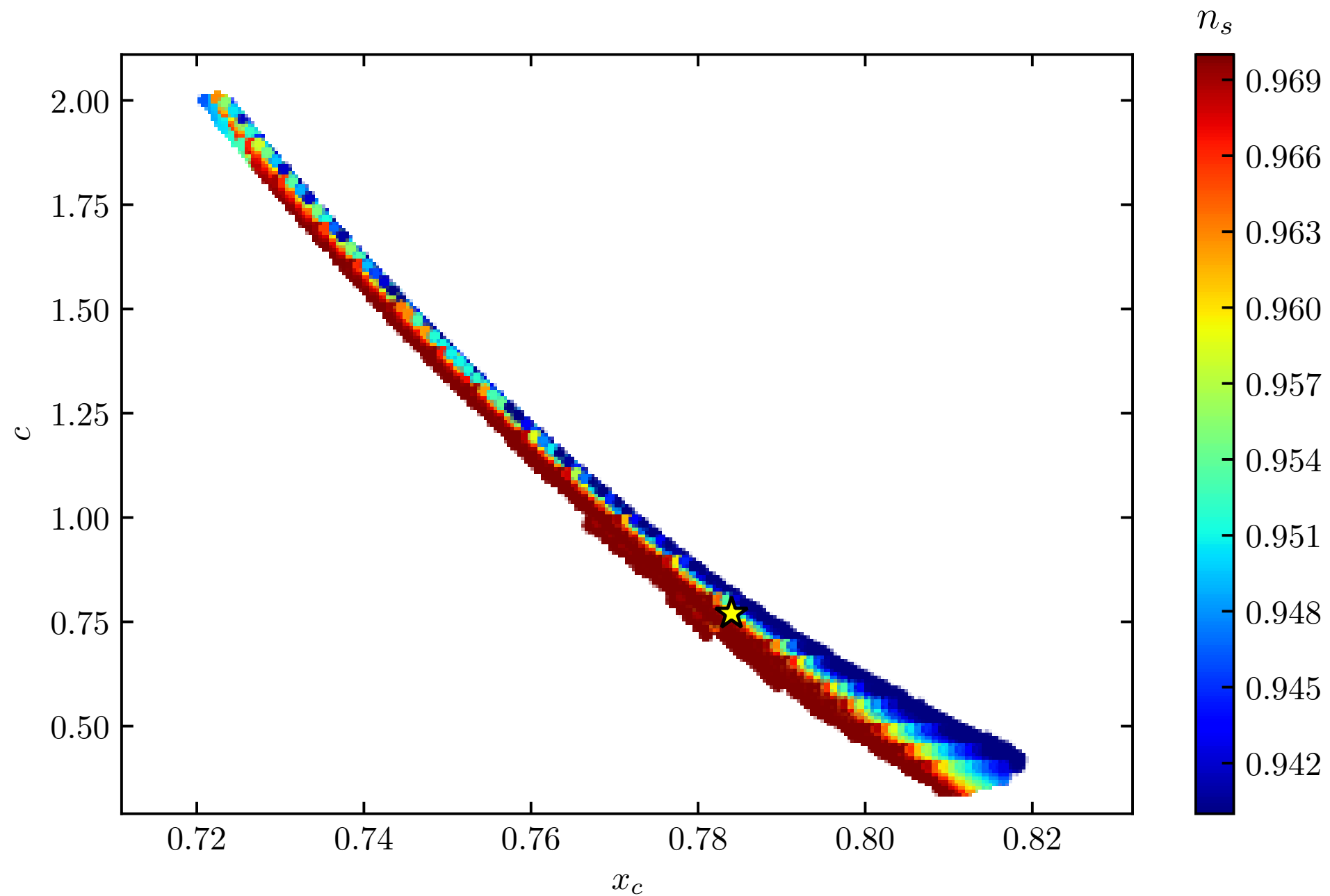
arXiv [1705.04861](#) by JME, J. GARCÍA-BELLIDO, E. RUIZ MORALES

or by e-mail

jose.ezquiaga@uam.es

Back-up slides

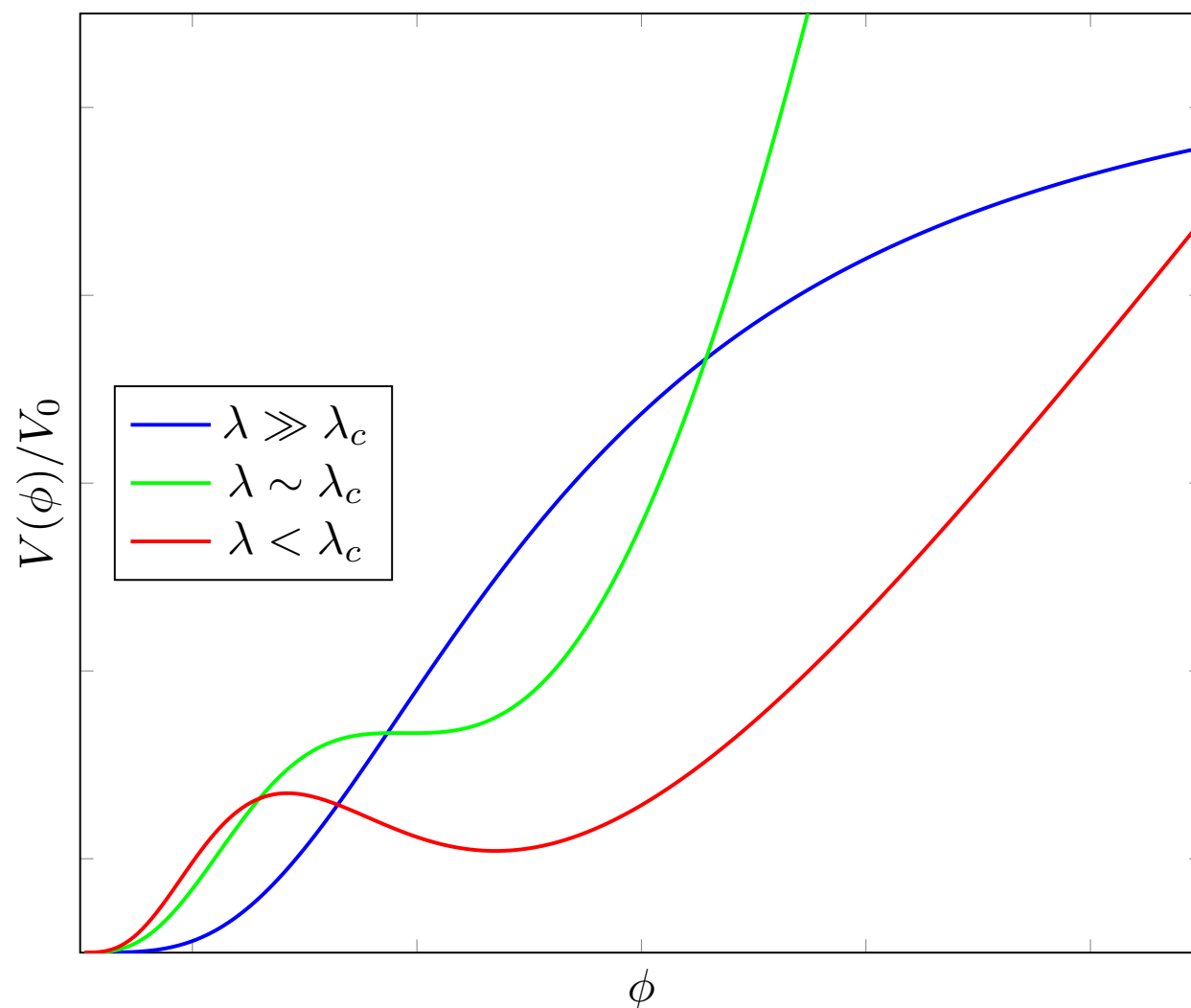
- Parameter space:



★ $(\beta = 10^{-5}, \Delta N = 33.5, x_c = 0.784, c = 0.77)$

- Inflation near the critical point: $\lambda(\phi) = \lambda_0 + b_\lambda \ln^2(\phi/\mu)$

$$S = \int d^4x \sqrt{-g} \left(\left(\frac{1}{2\kappa^2} + \frac{\xi}{2} \phi^2 \right) R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4} \lambda(\phi) \phi^4 \right)$$



- Potential can develop an inflection point

$$V(\varphi) = \frac{\lambda(\phi)\phi^4}{4(1 + \xi\phi^2)^2}$$

and it is very sensitive to the Higgs self-coupling w.r.t. its critical value

- Some remarks:

i) Most e-folds spent near the inflection point. r can be large

$$r = 16\epsilon$$

ii) Amplitude fixed by couplings at critical point

$$V(\phi) \rightarrow \lambda_c / \xi^2$$

with SM RGE giving $\lambda_c \sim 10^{-6}$ and thus $\xi \sim 10$

iii) SM parameters (mainly top mass) play crucial roll in determining λ and b_λ

