Effects of equation of state in the Standard Model on the primordial gravitational wave spectrum

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Work in progress, collaboration with Satoshi Shirai (Kavli IPMU)

Introduction

- Primordial GWs : important prediction of inflationary theory
- Tensor perturbations

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

contain two kinds of information :

$$h(t, \mathbf{k}) = h_{\rm prim}(\mathbf{k})\mathcal{T}(t, \mathbf{k})$$

$h_{\rm prim}(\mathbf{k})$: initial condition (primordial power spectrum)

$$\mathcal{P}_T(k) = \frac{k^3}{\pi^2}\sum_\lambda |h^\lambda_{\rm prim}|^2 = \left.\frac{2H^2_{\rm inf}}{\pi^2 M^2_{\rm Pl}}\right|_{k=aH}$$

… information about inflationary models

$\mathcal{T}(t, \mathbf{k})$: transfer function

… information about thermal history after inflation

- Accurate knowledge of $\mathcal{T}(t, \mathbf{k})$ is required in order to extract information about the primordial universe from observational results.

Standard Model

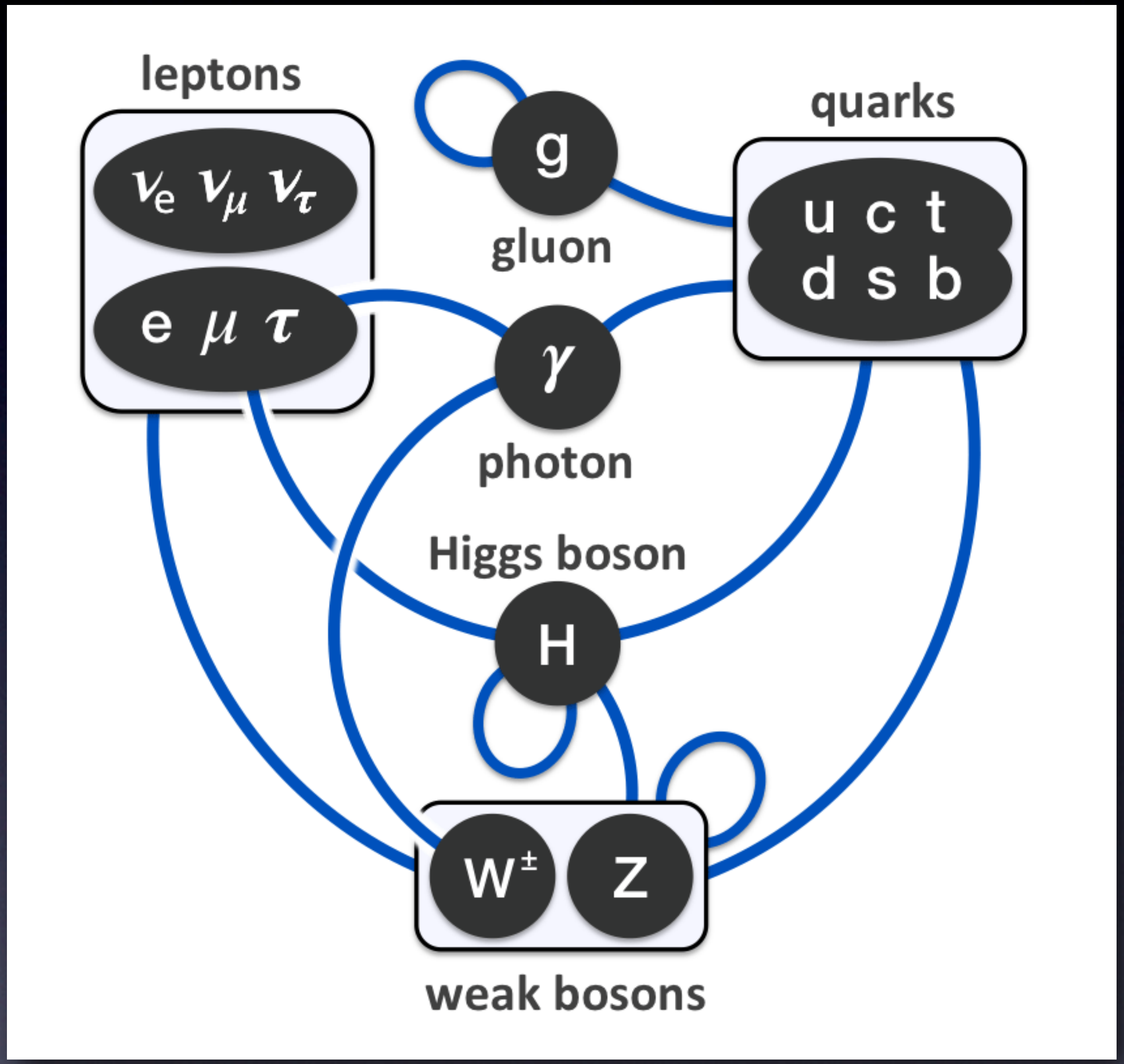
- 6 quarks, 6 leptons, gauge bosons (photons, gluons, weak bosons), Higgs boson

- A "benchmark" model to describe the properties of the primordial plasma

- Two phase transitions:

  - QCD phase transition
    $(T \sim 150\,\mathrm{MeV})$
    Hadrons ⇔ Quark-gluon plasma

  - Electroweak phase transition
    $(T \sim 160\,\mathrm{GeV})$
    Particles acquire mass due to the Higgs mechanism



https://en.wikipedia.org/wiki/Standard_Model

Previous research

PHYSICAL REVIEW D **73,** 123515 (2006)

Improved calculation of the primordial gravitational wave spectrum in the standard model

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- Change of relativistic d.o.f. affects the spectrum of inflationary GWs.
- Ideal gas was assumed.

$$g_{*\rho}(T) \equiv \frac{\rho(T)}{\left[\frac{\pi^2 T^4}{30}\right]} = \sum_i g_{*\rho,i}(T)\left(\frac{T_i}{T}\right)^4, \quad g_{*s}(T) \equiv \frac{s(T)}{\left[\frac{2\pi^2 T^3}{45}\right]} = \sum_i g_{*s,i}(T)\left(\frac{T_i}{T}\right)^3$$

$$g_{*\rho,i}(T) = g_i \frac{15}{\pi^4}\int_{m_i/T}^{\infty} \frac{(u^2 - m_i^2/T^2)^{1/2}}{e^u \pm 1} u^2 du, \quad g_{*s,i}(T) = g_i \frac{15}{\pi^4}\int_{m_i/T}^{\infty} \frac{(u^2 - m_i^2/T^2)^{1/2}}{e^u \pm 1}\left(u^2 - \frac{m_i^2}{4T^2}\right) du$$

- This assumption does not always hold true.
  There exist corrections due to particle interactions, which have to be quantitatively taken into account.

Equation of state of the universe and GW spectrum

After the GW mode enters the horizon ($t_{\rm hc}$), it evolves as $\rho_{\rm gw}(t) \propto a(t)^{-4}$
while the background energy density $\rho(t)$ evolves according to
the equation of state $w = p/\rho$.

Seto and Yokoyama, J. Phys. Soc. Jap. 72, 3082 (2003) [gr-qc/0305096]

$$\Omega_{\rm gw}(t,k) = \frac{1}{\rho(t)}\frac{d\rho_{\rm gw}(t,k)}{d\ln k} = \exp\left[\int_{a_{\rm hc}}^{a}(3w-1)d\ln a\right]\Omega_{\rm gw}(t_{\rm hc},k)$$

$\Omega_{\rm gw}(t_{\rm hc},k) \sim \mathcal{P}_T(k)$: primordial tensor power spectrum

In the radiation dominated universe, we have

$$\Omega_{\rm gw}(t,k) = \left(\frac{g_{*\rho,\rm hc}}{g_{*\rho}(T)}\right)\left(\frac{g_{*s,\rm hc}}{g_{*s}(T)}\right)^{-4/3}\Omega_{\rm gw}(t_{\rm hc},k)$$

A deviation from $w = 1/3$ or $g_{*\rho} = g_{*s} = \rm constant$ results in a deviation from a nearly flat spectrum due to $\mathcal{P}_T(k)$

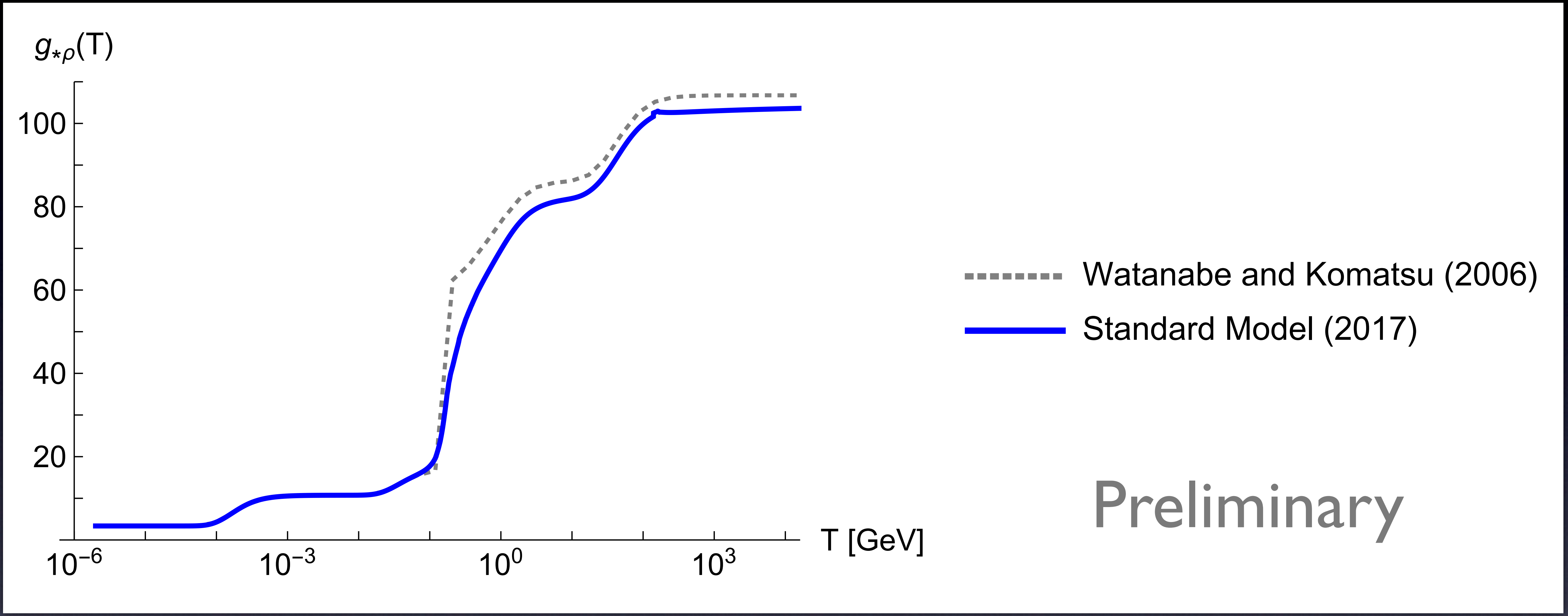
Calculation of the equation of state ("trace anomaly")

$$\Delta(T) \equiv \frac{\rho(T) - 3p(T)}{T^4} = T\frac{d}{dT}\left\{\frac{p(T)}{T^4}\right\} \quad p(T) = \lim_{V\to\infty}\frac{T}{V}\ln Z$$

$$Z = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\Phi \exp\left(-\int_0^{1/T}\int d^{d-1}x\mathcal{L}\right)$$: partition function

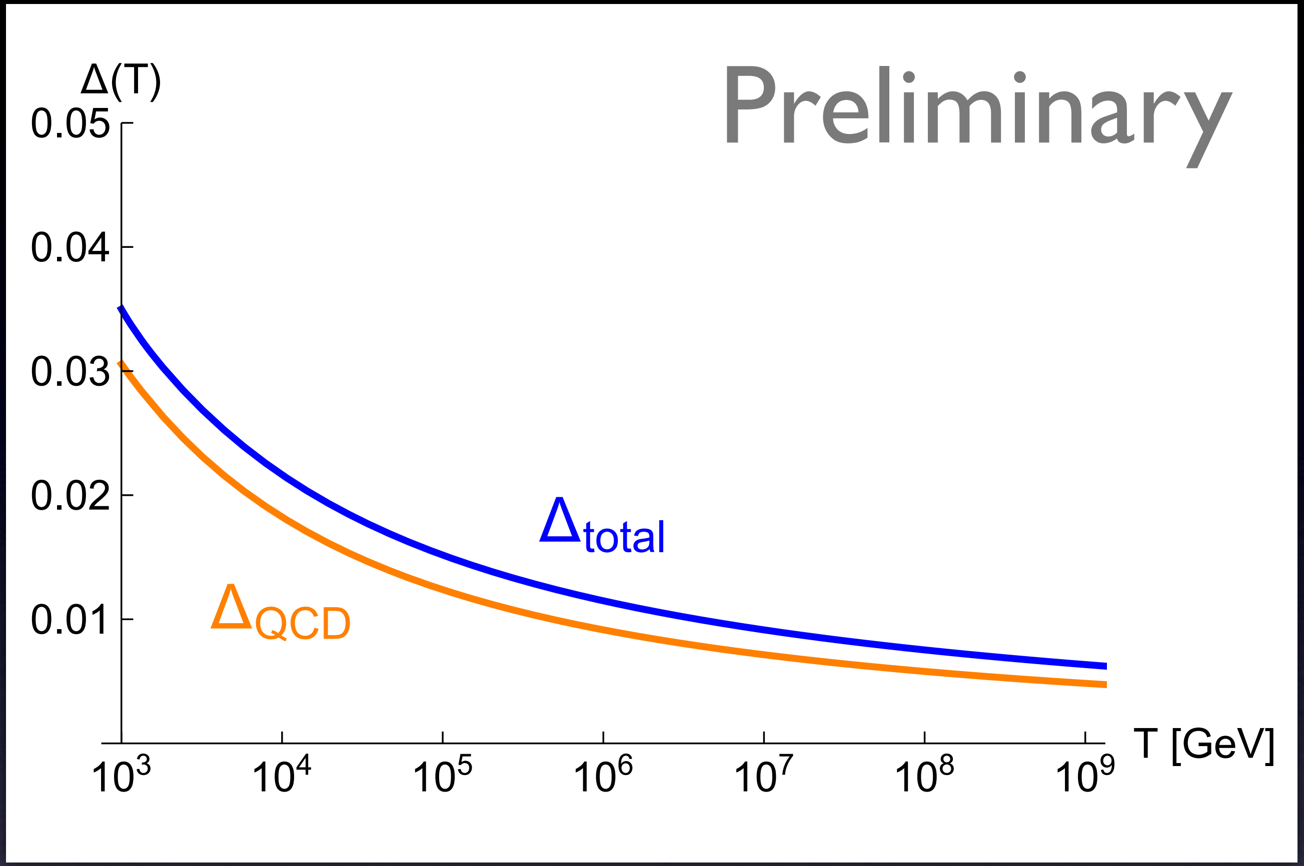
- Perturbative calculations
  - Perturbative QCD: up to $\mathcal{O}(g^6)$ Kajantie, Laine, Rummukainen and Schroder, PRD67, 105008 (2003) [hep-ph/0211321]
  - Electroweak: up to $\mathcal{O}(g^5)$ Gynther and Vepsäläinen, JHEP01 (2006) 060 [hep-ph/0510375]
- Non-perturbative calculations (QCD phase transition, electroweak phase transition)
  - Lattice QCD: Borsanyi et al., Phys. Lett. B370, 99 (2014) [1309.5258 [hep-lat]]
    Bazavov et al., PRD90, 094503 (2014) [1407.6387 [hep-lat]]
    Borsanyi et al., Nature 539 (2016) no.7627, 69 [1606.07494 [hep-lat]]
  - Electroweak crossover: Laine and Meyer, JCAP07 (2015) 035 [1503.04935 [hep-ph]]
    D'Onofrio and Rummukainen, PRD93, 025003 (2016) [1508.07161 [hep-ph]]
- We reconstruct $g_{*\rho}$ and g_{*s} by collecting all these results.

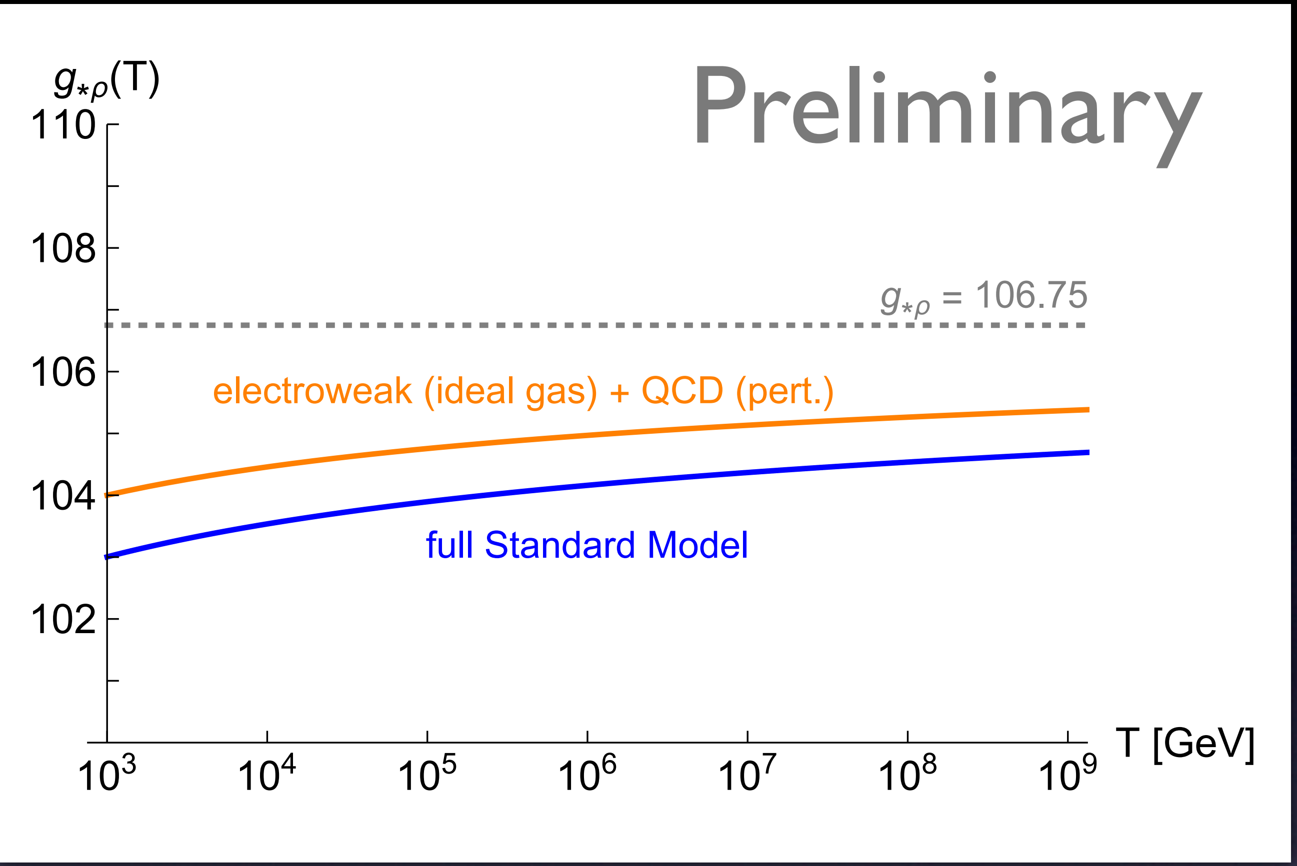
Effective relativistic d.o.f. in the SM



There is some deviation from the ideal gas result at high temperatures, which can affect the spectrum of primordial gravitational waves.

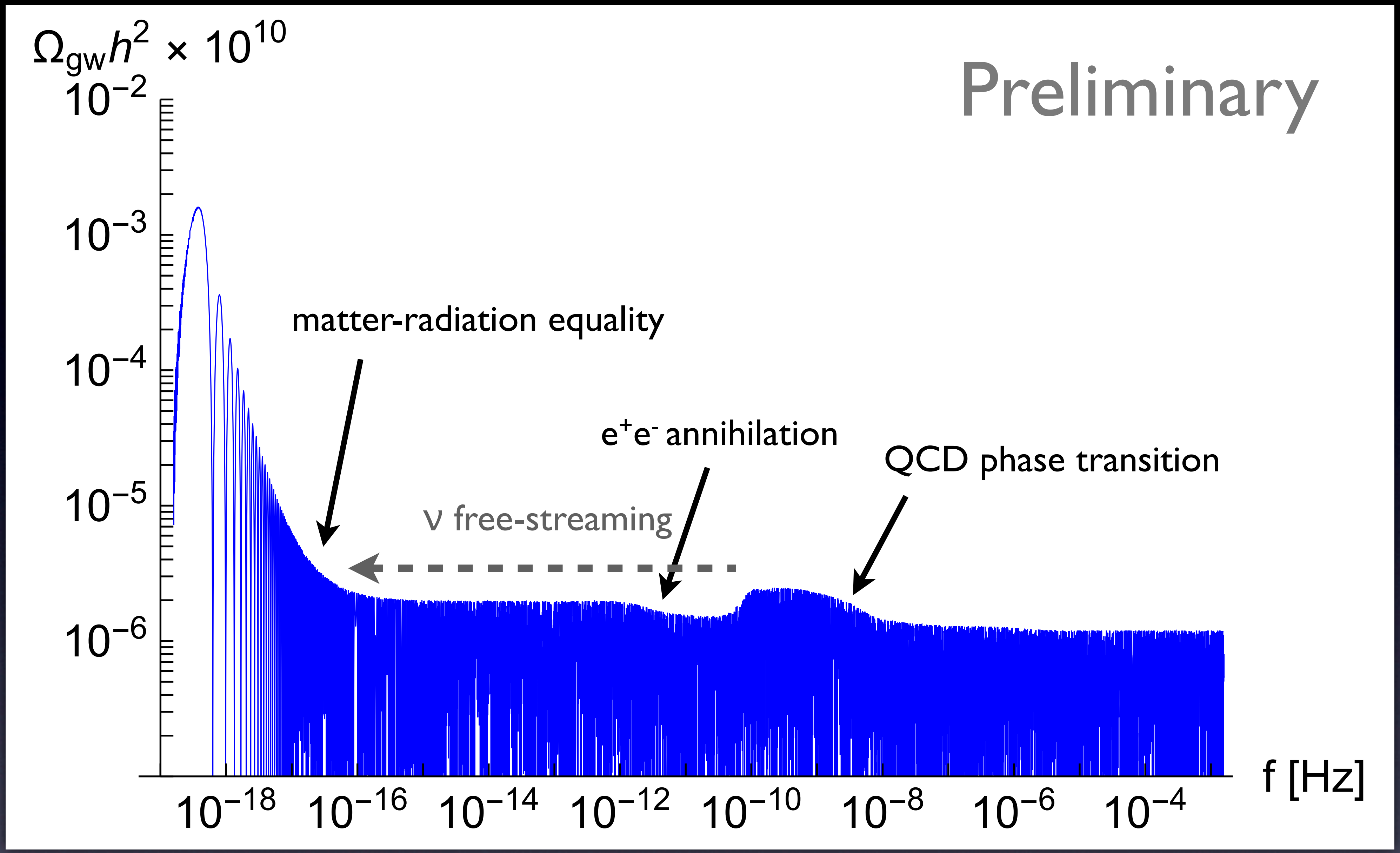
Behavior at higher temperatures





- Trace anomaly is dominated by the contribution from QCD sector.
- Convergence to the ideal gas result is fairly slow even at very high temperatures.

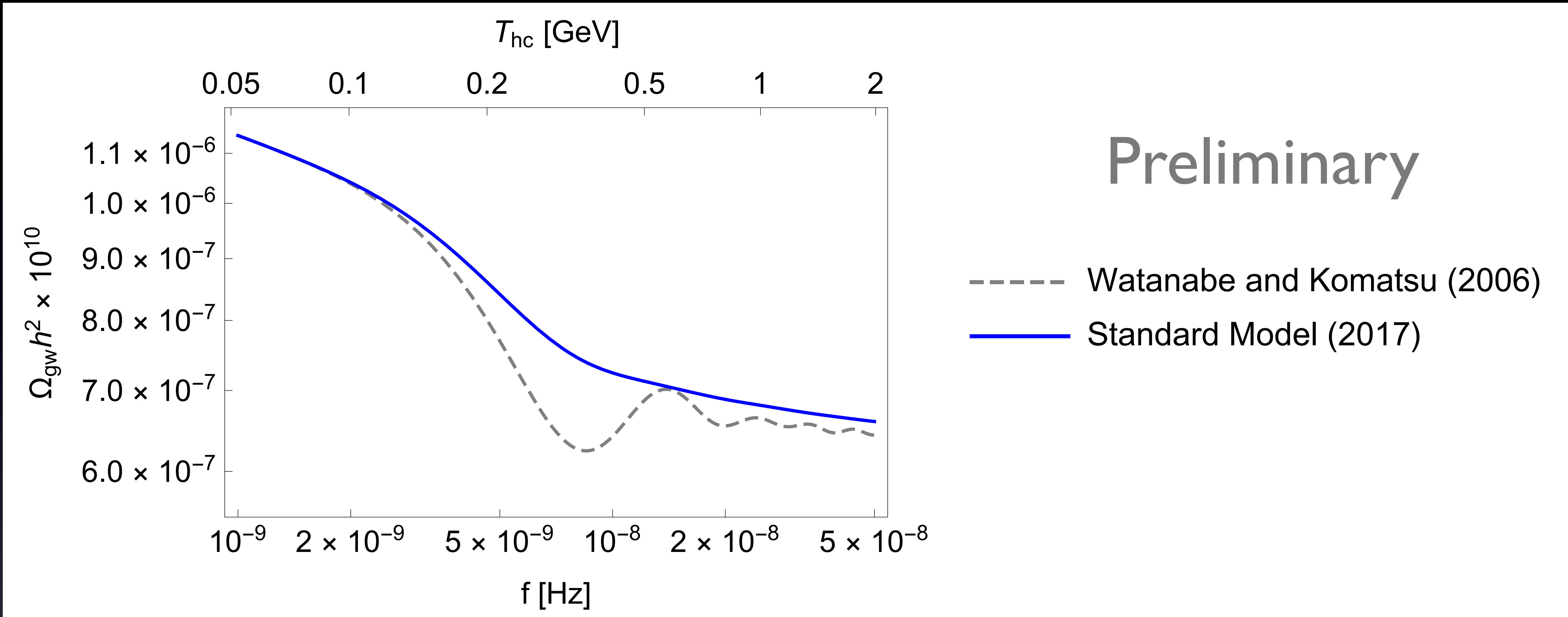
Gravitational wave spectrum



Basic features are similar to the previous results:
Various events in the early universe are imprinted on the spectrum of GWs.

$$f = \frac{k}{2\pi a_0} = \frac{H_{\rm hc}}{2\pi}\frac{a_{\rm hc}}{a_0} = 2.65\,{\rm Hz}\left(\frac{g_{*\rho,{\rm hc}}}{106.75}\right)^{1/2}\left(\frac{g_{*s,{\rm hc}}}{106.75}\right)^{-1/3}\left(\frac{T_{\rm hc}}{10^8\,{\rm GeV}}\right)$$

Effect of the QCD phase transition

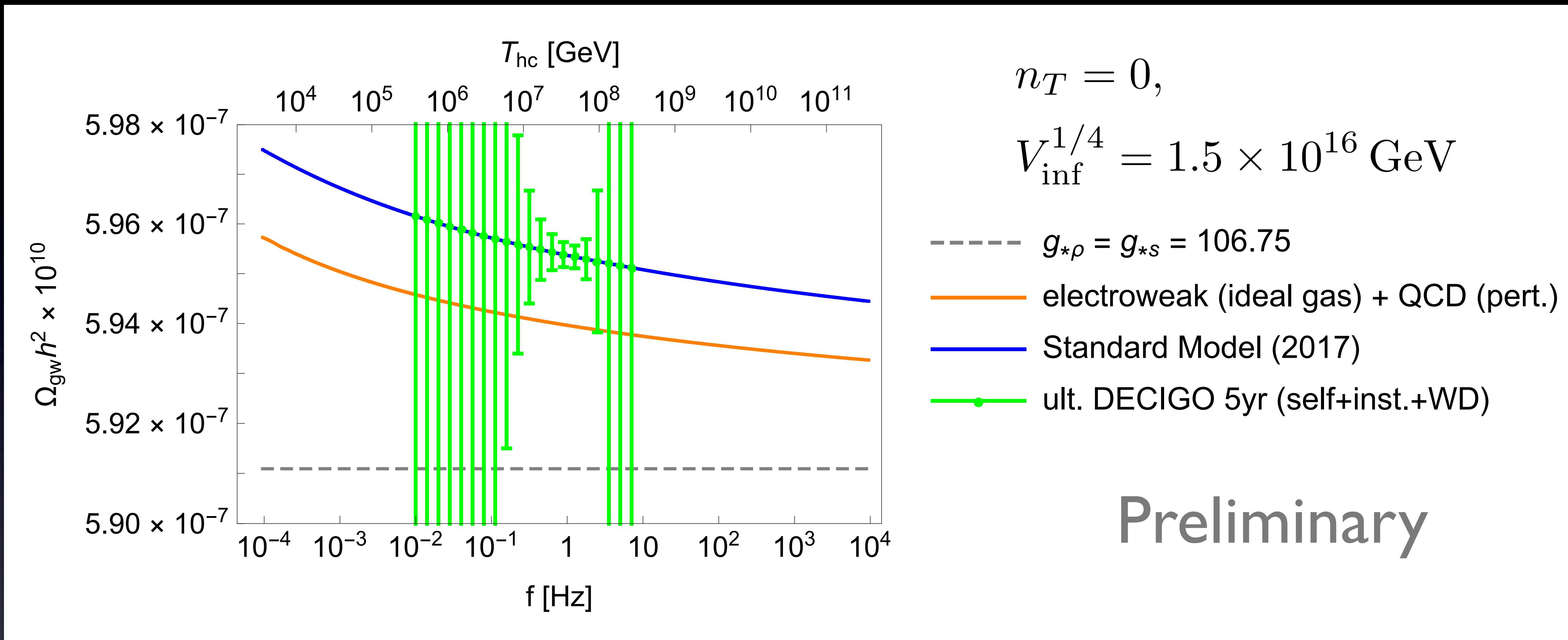


- Oscillatory features appear when $g_{*\rho}(T)$ and $g_{*s}(T)$ suddenly change.

  Watanabe and Komatsu, PRD73, 123515 (2006) [astro-ph/0604176]

- Such features are hardly observed in the updated results.
  (QCD phase transition is expected to be a mild crossover.)
- Difficult to observe: $\Omega_{\rm gw} h^2 \gtrsim 10^{-9}$ EPTA (present)
  $\Omega_{\rm gw} h^2 \gtrsim 10^{-14}$ SKA (future)
  at $f \simeq \mathcal{O}(10^{-9}\text{–}10^{-8})\,\mathrm{Hz}$

Spectrum at higher frequencies



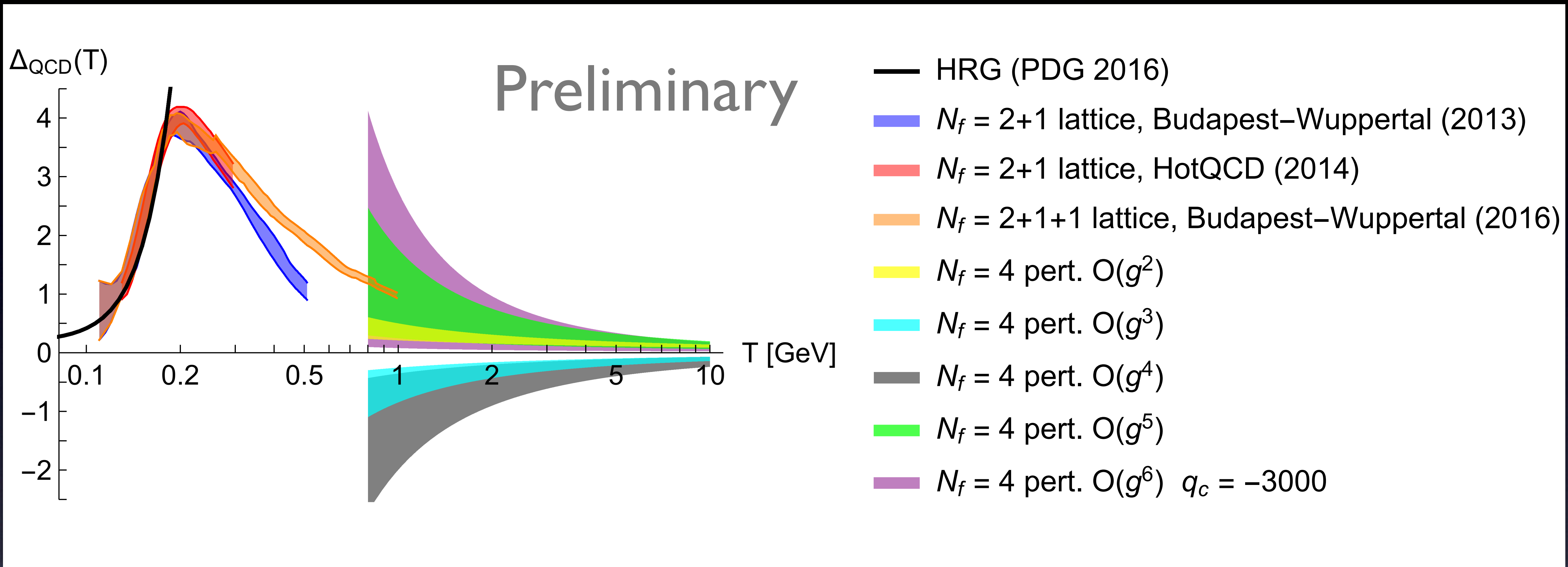
- Amplitude is about 1% larger than the result with constant $g_{*\rho},\ g_{*s}$.
- Non-trivial frequency dependence due to the running QCD gauge coupling.
- The effect becomes non-negligible in the ultimate sensitivity of DECIGO.

Conclusion

- We reconstructed the effective degrees of freedom (or equation of state) of radiations by taking account of particle interactions in the Standard Model.
- Deviations from ideal gas at high temperatures affect the spectrum of GWs at high frequencies.
- Other potential applications:
  - Spectra of other primordial GWs are subjected to similar corrections.
  - Effects on other cosmological relics (dark matter abundance, etc.).

Backup slides

Trace anomaly of strongly interacting particles



- HRG results agree with lattice results at $T \lesssim 180\,\mathrm{MeV}$.
- Lattice results in 4-flavor QCD are available up to $T < 1\,\mathrm{GeV}$, which can be matched to perturbative results.
- Poor convergence of perturbative calculations.

Estimation of effective degrees of freedom

Once we have $\Delta(T)$, we integrate it to obtain the pressure:

$$\frac{p(T)}{T^4} = \frac{p_0}{T_0^4} + \int_{T_0}^{T} \frac{dT'}{T'} \Delta(T')$$

Value of p_0/T_0^4 is specified at some low temperature.

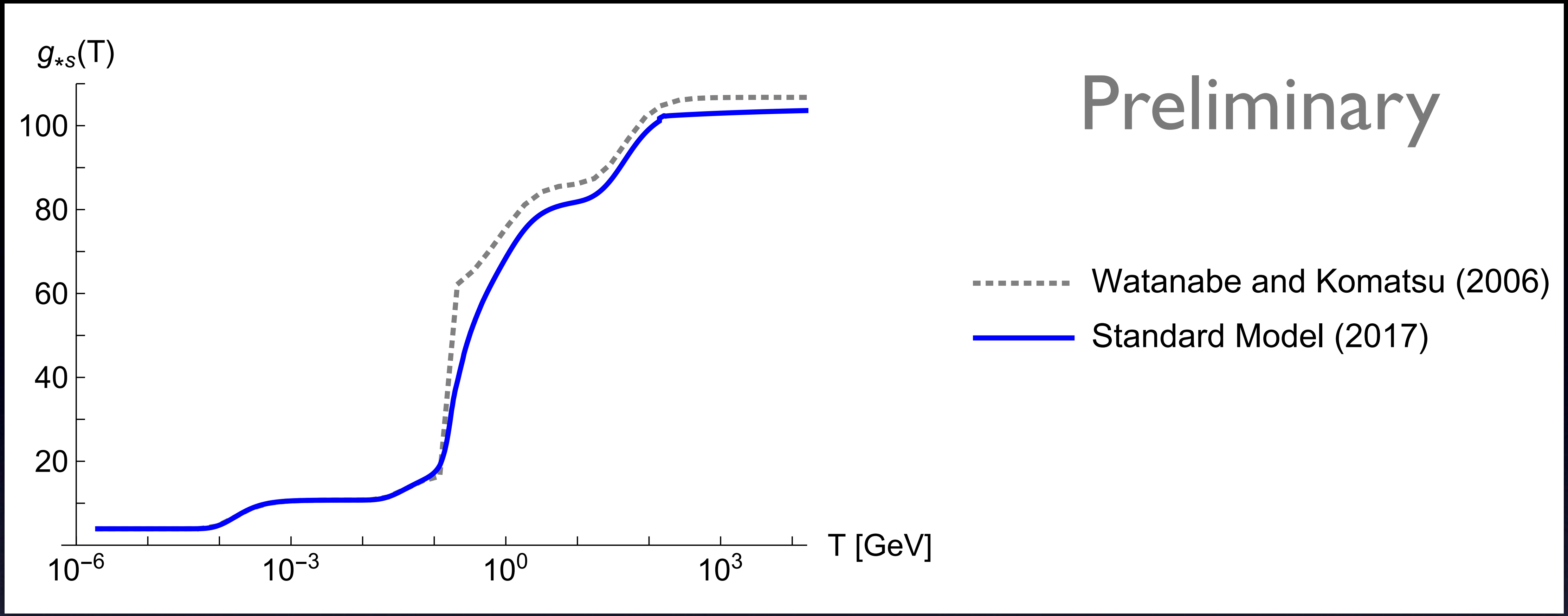
Energy density and entropy density are given by

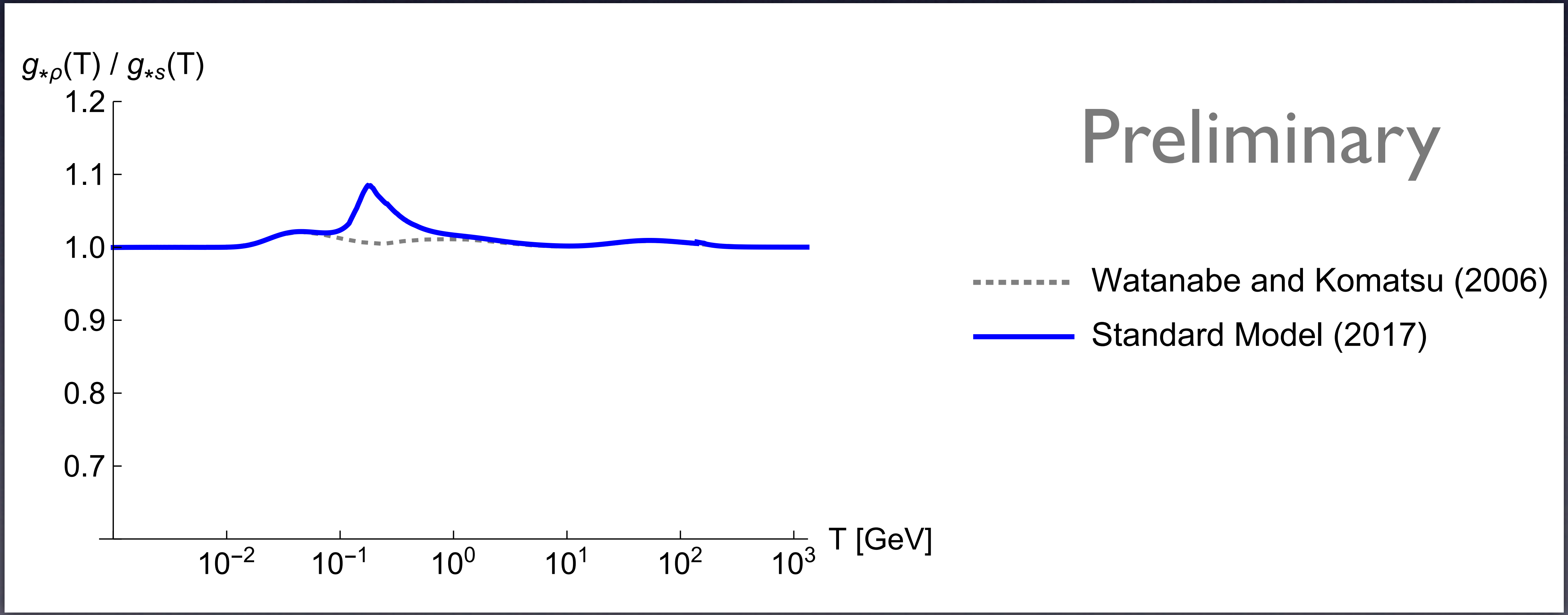
$$\rho(T) = T\frac{dp}{dT}(T) - p(T) = T^4 \left[\Delta(T) + \frac{3p(T)}{T^4}\right]$$

$$s(T) = \frac{\rho(T) + p(T)}{T} = T^3 \left[\Delta(T) + \frac{4p(T)}{T^4}\right]$$

$$\Rightarrow \quad g_{*\rho} = \frac{\rho(T)}{\left[\frac{\pi^2 T^4}{30}\right]}, \quad g_{*s} = \frac{s(T)}{\left[\frac{2\pi^2 T^3}{45}\right]}$$

Effective relativistic d.o.f. for the entropy density





Numerical calculation

Watanabe and Komatsu, PRD73, 123515 (2006) [astro-ph/0604176]
Kuroyanagi, Chiba and Sugiyama, PRD79, 103501 (2009) [0804.3249]

- Linear order Einstein equations for transverse-traceless part of metric perturbations

$$\ddot{h}_\lambda(t,\mathbf{k}) + 3H\dot{h}_\lambda(t,\mathbf{k}) + \frac{k^2}{a^2(t)}h_\lambda(t,\mathbf{k}) = 16\pi G\Pi_\lambda(t,\mathbf{k})$$

with initial conditions $h_\lambda(0,\mathbf{k}) = h_{\lambda,\mathrm{prim}}(\mathbf{k})$ and $\dot{h}_\lambda(0,\mathbf{k}) = 0$

$\lambda = +, \times$: two polarization states

- Anisotropic stress from free-streaming neutrinos

$$\Pi_\lambda(\tau,\mathbf{k}) = -4\rho_\nu(\tau)\int_{\tau_{\nu\,\mathrm{dec}}}^{\tau} d\tau' \frac{j_2[k(\tau-\tau')]}{k^2(\tau-\tau')^2}h_\lambda(\tau',\mathbf{k})$$

τ : conformal time

This leads to the damping of GW amplitude for the modes entering the horizon after the neutrino decoupling $T_{\nu\,\mathrm{dec}} \sim 2\,\mathrm{MeV}$

Weinberg, PRD69, 023503 (2004) [astro-ph/0306304]

- Spectrum of gravitational waves is given by

$$\Omega_{\mathrm{gw}}(t,\mathbf{k}) = \frac{1}{\rho_c(t)}\frac{d\rho_{\mathrm{gw}}(t,k)}{d\ln k} = \frac{k^3}{12\pi^2 H^2(t)}\sum_\lambda \left|\dot{h}_\lambda(t,\mathbf{k})\right|^2$$